GFT Template User's Manual v.1.0

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The ICAP/4 GFT Template is an implementation of the General Feedback Theorem (GFT) presented in [1], in which the GFT is derived by application of the Extra Element Theorem (EET) developed in [2]. Many examples of useful applications of the EET are available in a book [3], and extensive background material is presented in a series of Video CDs [4].

[1] R.D.Middlebrook, "The GFT: A General Yet Practical Feedback Theorem," to be published.

[2] R.D.Middlebrook, "Null Double Injection and the Extra Element Theorem," IEEE Trans. on Education, vol. 32, no. 3, Aug. 1989; pp. 167-180.

[3] Vatché Vorpérian, *Fast Analytical Techniques for Electrical and Electronic Circuits,* Cambridge Univ. Press, 2002.

[4] R.D.Middlebrook, *Structured Analog Design,* Video CDs, Ardem Associates, http://www.ardem.com; to be published.

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What the GFT is, and how the GFT Template works

1. What we know already

Electronic feedback systems are usually designed with the familiar single-loop block diagram of Fig. 1 in mind.

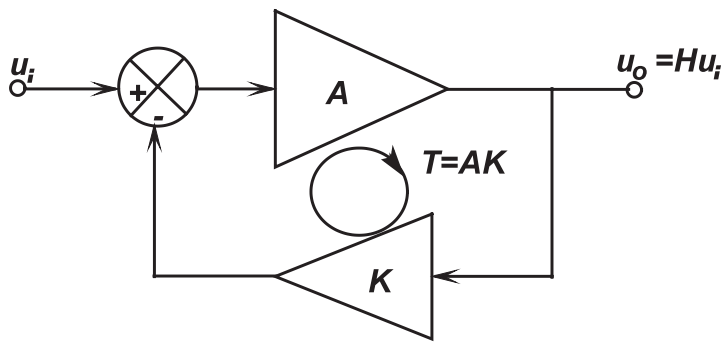


Fig. 1. Conventional single-loop block diagram of a feedback system.

This model predicts that the closed-loop gain $H \equiv u_o/u_i$ is given by

$$H = \frac{A}{1 + AK} \tag{1}$$

where A is the forward (open-loop) gain, K is the feedback ratio, and T is the loop gain defined by

$$T \equiv AK \tag{2}$$

Consider the simplified model of a typical two-stage transistor feedback amplifier shown in Fig. 2.

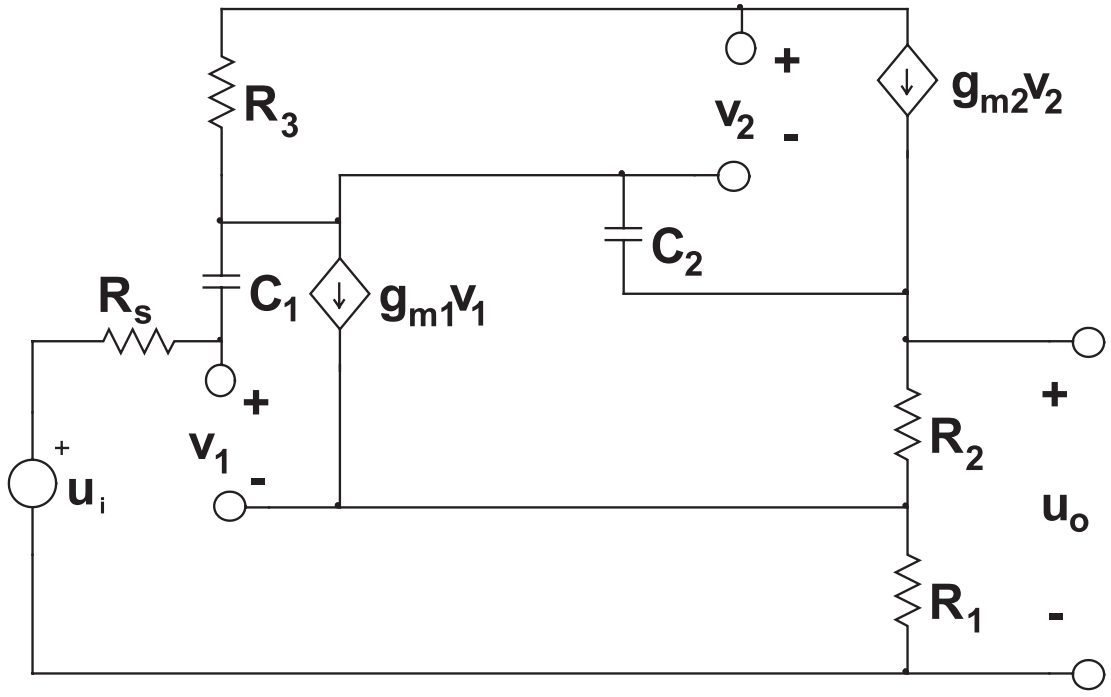


Fig. 2. A simple equivalent circuit model of a two-stage voltage-to-voltage feedback amplifier.

We can see immediately that this model is not completely represented by the block diagram of Fig. 1. Equation (1) says that if A goes to zero, H goes to zero; however, in Fig. 2 if g_{m2} and C_2 go to zero, the output u_o does not go to zero because the input u_i can reach the output by going through the feedback path in the "wrong" direction.

This *nonideality* of the real circuit is not accounted for in Fig. 1 because the blocks are defined to be unidirectional, which is the reason they are drawn as arrowheads rather than as rectangular boxes. However, this nonideality can be accommodated by augmenting the block diagram with another block as in Fig. 3, where H_0 represents the direct forward transmission.

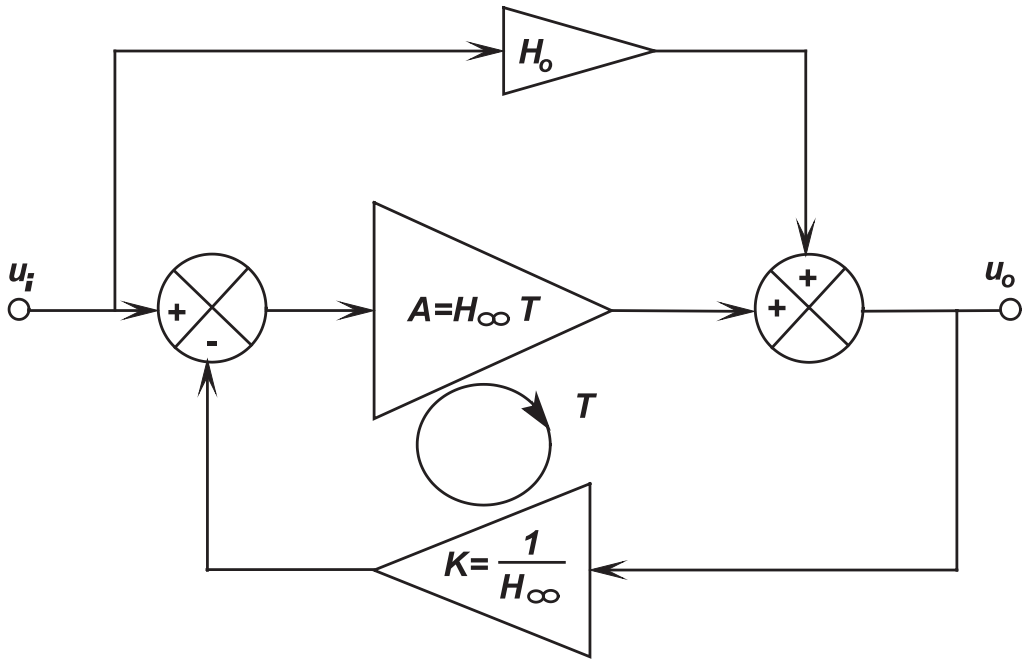


Fig. 3. The block diagram of Fig. 1 augmented by a block that represents direct forward transmission. It is identical to the "natural" block diagram that is the result of application of the GFT to an equivalent circuit model.

Figure 3 predicts the presence of an added term in the expression for the closed-loop gain H:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} \tag{3}$$

in which a new quantity H_∞ is introduced:

$$H_\infty \equiv \frac{1}{K} \tag{4}$$

The seemingly trivial substitution of H_∞ for 1/K is actually very valuable, because it enables an interpretation of (3) that discloses much useful information. Equation (3) says that the closed-loop gain H of a feedback system can be expressed as the weighted sum of two components, one containing H_∞ which would be the gain if the loop gain T were infinite, and the other containing H_0 which would be the gain if T were zero. Thus,

$$H_\infty \equiv H \,|\, T \to \infty \tag{5}$$

$$H_0 \equiv H \,|\, T \to 0 \tag{6}$$

can be considered as definitions. It follows that H is dominated by H_∞ when T is greater than 1, and is dominated by H_0 when T is less than 1.

The objective of designing a feedback system is to make T large enough that H is dominated by H_∞ and the H_0 contribution is negligible, over the useful frequency range.

A further interpretation of the expectedly dominant term in (3) is that H_∞ is the *ideal closed-loop gain* and

$$D \equiv \frac{T}{1+T} \tag{7}$$

is the *discrepancy factor* because it is the factor by which the actual closed-loop gain H deviates from the ideal closed-loop gain H_∞.

Another useful version of (3) arises from combining the two terms into one:

$$H = \frac{H_\infty T + H_0}{1+T} = H_\infty \frac{1+\frac{H_0}{H_\infty T}}{1+\frac{1}{T}}$$

or

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} \tag{8}$$

where

$$\frac{T_n}{T} = \frac{H_\infty}{H_0} \tag{9}$$

In the derivation of the GFT in [1], T_n is identified as the *null loop gain*, which in turn suggests the definition of a *null discrepancy factor*

$$D_n = 1+\frac{1}{T_n} \tag{10}$$

In summary, a "first level" transfer function (TF) such as the closed-loop gain H can be expressed in terms of three out of four "second level" TFs H_∞, T, T_n, and H_0. Because of the redundancy relation (9), the four versions each omits one of the four second level TFs. The two versions useful for the present purposes respectively omit T_n and H_0, and are:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty D + H_0(1-D) \tag{11a, 11b}$$

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n \tag{12a, 12b}$$

Although H_0 represents nonidealities physically identified in Fig. 3, their alternative representation by T_n has two advantages. First, in (12) D_n appears as a multiplier factor within H, and therefore for the nonidealities to be negligible D_n must be close to 1, or equivalently T_n must be much larger than 1; on the other hand in (11) H_0 appears in an additive term, and to be negligible, H_0 must be much smaller than $H_\infty T$ which is a more complicated comparison. Second, in symbolic circuit analysis, T_n is usually a much simpler and shorter calculation than is H_0.

The job of a *designer*, as distinct from that of an *analyst*, is to construct hardware that meets certain specifications within certain tolerances. If you are designing a feedback amplifier, you effectively proceed through four steps:

Design Step #1. Design the feedback network $K = 1/H_\infty$ so that H_∞ meets the specification;

Design Step #2. Design the loop gain so that T is large enough that the actual H meets the specification within the allowed tolerances;

Design Step #3. Since in any real system T drops below unity beyond some "crossover" frequency, design T to be large enough up to a sufficiently high frequency. In an ac coupled system, a dual condition occurs in the reverse frequency direction. Usually, #3 is the most difficult step.

Design Step #4. Design the null loop gain so that T_n is large enough, or the direct forward transmission H0 is small enough, that the actual H meets the specification within the allowed tolerances.

Although not explicitly stated, Design Step #3 includes the stability requirement. The discrepancy factor $D \approx 1$ when $T >> 1$, and $D \approx T$ when $T << 1$; the transition between these two ranges, which is not necessarily a monotonic decrease, occurs at the loop gain crossover frequency wc, the frequency at which $|T| = 1$. If the phase margin of T is less than about 60°, D begins to peak up in the neighborhood of the crossover frequency, and in the limit of zero phase margin the peak in D becomes infinite, which marks the onset of instability. Therefore, the discrepancy factor D must not be allowed to peak up beyond the tolerance limit of the specification, which is a much more stringent design requirement than mere "stability".

2. Injected test signals

Although the second level TFs are derived quantities defined in terms of the blocks in Fig. 3, in looking ahead to evaluating them on a circuit diagram we find it useful to see how they could be determined individually by direct virtual measurements on the block diagram of Fig. 3. We want to do this by signal injection, without disturbing the circuit configuration and therefore without disturbing the circuit determinant.

The most important second level TF (and also the *simplest*) is the ideal closed-loop gain H_∞. The actual closed-loop gain falls short of H_∞ because the error signal, which is the difference between the input signal u_i and the fedback signal Ku_o, is not zero when the loop gain T is not infinite. A test signal u_z can be injected so as to add to the error signal, as in Fig. 4.

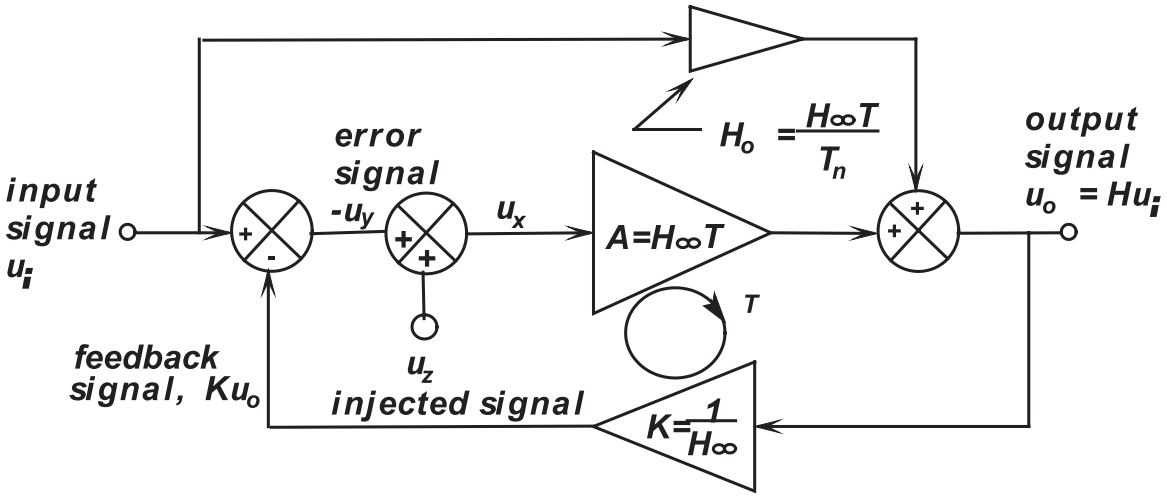


Fig.4.Injection of a test signal u_z so as to add to the error signal in the block diagram of Fig. 3.

