An invariance-based classification of physical theories

Erik Axelkrans*

February 5, 2026

Abstract

Contemporary theoretical physics encompasses a wide range of formalisms, often introduced and developed within distinct conceptual and mathematical frameworks. While invariance principles play a central role in individual theories, they are typically treated locally rather than as a basis for organizing theory space as a whole. This paper develops a systematic classification of established theory classes in fundamental and mathematical physics based on their primary invariance, understood as the invariant structure that defines theory identity under admissible changes of description.

By applying this classification across classical, relativistic, quantum, statistical, topological and information-theoretic frameworks, we find that many apparently disparate theories cluster around a relatively small set of invariance anchors. The resulting organization reveals a significant compression of theory space when compared to classifications based on formalism or dynamical content. We further show that well-known inter-theory relations—such as quantization, coarse-graining, renormalization group flow, duality and holographic correspondence—can be understood as representation-changing transformations that preserve the defining invariance of the associated theory classes.

The analysis is methodological rather than dynamical and does not propose new physical models or empirical predictions. Instead, it provides a structural taxonomy that clarifies relationships between existing theories and highlights unoccupied or sparsely populated regions of theory space. A brief interpretative discussion suggests that the observed structure admits a natural reading in terms of projection-based descriptions, although the classification itself remains independent of any specific ontological framework.

*Researcher, Gothenburg, Sweden.
Email: erik@axelkrans.se

1 Introduction

Contemporary theoretical physics comprises a remarkably diverse collection of formalisms, ranging from classical and relativistic field theories to quantum, statistical, topological and information-theoretic frameworks. These theories differ widely in their mathematical carriers—phase spaces, Hilbert spaces, operator algebras, fibre bundles, categories—and are often introduced in distinct historical and conceptual contexts. As a result, relationships between theories are frequently discussed in terms of specific models, dynamical assumptions or limiting procedures, rather than through a unified structural perspective.

A recurring theme across this diversity is the central role played by invariance [4, 5]. Lorentz, gauge, diffeomorphism, unitarity, scale and topological invariances each define what is regarded as physically meaningful within their respective domains. In practice, such invariances serve as stability criteria: they identify which features of a description remain unchanged under admissible transformations of coordinates, representations or scale and thereby delineate the equivalence classes that constitute a given theory.

Despite their foundational importance, invariance principles are typically treated locally, within the context of individual theories or narrow classes of models. What is less commonly addressed is the extent to which invariance itself may provide a unifying criterion for organizing physical theories across formal and conceptual boundaries. In particular, it is not obvious whether the apparent diversity of theories reflects a corresponding diversity of fundamentally distinct structures or whether many established theories may instead be understood as realizations of a smaller set of invariant principles expressed through different mathematical representations.

The aim of the present work is to explore this question by developing a systematic classification of established physical theory classes based on their *primary invariance*. Here, a theory class is characterized not by its specific equations or field content but by the invariant structure that defines its physical identity under admissible changes of description. Secondary invariances may be present and play important roles but the primary invariance is taken to be the defining feature without which the theory would no longer retain its characteristic form.

By applying this invariance-based classification across a broad range of theories in fundamental and mathematical physics, we find that many apparently disparate frameworks cluster naturally around a small number of invariance anchors. Moreover, well-known inter-theory relations—such as quantization, coarse-graining, renormalization group flow, duality and holographic correspondences—can be understood as transformations that alter representational structure while preserving the relevant invariance. This perspective reveals a non-trivial compression of theory space and highlights structural commonalities that are often obscured by differences in formalism.

The scope of this paper is deliberately limited. The analysis is confined to established theory classes in fundamental and mathematical physics and no new dynamical models or experimental predictions are proposed. The goal is instead taxonomic and structural: to provide a coherent organizational framework that clarifies how existing theories are related through shared invariance principles.

This approach should be distinguished from earlier multi-axial classifications, in which physical theories are characterized along several independent structural dimensions [2]. Here the focus is instead on identifying the dominant invariant structure that defines theory identity, leading to equivalence classes rather than coordinate-based descriptions.

