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Modeling financial sector joint tail risk in the euro area

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Abstract

We develop a novel high-dimensional non-Gaussian modeling framework to infer measures of conditional and joint default risk for numerous financial sector firms. The model is based on a dynamic Generalized Hyperbolic Skewed-t block-equicorrelation copula with time-varying volatility and dependence parameters that naturally accommodates asymmetries, heavy tails, as well as non-linear and time-varying default dependence. We apply a conditional law of large numbers in this setting to define joint and conditional risk measures that can be evaluated quickly and reliably. We apply the modeling framework to assess the joint risk from multiple defaults in the euro area during the 2008–2012 financial and sovereign debt crisis. We document unprecedented tail risks between 2011–2012, as well as their steep decline following subsequent policy actions.

Keywords: dynamic equicorrelation; generalized hyperbolic distribution; law of large numbers; large portfolio approximation.

JEL classification: G21, C32.

Non-technical summary

The issue of measuring and monitoring interconnected financial sector default risk has received considerable interest in the wake of the financial and sovereign debt crisis in the euro area. This paper introduces a novel empirical framework to estimate marginal, joint, and conditional probabilities of financial firm default from equity data and EDF-based risk measures. Such joint and conditional probabilities of default are informative for risk management and financial sector surveillance purposes. Furthermore, conditional probabilities shed light on the extent that financial sector firms are subject to contagion, and to what extent priced credit risks are affected by policy measures. Our methodology is novel in that our joint risk measures are derived from a multivariate framework that is based on a flexible yet parsimonious multivariate density that naturally accommodates skewed and heavy tailed changes in marginal risks as well as time variation in uncertainty and multivariate dependence.

We apply our novel framework by analyzing financial sector joint and conditional default risks of $N = 10$ and $N = 73$ financial sector firms located in the euro area, based on weekly data from January 1999 to September 2013. Using a limited dimension of $N = 10$ firms, we verify that simple equicorrelation dynamics closely track the average correlations from a full correlation model analysis. In addition, we verify that novel semi-analytical approximations to compute joint and conditional tail risk measures work quickly and reliably even if the cross-sectional dimension is as low as 10 firms.

In our final high-dimensional application, we document unprecedented joint tail risks for a set of large euro area financial sector firms during the financial and euro area sovereign debt crises. We also document a clear peak of financial sector joint default risk in the summer of 2012. Based on time variation in our joint risk measure we argue that three events collectively ended the most acute phase of extreme financial sector joint tail risks in the euro area. These are a speech by the ECB president in London on 26 July 2012, the announcement of the Outright Monetary Transactions (OMT) program on 2 August 2012, and the disclosure of the OMT details on 6 September 2012. Finally, we argue that the design and implementation of non-standard monetary policy measures and financial stability (tail) outcomes are strongly related. This finding suggests substantial scope for the coordination of monetary, macro-prudential, and bank supervision policies. This is relevant as both monetary policy as well as banking supervision are carried out jointly by the ECB since November 2014.

1 Introduction

In this paper we develop a novel high-dimensional non-Gaussian modeling framework to infer conditional and joint risk measures for many financial sector firms. The model is based on a dynamic Generalized Hyperbolic Skewed-t copula with time-varying volatility and dependence parameters. Such a framework naturally accommodates asymmetries, heavy tails, as well as non-linear and time-varying default dependence. To balance the need for parsimony as well as flexibility in a high-dimensional cross-section, we endow the dynamic model with a score-driven block-equicorrelation structure. Furthermore, we demonstrate that a conditional law of large numbers applies in our setting, which allows us to define risk measures that can be evaluated semi-analytically and within seconds. The modeling framework allows us to assess the joint risk from multiple financial firm defaults in the euro area during the financial and sovereign debt crisis. We document unprecedented tail risks between 2011–12, as well as a sharp decline in joint (but not conditional) tail risk probabilities following a sequence of announcements by the European Central Bank (ECB) that introduced its Outright Monetary Transactions (OMT) program.[1]

Since the onset of the financial crisis in 2007, financial stability monitoring has become a key priority for many central banks, which added to their respective monetary policy mandates; see, for example, Acharya et al. (2012), and Adrian et al. (2013). The new prudential responsibilities involve monitoring financial risks to and from a large system of interconnected financial intermediaries. The cross-sectional dimensions of these systems are typically high, even if attention is restricted to only large and systemically important institutions. Our modeling framework is directly relevant for such monitoring tasks. In addition, our framework is interesting for financial institutions and clearing houses that are required to actively set risk limits (often in real time) and maintain economic capital buffers to withstand bad risk outcomes due to exposures to a large number of credit risky counterparties. Finally, with the benefit of hindsight, evaluating the time variation in conditional and joint risks

[1] See ECB (2012) and Coeuré (2013). The OMT is a non-standard monetary policy measure within which the ECB could, under certain conditions, make purchases in secondary markets of bonds issued by euro area member states.

allows us to assess the impact of non-standard policy measures that central banks (or other actors) take on the risk of a simultaneous and widespread failure of financial intermediaries.

Our starting point for modeling time-varying joint and conditional risks is a dynamic copula framework, also considered by Segoviano and Goodhart (2009), Christoffersen, Errunza, Langlois, and Jacobs (2012), Oh and Patton (2014), and Lucas, Schwaab, and Zhang (2014). In each case, a collection of firms is seen as a portfolio of firms whose multivariate dependence structure is inferred from equity returns or CDS data. Our current framework extends the Lucas et al. (2014) approach in two ways. First, by considering grouped equicorrelation structures, our current framework allows us to fit a cross-sectional dimension much larger than, say, 15 firms, while retaining the ability to capture the salient data features such as skewness, fat tails, and time-varying correlations. We thus fix the drawback that parameter estimation breaks down when considering many firms in this class of models due to a well-known curse of dimensionality; see Engle and Kelly (2012) for a discussion. Second, we show how to evaluate joint and conditional risk measures within seconds in the current framework. Estimating time-varying parameters and portfolio risk measures is computationally relatively inexpensive because explicit expressions are available for the likelihood and the joint and conditional portfolio credit risk measures. Simulation-based methods are not required, but are used in our empirical study to provide points of comparison from a robustness perspective.

We apply our general framework by studying joint and conditional default probabilities for financial sector firms in the euro area, based on weekly data from January 1999 to September 2013. We consider two applications. First, using a limited sample of $N = 10$ firms, we verify that our dynamic correlations based on the block-equicorrelation assumption closely track the average correlations from a full-correlation-matrix model analysis. We demonstrate that the loss of cross-sectional dispersion in correlations hardly matters when evaluating joint credit risk measures, at least for our sample of firms. Finally, we verify that our semi-analytical approximations to compute joint and conditional risk measures already work well even if the cross-sectional dimension is as low as 10 firms.

We then turn to our final high-dimensional study of $N = 73$ firms. We document unprecedented joint tail risks for a set of large euro area financial sector firms during the financial and sovereign debt crises, and also establish a clear peak of financial sector joint default risk in the summer of 2012. Based on time variation in our joint risk measures we argue that three events collectively ended the most acute phase of extreme financial sector tail risks in the euro area. These events are a speech by the ECB president in London to do '*whatever it takes*' to save the euro on 26 July 2012, the announcement of the ECB's OMT program on 2 August 2012, and the disclosure of the OMT details on 6 September 2012. This is a startling finding, as the ECB's OMT program provides conditional and partial insurance to governments and not to financial sector firms. Conditional tail probabilities did not decline as much, indicating that risk spillovers might have remained a concern. Based on the OMT's strong impact on financial sector joint risks, we conclude that the design and implementation of unconventional monetary policies and financial stability (tail) outcomes are strongly related. This finding suggests substantial scope for the coordination of monetary, macro-prudential, and bank supervision policies. This is relevant as both monetary policy as well as banking supervision will be carried out jointly by the ECB as of November 2014.

Our study relates to several directions of current research. First, we draw from the growing literature on non-Gaussian dependence modeling as well as the literature on credit risk measurement and portfolio loss asymptotics. Time-varying parameter models for volatility and dependence were considered, inter alia, by Engle (2002), Creal, Koopman, and Lucas (2011), Christoffersen et al. (2012), Engle and Kelly (2012), and Creal and Tsay (2015). Similarly, credit risk models and portfolio tail risk measures were studied, for example, by Vasicek (1987), Lucas, Klaassen, Spreij, and Straetmans (2001, 2003), Gordy (2000, 2003), Koopman et al. (2011, 2012), and Giesecke, Spiliopoulos, Sowers, and Sirignano (2014). Combining results from both strands of literature allows us to obtain joint credit risk measures at a relatively high frequency, such as daily or weekly. Second, to balance the need for parsimony and flexibility, we consider a variant of the block equicorrelation structure for the covariance matrix in Engle and Kelly (2012); see also Christoffersen, Errunza, Jacobs, and Jin (2014), and Creal and Tsay (2015) for applications with DECO and grouped dependence structures, respectively. The version of the block equicorrelation model developed in this paper still allows us to draw on the machinery developed in the credit portfolio tail risk literature mentioned earlier. Third, an additional strand of literature investigates joint and conditional default dependence from a financial stability perspective, see, for example, Hartmann et al. (2007), Acharya et al. (2012), and Suh (2012). Finally, to introduce time-variation into our econometric model specification, we endow our model with observation-driven dynamics based on the score of the conditional predictive log-density. Score-driven time-varying parameter models are actively researched, see for example Creal et al. (2011), Creal et al. (2013), Harvey (2013), Oh and Patton (2014), Creal et al. (2014).[2]

The two papers that are most closely related to ours are Oh and Patton (2014), and Christoffersen, Jacobs, Jin, and Langlois (2014). Oh and Patton (2014) propose a class of dynamic copula factor models for high dimensions, which facilitates the estimation of a wide variety of systemic risk measures. Firms in their framework load on a common factor that has the skewed Student's t density of Hansen (1994). Additional idiosyncratic shocks are modeled by a symmetric-t distribution. They use their model for studying the risk from a large number of U.S. non-financial corporates. Time-varying dependence is modeled in a score-driven way, as in this paper. Our study differs in that we use the GHST distribution for both the marginal and the copula modeling; this means that our factor copula has a non-linear two factor rather than a linear one-factor structure. Moreover, our application focusses on the euro area financial sector during the financial and sovereign debt crisis, and on the impact evaluation of central bank unconventional monetary policy measures during that time. Finally, we introduce and discuss the efficient evaluation of semi-analytic risk measures. Christoffersen et al. (2014) study diversification benefits among U.S. corporates, and use Hansen's (1994) distribution for the innovations in univariate GARCH models. Their GHST copula is the same as used in our paper. Our paper differs in that we provide a score-driven approach to the modeling of dynamic dependence, which is particularly attractive in a non-Gaussian context when data is fat tailed and skewed; see Zhang, Creal, Koopman, and Lucas (2011) and Blasques, Koopman, and Lucas (2015). We do not rely exclusively

[2]We refer to `http://www.gasmodel.com` for an extensive enumeration of recent work in this area. Computer code for this paper will be available from this source.

on composite likelihood methods, but employ them only to provide alternative points of comparison from a robustness perspective. Composite likelihood techniques are feasible in high dimensions, but also statistically inefficient. Instead, we proceed by proposing block equicorrelation models, at least for settings where prior information is available on which subsets of firms are likely to move together as a group, and risks need to be evaluated in a computationally efficient way. We explicitly highlight what information is lost when moving from a full to an equicorrelation copula structure. Finally, as for Oh and Patton (2014), we introduce semi-analytic risk measures and focus our application on the euro area sovereign debt crisis and non-standard monetary policy measures.