The injected signal can be adjusted so as to null the error signal $-u_y$, which makes Ku_o exactly equal to u_i so that the closed-loop gain is 1/K which is H_∞. Thus, H_∞ can be directly measured under the *null double injection* condition shown in Fig. 5.

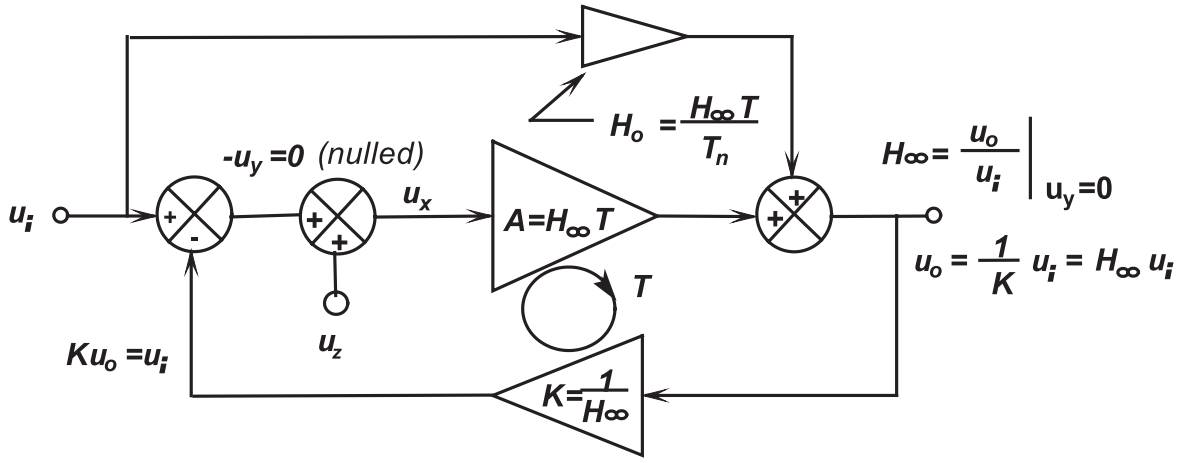


Fig. 5. Null double injection calculation of the ideal closed-loop gain H_∞ by mutual adjustment of the injected signal u_z and the input signal u_i to null the error signal $-u_y$, which simulates infinite loop gain.

The result can be written

$$H_\infty = \frac{u_o}{u_i}\bigg|_{u_y=0} \tag{13}$$

It's worth going through this process again. Initially, $u_z=0$ and the system is in the normal closed-loop condition with a given input signal u_i so that the output is $u_o=Hu_i$. As u_z is increased, the output rises, causing more signal to be fed back, and decreasing the error signal $-u_y$. When $-u_y$ has been reduced to zero, the output has risen to $H_\infty u_i$, the value it would have if the loop gain were infinite.

Note that H_∞ is determined under a condition of *simulated infinite loop gain*, in accordance with (5): when the error signal u_y is nulled u_x, which is then equal to u_z, drives the output to the value necessary to make Ku_o equal to u_i. Thus, the forward gain appears to be infinite, although the output is actually being driven by the "hidden" test signal u_z.

Null double injection is a very powerful tool, for numerical as well as for symbolic analysis. In general, two independent signals are mutually adjusted in order to null a designated dependent signal. The purpose is to determine a special value of a certain TF that exists under this null condition. In the present case, the purpose is to determine the special value of H, defined as H_∞, that exists under the condition of nulled error signal $-u_y$.

There are two important features of null double injection. One is that the system determinant is not disturbed, because only system signals are manipulated. The other is that determination of

the required answer, the TF under the specified null condition, is VERY EASY because we don't need to know the value of the injected test signal, we need to know only that the resulting null exists. In the present case, we use the existence of the null condition $-u_y=0$ to conclude that the fedback signal exactly equals the input signal, which immediately sets up the required result (13).

If you are not familiar with the term "null double injection," you may be surprised to learn that you've actually been doing the corresponding calculation all along. The conventional procedure for Design Step #1 is to *assume* that the error signal is vanishingly small, which makes the fedback signal approximately equal to the input signal, and hence makes the closed-loop gain approximately equal to 1/K. But this is exactly the same calculation that is the culmination of the null double injection procedure described above. So, you've actually been *doing* the null double injection calculation all along, by *assuming* infinite loop gain instead of *simulating* infinite loop gain, the result of which is H_∞ and not H.

If, as is commonly the case, K is a resistance divider, H_∞ is flat to infinite frequency even though the actual H falls off because of the declining discrepancy factor D=T/(1+T). At increasing frequencies, the injected test signal u_z has to be increased in order to maintain the output $H_\infty u_i$ flat in spite of decreasing loop gain.

All four second level TFs can be determined by various manipulations of the two injected signals in Fig. 3, without disturbing the circuit determinant.

After H_∞, the next most important second level TF is the loop gain T. From the definition (2), the loop gain is that experienced by a signal such as u_x that starts around the loop and arrives as a signal u_y at the other side of the injected test signal u_z, under the condition that the input signal u_i is zero, as illustrated in Fig. 6.

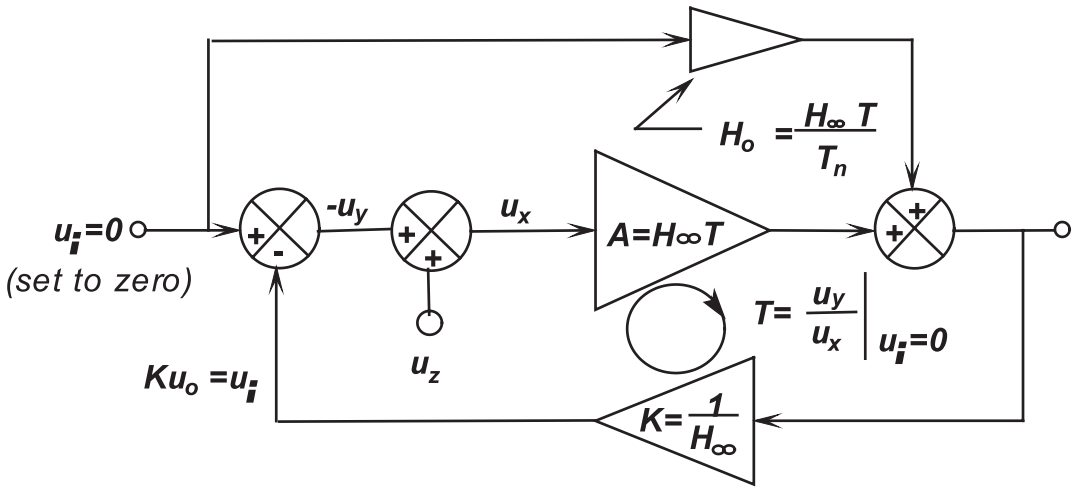


Fig. 6. Single injection calculation of the loop gain T.

The input signal u_i is not nulled, it is set to zero, and so the result emerges from a *single injection*, not a null double injection, calculation:

$$T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0} \tag{14}$$

To measure the direct forward transmission gain H_0, we invoke the interpretation (6) that H_0 is the gain when the loop gain T is zero. Just as H_∞ is measured by simulating infinite loop gain, H_0 is measured by simulating zero loop gain in Fig. 4. This is done by adjusting the injected test signal u_z, in the presence of u_i, to null u_x. This is shown in Fig. 7, whose interpretation is that if u_x is zero the only way the input signal u_i can reach the output u_o is through the H_0 block.

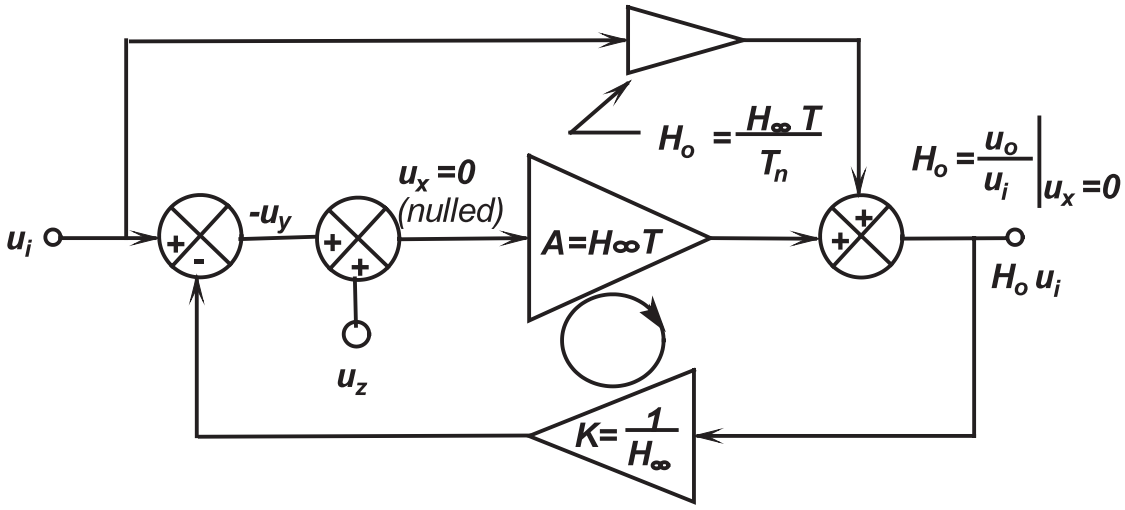


Fig. 7. Null double injection calculation of the direct forward transmission gain H_0 by mutual adjustment of the injected signal u_z and the input signal u_i to null u_x, which simulates zero loop gain.

So, the direct forward transmission TF emerges from another null double injection calculation as

$$H_0 \equiv \left.\frac{u_o}{u_i}\right|_{u_x=0} \tag{15}$$

It may not be immediately obvious, but the null loop gain T_n can also be found from Fig. 4 by yet another null double injection calculation in which u_z is adjusted in the presence of u_i to null the output signal u_o, as in Fig. 8.

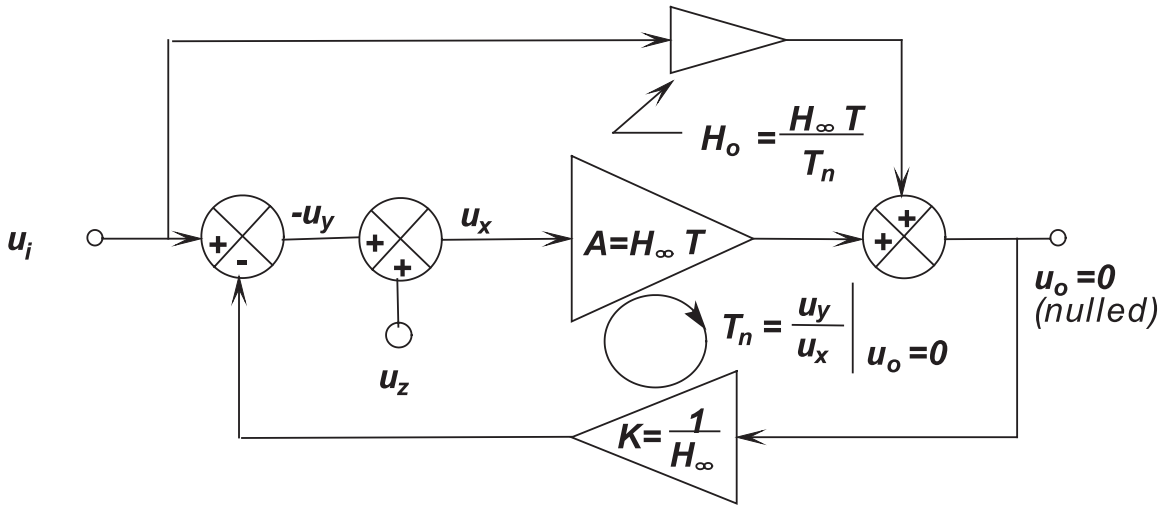


Fig. 8. Null double injection calculation of the null loop gain T_n by mutual adjustment of the injected signal u_z and the input signal u_i to null the output signal u_o.

The null loop gain T_n is then

$$T_n \equiv \frac{u_y}{u_x}\Big|_{u_0=0} \tag{16}$$

The interpretation is as follows. The output u_o is the sum of $Au_x = H_\infty Tu_x$ and $H_0 u_i$. If the output is nulled, $H_\infty Tu_x = -H_0 u_i$. But there is no fedback signal, so $-u_y = u_i$ and $H_\infty Tu_x = H_0 u_y$, which leads to (9), and so justifies the interpretation of T_n as the null loop gain.

All the familiar properties of feedback are expressed in (11) or (12), which relate to the block diagram of Fig. 3. The first level TF, the closed-loop gain H, is expressed in terms of three of the four second level TFs which can be determined from (13) through (16) by injection of a test signal u_z that sums with the error signal.

The next step is to relate the block diagram of Fig. 3 or 4 to an actual equivalent circuit model of the system under consideration. This is where difficulties begin to emerge: the conventional approach attempts to identify the three blocks as containing distinct parts of the circuit, but this is only true in simple cases. Usually interactions between blocks (loading) impose approximations upon the resulting block gains, especially at high frequencies which is just where accurate results are most desired. This approach is effectively dealt a *coup de grâce* by the presence of nonzero admittances across active-device dependent generators, which introduce not only local feedback paths but also an additional bidirectional signal path in the forward gain block A. Such nonzero

admittances result from C_1 and C_2 in Fig. 2, which create considerable analytical difficulties even in such a simple circuit.

