Bayesian statistics with R
7. Contrast scientific hypotheses with model selection

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Model selection

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- The proportion of explained variance R^2 is problematic, because the more variables you have, the bigger R^2 is.
- Idea: **penalize models with too many parameters**.

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A measure of goodness-of-fit of the model to the data: the more parameters you have, the smaller the deviance is (or the bigger the likelihood is).

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A penalty: twice the number of parameters K

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- Two models are difficult to distinguish if $\Delta\text{AIC} < 2$.

Bayesian version

- Deviance Information Criteria or DIC, a Bayesian method for model comparison that JAGS can calculate for (m)any models.

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- DIC is intended as a generalisation of AIC, and with little prior information, p_{D} should be approximately the true number of parameters.
- The model with the smallest DIC is estimated to be the model that would best predict a replicate dataset with same structure as that observed.

DIC in Jags

```
storks
#> Inference for Bugs model at "code/logistic.txt", fit using jags,
#>  2 chains, each with 2000 iterations (first 1000 discarded)
#>  n.sims = 2000 iterations saved. Running time = 0.059 secs
#>           mu.vect sd.vect    2.5%     25%     50%     75%   97.5%  Rhat n.eff
#> a           1.555   0.054   1.449   1.518   1.555   1.593   1.658 1.011   220
#> b.rain     -0.160   0.066  -0.283  -0.206  -0.162  -0.116  -0.026 1.004  1300
#> b.temp      0.035   0.062  -0.082  -0.008   0.033   0.075   0.165 1.007   330
#> deviance  204.639   2.446 201.823 202.855 203.982 205.837 210.943 1.006   260
#>
#> For each parameter, n.eff is a crude measure of effective sample size,
#> and Rhat is the potential scale reduction factor (at convergence, Rhat=1).
#>
#> DIC info (using the rule: pV = var(deviance)/2)
#> pV = 3.0 and DIC = 207.6
#> DIC is an estimate of expected predictive error (lower deviance is better).
```

Further reading

- Hooten, M.B. and Hobbs, N.T. (2015), A guide to Bayesian model selection for ecologists. Ecological Monographs, 85: 3-28. https://doi.org/10.1890/14-0661.1
- Conn, P.B., Johnson, D.S., Williams, P.J., Melin, S.R. and Hooten, M.B. (2018), A guide to Bayesian model checking for ecologists. Ecol Monogr, 88: 526-542. https://doi.org/10.1002/ecm.1314