Section 2 introduces our statistical framework, the dynamic Generalized Hyperbolic Skewed-t block equicorrelation model, and discusses parameter estimation. Section 3 demonstrates how a conditional law of large numbers can be applied to reliably and quickly compute portfolio risk measures in a GHST factor copula setting. Section 4 applies the modeling framework to the euro area financial sector during the financial and sovereign debt crisis. Section 5 concludes. A supplementary Web Appendix presents proofs and additional results.

2 Statistical model

2.1 The dynamic Generalized Hyperbolic Skewed t copula model

Following Lucas et al. (2014), we consider a copula model based on the Generalized Hyperbolic Skewed t (GHST) distribution. Let

$$y_{it} = (\varsigma_t - \mu_\varsigma)\gamma_i + \sqrt{\varsigma_t}\,\tilde{\Sigma}_{it}^{1/2}\epsilon_t, \qquad i = 1, \ldots, N, \tag{1}$$

where $y_t = (y_{1t}, \ldots, y_{Nt})'$ is a vector of firm-specific log asset values, $\varsigma_t \in \mathbb{R}^+$ is an inverse-Gamma distributed common risk factor that affects all firms simultaneously, $\varsigma_t \sim \mathrm{IG}(\frac{\nu}{2}, \frac{\nu}{2})$, $\gamma \in \mathbb{R}^N$ is a vector controlling the skewness of the copula, $\tilde{\Sigma}_{it}$ is the ith row of the GHST copula scale matrix $\tilde{\Sigma}_t \in \mathbb{R}^{N\times N}$, and $\epsilon_t \in \mathbb{R}^N$ is a vector of standard normally distributed risk factors. We assume that the two random variables ς_t and ϵ_t are independent and set

$\mu_\varsigma = \mathrm{E}[\varsigma_t] = \nu/(\nu-2)$, such that y_{it} has zero mean if $\nu > 2$. Matrix $\tilde{\Sigma}_t^{1/2}$ is the Choleski matrix square root of the scale matrix $\tilde{\Sigma}_t$.

In our copula framework, a firm defaults if its (unobserved) log asset value y_{it} falls below its default threshold y^*_{it}. The cross sectional dependence in defaults captured by equation (1) thus stems from two sources: common exposures to the normally distributed risk factors ϵ_t as captured by the time-varying matrix $\tilde{\Sigma}_t$; and an additional common exposure to the scalar risk factor ς_t. The former captures connectedness through correlations, while the latter captures such effects through the tail-dependence of the copula. To see this, note that if ς_t is non-random, the first term in (1) drops out of the equation and there is zero tail dependence. Conversely, if ς_t is large, all asset values are affected at the same time, making joint defaults of two or more firms more likely.

Earlier applications of the GHST distribution to financial and economic data include Mencía and Sentana (2005), Aas and Haff (2006), and Oh and Patton (2014). Alternative skewed t distributions have been proposed as well, such as Branco and Dey (2001), Gupta (2003), Azzalini and Capitanio (2003), and Bauwens and Laurent (2005); see also the overview of Aas and Haff (2006). The GHST distribution closely connects to a continuous-time finance literature utilizing Lévy processes for stock price processes and firm asset values; see Bibby and Sørensen (2003) for a survey.

We denote the one-year-ahead default probability for firm i at time t as p_{it}, such that

$$p_{it} = \Pr\left[y_{it} < y^*_{it}\right] = F_{it}(y^*_{it}) \Leftrightarrow y^*_{it} = F^{-1}_{it}(p_{it}), \tag{2}$$

where F_{it} is the univariate GHST cumulative distribution function (cdf) of y_{it}. In our application, we assume that we observe p_{it} as the expected default frequency of firm i reported at time t by Moody's Analytics (formerly Moody's KMV). Instead of focusing on the individual default probabilities p_{it}, our focus is on the time-varying *joint* probabilities, such as $\Pr\left[y_{it} < y^*_{it}, y_{jt} < y^*_{jt}\right]$, and on conditional probabilities such as $\Pr\left[y_{it} < y^*_{it} | y_{jt} < y^*_{jt}\right]$, for firms $i \neq j$. Below, we first develop a dynamic version of the GHST copula model. Then we consider a dynamic (block)-equicorrelation version of the model in the spirit of Engle and

Kelly (2012), which turns out to be particularly useful to study joint and conditional default probabilities in a parsimonious way for large dimensional systems.

2.2 The dynamic GHST model

To describe the dynamics of the scale parameter $\tilde{\Sigma}_t$ in the GHST model (1), we use the generalized autoregressive score (GAS) dynamics as proposed in Creal et al. (2011, 2013); see also Harvey (2013). These dynamics easily adapt to the skewed and fat-tailed nature of the GHST density and improve the stability of dynamic volatility and correlation estimates; see Blasques et al. (2015). Our version of the model is different from that of Lucas et al. (2014) due to a different parameterization. We consider the scale matrix (rather than the covariance matrix) in order to fully employ the block equicorrelation structure later on for our conditional law of large numbers result.

To derive the score dynamics for the GHST model, we need the conditional density of y_t, which we parameterize as

$$p(y_t;\tilde{\Sigma}_t,\gamma,\nu) = \frac{2\,(\nu/2)^{\nu/2}}{\Gamma(\nu/2)\;|2\pi\,\tilde{\Sigma}_t|^{1/2}} \cdot \frac{K_{(\nu+N)/2}\left(\sqrt{d(y_t)\cdot d(\gamma)}\right)\, e^{\gamma'\tilde{\Sigma}_t^{-1}(y_t-\tilde{\mu})}}{\left(d(y_t)/d(\gamma)\right)^{(\nu+N)/4}}, \tag{3}$$

$$d(y_t) = \nu + (y_t-\tilde{\mu})'\tilde{\Sigma}_t^{-1}(y_t-\tilde{\mu}), \tag{4}$$

$$d(\gamma) = \gamma'\tilde{\Sigma}_t^{-1}\gamma, \quad \tilde{\mu} = -\frac{\nu}{\nu-2}\gamma, \tag{5}$$

where $K_a(b)$ is the modified Bessel function of the second kind; see Bibby and Sørensen (2003). The parameters $\gamma = (\gamma_1,\ldots,\gamma_N)' \in \mathbb{R}^N$ and $\nu \in \mathbb{R}^+$ are the skewness and degrees of freedom (or kurtosis) parameter, respectively, while $\tilde{\mu}$ and $\tilde{\Sigma}_t$ denote the location vector and scale matrix, respectively. Note that if y_t has a multivariate GHST distribution with parameters $\tilde{\mu}$, $\tilde{\Sigma}$, γ, and ν as given in (3), then $Ay_t + b$ for some matrix A and vector b also has a GHST distribution, with parameters $A\tilde{\mu}+b$, $A\tilde{\Sigma}A'$, $A\gamma$, and ν. In particular, the marginal distributions of y_{it} also have a GHST distribution. The GHST density (3) nests the symmetric-t ($\gamma = 0$) and multivariate normal ($\gamma = 0$ and $\nu \to \infty$) distributions as special cases.

We parameterize the time-varying matrix $\tilde{\Sigma}_t$ as in Engle (2002), i.e.,

$$\tilde{\Sigma}_t = D(f_t)\tilde{R}(f_t)D(f_t), \tag{6}$$

where f_t is a vector of time varying parameters, $D(f_t)$ is a diagonal matrix holding the scale parameters of y_{it}, and $\tilde{R}(f_t)$ captures the dependence parameters. In our current copula set-up, we use univariate models for $D(f_t)$, and the multivariate model for $\tilde{R}(f_t)$. The Web Appendix provides further details on our univariate volatility modeling approach for $D(f_t)$. In the remainder of this section, we concentrate on the matrix $\tilde{R}(f_t)$.

Following Creal et al. (2011, 2013), we endow f_t with score-driven dynamics using the derivative of the log conditional observation density (3). The transition dynamics for f_t are given by

$$f_{t+1} = \tilde{\omega} + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j}, \tag{7}$$

$$s_t = \mathcal{S}_t \nabla_t, \quad \nabla_t = \partial \ln p(y_t|\mathcal{F}_{t-1}; f_t, \theta)/\partial f_t, \tag{8}$$

where $\tilde{\omega} = \tilde{\omega}(\theta)$ is a vector of fixed intercepts, $A_i = A_i(\theta)$ and $B_j = B_i(\theta)$ are fixed parameter matrices that depend on the vector θ containing all time invariant parameters in the model, and $\mathcal{S}_t$ a scaling function.

The key element in (7) is the scaled score s_t. If y_t has a zero mean GHST distribution $p(y_t; \tilde{\Sigma}_t, \gamma, \nu)$ and the time-varying scale matrix is driven by (7)-(8), then the score is given by

$$\nabla_t = \Psi_t' H_t' \mathrm{vec}\left(w_t \cdot (y_t - \tilde{\mu})(y_t - \tilde{\mu})' - 0.5\tilde{\Sigma}_t - \gamma(y_t - \tilde{\mu})' - \check{w}_t \cdot \gamma\gamma'\right), \tag{9}$$

where

$$
\begin{aligned}
w_t &= \frac{\nu+N}{4d(y_t)} - \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(y_t)/d(\gamma)}}, \\
\check{w}_t &= \frac{\nu+N}{4d(\gamma)} + \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(\gamma)/d(y_t)}}, \\
H_t &= \tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}, \qquad \Psi_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t},
\end{aligned}
$$

and where $k_\nu(\cdot) = \ln K_\nu(\cdot)$ with first derivative $k'_\nu(\cdot)$. The matrices Ψ_t and H_t are time-varying and parameterization-specific; both matrices depend on f_t but not on the data y_t. We refer to the Web Appendix for a derivation of (9).

Equation (8) reveals the key feature of the score-driven specification. In essence, the score-driven mechanism takes a Gauss-Newton improvement step for the scale matrix to better fit the most recent observation. Equation (8) shows that f_t reacts to deviations between $\tilde{\Sigma}_t$ and the observed $(y_t - \tilde{\mu})(y_t - \tilde{\mu})'$. The reaction is asymmetric if $\gamma \neq 0$, in which case there is also a reaction to the level $(y_t - \tilde{\mu})$ itself. The reaction to $(y_t - \tilde{\mu})(y_t - \tilde{\mu})'$ is modified by the weight w_t. If $\nu < \infty$, the GHST distribution is fat tailed and the weight decreases in the Mahalanobis distance $d(y_t)$; compare the discussion for the symmetric Student's t case in Creal et al. (2011). This feature gives the model a robustness flavor in that incidental large values of y_t have a limited impact on future volatilities and correlations. The remaining expressions for H_t and Ψ_t only serve to transform the dynamics of the covariance matrix in (6) into the dynamics of the unobserved factor f_t.

To scale the score in (8) we set the scaling matrix $\mathcal{S}_t$ equal to the inverse conditional Fisher information matrix of the symmetric Student's t distribution; see the Web Appendix for details. Zhang et al. (2011) demonstrate that this choice of scaling matrix results in a stable model that outperforms alternative models if the data are fat-tailed and skewed. We do not use the inverse conditional Fisher information matrix of the multivariate GHST distribution because it is not available in closed form.

2.3 Dynamic block-equicorrelation structure

As we want to use our model in the context of a large cross-sectional dimension to describe the joint (tail) risk dynamics in a large system of financial institutions, we refrain from modeling all dependence parameters in $\tilde{R}(f_t)$ individually. Instead, we adopt the approach of Engle and Kelly (2012) and impose a block-equicorrelation structure on the matrix $\tilde{R}(f_t)$. By limiting the number of free parameters, we facilitate the estimation process to a large extent while retaining the ability to capture dynamic patterns in the dependence structure among financial firms, particularly in times of stress.