The General Feedback Theorem (GFT) avoids all these difficulties by bypassing the initial step of trying to relate each block individually to distinct parts of the circuit. Instead, the blocks in Fig. 3 are evaluated from (13) through (16) by test signals injected into the complete circuit whose determinant remains undisturbed at all times.

3. The General Feedback Theorem (GFT)

In the "bottom up" approach to feedback analysis described above, the starting point is a block diagram intended to preserve the concept of a single loop describing a forward path and a feedback path. The accuracy of the results depends on how well the blocks represent the actual circuit configuration.

In contrast, the GFT provides a "top down" approach in which the starting point is the actual equivalent circuit model and a block diagram is the *result*. This block diagram is exactly the same as in Fig. 3, which therefore may be called a "natural" block diagram because it represents any transfer function of any linear circuit model, regardless of whether there is any identifiable "feedback path" or not.

Since the two approaches have the same block diagram, they also have the same relations (11) and (12) between H, the first level TF, and the four second level TFs H_∞, T, T_n, H_0, repeated here:

$$H_1 = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n \qquad \text{(17a, 17b)}$$

$$H_2 = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty D + H_0 (1-D) \qquad \text{(18a, 18b)}$$

The designators 1 and 2 are used to distinguish the results calculated from different arrangements of the second level TFs, or different "dissections" of the first level TF H, and henceforth the unadorned H will be reserved for the first level TF measured directly with no test signal injection, as would be done in a normal ICAP/4 calculation. The order of the two forms is reversed because H_1 is the form that first emerges from the GFT derivation and therefore has the most direct relationship to the equivalent circuit model. Consequently, H_1 is designated the "natural" form of the GFT and H_2, and others still to be introduced, are designated as "derived" forms. Likewise, the redundancy relation (9) is also reversed:

$$\frac{H_0}{H_\infty} = \frac{T}{T_n} \qquad (19)$$

In the previous section, it was shown how the four second level TFs could in principle be evaluated by injection of a test signal u_z into the block diagram. The difficulties arise when we try to implement these calculations on the actual circuit diagram instead of on the block diagram. In particular, is u_z a current or a voltage source?

The answer is that, in general, u_z is both a current and a voltage source, and the GFT provides extensions of (13) through (16) by which the four second level TFs can be derived from 12 third level measurements made under various single, null double, and double null triple, injection conditions established on the circuit model.

These extension formulas are needed if you want to do symbolic analysis of the circuit model . However, if you want only numerical results, you can ignore these formulas — the ICAP/4 GFT Template does it all for you, so the extension formulas are not displayed here, but are listed in full in the Appendix.

The only decisions you have to make are which is the appropriate test signal injection configuration, and how to connect it to the circuit. After the program has been run, the second level TFs are available for display in Intuscope, as are the discrepancy factors D from (7) and D_n from (10), and the calculated closed-loop gains H_1 from (17) and H_2 from (18). The derived results H_1 and H_2 should be the same, and as a final check both should be the same as H measured directly on the circuit without any injected test signals, which is also available in Intuscope.

The GFT is completely general, the only condition being that it applies to a linear system model. Any number of injected test currents and/or voltages can be employed at the same or different places in the circuit model. The GFT Template incorporates up to two injected test signals in any combination. A "test signal configuration" is defined by what test signals are employed, and where they are injected into the circuit model.

A particular test signal configuration produces a corresponding set of second level TFs H_∞, T, T_n, H_0, which, when inserted into the GFT (17) or (18), gives the first level TF H. Although all test signal configurations give the *same* H, *the sets* H_∞, T, T_n, H_0 *may be different.*

The objective of establishing the block diagram of Fig. 3 is to interpret each block in terms of distinct circuit functions which in turn can be identified in terms of circuit elements. If you make an *arbitrary* choice of injection configuration, the resulting set of H_∞, T, H_0, T_n won 't mean anything and so will be useless for informed design-oriented analysis, even though if you insert it into the GFT you get the "correct" answer. For example, the result for T may be completely unrecognizable as the "loop gain" you thought you had built into the circuit.

Therefore, the crucial step in applying the GFT is to choose a test signal injection configuration that *does* produce the set H_∞, T, T_n, H_0 with the desired interpretation. In many cases, the desired result is that H_∞ be equal to 1/K, the reciprocal of the feedback network

transmission, and the corresponding H_∞, T, T_n, H_0 set is referred to as the *principal set*, which contains the *principal loop gain* T. Other sets, containing different T's, are not "wrong," they just aren't useful for the same purpose.

So, how do we choose an injection configuration so that H_∞ equals 1/K? This is simply an extension of the process set up in Fig. 5 that led to (13).

The starting point is an extension of the block diagram of Fig. 3 to incorporate two things: identification of the generalized signals u as being currents, or voltages, or both; second, recognition that in general the A and K boxes are each bidirectional and so are drawn as rectangles instead of arrowheads, as in Fig. 9.

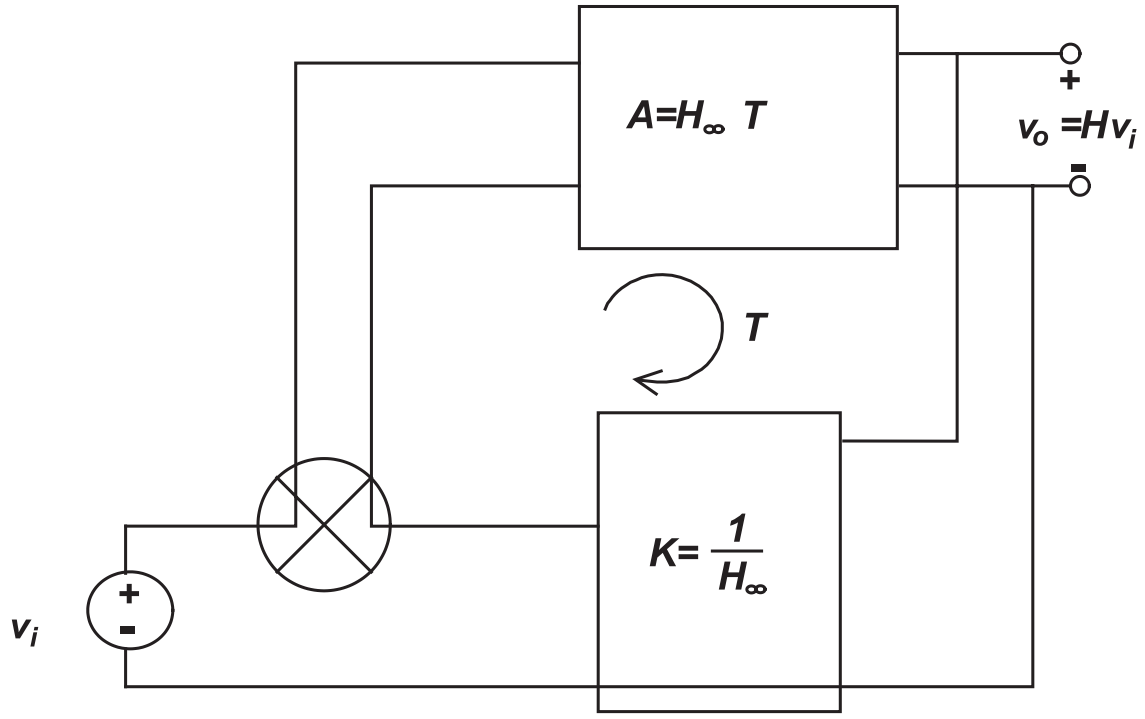


Fig. 9. Selection of a test signal injection configuration that sets up H_∞=1/K. Step 1: Identify the feedback network in an equivalent circuit model.

The input and output signals have been rewritten as v_i and v_o so that $H=v_o/v_i$ is a voltage-to-voltage gain. You may wish to visualize the two blocks as containing the circuit of Fig. 2, but to make the discussion more concrete only the feedback divider will be shown explicitly, as in Fig. 10.

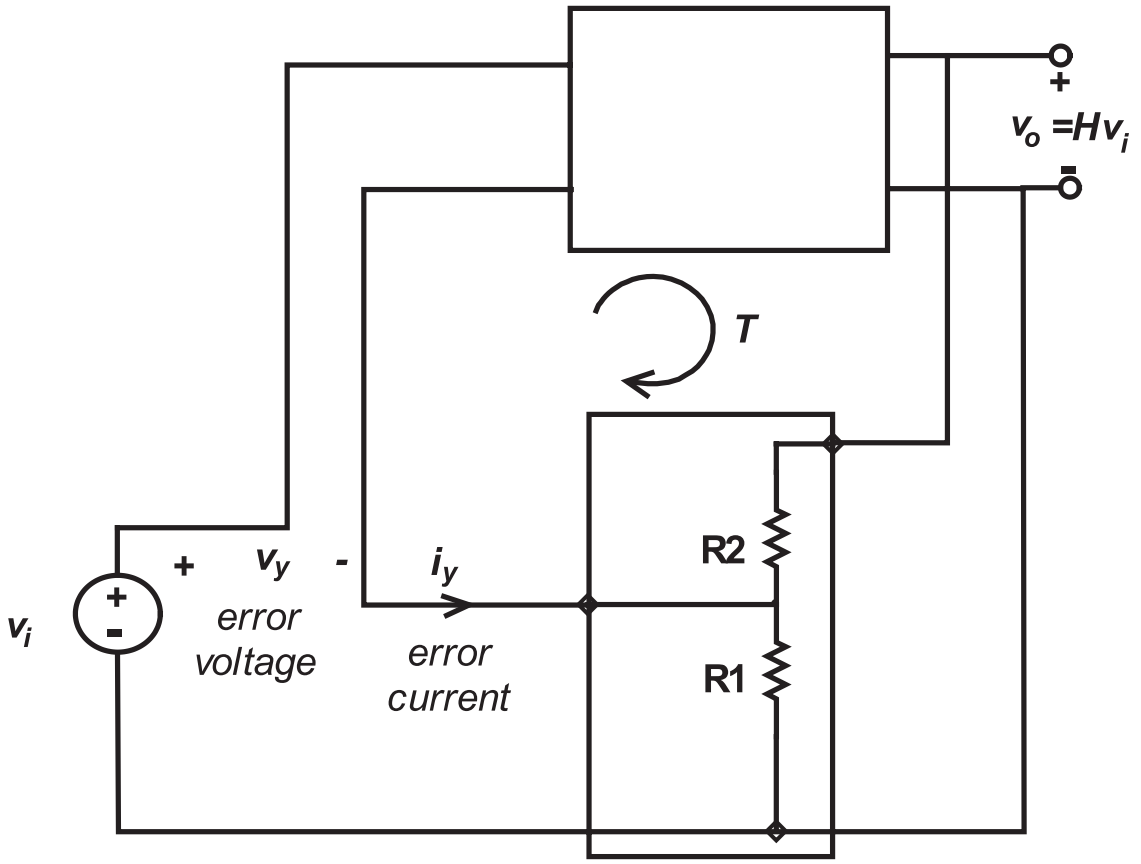


Fig. 10. Step 2: The error signal u_y has to be identified as both a voltage v_y and a current i_y.

The criterion is that the test signal is to be adjusted so that the error signal is nulled, which makes the fedback signal exactly equal to the input signal, which in turn sets up $H = H_\infty$. In Fig. 10, the fedback signal is $[R1/(R1+R2)]v_o$, so for this to be equal to the input voltage v_i, the error signal must be identified as the voltage v_y shown in Fig. 10.

However, it is not sufficient for v_y to be nulled. For the fedback voltage to be $[R1/(R1+R2)]v_o$, there must be no current drawn from the divider tap. Therefore, the error signal must also be identified as the current i_y shown in Fig. 10.

The bottom line is that, in a circuit such as Fig. 10, to set up H_∞ to be equal to (R1+R2)/R1, both the error voltage v_y and the error current i_y must be nulled. This requires that the injected test signal u_z also be both an injected current j_z and an injected voltage e_z, as in Fig. 11.

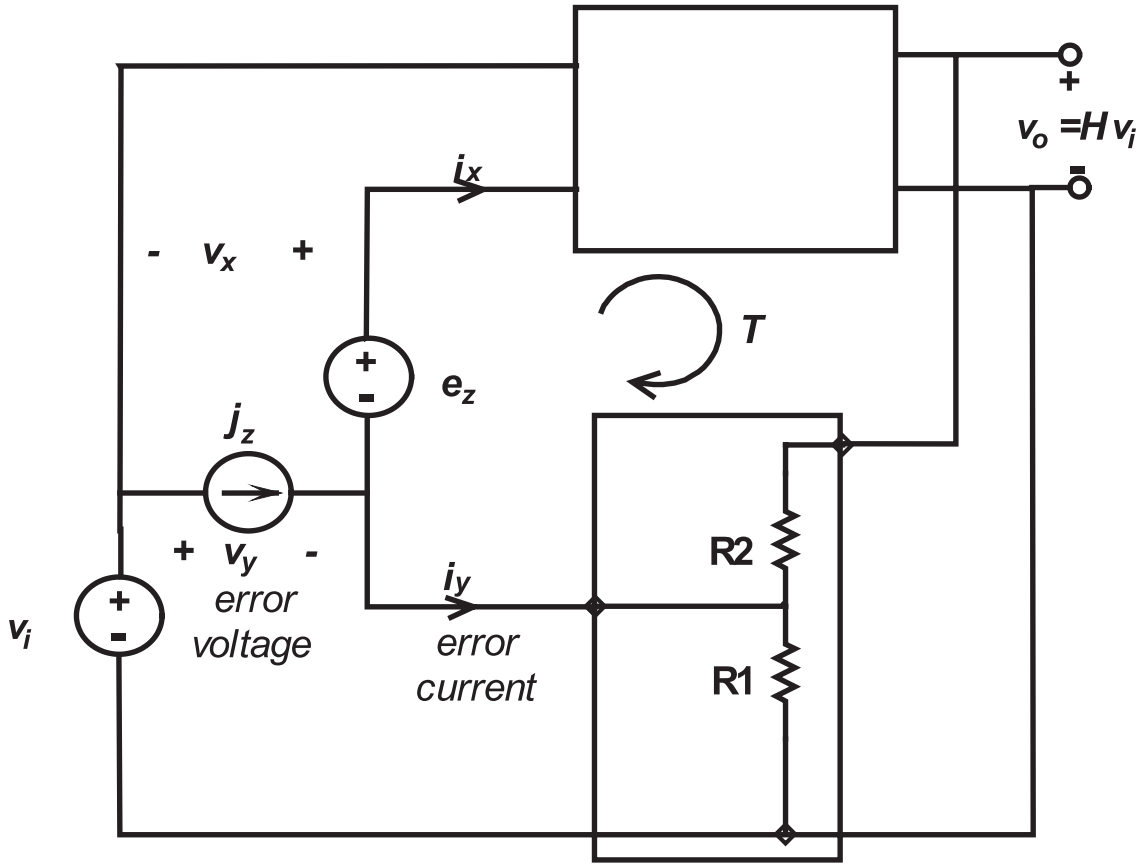


Fig. 11. Step 3: The injected test signal u_z has to be both a voltage e_z and a current j_z, both of which have to be mutually adjusted relative to u_i to null both error signals v_y and i_y. This is a double null triple injection condition.