The paper is organized as follows: In Section 2 we clarify the notion of invariance employed in this work and distinguish it from narrower symmetry concepts. Section 3 describes the classification method and criteria used to identify primary invariances. Section 4 presents a comprehensive invariance table covering a wide range of theory classes. In Section 5 the resulting clustering of theories is analyzed and visualized. Section 6 discusses inter-theory morphisms that preserve invariance while changing representation. Section 7 identifies unoccupied or sparsely populated invariance classes suggested by the classification. In Section 8 we offer a cautious interpretative discussion of the observed structure. Section 9 concludes with a brief summary and outlook.

2 Invariance, symmetry and scope

The term *invariance* is used throughout this work in a deliberately broad sense. By invariance we mean the preservation of physically relevant structure under admissible changes of description, representation or scale. Such changes may include coordinate transformations, field redefinitions, changes of basis, coarse-graining procedures or other transformations that are regarded as physically non-essential within a given theoretical context. An invariant structure, in this sense, characterizes what remains unchanged across equivalent descriptions of the same physical content.

This notion of invariance should be distinguished from but includes as a special case, the more familiar concept of symmetry as realized by group actions. Lorentz invariance, gauge invariance and internal symmetry groups are central examples of group-based symmetries. However, many physically significant invariances are not naturally expressed as global or local symmetry groups. Scale invariance under renormalization group flow, topological invariance under continuous deformations, functorial invariance in categorical formulations and information-theoretic equivalence in holographic or entanglement-based descriptions, all represent forms of structural preservation that extend beyond the standard symmetry paradigm.

In practical terms, invariance functions as a criterion for physical equivalence. Two descriptions are considered to represent the same theory if they differ only by transformations that leave the relevant invariant structure unchanged. For example, canonical transformations preserve the symplectic structure of classical phase space; unitary transformations preserve transition probabilities in quantum mechanics; gauge transformations preserve physical observables; diffeomorphisms preserve relational geometric structure in general relativity; and renormalization group transformations preserve universal behavior across scales. In each case, invariance delineates the boundary between representational freedom and physical distinction.

Within this framework, we introduce the notion of a *primary invariance*. The primary invariance of a theory class is the invariant structure that defines its physical identity: if this invariance is broken, the theory no longer belongs to the same class. Additional invariances may be present and may play essential roles in specific formulations or regimes but they are regarded here as secondary. The distinction between primary and secondary invariances is not intended to imply a hierarchy of importance but rather to provide a practical criterion for classification.

The scope of the present analysis is intentionally restricted. We consider only established theory classes in fundamental and mathematical physics, including classical, relativistic, quantum, statistical, topological and information-theoretic frameworks. Areas such as chemistry, biology and theories of cognition or consciousness are excluded, not because the invariance-based perspective could not be extended to them but because the present work aims to remain within domains where the relevant structures are already well formalized and broadly accepted.

No assumptions are made concerning the ontological origin of invariance or its possible emergence from deeper levels of description. The focus is exclusively on the organizing role that invariance plays across existing theories. The classification developed in the following sections is therefore intended as a structural taxonomy, rather than as a proposal for a new fundamental dynamics or a revision of established physical principles.

3 Method: classification by primary invariance

The classification developed in this work is based on a single guiding principle: physical theory classes are organized according to the invariant structure that defines their physical identity under admissible changes of description. The goal is not to derive new theories or to privilege particular formalisms but to provide a consistent and transparent method for comparing and grouping existing frameworks across disparate domains of physics.

3.1 Theory classes

Throughout this paper, the term *theory class* refers to a family of formulations that share the same defining invariant structure, even if they differ in mathematical realization, dynamical details or domain of application. For example, quantum mechanics may be formulated in terms of Hilbert spaces, operator algebras or path integrals, yet these formulations are commonly regarded as belonging to the same theory class insofar as they preserve the same fundamental probabilistic structure under admissible transformations.

The classification is therefore intentionally coarse-grained. Individual models, specific Lagrangians or particular choices of fields and interactions are not treated as distinct entries. Instead, the focus is on structurally stable categories that have proven robust across multiple formulations and contexts.

3.2 Primary and secondary invariances

For each theory class, a distinction is made between *primary* and *secondary* invariances. The primary invariance is defined as the invariant structure without which the theory would no longer retain its characteristic physical content. If this invariance is violated, the resulting framework is regarded as belonging to a different theory class.