For the single block equicorrelation model, we assume that $\tilde{\Sigma}_t$ takes the form

$$\tilde{\Sigma}_t = (1 - \rho_t^2)\mathrm{I}_N + \rho_t^2 \ell_N \ell_N', \tag{10}$$

where $\rho_t = (1 + \exp(-f_t))^{-1} \in (0, 1)$, and ℓ_N is a $N \times 1$ vector of ones. In this case, the expressions for Ψ_t and H_t simplify. In particular,

$$\Psi_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = (\ell_{N^2} - \mathrm{vec}(\mathrm{I}_N)) \frac{2\exp(-f_t)}{(1 + \exp(-f_t))^3}, \tag{11}$$

which implies that the score ∇_t reduces to a scalar process over time. We refer to the Web Appendix for a derivation. We can easily generate the equicorrelation structure (10) from model (1) by specifying

$$y_{it} = (\varsigma_t - \mu_\varsigma)\, \gamma_i + \sqrt{\varsigma_t} \left(\rho_t \, \kappa_t + \sqrt{1 - \rho_t^2} u_{it} \right), \tag{12}$$

where κ_t and u_{it} are two independent standard normal random variables. Note that (12) is a special case of (1). The logistic parameterization $\rho_t = (1 + \exp(f_t))^{-1}$ forces the correlation parameter to be in the unit interval, irrespective of the value of $f_t \in \mathbb{R}$. While the original equicorrelation specification of Engle and Kelly (2012) also allows for (slightly) negative equicorrelations, such values are typically unrealistic in the type of applications we consider later on. The parameterization with equicorrelation parameter $\rho_t^2 > 0$ therefore suffices for our current purposes.

A 2-block equicorrelation model may be considered in settings for which the restriction of a single correlation parameter characterizing the entire scaling matrix $\tilde{\Sigma}_t$ might be too restrictive empirically. For example, we might want to allow for different dependence between financial firms in stressed and non-stressed countries in the context of the euro area sovereign debt crisis. For two blocks containing N_1 and N_2 firms, respectively, the 2-block equicorrelation specification is given by

$$\tilde{\Sigma}_t = \begin{bmatrix} (1-\rho_{1,t}^2)\mathrm{I}_{N_1} & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_{N_2} \end{bmatrix} + \begin{pmatrix} \rho_{1,t}\ell_{N_1} \\ \rho_{2,t}\ell_{N_2} \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell'_{N_1} & \rho_{2,t}\ell'_{N_2} \end{pmatrix}. \tag{13}$$

The 2-block equicorrelation structure (13) differs from the setup in Engle and Kelly (2012) in that there is a direct relation between the equicorrelation in the off-diagonal blocks and the diagonal blocks. The main advantage of this specification is that conditional on ς_t equation (13) preserves the Vasicek (1987) single factor credit risk structure of y_t. In particular, specification (12) depends on two common factors only, namely κ_t and ς_t, and conditionally on ς_t the model is linear. We use this feature extensively to compute joint and conditional risk measures fast and reliably in Section 3 in settings where standard simulation methods quickly become inefficient.[3] If $\tilde{\Sigma}_t$ is given by (13) with $\rho_{j,t} = (1+\exp(-f_{j,t}))^{-1}$ for $j = 1, 2$, then the time varying factor $f_t = (f_{1,t}, f_{2,t})' \in \mathbb{R}^{2\times 1}$ follows (9), with

$$\Psi_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t}\frac{\mathrm{d}\rho_t'}{\mathrm{d}f_t}, \tag{14}$$

$$\frac{\mathrm{d}\rho_t'}{\mathrm{d}f_t} = \begin{pmatrix} \frac{\exp(-f_{1,t})}{(1+\exp(-f_{1,t}))^2} & 0 \\ 0 & \frac{\exp(-f_{2,t})}{(1+\exp(-f_{2,t}))^2} \end{pmatrix},$$

$$\frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t} = \left(\mathrm{vec}\begin{pmatrix} \mathrm{I}_{N_1} & 0 \\ 0 & 0 \end{pmatrix}, \mathrm{vec}\begin{pmatrix} 0 & 0 \\ 0 & \mathrm{I}_{N_2} \end{pmatrix} \right) \cdot \begin{pmatrix} -2\rho_{1,t} & 0 \\ 0 & -2\rho_{2,t} \end{pmatrix}$$
$$+ \left(\begin{pmatrix} \rho_{1,t}\ell_{N_1} \\ \rho_{2,t}\ell_{N_2} \end{pmatrix} \otimes \mathrm{I}_N + \mathrm{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_{N_1} \\ \rho_{2,t}\ell_{N_2} \end{pmatrix} \right) \cdot \left(\begin{pmatrix} \ell_{N_1} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \ell_{N_2} \end{pmatrix} \right),$$

[3] We demonstrate in Section 4 that the tail risk measurements obtained from applying (10) or (13) are close to those based on a full correlation matrix analysis.

where $\rho_t = (\rho_{1,t}, \rho_{2,t})'$ and $N = N_1 + N_2$.[4]

The m-block equicorrelation structure is a straightforward generalization of the 2-block case (14). Rather than providing the (lengthy) expressions in the main text, we refer to our Web Appendix for the precise formulations. These formulations are used in our empirical analysis in Section 4, where we also consider a 3-block equicorrelation specification.

2.4 Parameter estimation

We can estimate the static parameters θ of the dynamic GHST model through standard maximum likelihood procedures. Parameter estimation is straightforward as the likelihood function is known in closed form using a standard prediction error decomposition. Deriving the asymptotic behavior for time varying parameter models with GAS dynamics is non-trivial. We refer to Blasques et al. (2012, 2014) for details.

We split the estimation problem into two parts by adopting a copula perspective. As a result, the number of parameters that need to be estimated in each step is reduced substantially. In addition, the copula perspective has the advantage that we can add more flexibility to modeling the marginal distributions. For example, when working with a multivariate GHST density, all marginal distributions must have the same kurtosis parameter ν. By adopting a copula perspective, we can relax this restriction considerably.

The two stages of the estimation process can be summarized as follows. In a first step, we estimate univariate dynamic GHST models using the equity returns for each firm i; see the Web Appendix for details. Based on the estimated univariate models with parameters $\tilde{\mu}_{it}$, $\tilde{\sigma}_{it}$, γ_i, and ν_i, we transform the observations into their probability integral transforms $u_{it} \in [0, 1]$. In the second step, we estimate the matrix $\tilde{\Sigma}_t = \tilde{R}_t$ as parameterized in Section 2.3, using the probability integral transforms u_{it} constructed in the first step. The GHST copula parameters are 0, $\tilde{R}_t$, $\gamma \cdot (1, \ldots, 1)'$ for $\gamma \in \mathbb{R}$, and ν, respectively. The static parameter vector θ includes γ, ν, and $\tilde{\omega}$, A_j, and B_j of the dynamic equation (7).

[4] We can introduce further flexibility to the model by extending the support of $\rho_{1,t}$ and $\rho_{2,t}$ from $(0, 1)$ to $(-1, 1)$ by defining $\rho_{j,t} = (\exp(f_{j,t}) - 1)/(\exp(f_{j,t}) + 1)$ for $j = 1, 2$. The advantage of this extension is that the correlations between blocks can now become negative, whereas the within block correlation remains positive. This extension is not needed for our empirical application in Section 4, however.

3 Joint and conditional risk measures

This section defines joint and conditional risk measures and demonstrates how to compute these efficiently and reliably based on the application of a conditional law of large numbers (cLLN). Using a cLLN in the credit risk context was popularized by Vasicek (1987) and studied further in for example Gordy (2000, 2003) and Lucas et al. (2001, 2003).

Conceptually, the simplest way to compute joint and conditional default probabilities is based on Monte Carlo simulations of firms' asset values. For example, one can generate many paths for the joint evolution of $(y_{1t}, \ldots, y_{Nt})$ and check how many simulations lie in a joint distress region of the type $\{y_t \,|\, y_{jt} < y^*_{jt} \; \forall j \in J\}$ for some set of firms $J \subset \{1, 2, \ldots, N\}$, where y^*_{jt} denotes the default threshold for firm j at time t. Such a simulation-based approach quickly becomes inefficient if the cross sectional dimension of the data and the number of firms considered in the set J become large: because marginal default probabilities are typically small, we need a large number of simulations to obtain a sufficient number of realizations of joint defaults, particularly if 3, 4, or even more joint defaults are considered. A partial remedy could be to use simulations based on importance sampling methods as in Glassermann and Li (2005), but the computational burden would remain high compared to the simple semi-analytic approach proposed in this section.

The semi-analytic cLLN approximation we propose is based on the observation that (12) is a non-linear 2-factor model. More specifically, conditional on the common factor ς_t model (12) simplifies to a (heterogeneous) Gaussian 1-factor model as in Vasicek (1987). This holds even if we replace ρ_t in (12) by ρ_{it} using the block-equicorrelation structure from Section 2. We exploit this feature to obtain reliable alternative risk measures that can be evaluated semi-analytically.

We define our joint tail risk measure (JRM) as the time-varying probability that a certain fraction of firms defaults over a pre-specified period. Let $D_{N,t}$ denote the fraction of firms defaulting over period t, e.g., $D_{N,t} = 5\%$, with

$$D_{N,t} = \frac{1}{N} \sum_{i=1}^{N} 1\{y_{it} < y^*_{it}\}. \tag{15}$$

Since the indicators $1\{y_{it} < y_{it}^*\}$ are conditionally independent given κ_t and ς_t, we can apply a conditional law of large numbers to obtain

$$D_{N,t} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{E}[1\{y_{it} < y_{it}^*\} \mid \kappa_t, \varsigma_t] = \frac{1}{N}\sum_{i=1}^{N} \mathrm{P}[y_{it} < y_{it}^* \mid \kappa_t, \varsigma_t] \equiv C_{N,t}, \tag{16}$$

for large N, as under standard regularity conditions $|D_{N,t} - C_{N,t}| \stackrel{a.s.}{\to} 0$ for $N \to \infty$; see for example Vasicek (1987), Gordy (2000, 2003), and Lucas et al. (2001). Note that

$$\mathrm{P}[y_{it} < y_{it}^* | \kappa_t, \varsigma_t] = \Phi\left(\frac{y_{it}^* - (\varsigma_t - \mu_\varsigma)\,\gamma_i - \sqrt{\varsigma_t}\,\rho_t \kappa_t}{\sqrt{\varsigma_t\,(1-\rho_t^2)}}\right), \tag{17}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution. Also note that $C_{N,t}$ is a function of the random variables κ_t and ς_t only, and not of u_{it} in (12). We now define the joint tail risk measure as

$$\mathrm{P}(D_{N,t} > \bar{c}) \approx \mathrm{P}(C_{N,t} > \bar{c}) = \mathrm{P}(C_{N,t}(\kappa_t, \varsigma_t) > \bar{c}) \equiv p_t, \tag{18}$$

i.e., we approximate the probability that the fraction of credit portfolio defaults $D_{N,t}$ exceeds the threshold $\bar{c} \in [0,1]$ by the quantity p_t. Following Vasicek (1987) and using the 1-factor structure of (12) for given ς_t, we note that $C_{N,t}$ is monotonically decreasing in κ_t for $\rho_{i,t} > 0$. This is intuitive: an increase in κ_t (for instance due to improved business cycle conditions) implies less defaults in the portfolio and vice versa. We exploit this to efficiently compute unique threshold levels $\kappa_{N,t}^*(\bar{c}, \varsigma^{(g)})$ for a number of grid points $\varsigma^{(g)}$, $g = 1, \ldots, G$. This can be done by solving the equations $C_{N,t}(\kappa_{N,t}^*(\bar{c}, \varsigma^{(g)}), \varsigma^{(g)}) = \bar{c}$ numerically for the threshold values $\kappa_{N,t}^*(\bar{c}, \varsigma^{(g)})$ for each grid point $\varsigma^{(g)}$ and time t. Given a grid of threshold values, we can then use standard numerical integration techniques to efficiently compute the joint default probability

$$p_t = \mathrm{P}(C_{N,t} > \bar{c}) = \int \mathrm{P}(\kappa_t < \kappa_{t,N}^*(\bar{c}, \varsigma_t))p(\varsigma_t)\mathrm{d}\varsigma_t. \tag{19}$$

