Homework 1
Introduction to Computer Graphics
Due September 29, 2011, in class at 1:30 pm

1. Let a = [2, 3, 1], b = [-1, 3, 4], and c = [0, 5, 7] be points on a plane Q
a. Find a normal vector of plane Q. [5 points].

b. Find the implicit equation of plane Q and verify that a, b, and c lie on the plane using the equation. [10 points]

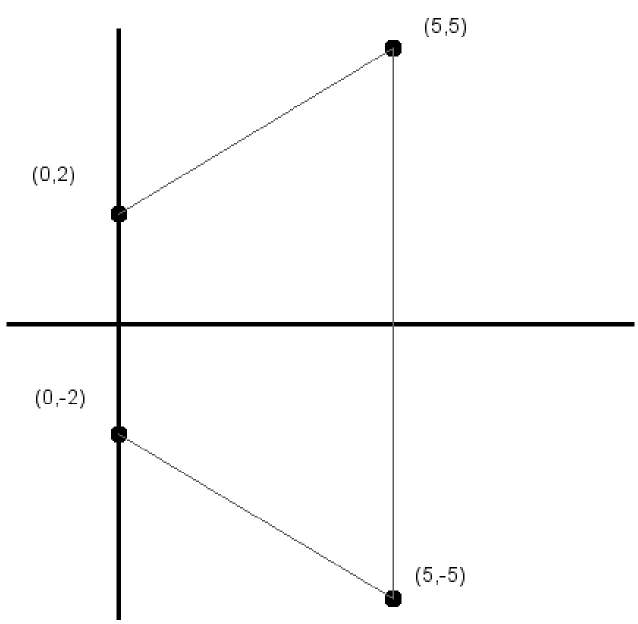
c. Find the parametric equation $P(s,t)$ of the plane Q. What are the (s,t) values for points a, b, c? [10 points].

2. Consider a triangle with point a = [-2, 4], b = [4, 3] and c = [3, 5], with color C_a = [121, 0, 0], C_b = [0, 209, 0] and C_c = [0, 0, 77] respectively. (The color is in the form of [r, g, b], i.e. the first index is the red component, second index is the green component and the third index is the blue component). Suppose we have a point p = [1, 4].
a. Show that point p is inside the triangle. [10 points]

b. Find the color of point p by linear interpolation of the colors at the three corners a, b, and c of the triangle. [5 points]

3. Let there be viewing area with a viewing line ranging from (0.0, -2.0) to (0.0, 2.0) and a far-clipping line ranging from (5.0, -5.0) to (5.0, 5.0).

Use the following image as a reference.



a. Project the 2D points (3.5, 4.0) and (1.0, 3.0) onto the line: $x = 0$. Use perspective projection (such that (5,5) maps to (0,2) and (5,-5) maps to (0,-2)) [5 points]

b. Which points fall onto the viewing line (from (0,-2) to (0.2))? [5 points]

4. Suppose you have a camera located at [0, 1, 0], pointing at the center of an object, with the center located at [0, 0, 0]. The camera also has an up vector of [1, 1, 0]. Derive the transformation matrix to transform a point in world coordinate to the camera coordinate specified above. [10 points]

5. Transform the points: (0.5, 2.0, 3.0), (1.0, 3.5, 7.0), and (2.0, 0.0, 1.0) by rotation of 90 degrees around the x-axis, translation of (1.0, 0.0, 0.0), and scale of 2.0 along all axes in that order (as applied to the vertices). Show ALL of your steps. [10 points]

6. a. Give a single 3 x 3 matrix which rotates a point by angle ϕ about the x-axis, then by θ about the y-axis, and finally by ψ about the z-axis (rotate around x-axis, then y-axis, then z-axis). Simplify your answer. [10 points]

For your reference, the rotation matrices are as follows,

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b. Find the inverse of the rotational matrix found in part a. [5 points]

7. Using the Bezier Spline formula, let the control vector be

$$\begin{pmatrix} 0.0 & 0.0 & 0.0 \\ 0.25 & 0.0 & 1.0 \\ 0.5 & 0.0 & 0.5 \\ 1.0 & 1.0 & 2.0 \end{pmatrix}$$

a. Find the formula $p(u)$ for this Bezier Spline. [10 points]

b. Find the points on the Bezier Spline at $u = 0.2$, $u = 0.5$, and $u = 0.6$. [5 points]

(See Lecture 6 for Bezier Spline Formula)