The result is

$$H_\infty = \left.\frac{v_o}{v_i}\right|_{i_y, v_y = 0} = H_{i_y v_y} \tag{20}$$

where the second version is merely a shorthand version of the first, and the test signal injection conditions have been purposely set up to make H_∞ equal to the required (R1+R2)/R1.

We see that a test signal may be both a current and a voltage source, and that both test sources need to be adjusted relative to a given input signal in order to null both an error voltage and an error current. This is a condition of *double null triple injection*, in which three independent sources are mutually adjusted to null two dependent signals. This is just as easy as a null double

injection calculation. You don't need to know what the values of the sources are, you need to know only that the two nulls exist, because this is equivalent information and is sufficient to deliver the required answer.

The procedure for using the GFT Template will be illustrated below through a series of examples, the first of which is taken from [1].

3.1 Example 1: 2-stage voltage-to-voltage feedback amplifier

The circuit of Fig. 2 is redrawn in Fig. 12 with a slight modification. Each FET model is replaced with an equivalent model that represents either an FET or a BJT, depending respectively on whether $\alpha = 1$ or $\alpha < 1$.

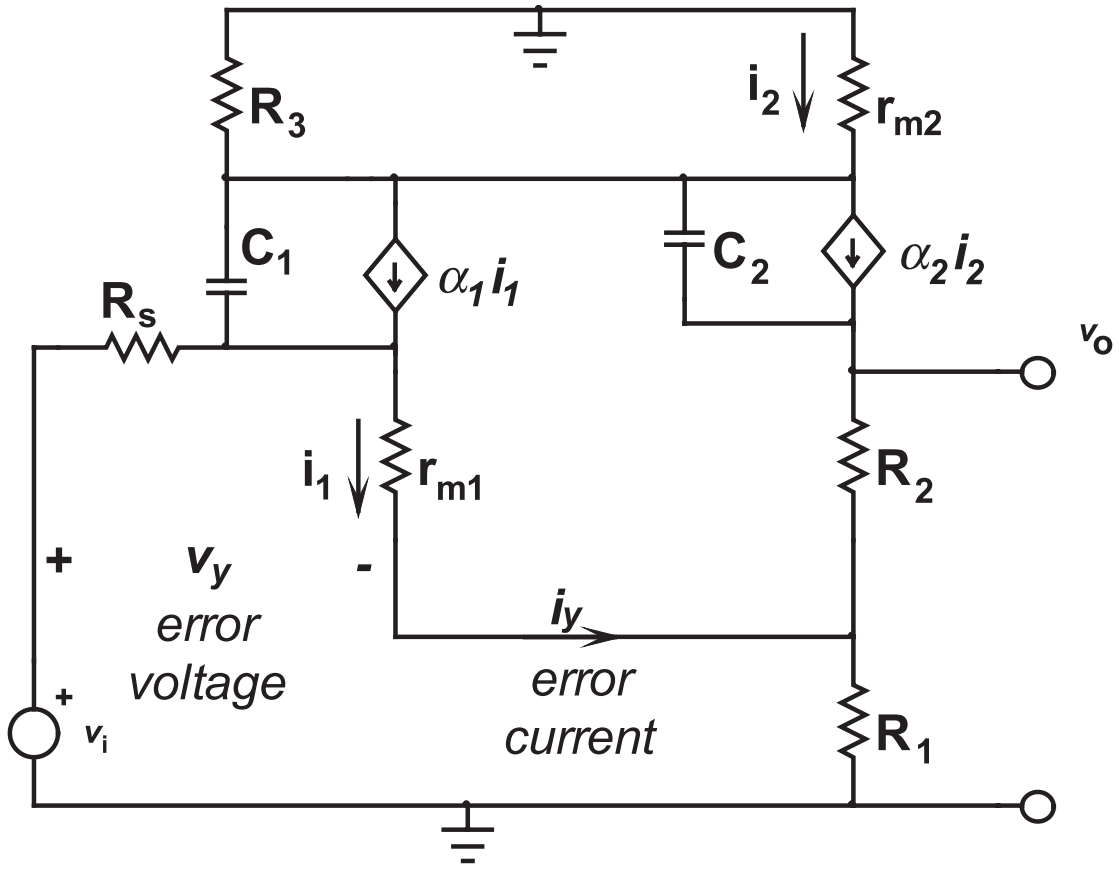


Fig. 12. The equivalent circuit model of Fig. 2 redrawn to resemble the model of Fig. 11, with the error signals v_y and i_y identified.

Draw this circuit in the SpiceNet window, and go to Parts→PartBrowser→Part Types→GFT. Under Sub Types, select the one that corresponds to the required transfer function H in this circuit, which is Voltage Gain (voltage-to-voltage).

We already know that the principal set of second level TFs H_∞, T, H_0, T_n (the set with H_∞ made equal to 1/K) is produced by both current and voltage test signal injection in the configuration of Fig. 11, so in the Voltage Gain Parts List select GFTv, coupled voltage, current injection. Place the icon in the SpiceNet circuit diagram. In the Subcircuit properties window, re-name the Ref Des as xgft.

Connect the necessary wires, either directly, or by use of Continuations as in Fig. 13. Note that the u_i pair in the icon *replaces* the normal input signal source. This is because the GFT Template has to adjust the input signal together with the nulling conditions in order to measure the 12 third level TFs. The zero voltage source V3 is there merely to preserve the distinction between the nodes uu_i and W. The element values are those from the example in [1], with α=1 and Rs=1n, essentially zero.

The model of Fig. 13 is ready made in spice8\circuits\gft\figure13.dwg

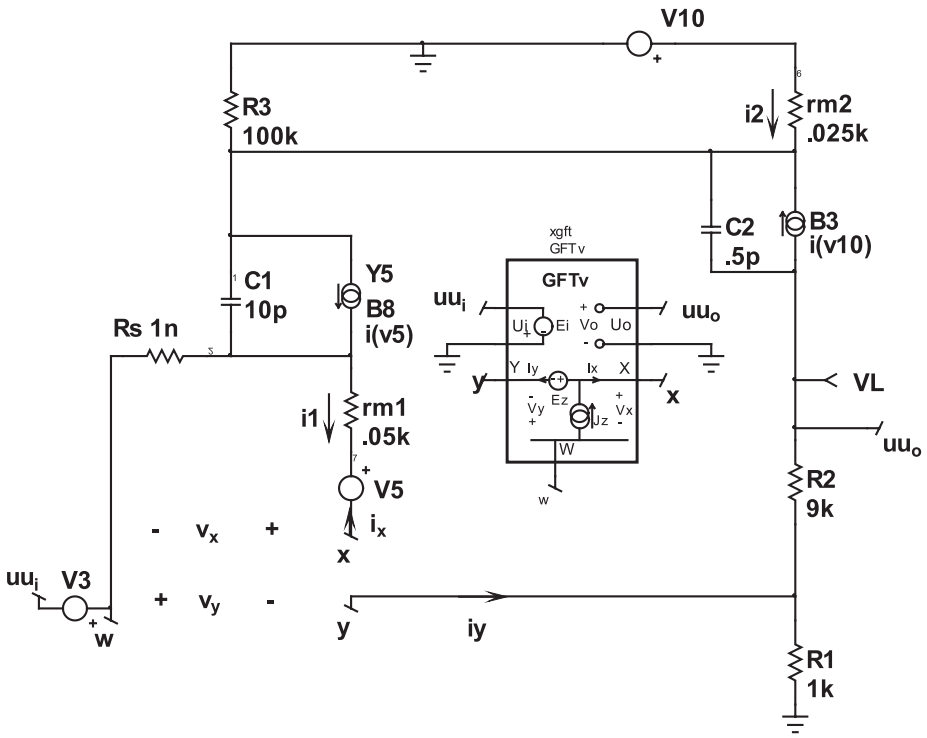


Fig. 13. Insertion of the GFT Template icon GFTv into the model of Fig. 12. Running the program simulates twelve second level TFs and calculates the four second level TFs H_∞, T, T_n, H_0, together with the simulation of the normal closed-loop first level TF H.

You are now ready to run the GFT Template. Open Intuscope, and go back to the SpiceNet window. Click Actions→ICAPS... Select Config 1 + Setup 1, and in the Mode area select 'GFT'. Make sure the Script box is checked in the Data Reduction area. Click Simulate Selections, and answer Yes to the query. The simulation now runs.

Go to Intuscope. In the Add Waveforms window, select 'result' in the Type box in the Source area. All the TFs appear in the Y Axis box, and can be displayed in dB and phase on the Graph. Alternatively, all the TFs in (17) and (18) can be displayed in both dB and phase by clicking Calculator→GFT.

Some of the results are shown in Fig. 14. The first result to check is that H_∞ is flat at 20dB at all frequencies, confirming that H_∞ = 1/K=(R1+R2)/R1 and that the H_∞, T, T_n, H_0 set is indeed the *principal set*, so the proper test signal injection configuration was in fact chosen. If H_∞ is not equal to the expected result, something is wrong and all the results should be ignored until the problem is fixed.

Next, note that five other TFs are displayed, but only three are visible. This confirms that the derived first level TF H_1 and H_2 calculated from (17) and (18), respectively, are indistinguishable from the directly measured H. This Figure appears in [1], together with interpretation and discussion of the agreement with the symbolic results.

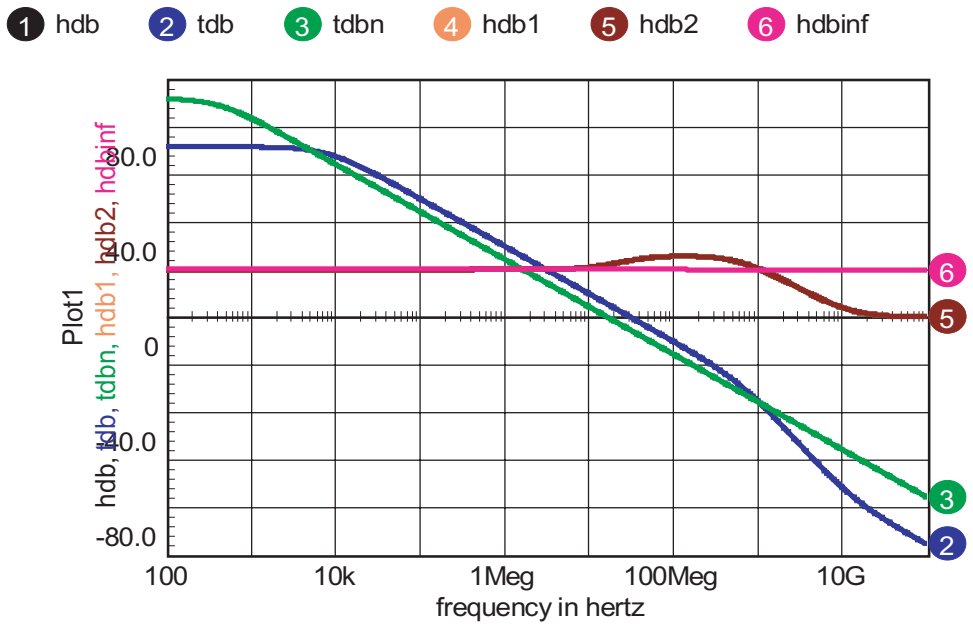


Fig. 14. Some of the Intuscope results for Fig. 13. Check that H_∞=1/K=(R1+R2)/R1, as expected. The calculated closed-loop gain H_1 and H_2 are indistinguishable from the directly simulated H. The loop gain T crosses over on a single -20dB/dec slope. The null loop gain T_n is not negligibly large at all frequencies.

At first sight you might think that the apparent peaking in H results from inadequate phase margin, but this is not the case. The correct explanation is that the crossover frequency of T_n is lower than that of T, which causes D_n to increase before D decreases, as seen in the constituents of H_1 shown in Fig. 15.