Our second measure is a conditional tail risk measure (CRM). Let

$$D_{N-1,t}^{(-i)} = \frac{1}{N-1}\sum_{j\neq i} 1[y_{jt} < y_{jt}^{*} \mid \kappa_t, \varsigma_t] \approx \frac{1}{N-1}\sum_{j\neq i} \mathrm{P}[y_{jt} < y_{jt}^{*} \mid \kappa_t, \varsigma_t] \equiv C_{N-1,t}^{(-i)},$$

where $D_{N-1,t}^{(-i)}$ is the fraction of defaulted companies excluding firm i, which we approximate using the cLLN by $C_{N-1,t}^{(-i)}$. We define the CRM as the probability of $C_{N-1,t}^{(-i)}$ exceeding $\bar{c}^{(-i)}$ conditional on the default of firm i, i.e.,

$$\begin{aligned} \mathrm{P}\Big(C_{N-1,t}^{(-i)} > \bar{c}^{(-i)} \Bigm| y_{it} < y_{it}^{*}\Big) &= p_{it}^{-1} \cdot \mathrm{P}(C_{N-1,t}^{(-i)} > \bar{c}^{(-i)} \ , \ y_{it} < y_{it}^{*}) \\ &= p_{it}^{-1} \cdot \int \mathrm{P}\left(\kappa_t < \kappa_{N-1,t}^{*}(\bar{c}^{(-i)}, \varsigma_t) \ , \ y_{it} < y_{it}^{*} \bigm| \varsigma_t\right) \ p(\varsigma_t)\mathrm{d}\varsigma_t \\ &= p_{it}^{-1} \cdot \int \Phi_2(\kappa_{N-1,t}^{*}(\bar{c}^{(-i)}, \varsigma_t) \ , \ y_{it}^{**}(\varsigma_t) \ ; \ \rho_t) \ p(\varsigma_t)\mathrm{d}\varsigma_t, \end{aligned} \tag{20}$$

where $y_{it}^{**}(\varsigma_t) = (y_{it}^{*} - (\varsigma_t - \mu_\varsigma)\gamma_i)/\sqrt{\varsigma_t}$, and $\Phi_2(\cdot, \cdot; \rho_t)$ denotes the cumulative distribution function of the bivariate normal with standard normal marginals and correlation parameter ρ_t. To obtain the last equality in (20), note that the GHST distribution becomes Gaussian conditional on ς_t. The conditional probability (20) is a time-varying higher-frequency extension of the multivariate extreme spillovers measure of Hartmann et al. (2004, 2007).

Both the joint probability (19) and the conditional probability (20) can be computed quickly, using simple 1-dimensional numerical integration techniques.[5] In addition, the model is easily extended to fit the m-block equicorrelation structure explained in Section 2.3. The fact that the $\rho_{i,t}$ parameters are different between blocks does not distort the one-factor structure à la Vasicek (1987) of the current set-up, and all computations remain of similar structure and speed.

[5] It is straightforward to add exposure weights e_i to the definition of $D_{N,t}$ in (15). The computations in that case remain equally efficient. If the exposures are very unevenly distributed, however, the approximation error of the cLLN in (16) might increase. To mitigate such an effect, one could try to implement a second-order expansion using a conditional central limit theorem rather than a cLLN only.

4 Euro area financial sector joint tail risk

We apply our model to a high-dimensional dataset of $N = 73$ euro area financial firms. We first present a preliminary analysis for a subsample of $N = 10$ large firms headquartered in a different EU countries. We then present the results for the entire dataset. This allows us to benchmark the results for the dynamic GHST block equicorrelation model against a model with a fully specified time varying correlation matrix and to investigate the sensitivity of joint and conditional tail risk measures to the (block) equicorrelation assumption.

4.1 Equity and EDF data

Our equity data come from Bloomberg. We use 73 listed financial firms that are located in 11 euro area countries: Austria (AT), Belgium (BE), Germany (DE), Spain (ES), Finland (FI), France (FR), Greece (GR), Ireland (IE), Italy (IT), the Netherlands (NL), and Portugal (PT). Firms were selected if (i) they were financial firms headquartered in the euro area, and (ii) were listed as of 2011Q1 as a component of the STOXX Europe 600 index. For each firm, we construct recursively demeaned weekly equity returns. The sample comprises large commercial banks as well as large financial non-banks such as insurers and investment companies. The total panel covers 762 weeks from January 1999 to August 2013. The panel is unbalanced in that some data are missing in the first part of the sample. We assume that this is not related to the volatility dynamics or the credit risk mechanism in the data. The scores in equation (9) automatically correct for the unbalancedness of the data.

For the marginal default probabilities p_{it}, we use one year ahead expected default frequencies (EDF) obtained from Moody's Analytics. EDFs are widely-used measures of time-varying one-year-ahead marginal default probabilities (see Duffie et al., 2007). We do not use the EDFs to estimate the dependence structure. Rather, we only use the EDFs to calibrate the model at any time to current market perceptions of one-year-ahead marginal default risk conditions for each firm in the sample. The copula as estimated from the equity data subsequently takes care of the dependence structure when computing the joint and conditional tail risk measures.

4.2 Small-sample study of 10 banking groups

Before presenting the results for the full sample of $N = 73$ institutions, we first study a geographically diversified sub-sample of ten large financial firms from ten euro area countries. We do this to study how the equicorrelation assumption may affect our joint and conditional risk measures compared to a model with an unrestricted correlation structure. The subsample contains no missing observations and consists of Erste Bank Group (AT), Dexia (BE), Deutsche Bank (DE), Santander (ES), BNP Paribas (FR), National Bank of Greece (GR), Bank of Ireland (IE), UniCredito (IT), ING (NL), and Banco Comercial Portugues (PT). We distinguish firms across different countries given the interdependence of bank risk and sovereign risk as an important feature of the euro area sovereign debt crisis; see for example ECB (2012, 2014).

4.2.1 Dependence modeling

This section compares correlation estimates across four different dependence models. Since we use a copula approach, the models share the same structure for the univariate volatilities. The descriptive statistics in the Web Appendix reveal that the return data are significantly negatively skewed and fat-tailed. We therefore use the dynamic GHST model for the marginals.

We consider three score-driven equicorrelation specifications with 1, 2, and 3 blocks, respectively. The 2-block model distinguishes firms in countries that experienced pronounced stress during the sovereign debt crisis (Greece, Ireland, Italy, Portugal, Spain) and firms headquartered in non-stressed countries; see Eser and Schwaab (2015) for a similar grouping of countries. The 3-block model further distinguishes firms in smaller stressed countries that entered the sovereign debt crisis earlier (Greece, Portugal, Ireland), and larger stressed countries that entered the sovereign debt crisis at a relatively later stage (Spain and Italy); see ECB (2014). As a benchmark, we consider a model with a full correlation matrix with DCC dynamics as in Christoffersen et al. (2014), which we estimate through composite likelihood methods. As in the block-equicorrelation specifications, the DCC full correlation

Table 1: Parameter estimates for the copula models

Parameter estimates for multivariate GAS-GHST models for financial firms' equity returns. The left-hand and right-hand blocks refer to the copula models for $N = 10$ and $N = 73$ firms, respectively. Univariate GAS-GHST models are used to model the marginal volatilities. The 2-block model distinguishes between firms from stressed countries, i.e., Greece, Portugal, Spain, Italy, Ireland, and firms from the remaining non-stressed countries in the euro area, i.e., Austria, Belgium, France, Germany, and the Netherlands. The 3-block model further distinguishes between financial firms from smaller stressed countries (Greece, Portugal, Ireland), larger stressed countries (Spain, Italy), and non-stressed countries. Standard errors are in parentheses.

	10 firms				73 firms		
	GAS-BEq[1]	GAS-BEq[2]	GAS-BEq[3]	DCC-Full	GAS-BEq[1]	GAS-BEq[1]-t	GAS-BEq[3]
A	0.202	0.111	0.093	0.021	0.544	0.261	0.165
	(0.058)	(0.022)	(0.015)	(0.003)	(0.057)	(0.031)	(0.020)
B	0.983	0.991	0.993	0.966	0.984	0.980	0.992
	(0.015)	(0.007)	(0.005)	(0.003)	(0.011)	(0.011)	(0.005)
ω_1	0.569	0.764	0.713		0.179	0.333	0.167
	(0.210)	(0.284)	(0.293)		(0.236)	(0.171)	(0.193)
ω_2		0.201	0.870				0.459
		(0.318)	(0.451)				(0.191)
ω_3			-0.341				-0.387
			(0.446)				(0.335)
ν	17.106	17.683	18.571	14.784	32.534	34.802	31.359
	(2.012)	(2.135)	(2.705)	(1.791)	(1.809)	(2.152)	(2.673)
γ_1	-0.230	-0.237	-0.250	-0.294	-0.586		-0.590
	(0.092)	(0.106)	(0.117)	(0.058)	(0.054)		(0.098)
γ_2		-0.307	-0.272				-0.433
		(0.101)	(0.132)				(0.068)
γ_3			-0.352				-0.574
			(0.126)				(0.087)

matrix model uses a common scalar skewness parameter γ in the GHST copula. For $N = 10$ firms the model with full correlation matrix contains 45 pairwise correlation coefficients, and thus 45 dynamic factors. Correlation targeting is used to estimate the intercepts in the transition equation for the correlations.

The first columns in Table 1 report parameter estimates and model selection criteria for the 1, 2, and 3-block equicorrelation models for $N = 10$. The correlation dynamics are highly persistent in all specifications given the high values of B, or of $A + B$ in case of the DCC specification. The unconditional correlation levels as captured by the parameters ω_i are higher for firms in non-stressed countries. For $N = 10$, both the degrees of freedom parameter and the common skewness parameter of the GHST copula have the same sign and similar magnitudes for the block equicorrelation models (BEq[m]) and the DCC specification.

Figure 1 plots the estimated dynamic correlations. The top left panel plots the single block equicorrelation along with the average pairwise correlations of each firm with the other (nine) firms. The latter correlation estimates are based on the full-correlation model with 45 time varying parameters. The correlation estimates reveal a pronounced commonality in the correlation dynamics. This is intuitive, as we model firms from the same industry which operate in a single currency area and are subject to similar regulatory requirements.[6] All correlations tend to increase over the sample period, possibly reflecting gradual financial integration and economic convergence in the euro area following the inception of the euro in 1999. All correlations remain elevated during the global financial crisis from 2008–2010, and peak at a time when Greece, Ireland, and Portugal needed the assistance of third parties, such as the EU and the IMF in mid-2010, see ECB (2014). Such shared correlation dynamics can be captured simply and conveniently by block equicorrelation structures.

We can capture a larger share of the cross-sectional dispersion in correlations when we allow for multiple blocks. The top right panel of Figure 1 plots the dynamic correlation estimates for two groups. The first group contains the Bank of Ireland, Banco Comercial Portugues, Santander, UniCredito and National Bank of Greece. The second group includes BNP Paribas, Deutsche Bank, Dexia, Erste Bank Group, and ING. The overall dependence dynamics are similar.[7] The bottom left panel plots the correlation estimates for the 3-block model, which allows for further cross-sectional dispersion in correlation estimates. Finally, the bottom right panel compares the average correlations across models. In addition, we provide average correlations estimated from a 52-week rolling window. The average correlations are similar across all models. Only relatively minor deviations are observed. We conclude that despite its restrictive nature, the equicorrelation model reliably estimates the salient trends in average correlation dynamics.

Although the block equicorrelation models work well in capturing the average correla-

[6]A principal components analysis of the 45 correlation pairs from the full-correlation model suggest that the first three components explain 55.6%, 16%, and 8.5% of the total variation, respectively. The first two factors therefore explain approximately 72% of the total variation in correlations.

[7]Perhaps surprisingly, the correlation between firms from non-stressed countries lies above that of firms in stressed countries, also before the financial and sovereign debt crisis starting 2008. It is therefore probably related to different degrees of financial integration, rather than to shared exposure to heightened market turmoil in a crisis.

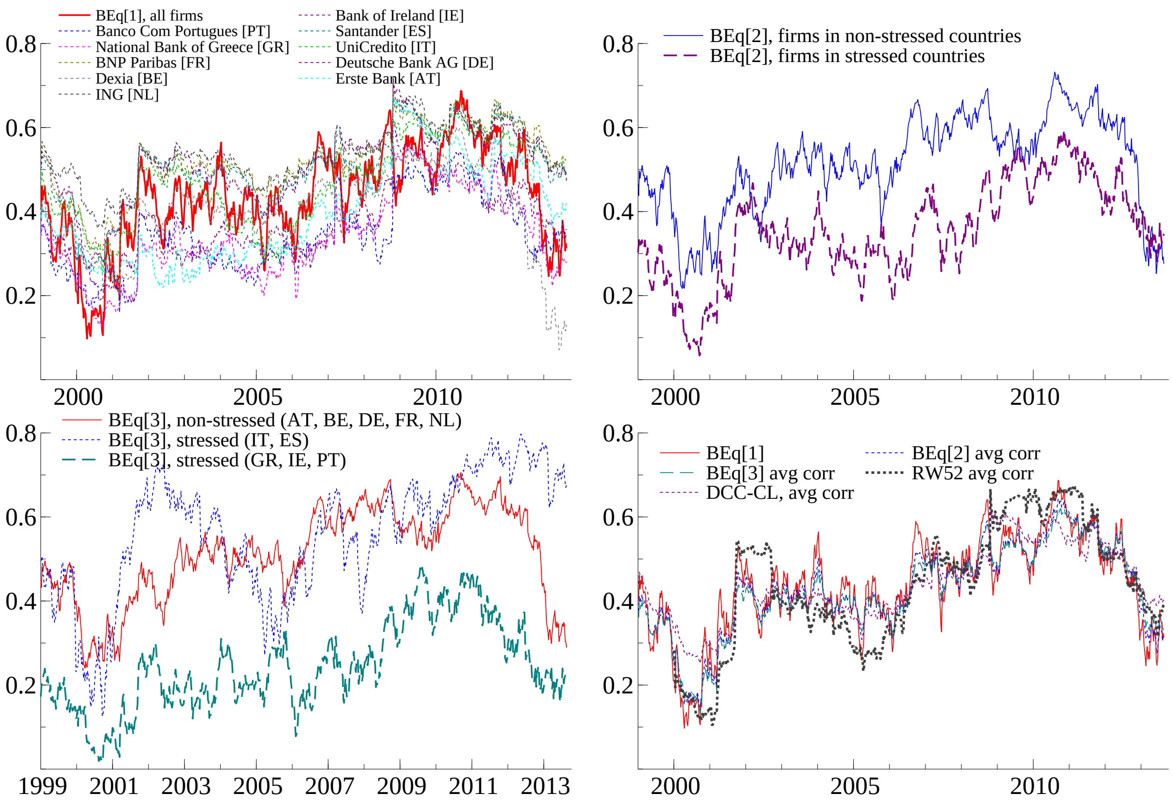

Figure 1: Filtered correlation estimates for $N = 10$ firms

The top left panel plots the correlation estimates based on the BEq[1] model, along with the average pairwise correlation of each firm with the other nine firms. The latter are based on the specification with full correlation matrix with 45 time varying parameters and DCC dynamics. The top right and bottom left panels plot the block equicorrelation estimates based on the 2-block (BEq[2]) and 3-block (BEq[3]) model specification, respectively. The bottom right panel compares the average correlation estimates from the 1-block, 2-block, and 3-block models to the average (over 45 pairs) correlation estimates from a GHST full-correlation DCC model, as well as the average correlation based on a 52-week rolling window.

tions, a substantial share of the cross-sectional dispersion in correlations may be lost when using block equicorrelation structures; see the top left panel of Figure 1. This may or may not matter when evaluating joint and conditional credit risk measures. To investigate how much cross-sectional dispersion in correlations is lost, the left panel of Figure 2 plots R^2 statistics that correspond to repeated *cross-sectional* regressions of 45 correlation pairs from the GHST full-correlation-matrix model on a constant and the corresponding correlation estimates from a 2-block and 3-block equicorrelation model, respectively. By construction, the 1-block equicorrelation model is unable to account for any cross-sectional variation in correlations, as it collapses the latter to a single number (i.e., a regression on a constant). For the current sample, the 2-block and 3-block equicorrelation models are able to account

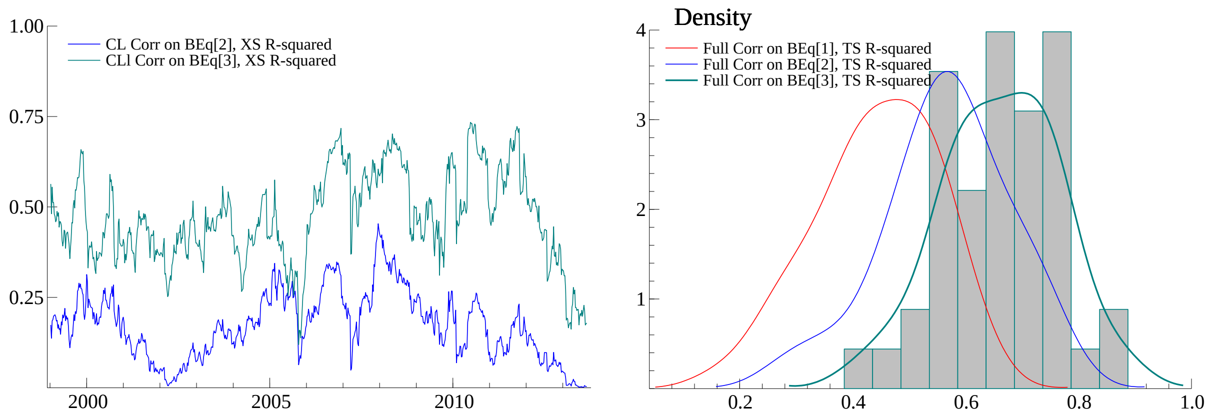

Figure 2: R-squared statistics

The left panel plots R^2 statistics corresponding to repeated (over time) *cross-sectional* regressions of 45 correlation pairs from the $N = 10$ GHST full-correlation-matrix model on a constant and the corresponding correlation estimates from a 2-block, and 3-block equicorrelation model. The right panel plots Kernel density estimates and a histogram of 45 R^2 statistics corresponding to *time series* regressions with $T = 762$ observations each of correlations from a GHST full-correlation-matrix model on the corresponding correlation estimates from a 1-block, 2-block, and 3-block equicorrelation model, respectively. The histogram refers to R^2s from the 3-block equicorrelation model.

for approximately 20% and 50% of the cross-sectional dispersion in correlations.

The right panel of Figure 2 plots kernel density estimates and a histogram of *time-series* R^2 statistics. These correspond to a histogram of 45 time series regressions, with $T = 762$ weekly observations each, of GHST full-correlation matrix model correlations on the corresponding estimates from a 1-block, 2-block, and 3-block equicorrelation specification, respectively. The 1-block, 2-block, and 3-block equicorrelation models are able to account for approximately 30-50%, 40-60%, and 50-70% of the time series variation in the full correlation estimates, respectively. Both findings confirm that the simplified equicorrelation assumption can capture a large part of the variation in correlations. In the next section we highlight that the percentage of variation captured by the BEq[m] models appears adequate for reliable estimates of our systemic risk measures.

4.2.2 Joint and conditional probabilities

This section compares joint and conditional default probabilities as defined in Section 3 and as implied by different copula model specifications. We argue that the cLLN works well even when the data dimension is very small. This is because it eliminates a source of risk (idiosyncratic risk) that does not matter in the tail of the credit loss distribution. Only κ_t and ς_t are common to all firms and drive extreme default clustering. In addition, block equicorrelation and full correlation models lead to approximately similar patterns for joint and (to a lesser degree) also for conditional tail probability estimates.

We compare the full-correlation specification with a 1-block and 3-block copula model. In addition, we assess the adequacy of the cLLN approximation by comparing it to a simulation-based approach to compute joint and conditional default probabilities. The default thresholds are obtained by inverting the GHST distribution function at the observed EDF levels. For the simulation-based computations, we use 500,000 simulations, each at time t, where we save the number and the identities of the defaulted firms in each simulation.

The top left and right panels in Figure 3 refer to the probability of observing three or more defaults (out of ten firms) over a one-year-ahead horizon. The left panel compares the 1-block specification with the full-correlation matrix outcome; the right panel considers the 3-block specification. Risk measures are either simulated or computed semi-analytically based on equation (19). As an important finding, the joint default probabilities are similar in each of the six cases. The losses from moving from a full correlation matrix to a much simpler equicorrelation structure are generally small. Also the losses from applying the cLLN approximation vis-á-vis the simulation approach are small in these cases, even though $N = 10$ is far from infinity.

The bottom panels in Figure 3 plot the conditional probability of 2 or more (out of 9 possible) credit events given a credit event for a specific financial firm, averaged over all ten firms. Again, the left and right panels refer to the 1-block and 3-block model, respectively. In the 1-block case, the loss in fit from moving from a full-correlation-matrix model to the equicorrelation model is more pronounced. This is in line with the R^2 results shown in Figure 2. The cLLN approximation, however, works relatively well. In the 3-block case, the respective loss in fit is less pronounced. Here, however, the cLLN approximation appears not to work as well. This reveals an interesting trade-off when choosing the number of blocks. On the one hand, increasing the number of blocks can help in capturing more of the cross-sectional dispersion in correlations and increases the fit in the time dimension, see Figure

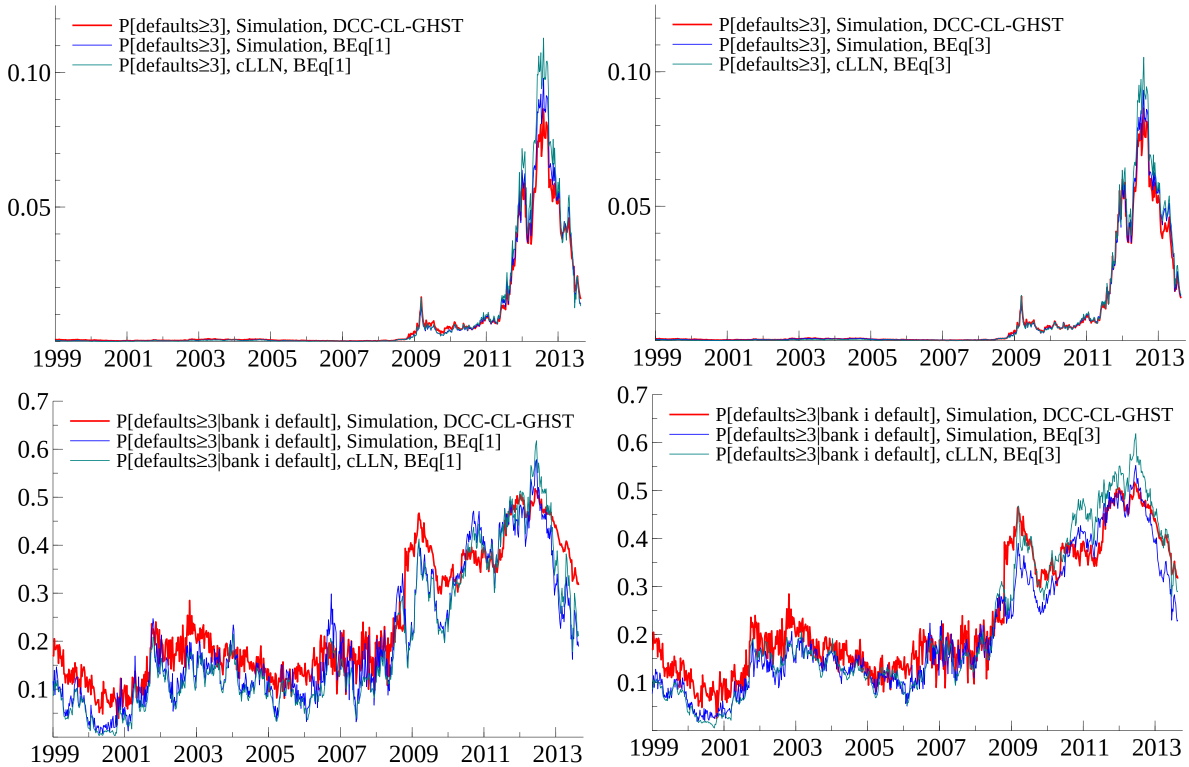

Figure 3: Joint and conditional probabilities

The two top panels plot the joint default probability (19) of three or more financial firm defaults out of ten. The two bottom panels plot the (average) conditional probability (20), which is the probability of 2 or more (out of 9 possible) defaults given a default of a specific financial firm, averaged over all ten firms. The probabilities are either computed using 500,000 simulations at each point in time t, or alternatively using the cLLN approximation as discussed in Section 3 for block equicorrelation models. The left (top and bottom) panels are based on the estimated 1-block equicorrelation model, while the right (top and bottom) panels are based on the estimated 3-block equicorrelation model.