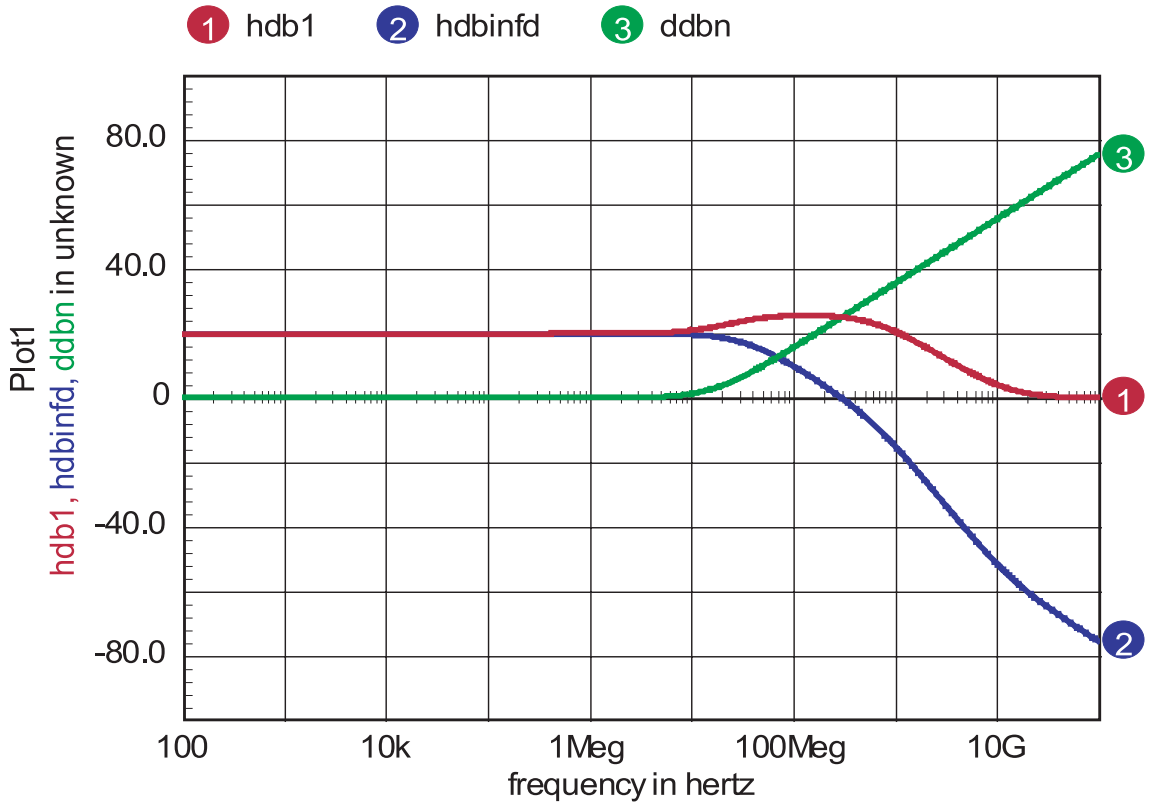


Fig. 15. The closed-loop gain H dissected into the product of two constituents $H_\infty D$ and D_n, according to (17b).

The constituents $H_\infty D$ and $H_0 D_o$ of H_2 in Fig. 16 confirm that the nonidealities, whether represented by H_0 or by T_n, take over above the null loop gain crossover frequency.

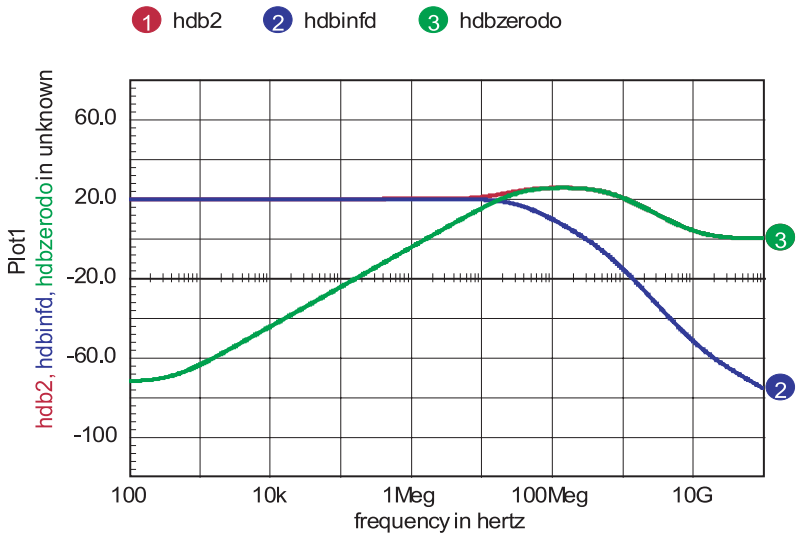


Fig. 16. The closed-loop gain H dissected into the sum of two constituents $H_\infty D$ and $H_0(1-D)$, according to (18b).

4. Manipulations of the natural block diagram

Equations (17) and (18) describe the closed-loop gain H dissected into two different sets of constituents, and correspond to the natural block diagram of Fig. 3. Other dissections are possible, a common one being the elimination of the H_0 block by modifying one or both of the remaining blocks in such a way as to preserve the same answer for H.

One possible choice is to keep the K block the same and incorporate the H_0 block (in terms of T_n) into a modified forward gain block, as shown in Fig. 17.

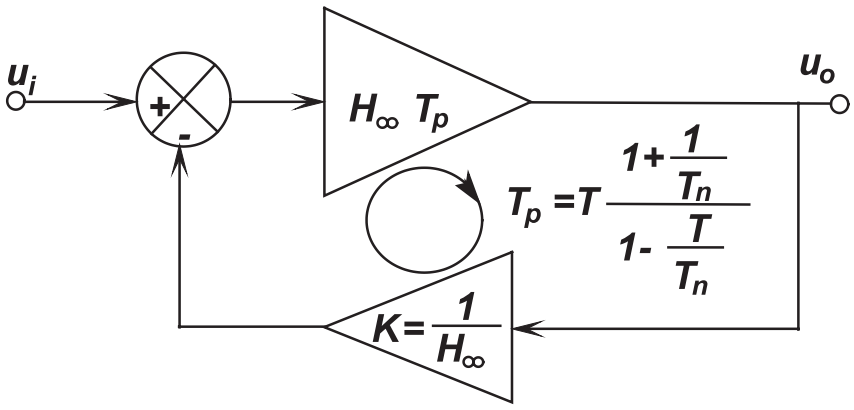


Fig. 17. Absorption of the block H_0 into a pseudo forward gain block that creates a pseudo loop gain T_p that contains both T and T_n.

This model has a closed-loop response

$$H_3 = H_\infty \frac{T_P}{1+T_P} = H_\infty D_P \tag{21}$$

where T_p is a "pseudo loop gain" defined by equating (21) with the GFT of (17),

$$T_P \equiv T \frac{1+\frac{1}{T_n}}{1-\frac{T}{T_n}} \tag{22}$$

and

$$D_P \equiv \frac{T_P}{1+T_P} \tag{23}$$

Graphs of T_p and D_p are shown in Fig. 18.

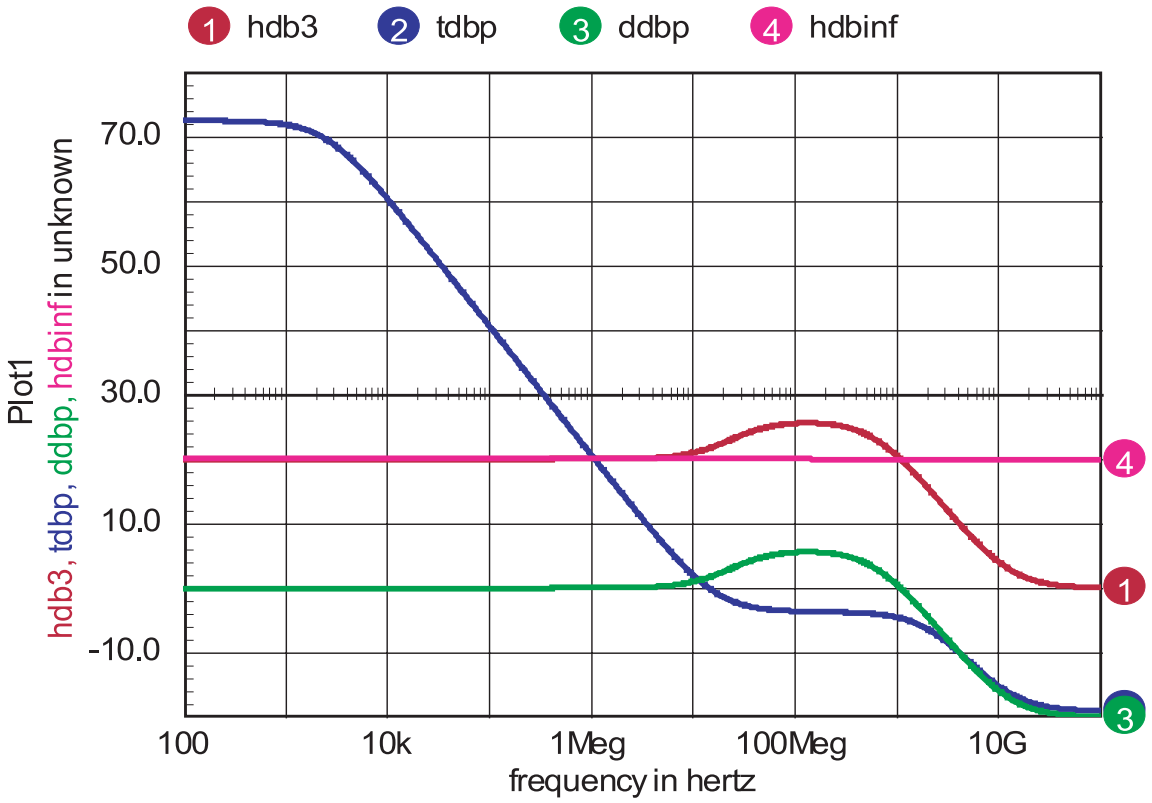


Fig. 18. The closed-loop gain H dissected into the product of two constituents H_∞ and $D_p = T_p/(1+T_p)$. The pseudo loop gain T_p has poor gain and phase margins that do not imply imminent instability.

The reason we want to dissect the closed-loop gain H into its constituent parts is to be able to adjust the circuit element values in order to meet the specifications and optimize the design. This is usually an iterative procedure, lubricated by both symbolic and numerical analysis, as done for Example 1 of Fig. 13 here and in [1]. It is obviously advantageous to have the second level TFs expressed in terms of as few as possible intuitive single-path signal flows. This is the case for H, T, and T_n, and so the GFT form H_1 of (17) has the closest relation to the equivalent circuit elements. However, H_0 involves certain loop gains as well as single-path signal flows from u_i to u_o (see Appendix), and so the GFT form H_2 of (18) is not quite so closely related to the circuit elements.

The GFT form H_3 of (21) , which describes the apparently simpler block diagram of Fig. 17, is even further removed from intuitive interpretation in terms of circuit elements, which is not surprising considering the tortuous definition (22) of the pseudo loop gain T_p. Thus, although in Fig. 18 the results for H and H_3 are correct, it is hard to relate the shapes of T_p and D_p to the physical operation of the circuit.

Although the ICAP/4 GFT Template makes it very easy and fast to graph all sorts of TFs, you have to be very clear as to what are the meanings of the second level TFs you are calculating. ICAP/4 Intuscope shows that the loop gain T displayed in Fig. 14 has a very docile phase margin of 87° and a gain margin of 52dB, far from instability; on the other hand the pseudo loop gain T_p displayed in Fig. 18 has a much smaller phase margin of 45° and a gain margin of only 0.2dB, which might be taken incorrectly as a red flag that the system is on the brink of instability.

The conclusion is that you are probably better off to deal with the natural block diagram of Fig. 3 rather than the deceptively simpler one of Fig. 17. A block diagram with fewer blocks implies that the blocks are described by more complicated expressions, which can be counterproductive in design-oriented analysis. Indeed, the opposite is one of the principles of Design-Oriented Analysis: “divide and conquer,” or, make more simpler blocks rather than fewer complicated blocks.

In the SpiceNet PartBrowser there is a GFT Sub Type called Loop Gain, which is a short cut to obtaining only the T result.

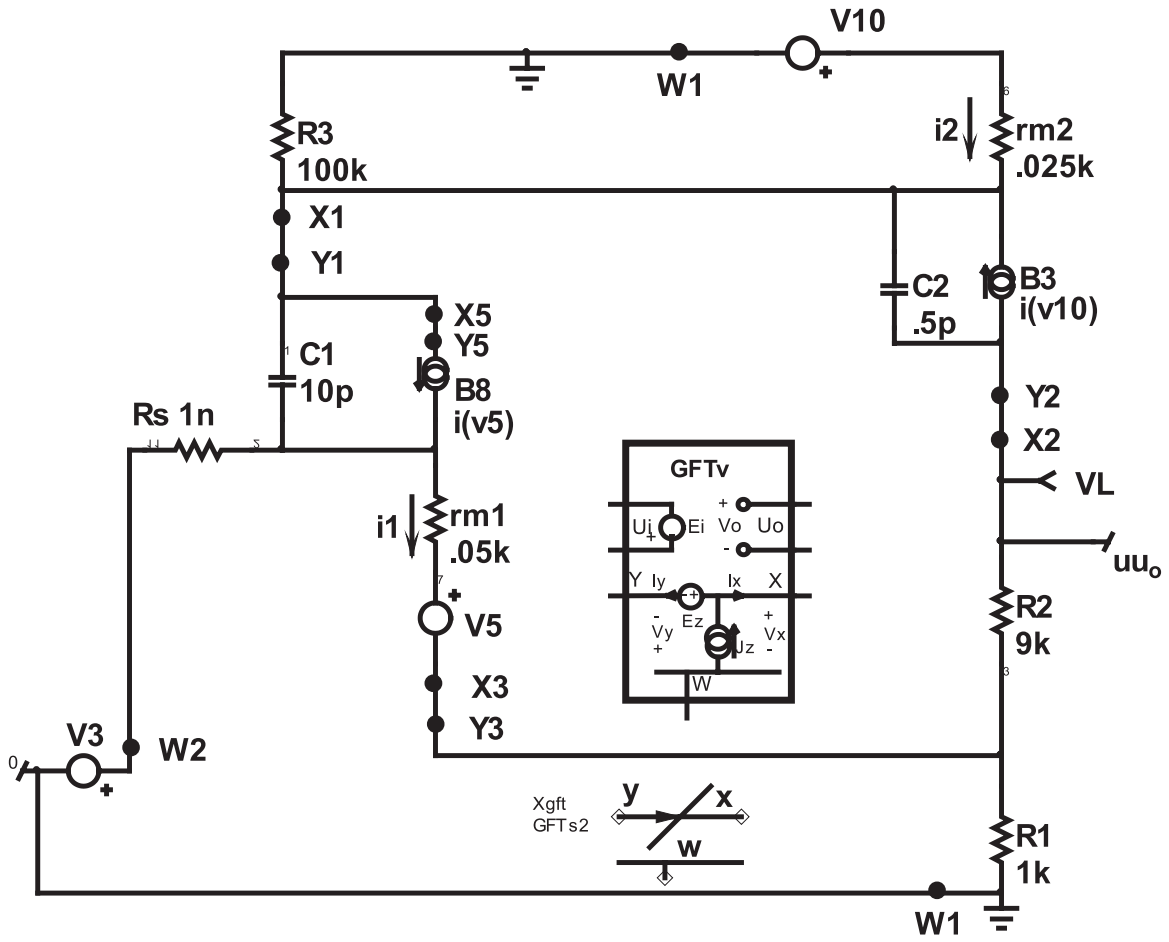


Fig. 19. The model of Fig. 13 showing other possible injection configurations WXY, and an optional icon GFTs that provides calculations only of T.