2. On the other hand, increasing the number of blocks also means less firms in each block, which in turn implies that the cLLN approximation does not work as well within each block.[8] Finally, the simulation based approach can suffer from sizable simulation noise in non-crisis periods (when marginal default probabilities are low, see bottom panels before 2007). No such problems are encountered for the cLLN based approximation.

[8]Recall that the 3-block model contains 5, 3,and 2 firms in each block. The cLLN hardly applies in the latter two cases, particularly given the low loading to the common κ_t for the stressed small countries shown in Figure 1.

4.3 All 73 euro area financial firms

Given the encouraging preliminary results for the subsample of $N = 10$ institutions, we now turn to the joint and conditional tail risk measures based on the full panel of 73 large financial sector firms. The sample contains commercial banks as well as financial non-banks such as insurers and investment companies. All are listed at a stock exchange.[9] Based on descriptive statistics as presented in the Web Appendix, univariate GHST models seem appropriate given the skewness and kurtosis features of the equity return data.

4.3.1 Dependence modeling

Figure 4 plots the correlation estimates. For parameter estimates, we refer to Table 1. We use the same model specifications as described in Section 4.2, but now estimated on the full sample of $N = 73$ firms. The equicorrelation estimates (top left) range from low values of approximately 0.1 in 2000 to values as high as 0.6 towards the start of the sovereign debt crisis in 2010. Using the two block structure (top right), there appear to be little differences between financial firms from stressed versus non-stressed countries, except during the peak of the sovereign debt crisis between 2011–12. The bottom left panel of Figure 4 introduces further heterogeneity by modeling the dynamic correlation for firms from Greece, Ireland, and Portugal on the one hand, and Italy and Spain on the other hand. Again, the rationale for this grouping is that Ireland, Greece, and Portugal entered the euro area sovereign debt crisis earlier, and were relatively more stressed, compared with Spain and Italy. This distinction may matter for our inference on financial sector tail risks around 2012. We obtain a lower correlation level among financial institutions from the first group of smaller stressed countries. The correlation rises after the financial crisis up to the start of 2010, and then decreases to pre-crisis levels around 2012. By contrast, the correlation for financial firms from Italy and Spain starts to rise earlier, peaks higher, and remains high until the end of

[9] Freezing the set of firms as the constituents of a broad based equity index in 2011Q1 means that we may underestimate total euro area financial sector risk prior to this date (due to sample selection; weaker firms may have dropped out of the index by then). While this concern should be kept in mind, it is unlikely to be large, as most financial institutions under stress during the financial crisis continued to operate, also due to substantial government aid and the extension of public sector guarantees. Sample selection is no issue after 2011Q1.

the sample. If we consider the average correlations across all $73(73-1)/2 = 2628$ pairs in the bottom right panel, the picture emerging from all three model specifications is similar.

Our equity panel dataset is characterized by a substantial amount of common variation across correlations. A principle components analysis (PCA) of 52-week rolling window correlations suggest that the first three components explain approximately 43%, 22% and 12% of the total variation in pair-wise correlations, respectively. This suggests that a 3-block model specification — which allows for 6 different correlations at each point in time — should capture a substantial share (approximately 60-75%) of the common time series variation of all 2628 full correlations. This is the case. We refer to the Web Appendix for goodness-of-fit statistics from regressing full correlation estimates on BEq[m]-estimates. Full correlation estimates are obtained from either rolling window correlations or based on the maximization of a composite likelihood as in Christoffersen et al. (2012).

4.3.2 Joint and conditional tail risk

This section presents our joint and conditional tail risk estimates for financial sector firms in the euro area. For the joint default probability, we consider the probability that $\bar{c} = 7/73 \approx 10\%$ or more of currently active financial institutions experience a credit event over the next 12 months. The probability of such widespread and massive failure should typically be very small during non-crisis times. We plot the result in the upper panel in Figure 5.

Different copula specifications yield strikingly similar results. As the joint probability moves relatively little before 2008, we only plot it over the period 2006–2013. For a comparison of joint and conditional risk outcomes for a variety of GHST and symmetric-t copula specifications we refer to the Web Appendix.[10] The joint probability moves sharply upwards after the bankruptcy of Lehman Brothers in September 2008, and reaches a first peak during the Irish sovereign debt crisis in the Spring of 2009. It remains approximately constant thereafter until the onset of the sovereign debt crisis in early 2010. It reaches a first peak in late 2011, followed by a second and final peak mid-2012.

[10]The likelihood of the single block equicorrelation model drops by approximately 48.4 points if the restriction of symmetry is imposed. This is statistically significant.

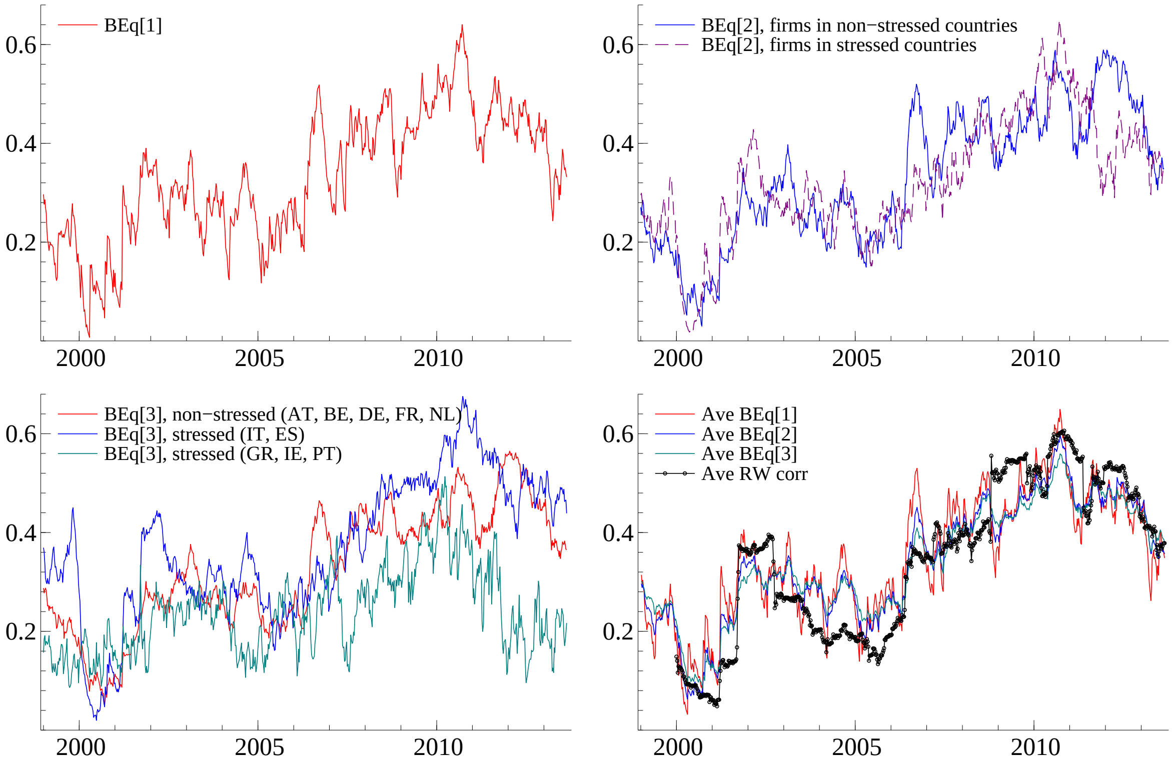

Figure 4: Filtered correlation estimates for all $N = 73$ financial firms

The panels plot GHST block equicorrelation estimates from different block equicorrelation models: 1-block (top left), 2-block (top right) and 3-block (bottom left). The fourth panel benchmarks the block equicorrelation estimates to the average correlation taken across $73(73-1)/2 = 2628$ pairwise estimates based on a 52 weeks rolling window.

The vertical lines in the top panel of Figure 5 indicate a number of relevant policy announcements. In late 2011, the announcement and implementation of two 3-year Long Term Refinancing Operations (3y-LTROs, allotted in December 2011 and February 2012) had a visible but temporary impact on financial sector tail risk.[11] In particular, the announcement and allotment of two 3y-LTRO in December 2011 and February 2012 appear to have lowered financial sector joint tail risk, temporarily, for a few months. In the first half of 2012, financial sector joint tail risk picked up again and rose to unprecedented levels until July 2012. The three vertical lines from July to September 2012, together with the time variation in the joint default probability, strongly suggest that three events collectively ended the most acute phase of extreme financial sector joint tail risks. These are a speech by the ECB president

[11]See ECB (2011) for the official press release and monetary policy objectives.

Figure 5: Joint and average conditional tail probabilities

The top panel plots the joint tail probability that 7 or more financial firms ($\bar{c} = 7/73 \approx 10\%$) experience a credit event over a 12 months ahead horizon, at any time t. The bottom panel plots the average conditional risk measure, the average (over all 73 financial firms) conditional probability that $\bar{c} = 7/73 \approx 10\%$ or more financial firms default given a default for a given firm i. In each case, computations are based on a GAS-BEq[1], GAS-BEq[3], and DCC-CL full correlation model, respectively. Computations rely on the conditional law of large numbers approximation for the GHST BEq[m] models, and on simulation for the DCC-composite likelihood model.

The vertical lines in the top panel indicate the following events: (a) the allotments of two three-year long term refinancing operations by the ECB on 21 December 2011 and 29 February 2012, (b) a speech by the ECB president ("whatever it takes") on 26 July 2012, (c) the OMT announcement on 2 August 2012, and (d) the announcement of the OMT technical details on 6 September 2012.

in London to do '*whatever it takes*' to save the euro on 26 July 2012, the announcement of the ECB's Outright Monetary Transactions program on 2 August 2012, and especially the disclosure of the OMT details on 6 September 2012. The joint default probability decreases sharply, all the way until the end of the sample. The OMT is a program within which the ECB can purchase ("outright transactions") bonds issued by euro area member states in secondary sovereign bond markets, under certain conditions. We refer to the ECB (2012) and Coeuré (2013) for details. No purchases have yet been undertaken by the ECB within the OMT. Instead, the mere announcement of the program was sufficient to trigger the substantial decline in financial sector joint tail risk.

The bottom panel of Figure 5 presents the average (across institutions) conditional tail probability for the 73 firms in our sample. Again, different copula model specifications yield similar results, although there are some discrepancies in the first third of the sample leading up to the financial crisis. Conditional probability estimates are also more sensitive to the choice of skewed or non-skewed copula; see the Web Appendix for details. Also, the DCC based conditional measures appear to be much less responsive to the different historical events. This is in line with the well-known phenomenon that the dynamics of DCC models become relatively flat in high dimensions. The equicorrelation model is much less susceptible to this bias. Though the composite likelihood approach for estimation partly corrects the potential bias in the DCC estimates, Figure 5 illustrates that the equicorrelation model pick up more of the dynamics in the series.

At the start of the sample, there is little evidence of systemic clustering on average with low levels of the CRM between 10%–40%. The conditional probability rises following the Lehman Brothers bankruptcy to levels of approximately 60%, and then to approximately 80% around the peak of the sovereign debt crisis. Such high levels of conditional tail probabilities signal strong interconnectedness among euro area financial institutions. Interestingly, the conditional tail probability is still quite high towards the end of the sample, despite the collapse in joint risk as shown in the top panel. The OMT announcements apparently did not lower market perceptions of conditional (or contagion) risks in the euro area financial system as a whole to a comparable extent.

As a final result, unconventional monetary policy measures, such as the 3y-LTROs and the OMT announcements, and financial stability tail risk outcomes appear strongly related. This suggests substantial scope for the coordination of monetary, macro-prudential, and bank supervision policies. This is relevant as both monetary policy as well as prudential policies and banking supervision will be carried out jointly within the ECB as of November 2014.

5 Conclusion

We developed a novel modeling framework for estimating joint and conditional tail risk probabilities over time in a financial system that consists of numerous financial sector firms. For this purpose, we used a copula approach based on the generalized hyperbolic skewed Student's t (GHST) distribution, endowed with score-driven dynamics. Parsimony and flexibility were traded off by using a dynamic block equicorrelation structure. Using this structure, we were able to implement efficient approximations based on a conditional law of large numbers to compute joint and conditional tail probabilities of multiple defaults for a large set of firms. An application to euro area financial firms from 1999 to 2013 revealed unprecedented joint default risks between 2011–12. We also document the collapse of these joint default risks (but not conditional risks) after a sequence of announcements pertaining to the ECB's Outright Monetary Transactions program in August and September 2012.

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Web Appendix to Modeling financial sector joint tail risk in the euro area

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Appendix A: Univariate volatility models

This appendix summarizes our univariate marginal modeling strategy. We estimate univariate dynamic GHST models using the equity returns for each firm i based on the maximum likelihood approach. Using the estimated univariate models with parameters are $\tilde{\mu}_{it}$, $\tilde{\sigma}_{it}$, γ_i, and ν_i, we transform the observations into their probability integral transforms $u_{it} \in [0,1]$. The univariate GHST density is given by

$$\begin{aligned}
p(y_t; \tilde{\sigma}_t^2, \gamma, \nu) &= \frac{\nu^{\frac{\nu}{2}} 2^{1-\frac{\nu+1}{2}}}{\Gamma(\frac{\nu}{2}) \pi^{\frac{1}{2}} \tilde{\sigma}_t} \cdot \frac{K_{\frac{\nu+1}{2}}\left(\sqrt{d(y_t)(\gamma^2)}\right) e^{\gamma(y_t - \tilde{\mu}_t)/\tilde{\sigma}_t}}{d(y_t)^{\frac{\nu+1}{4}} \cdot (\gamma^2)^{-\frac{\nu+1}{4}}}, \\
d(y_t) &= \nu + (y_t - \tilde{\mu}_t)^2 / \tilde{\sigma}_t^2, \\
\tilde{\mu}_t &= -\frac{\nu}{\nu-2} \tilde{\sigma}_t \gamma, \quad \tilde{\sigma}_t = \sigma_t T, \\
T &= \left(\frac{\nu}{\nu-2} + \frac{2\nu^2\gamma^2}{(\nu-2)^2(\nu-4)} \right)^{-1/2},
\end{aligned}$$

where $\tilde{\sigma}_t$ and σ_t are the univariate analogues to the Choleski square roots of the scale and covariance matrix, respectively. We here chose $\sigma_t = \sigma(f_t) = \exp(f_t/2)$. For this specification, the score driven dynamics are given by

$$\begin{aligned}
f_{t+1} &= \tilde{\omega} + \sum_{i=0}^{p-1} A_i s_{t-i} + \sum_{j=0}^{q-1} B_j f_{t-j} + C(s_t - s_t^{\tilde{\mu}}) 1\{y_t < \tilde{\mu}_t\}, \\
s_t &= \mathcal{S}_t \nabla_t, \quad \nabla_t = \partial \ln p(y_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, \\
s_t^{\tilde{\mu}} &= \mathcal{S}_t \nabla_t^{\tilde{\mu}}, \quad \nabla_t^{\tilde{\mu}} = \partial \ln p(\tilde{\mu}_t | \mathcal{F}_{t-1}; f_t, \theta) / \partial f_t, \\
\nabla_t &= \Psi_t H_t \left(w_t \cdot y_t^2 - \tilde{\sigma}_t^2 - \left(1 - \tfrac{\nu}{\nu-2} w_t\right) \tilde{\sigma}_t \gamma y_t \right), \\
w_t &= \frac{\nu+1}{2d(y_t)} - \frac{k'_{0.5(\nu+1)}\left(\sqrt{d(y_t) \cdot (\gamma^2)}\right)}{\sqrt{d(y_t)/(\gamma^2)}}, \\
\Psi_t &= \frac{\partial \sigma_t}{\partial f_t}, \quad H_t = T \tilde{\sigma}^{-3}.
\end{aligned}$$

The above score dynamics allow for a leverage effect, see, for example, Rodriguez and Ruiz (2012) for a survey and Lucas, Schwaab, and Zhang (2015). Univariate filtered volatilities for N=10 and for N=73 firms can be obtained in this way as well.

Appendix B: The dynamic GHST score

This appendix derives expression (9) for the GHST dynamic score, and discusses the choice of scaling matrix in (8).

We use the following two matrix calculus results for an invertible matrix X, and column vectors α and β.

$$\frac{\partial(\alpha' X^{-1}\beta)}{\partial \mathrm{vec}(X)} = -((X')^{-1} \otimes X^{-1})\mathrm{vec}(\alpha\beta'), \quad \frac{\partial(\log|X|)}{\partial \mathrm{vec}(X)} = ((X')^{-1} \otimes X^{-1})\mathrm{vec}(X).$$

We denote $k_\lambda(\cdot) = \ln K_\lambda(\cdot)$, with first derivative $k'_\lambda(\cdot)$. Based on these results one obtains

$$\begin{aligned}
\nabla_t &= \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t}\frac{\partial \ln p(y_t; \tilde{\Sigma}_t, \gamma, \nu)}{\partial \mathrm{vec}(\tilde{\Sigma}_t)} \\
&= \Psi'_t \left(-w_t \frac{\partial d(y_t)}{\partial \mathrm{vec}(\tilde{\Sigma}_t)} + \breve{w}_t \frac{\partial d(\gamma)}{\partial \mathrm{vec}(\tilde{\Sigma}_t)} + \frac{\partial(\gamma'\tilde{\Sigma}_t^{-1}(y_t - \tilde{\mu}))}{\partial \mathrm{vec}(\tilde{\Sigma}_t)} - \frac{1}{2}\frac{\partial(\log(|\tilde{\Sigma}_t|))}{\partial \mathrm{vec}(\tilde{\Sigma}_t)} \right) \\
&= \Psi'_t H_t \mathrm{vec}\left(w_t(y_t - \tilde{\mu})(y_t - \tilde{\mu})' - \breve{w}_t\gamma\gamma' - \gamma(y_t - \tilde{\mu})' - 0.5\tilde{\Sigma}_t \right),
\end{aligned}$$

with $H_t = \tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}$, and

$$w_t = \frac{\nu + N}{4d(y_t)} - \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(y_t)/d(\gamma)}}, \quad \breve{w}_t = \frac{\nu + N}{4d(\gamma)} + \frac{k'_{0.5(\nu+N)}\left(\sqrt{d(y_t)d(\gamma)}\right)}{2\sqrt{d(\gamma)/d(y_t)}}.$$

To scale the score updates, we set the scaling matrix $\mathcal{S}_t$ equal to the inverse conditional Fisher information matrix of the symmetric Student's t distribution,

$$\mathcal{S}_t = \left\{ \Psi'_t(\tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1})'[gG - \mathrm{vec}(\mathrm{I})\mathrm{vec}(\mathrm{I})'](\tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1})\Psi_t \right\}^{-1}, \tag{B.1}$$

where $g = (\nu + N)/(\nu + 2 + N)$, and $G = \mathrm{E}[x_t x'_t \otimes x_t x'_t]$ for $x_t \sim \mathrm{N}(0, \mathrm{I}_N)$. We do not use the inverse conditional Fisher information matrix of the multivariate GHST distribution because it is not available in closed form.

As a practical hint, researchers may also consider working with the simpler expression

$$\mathcal{S}_t = \left\{ \Psi_t' (\tilde{\Sigma}_t^{-1} \otimes \tilde{\Sigma}_t^{-1}) \Psi_t \right\}^{-1}, \tag{B.2}$$

which takes similar values as (B.1) but does not require the repeated multiplication of large matrices of dimension $N^2 \times N^2$. The information theoretic optimality results for score-driven models derived in Blasques, Koopman, and Lucas (2015) depend on the properties of the score, and apply in either case.

Appendix C: The GHST multi-block equicorrelation model

This appendix derives the score updating equation for the m-block equicorrelation model, BEq[m].

We assume that N firms can be divided up into m different groups. There are n_i firms within each group $i = 1, \ldots, m$. Firms have equicorrelation ρ_i^2 within each group, and $\rho_i \cdot \rho_j$ between groups i and j, $j = i+1, \ldots, m$. The BEq[m] model proposed here differs from Engle and Kelly (2012) in that there is a direct relation between the equicorrelation in the off-diagonal blocks and the diagonal blocks. We impose this restriction to maintain the factor copula structure that permits the quick computations of the joint and conditional risk measures as discussed in Section 3 of the main paper.

In the dynamic GHST m-block equicorrelation case the scale matrix at time t is given by

$$\tilde{\Sigma}_t = \begin{pmatrix} (1-\rho_{1,t}^2)\mathrm{I}_1 & \ldots & \ldots & 0 \\ 0 & (1-\rho_{2,t}^2)\mathrm{I}_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & (1-\rho_{m,t}^2)\mathrm{I}_m \end{pmatrix} + \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \cdot \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix}', \qquad \text{(C.1)}$$

where I_i is an $n_i \times n_i$ identity matrix, $\ell_i \in \mathbb{R}^{n_i \times 1}$ is a column vector of ones, and $|\rho_{it}| < 1$. The block structure of the matrix allows us to obtain analytical solutions for the determinant,

$$\det(\tilde{\Sigma}_t) = \left[1 + \frac{n_1\rho_{1,t}^2}{1-\rho_{1,t}^2} + \cdots + \frac{n_m\rho_{m,t}^2}{1-\rho_{m,t}^2}\right](1-\rho_{1,t}^2)^{n_1}\cdots(1-\rho_{m,t}^2)^{n_m}, \qquad \text{(C.2)}$$

see Harville (2008). This matrix determinant result further facilitates the computation of the likelihood and the score steps in high dimensions.