Figure 19 shows the same circuit as Example 1, but with the Loop Gain icon GFTs alternative to the GFTv icon. There are no input and output signal connections to the Loop Gain icon, so don't forget to set uu_i=0 by connecting uu_i to ground. The injection configuration is the same as that in Fig. 13 if WXY are identified as $W_2X_3Y_3$ (you have to break the X_3Y_3 connection in order to insert either the GFTv or the Loop Gain icon, of course).

To run this configuration, select GFTs instead of GFT in the Simulation Control window, and in IntuScope select Calculator GFT T. The dB and phase of T are displayed, plus the phase and gain margins are explicitly identified. The results for T are the same as when the GFTv icon is employed.

As already mentioned, different injection configurations produce different sets of second level TFs H_∞, T, T_n, H_0, although in some cases T may be the same. For example, if the GFTv or the Loop Gain icon is inserted in Fig. 19 at $W_1X_1Y_1$ or at $W_1X_2Y_2$, T remains the same as for the original insertion point $W_2X_3Y_3$ because the test signals e_z and j_z are being injected inside the major loop but outside the minor loops in such a way that the major loop is broken if either e_z is opened or j_z is shorted. Thus, T is still the *principal* loop gain. However, the other second level TFs are different, and in particular H_∞ is no longer flat at all frequencies.

On the other hand, if the GFTv or Loop Gain icon is inserted in Fig. 19 at $W_1X_5Y_5$, none of the second level TFs is the same because the test signal injection is now inside a minor loop.

4.1 Example 2: Output impedance of the 2-stage amplifier

In the PartBrowser GFT Sub Types, besides the GFTv there are three others, GFTi, GFTg, and GFTh, for use when the first-level TF H of interest is a current gain, a transadmittance, or a transimpedance. The procedure for use is exactly the same as for the voltage gain.

There are other uses for the Sub Types, which is to calculate TFs other than the one that describes the "gain" of the system. For example, the circuit of Fig. 13 represents a voltage-to-voltage amplifier, but you may also be interested in its output impedance Z_o. Output impedance is a special case of transimpedance, and the GFTh icon can be employed with the "output" signal still being the output voltage, but the "input" signal is an independent current driven into the output terminals.

Figure 20 shows the first level TF H set up to represent the output impedance Z_o, which is ready made in spice8\circuits\gft\figure20.dwg

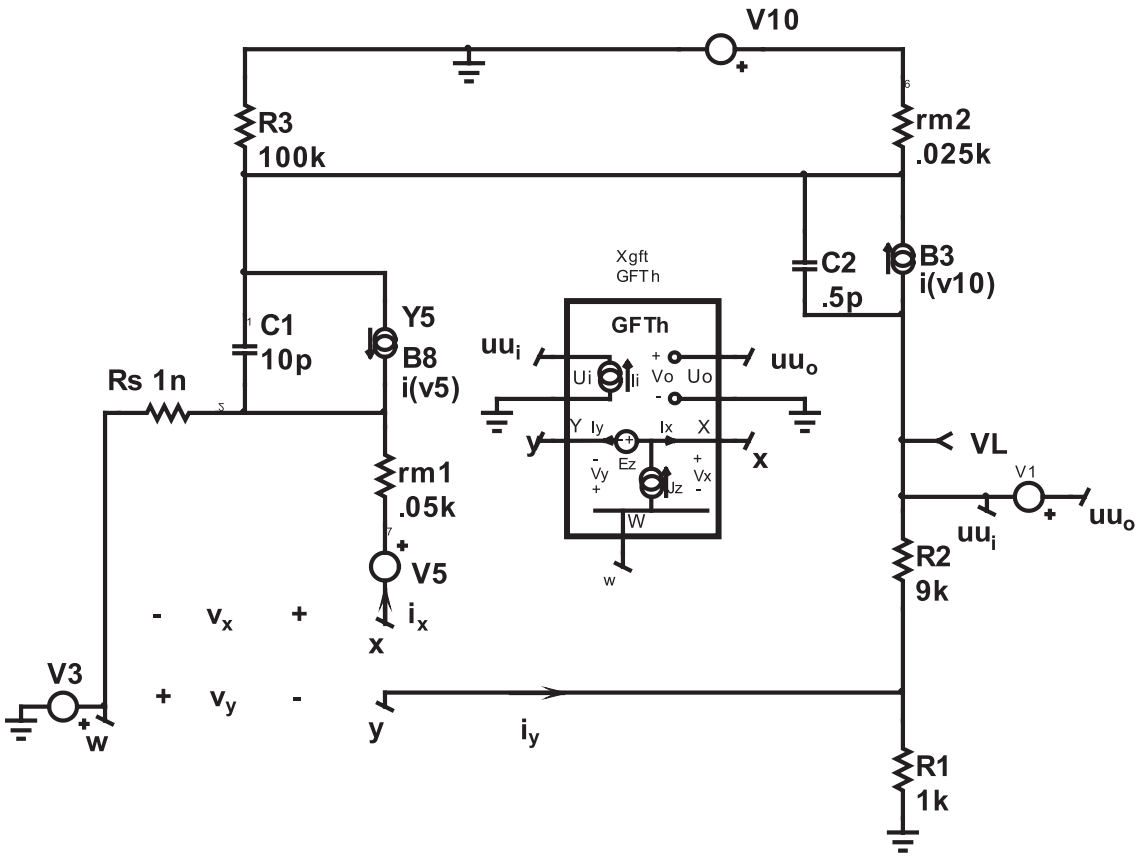


Fig. 20. Replacement of the GFT Template icon GFTv in the model of Fig. 13 by GFTh for calculation of the output impedance as a self-impedance H whose "output" is the voltage uu_o and whose "input" is the current uu_i.

The injection configuration is the same, because we want T still to be the principal loop gain. and the only differences from Fig. 12 are that the icon is GFTh instead of GFTv (don't forget to rename it xgft), and uu_i is connected to uu_o with the original uu_i grounded. The zero voltage source V6 is there to preserve the distinction between the uu_i and uu_o terminals.

The H_∞ for Fig. 20 is zero, as can be seen by doing the null double injection calculation for the output impedance with i_y and v_y nulled: if v_y is nulled, there is no voltage across R1; if i_y is also nulled, there is no current in R2, and so vL is also zero. Thus, e_z and j_z have been adjusted so that the entire independent driving current u_i goes "backwards" into the forward path, leaving none to go down the feedback divider. The result $H_\infty = H|T\rightarrow\infty = 0$ is consistent with the basic interpretation that voltage feedback reduces an amplifier's output impedance, so infinite loop gain would reduce the output impedance to zero.

Consequently, H_1 of (17) cannot be used, but H_2 of (18) is available, and reduces to

$$H_2 = H_0 \frac{1}{1+T} = H_0 D_0 \qquad (24)$$

in which $H_0 = H|T \to 0$ is interpreted to be the open-loop output impedance . At low frequencies, by inspection of Fig. 20, this is seen to be

$$H_0 = R2 + R1 // rm1 = 9.048k \qquad (25)$$

The dissected constituents are shown in Fig. 21, in which the calculated H_2 is seen to be identical to the directly measured H.

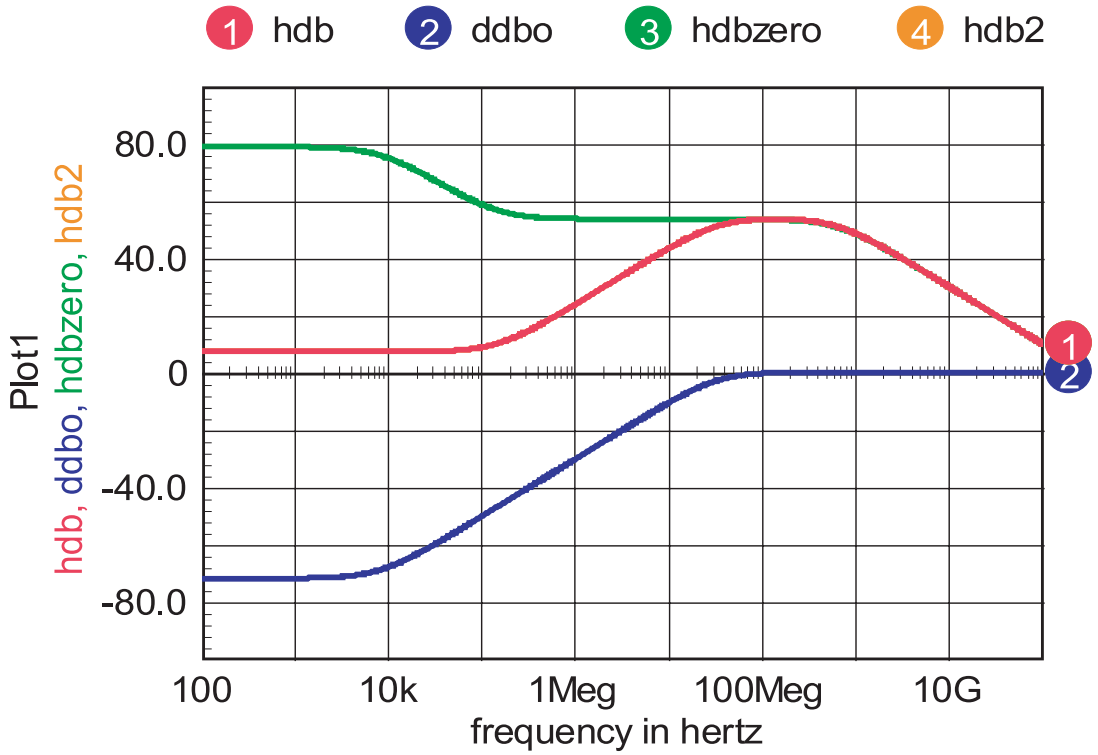


Fig. 21. Dissection of the output impedance H into the product of two components H_0 and (1-D) according to (24). The H_∞ term is zero.

The vertical axis is dB relative to 1 ohm, and at 100Hz H_0 is displayed on Intuscope as 79.13dB⇒9,047k, in complete agreement with the expected value. Because of the high loop gain, the closed-loop value at low frequencies is much smaller, displayed on Intuscope as 7.51dB⇒2.4 ohm.

4.2 Example 3: Emitter/source follower

In some circuits, the desired H_∞ requires that only i_y or v_y be nulled, not both. An example is the emitter/source follower of Fig. 22., which is ready made in spice8\circuits\gft\figure22.dwg.

The inductance typically arises from the output impedance of a driver emitter/source follower in a Darlington configuration. This circuit is well-known to be potentially unstable for certain values of load capacitance CL.

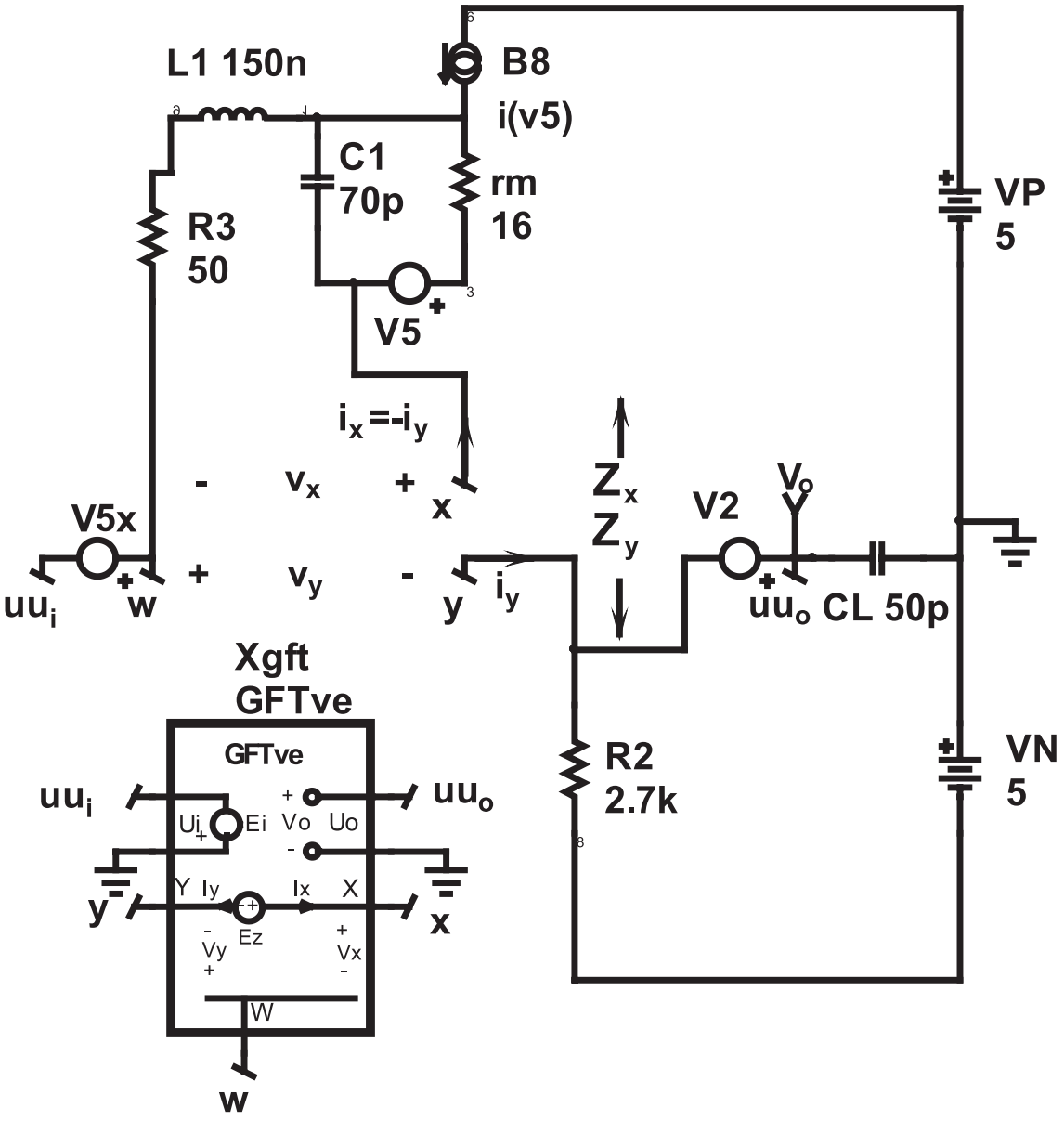


Fig. 22. A source follower circuit, with 1/K=1. The error voltage v_y appears between input and output, so nulling v_y is sufficient to set up the desired H=1/K=1, and the single injection icon GFTve is appropriate.