We assume that y_t follows a GHST distribution with time-varying scale matrix $\tilde{\Sigma}_t$ (C.1). Factor f_t is an $m \times 1$ vector such that $\rho_i = (1+\exp(-f_{it}))^{-1}$ for $i = 1, 2, \cdots, m$. It follows that

$$\tilde{\Psi}_t = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial f_t} = \frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_t} \frac{\mathrm{d}\rho_t'}{\mathrm{d}f_t},$$

$$\frac{\mathrm{d}\rho_t'}{\mathrm{d}f_t} = \begin{bmatrix} \frac{\exp(-f_{1,t})}{(1+\exp(-f_{1,t}))^2} & 0 & \dots & 0 \\ 0 & \frac{\exp(-f_{2,t})}{(1+\exp(-f_{2,t}))^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\exp(-f_{m,t})}{(1+\exp(-f_{m,t}))^2} \end{bmatrix},$$

$$\frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)}{\partial \rho_t'} = \left[\mathrm{vec} \begin{pmatrix} \mathrm{I}_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}, \cdots, \mathrm{vec} \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathrm{I}_m \end{pmatrix} \right] \cdot \begin{pmatrix} -2\rho_{1,t} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -2\rho_{2,t} \end{pmatrix}$$

$$+ \left[\begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \otimes \mathrm{I}_N + \mathrm{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \right] \cdot \left[\begin{pmatrix} \ell_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \ell_{n_m} \end{pmatrix} \right].$$

To derive this result we used (B), the fact that

$$\frac{\partial \mathrm{vec}(\tilde{\Sigma}_t)'}{\partial \rho_{it}} = -2\rho_{it} \cdot \mathrm{vec} \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \mathrm{I}_i & 0 \\ 0 & 0 & \dots & 0 \end{bmatrix} + \left[\begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \otimes \mathrm{I}_N + \mathrm{I}_N \otimes \begin{pmatrix} \rho_{1,t}\ell_1 \\ \rho_{2,t}\ell_2 \\ \vdots \\ \rho_{m,t}\ell_m \end{pmatrix} \right] \cdot \begin{pmatrix} 0 \\ \vdots \\ \ell_{n_i} \\ 0 \end{pmatrix}, \tag{C.3}$$

and combined expression BlockGAS2 across the m groups.

Appendix D: Data summary statistics

Tables D.1 and D.2 provide sample summary statistics for our small-scale and large-scale studies, respectively.

Table D.1: Sample descriptive statistics, 10 firms

The table reports descriptive statistics for $N = 10$ weekly equity returns between January 1999 and August 2013. All excess kurtosis and skewness coefficients are significantly different from 0 at the 5% significance level.

	Median	Std.Dev.	Skewness	Kurtosis	Minimum	Maximum
Bank of Ireland	0.004	0.099	-0.614	14.736	-0.658	0.581
Banco Comercial Portugues	0.004	0.049	-0.375	6.753	-0.284	0.230
Santander	0.002	0.049	-0.558	7.289	-0.261	0.212
UniCredito	0.001	0.084	-0.235	15.968	-0.705	0.613
National Bank of Greece	0.003	0.061	-1.253	12.909	-0.475	0.292
BNP Paribas	0.002	0.056	-0.364	10.344	-0.367	0.339
Deutsche Bank	0.000	0.059	-0.489	16.145	-0.529	0.398
Dexia	0.008	0.087	-0.569	13.735	-0.529	0.494
Eerste Group Bank	0.002	0.061	-1.143	16.100	-0.552	0.296
ING	0.004	0.069	-1.319	13.358	-0.546	0.290

Table D.2: Sample summary statistics; 73 firms

The table reports sample moments across all 73 institutions included in the empirical application. For example, the row labeled '*skewness*' contains the mean and standard deviation of the skewness statistic across all 73 firms, followed by the minimum, 25th, 50th, and 75th percentile, and the maximum. The first row reports the number of time series observations per firm.

	Mean	Std. Dev.	Minimum	25%	50%	75%	Maximum
T_i	717	116	196	762	762	762	762
Std. Dev.	0.06	0.02	0.03	0.04	0.05	0.06	0.11
Skewness	-0.81	1.94	-12.06	-0.84	-0.41	-0.22	1.89
Kurtosis	18.05	38.47	3.69	6.77	10.13	14.26	266.26

Appendix E: R^2 statistics for $N = 73$ case

Figure E.1 reports cross-sectional (left) and time-series R-squared statistics (right) for our large-dimensional $N = 73$ case.

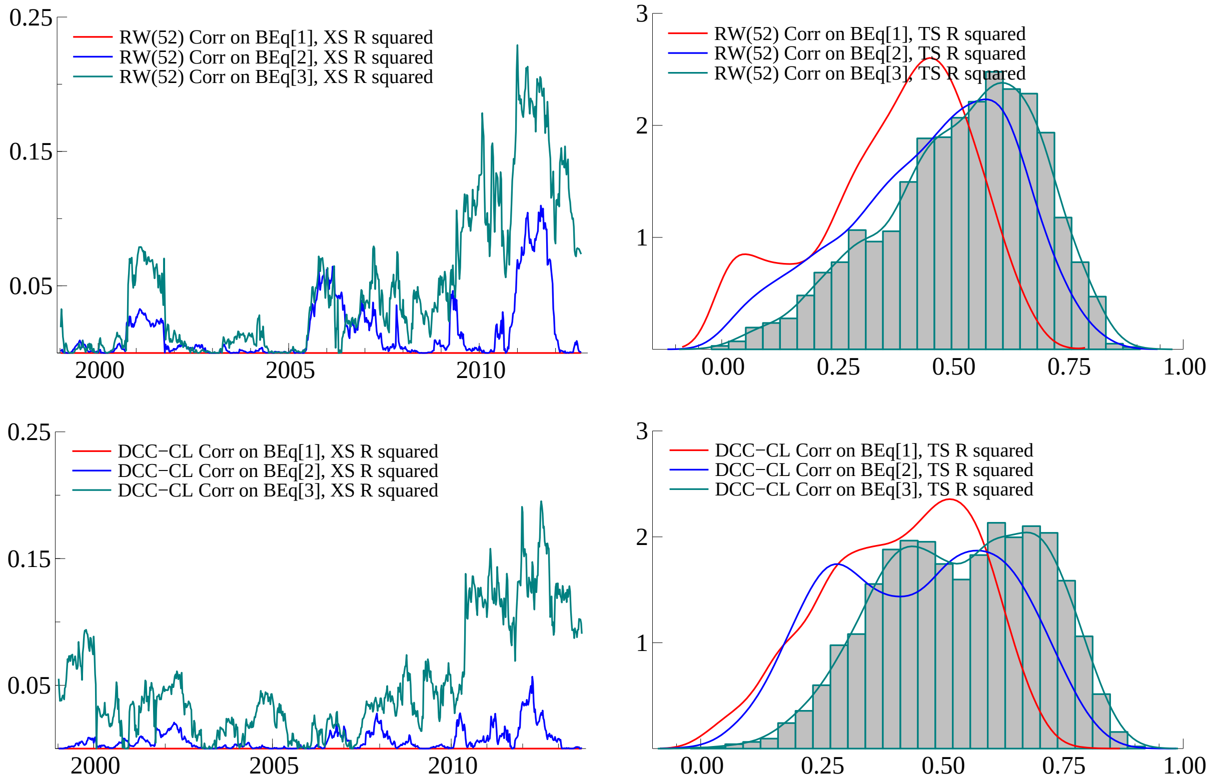

Figure E.1: R-squared statistics

The upper left panel plots R-squared statistics corresponding to repeated (over time) *cross-sectional* regressions of 2628 correlation pairs from the $N = 73$ rolling window correlation (window size 52) model on a constant and the corresponding correlation estimates from a 1-block, 2-block, and 3-block equicorrelation model. The upper right panel plots Kernel density estimates and a histogram of 2628 R-squared statistics corresponding to *time series* regressions with $T = 762$ observations each of rolling window correlation (window size 52) on the corresponding correlation estimates from a 1-block, 2-block, and 3-block equicorrelation model, respectively. The histogram pertains to R-squareds from the 3-block equicorrelation model.
The bottom two panels are equivalent to the respective above panels, but use the 2628 correlation pairs from the $N = 73$ DCC-CL-GHST model instead of rolling window correlations as the left-hand-side variable.

Appendix F: The GHST vs symmetric-t model

Figure F.1 provides estimates of the joint tail risk probability and average conditional tail probability based on different copula model specifications. While the joint probability estimates are fairly invariant to moving from a GHST to symmetric-t dependence, the conditional probabilities are not. The negative skewness implies considerably higher conditional probabilities on average.

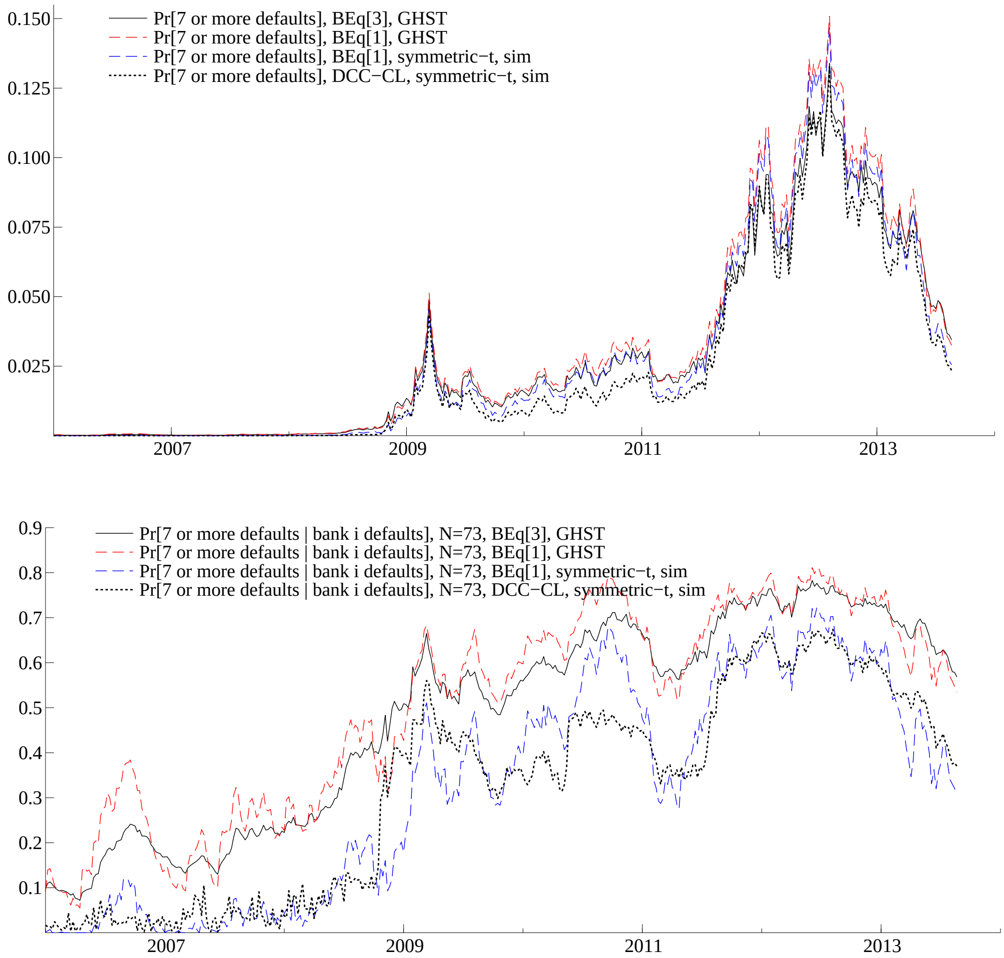

Figure F.1: GHST vs. symmetric-t estimates

Estimates of the joint tail risk probability (top) and average conditional tail probability (bottom) based on different copula model specifications. We provide two specifications of the GHST and symmetric-t copula, respectively. The estimation sample is from Jan 1999 – Oct 2013.

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