Even though we don't usually think of this as a "feedback circuit," the GFT of (17) or (18) gives the gain H in terms of a "loop gain" T from which stability can be assessed. As usual, the crucial question is what test signal injection configuration to adopt.

The purpose of an emitter/source follower is to provide unity voltage gain at all frequencies, so $H_\infty = 1$. However, because of finite transistor transconductance gm and various capacitances, the discrepancy factors D and maybe D_n are not unity, and the actual voltage gain H is less than H_∞.

For an emitter/source follower, the error voltage is the difference between the input and output voltage, because the fedback voltage is equal to the output voltage for $H_\infty = 1$. Therefore, the error voltage v_y is identified as in Fig. 22 and the test voltage must be injected between Y and X so as to drive the transistor with the voltage v_x. Adjustment of the injected voltage to null v_y is sufficient to raise the output voltage to be equal to the input voltage, and so i_y need not be nulled. In fact, it had better not be nulled, because i_y is the only current that produces the output voltage!

The appropriate GFT icon is GFTve, providing single voltage injection to a circuit in which the first level TF H is a voltage-to-voltage gain, and is already installed in Fig. 22.

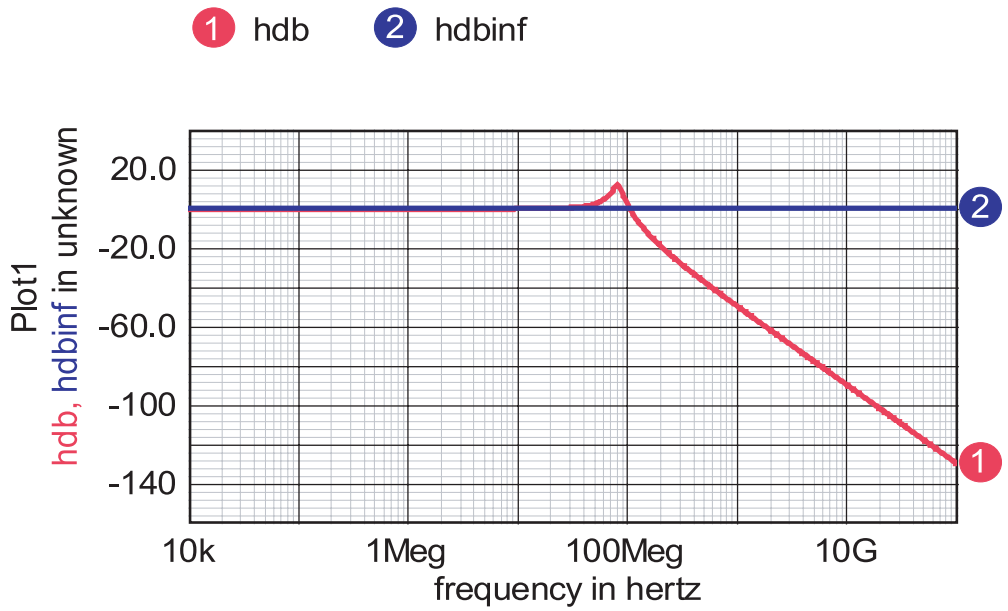


Fig. 23. Some of the Intuscope results for Fig. 22. Check that H_∞=1/K=1, as expected. The closed-loop gain H exhibits a considerable peak.

After you run the GFT Template, as always the first result to check is that H_∞ is in fact the intended value. Figure 23 shows that indeed H_∞=0dB at all frequencies, but that H peaks up before falling off. This is a red flag indicating inadequate phase margin, so we want to look at the magnitude and phase of the loop gain T, as in Fig. 24.

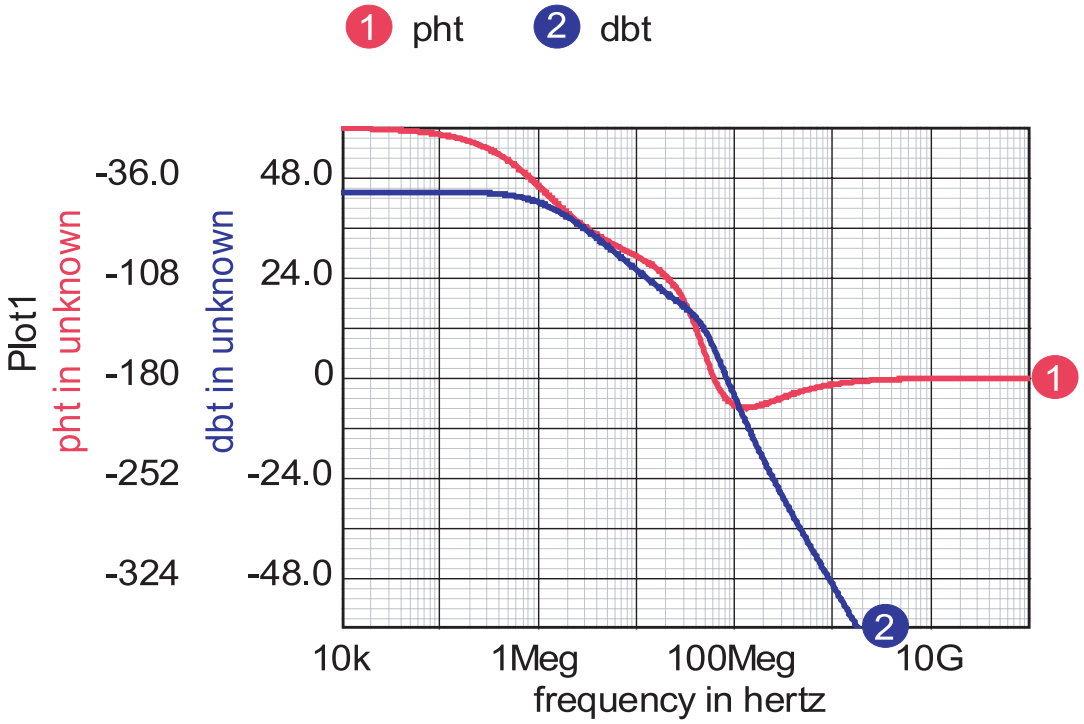


Fig. 24. The loop gain T for the source follower of Fig. 22 has negative phase and gain margins, and so is unstable.

ICAP/4 Intuscope automatically presents the phase and magnitude margins as ØM = -15° and TM = -7.0dB. These are both negative, and the circuit is actually unstable.

One of the main benefits of the GFT is that it enables you to relate the numerical results to the circuit structure and element values, one of the important steps in the design-oriented analysis strategy. So, let's examine the second level TFs T and T_n.

We can dismiss T_n quickly, because it's infinite. The condition in Fig. 22 to set up the null loop gain T_n is that there is a given input voltage uu_i, and the injected voltage is adjusted to null the output voltage uu_o, which makes $v_y = uu_i$. Also, $i_y=0$ and $i_x=-i_y=0$, so there is no current driving the transistor, and so $v_x=0$. Hence, $T_n=v_y/v_x=uu_i/0=\infty$.

The condition in Fig. 22 to set up the loop gain T is that the input uui is set to zero, and the injected voltage establishes v_x and v_y. For single voltage injection $i_x=-i_y$ so $T=v_y/v_x=Z_y/Z_x$ where Z_x and Z_y are respectively the impedances that the injected voltage source looks into at the X and Y nodes. At zero frequency T=R1/rm =2,700/16=169⇒44.5dB, in agreement with Fig. 24.

As frequency increases, |T| goes into a single negative slope (-20dB/dec) at the R1CL corner, and eventually displays a double negative slope due to L1 and CL with an asymptotic phase of

-180°. In between, however, T encounters a rather complicated transition consisting of a quadratic pair of poles at the L1Cd resonance of 49MHz and a zero at the transistor ft=1/2πCdrm= 140MHz. Thus, there is a region of triple negative slope between 49MHz and 140MHz that causes the phase lag to exceed 180°. The |T| crossover frequency of 85MHz is within this range, and the phase margin is negative.

The knowledge that $T=Z_y/Z_x$ provides insight into ways to make the circuit stable. For example, beyond the R_1CL corner, $Z_y=1/sCL$ and Z_x is independent of CL, therefore |T| is inversely proportional to CL but /T stays the same. Therefore, increasing CL lowers |T| and moves the crossover frequency lower such that the corresponding phase lag is less. Figure 25 shows the results of increasing CL from 50p to 250p.

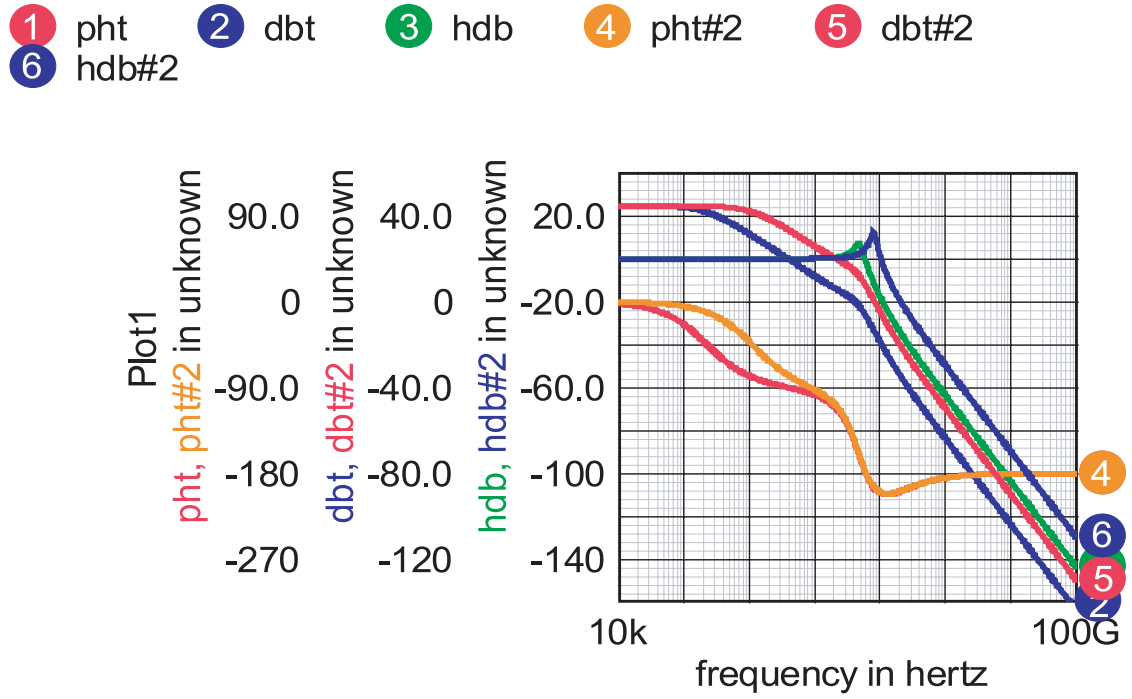


Fig. 25. Increase of the load capacitance CL from 50p to 250p causes a 14dB drop in |T| in the neighborhood of crossover, but no change in /T in the same neighborhood. The circuit is now stable.

The |T| graph is 250/50=5⇒14dB lower, and the crossover frequency decreases to 42MHz where the phase margin is +33°, and the gain margin is +7.0dB so that stability is achieved. There is still peaking in H, but less than originally.

APPENDIX

What the GFT is, and how the GFT Template works

The General Feedback Theorem (GFT) is a general network theorem that applies to any transfer function (TF) of a linear system model. The purpose of the GFT is to dissect H, the "first level" TF of interest, into several "second level" TFs H_∞, T, T_n, and H_0 so that insight can be gained in to how the circuit elements contribute to the result.

The GFT is derived in [1] as an application of the Extra Element Theorem (EET), and has two basic versions:

$$H_1 = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} \tag{A.1}$$

$$H_2 = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} \tag{A.2}$$

There is a redundancy relation between the four second level TFs:

$$\frac{H_0}{H_\infty} = \frac{T}{T_n} \tag{A.3}$$

The "natural" block diagram of Fig. A.1 represents either of the two basic versions of the GFT, in which $H = u_o/u_i$ is the first level TF of interest. The input and output signals u_i and u_o can represent any combination of voltages, currents, torque, angular velocity, etc.

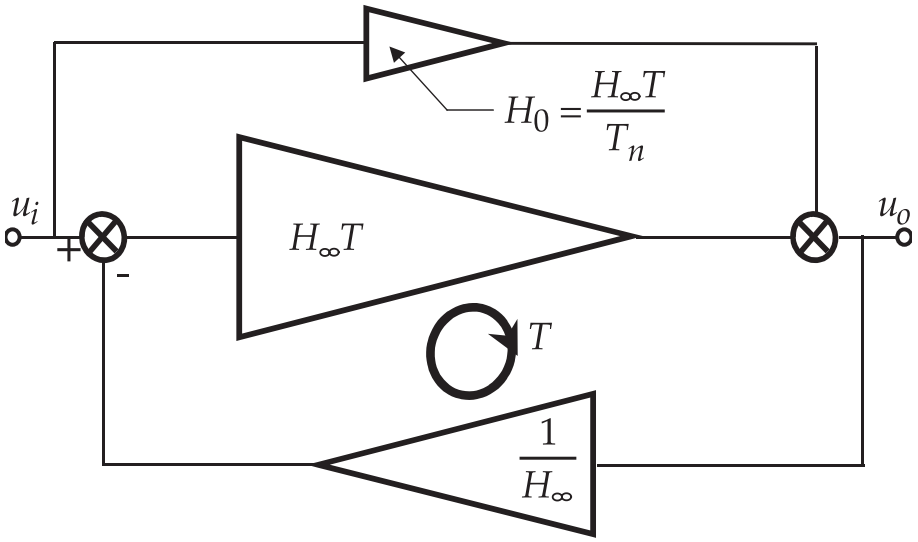


Fig. A.1 The natural block diagram for any TF H dissected according to the GFT into three out of four second level TFs H_∞, T, T_n, H_0 that are calculated directly from the circuit model.

The natural block diagram obviously resembles a single loop feedback system, and is one of the most common applications of the GFT.

The value of the GFT lies in the relationship between the second level TFs H_∞, T, T_n, H_0 and the circuit elements. This is determined by the results of certain "third level" calculations on the circuit model in the presence of one or more injected test signals. The "test signal configuration" is defined by the number and type of injected signals, and where they are injected. Different test signal configurations produce different sets of H_∞, T, T_n, H_0.

The GFT Template is limited to two test signals, and the typical configuration of Fig. A.2 shows a voltage e_z and a current j_z injected at the same nodes WXY, although in other configurations these nodes are not necessarily the same for each test signal.

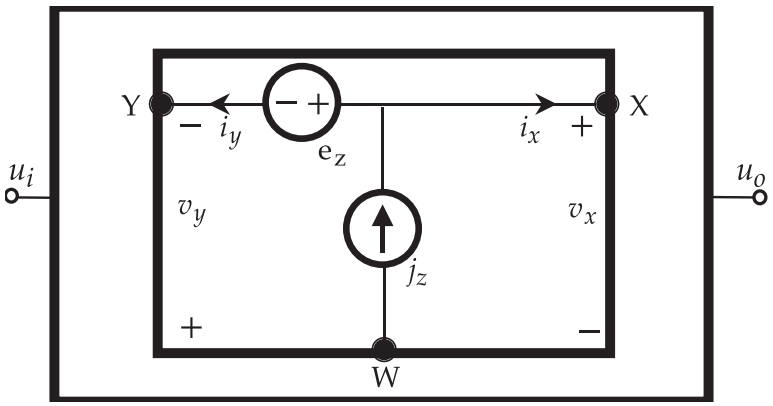


Fig. A.2 One example of injection of test signals e_z and j_z for determination of the third level null double injection and double null triple injection TFs needed to calculate the second level TFs.

The third level TFs simulated for the test signal configuration of Fig. A.2 are:

$$H_{i_y v_y} = \left.\frac{u_o}{u_i}\right|_{i_y, v_y = 0} \tag{A.4}$$

and similarly for H_{vyix}, H_{iyvx}, and H_{ixvx};

$$T_{i_v_y} \equiv \left.\frac{i_y}{i_x}\right|_{u_i, v_y = 0} \tag{A.5}$$

and similarly for T_{i_vx}, T_{v_iy}, and T_{v_ix};

$$T_{ni_v_y} \equiv \left.\frac{i_y}{i_x}\right|_{u_0, v_y = 0} \tag{A.6}$$

and similarly for T_{ni_vx}, T_{nv_iy}, and T_{nv_ix}.

The interpretations are as follows. Although there are three independent sources u_i, e_z, and j_z in Fig. A.2, only u_i appears explicitly. The constraint u_i,u_y=0 in (A5) means that u_i is set to zero, and e_z and j_z are mutually adjusted so as to null the dependent signal v_y; T_{i_vy} is the ratio i_y/i_x under the *null double injection* (ndi) condition of nulled v_y.

The constraint u_o,u_y=0 in (A6) means that ui is restored, and all three independent sources u_i, e_z, and j_z are mutually adjusted so as to null the two dependent signals u_o and v_y; T_{ni_vy} is the ratio i_y/i_x under the *double null triple injection* (dnti) condition of nulled u_o and v_y.

All four T's are ndi calculations; all four Tn's and all four H's are dnti calculations.

There is lengthy discussion in [1 - 4] to support the contention that ndi and dnti symbolic calculations are easy to make, in fact, easier than familiar single injection calculations. All are *single path* calculations: you never have to know what the independent source mutual adjustments are, you just follow the starting signal to the ending signal, incorporating along the way the condition that certain signals are nulled. Often, if one signal is nulled, others are too, so the null "propagates." Any element that supports a nulled signal does not contribute to the result, which is therefore not only easier to obtain, but *simpler* than if a null does not exist.

The GFT Template simulates null conditions by making the null self-adjusting. Only one independent source is retained, and the others are replaced by dependent sources driven by the signals to be nulled. For example, the dnti simulation H_{iyvy} can be implemented as in Fig. A.3, in which u_i is the retained independent source. The higher the "nulling gain," the better the resulting null. The default value is 1G.

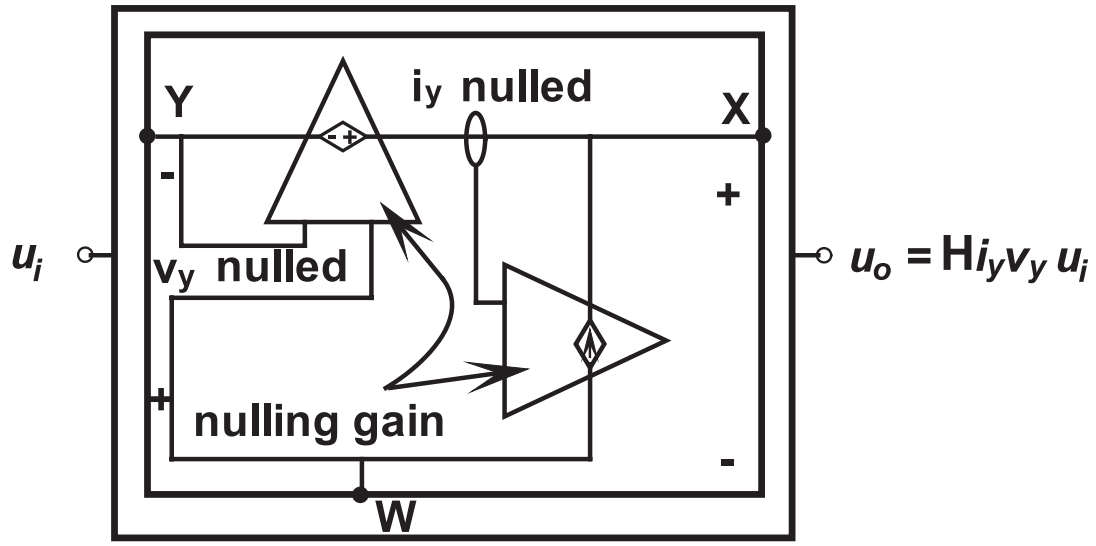


Fig. A.3 Typical GFT Template algorithm for creation of self-adjusting nulls by use of high-gain "nulling amplifiers."

In principle, it doesn't matter which independent signal is retained, or which signal is nulled by which nulling gain. Also, because of the happy circumstance that the nulling gains have infinite bandwidth, the nulling polarity doesn't matter and no system instability is introduced.

The GFT Template simulates the twelve third level TFs in succession without interruption, by invoking the "alter" feature to sequence the required ndi and dnti conditions. All twelve TFs, along with the first level TF H simulated with no test signal injection, are available for magnitude and phase display in Intuscope.

The GFT Template then calculates the second level TFs H_∞, T, T_n, H_0 according to the GFT definitions derived in [1], and makes them also available for display in Intuscope:

$$H_\infty = H_{i_y v_y} \tag{A.7}$$

$$\frac{1}{T} = \frac{1}{T_{i_v_y}} + \frac{1}{T_{v_i_y}} + \frac{1}{T_{iv}} \tag{A.8}$$

$$\frac{1}{T_n} = \frac{1}{T_{ni_v_y}} + \frac{1}{T_{nv_i_y}} + \frac{1}{T_{niv}} \tag{A.9}$$

$$H_0 = H_{v_y i_x} \frac{T}{T_{i_v_y}} + H_{i_y v_x} \frac{T}{T_{v_i_y}} + H_{i_x v_x} \frac{T}{T_{iv}} \tag{A.10}$$

where

$$T_{iv} \equiv T_{i_v_y} T_{v_i_x} \tag{A.11}$$

$$T_{vi} \equiv T_{v_i_y} T_{i_v_x} \tag{A.12}$$

$$T_{niv} \equiv T_{ni_v_y} T_{nv_i_x} \tag{A.13}$$

$$T_{nvi} \equiv T_{nv_i_y} T_{ni_v_x} \tag{A.14}$$

T_{vi} and T_{nvi} do not appear in (A.7) through (A.10) because T_{iv} and T_{vi} should be equal and T_{niv} and T_{nvi} should be equal, so (A.12) and (A.14) exist merely as computation checks. Also, H_0 is calculated from the redundancy relation (A.3), and should be equal to H_0 calculated from (A.10).

The GFT Template Parts Browser, under GFT, lists four Subtypes, one for each of the four first level TFs voltage gain, current gain, transimpedance, and transadmittance. Each of these has an icon that requires connection to the input and output, and a selection from six combinations of injected test signals. The one shown in Fig. A.2 represents Coupled voltage, current injection. Dual voltage, current is the same, except that the WXY connections are different for the two test sources. Dual voltage, voltage and Dual current, current have two like sources with different WXY connections. There are also icons for Single voltage and Single current injection.

The third level TFs required for the Single current test signal configuration are directly equal to the corresponding second level TFs:

$$H_\infty = H_{i_y} \equiv \left.\frac{u_o}{u_i}\right|_{i_y=0} \tag{A.15}$$

$$T = T_i \equiv \left.\frac{i_y}{i_x}\right|_{u_i=o} \tag{A.16}$$

$$T_n = T_{ni} \equiv \left.\frac{i_y}{i_x}\right|_{u_0=o} \tag{A.17}$$

$$H_0 = H_{i_x} \equiv \left.\frac{u_o}{u_i}\right|_{i_x=0} \tag{A.18}$$

The third level TFs required for the Single voltage test signal configuration are the same, except with v in place of i.

Regardless of which test signal configuration is employed, the result is a set of second level TFs H_∞, T, T_n, H_0. The GFT Template then does various postprocessing calculations that define some additional quantities useful in presenting various forms of the final result for H. These include a "pseudo loop gain" T_p into which both T and T_n are absorbed, and three "discrepancy factors" each of which is uniquely determined by T, T_n, or T_p. The postprocessing calculations can be viewed in spice8\script\GFT.scp.

Finally, the GFT produces three versions of H, with numerical designators, to indicate which form of the GFT is its origin.

$$H_1 = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty DD_n \tag{A.19}$$

$$H_2 = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T} = H_\infty D + H_0 D_0 \tag{A.20}$$

$$H_3 = H_\infty \frac{T_P}{1+T_P} = H_\infty D_P \tag{A.21}$$

where

$$\frac{1}{T_P} \equiv \frac{T_n}{1+T_n}\left(\frac{1}{T} - \frac{1}{T_n}\right) \tag{A.22}$$

$$D \equiv \frac{T}{1+T} \tag{A.23}$$

$$D_0 \equiv \frac{1}{1+T} \tag{A.24}$$

$$D_n \equiv \frac{1+T_n}{T_n} \qquad \text{(A.25)} \qquad\qquad D_P \equiv \frac{T_P}{1+T_P} \qquad \text{(A.26)}$$

Reminder

Different test signal configurations produce different sets of the second level TFs H_∞, T, T_n, H_0. Even the same test sources, for example the Coupled voltage, current icon of Fig. A.2 produces different sets for different connection nodes WXY, emphasized by writing (A.1) as

$$H = H_{\infty 1}\frac{1+\dfrac{1}{T_{n1}}}{1+\dfrac{1}{T_1}} = H_{\infty 2}\frac{1+\dfrac{1}{T_{n2}}}{1+\dfrac{1}{T_2}} = \ldots \qquad \text{(A.27)}$$

Although the constituent TFs may be different in each set, when they are assembled into (A.27) they all miraculously produce the same H (if everything works properly; see Caveats below).

Therefore, even though the GFT Template does all the work, *you* are responsible for choosing an injection configuration such that H_∞, T, T_n, H_0 have useful interpretations with respect to circuit performance; otherwise, they are useless in design-oriented circuit analysis and you might as well resort to the try-it-and-see approach.

Choice of injection configuration is the crucial step in implementing the GFT, which is why it is emphasized in the preceding Examples.

Caveats

Some second level TFs H_∞, T, T_n, H_0 may be zero or infinite. Because the GFT Template algorithm employs nulling gains that are necessarily finite, a zero or infinite result cannot be produced although the incorrect Intuscope graph may superficially look legitimate.

Fortunately, it is easy to determine in advance whether a certain TF is zero or infinite, and then you can avoid using a form of the result that depends upon that incorrect TF.

Example 2 exhibits a case in which $H_\infty = 0$, and the reason is described. The version (A.20) for H_2 has to be used, and (A.19) for H_1 gives a wrong answer because both H_∞ and T_n, which is actually infinite, are simulated incorrectly. Likewise, the simulation of H_0 from (A.10) is wrong, and does not agree with the calculation from (A.3), as both should be zero. In Example 3, T_n is infinite, for the reason described, and so $D_n = 1$.

In Example 1 T_{i_vx}, T_{v_ix}, T_{ni_vx}, T_{nv_ix} are all infinite, which can be seen by following arguments similar to that in Example 3. Consequently, all four simulations give incorrect results, and T_{iv} is not equal to T_{vi} and T_{niv} is not equal to T_{nvi} as demanded by (A.11) through (A.14). However, none matters because the T_{iv} and T_{niv} contributions to T and T_n in (A.8) and (A.9) are negligible.