Secondary invariances are those that are commonly present and often physically significant but are not uniquely defining. These may include additional symmetries, conservation laws or structural constraints that arise in particular formulations, regimes or limits. The distinction between primary and secondary invariances is pragmatic rather than absolute and is introduced solely to support a clear and workable classification scheme.

3.3 Criteria for admissible transformations

Invariance is always defined relative to a class of transformations that are regarded as physically admissible within a given theoretical context. Such transformations typically include but are not limited to:

- coordinate and frame transformations,
- changes of basis or representation,
- field redefinitions and gauge transformations,
- coarse-graining procedures and changes of scale,
- reformulations in alternative mathematical languages.

Transformations that alter the invariant structure identified as primary are not considered admissible for the purpose of classification, as they lead to a change of theory class rather than a re-description of the same physical content.

3.4 Handling overlapping and multi-invariance theories

Some well-established frameworks exhibit more than one prominent invariance and may therefore appear to straddle multiple categories. Quantum field theory is a prominent example, combining Lorentz covariance, unitarity and, in many cases, gauge invariance. In such cases, the classification does not enforce exclusivity. Instead, theories may be associated with multiple invariance anchors, reflecting their hybrid structural character.

These overlaps are not treated as ambiguities or shortcomings of the method but as informative features. They often signal regimes where different theoretical perspectives intersect or where multiple invariant structures coexist and jointly constrain the theory.

3.5 Scope and limitations

The classification presented here is not claimed to be unique or final. Alternative criteria or finer-grained distinctions may be appropriate for other purposes. The present method is intended to balance generality with clarity, capturing the dominant invariant structures that organize contemporary theoretical physics without becoming entangled in model-dependent details.

With these methodological choices in place, we proceed in the next section to present a comprehensive invariance table covering a broad range of established theory classes.

4 Comprehensive invariance table

In this section we present a comprehensive table classifying a broad range of established theory classes according to their primary invariance. The purpose of the table is not to provide an exhaustive survey but to offer a structural overview of how contemporary theoretical physics organizes around a limited set of invariant principles.

Table 1 summarizes the classification developed in this work. Each entry in the table corresponds to a theory class as defined in Section 3. That is, the listed classes represent families of formulations that share the same defining invariant structure, even when realized through different mathematical carriers or applied in different physical contexts. The emphasis is therefore on conceptual and structural stability rather than on technical detail.

For each theory class, a primary invariance is identified together with a small number of secondary invariances that are commonly present. The ordering of entries in the table carries no implication of fundamentality, historical priority or empirical status. Classical, quantum, relativistic, statistical, topological and information-theoretic frameworks are treated on equal footing, insofar as they admit a clear characterization in terms of invariant structure. Similarly, the absence of a particular framework from the table should not be interpreted as a judgment on its validity or importance but reflects the deliberate restriction to well-established theory classes within fundamental and mathematical physics.

Some theory classes naturally appear in proximity to more than one invariance anchor. As discussed in Section 3, such overlaps are retained rather than eliminated, as they often highlight structurally significant intersections between different theoretical perspectives. In this sense, the table should be read not as a partition of theory space into disjoint categories but as a map indicating regions of shared structural identity.

The table itself is presented in the following pages. In Section 5 we analyze the resulting distribution of theory classes and show that the classification exhibits a pronounced clustering around a small number of primary invariances. This observation forms the basis for the subsequent discussion of inter-theory relations and their structural interpretation.

Theory class	**Primary invariance**	**Secondary invariances (typical)**
Classical Newtonian mechanics	Galilean invariance	Time and space translations; spatial rotations
Lagrangian mechanics	Action (variational) invariance	Coordinate covariance; Noether symmetries
Hamiltonian mechanics	Canonical (symplectic) invariance	Liouville measure; time translation (autonomous systems)
Classical field theory	Field-redefinition covariance	Spacetime symmetries; internal symmetries
Continuum mechanics	Frame indifference	Euclidean invariance; balance-law symmetries
Classical electromagnetism	U(1) gauge invariance	Lorentz invariance; charge conservation
Special relativity	Lorentz invariance	Poincaré translations; invariant interval
General relativity	Diffeomorphism invariance	Local Lorentz invariance; Bianchi identities
Metric gravity theories	Metric covariance	Diffeomorphism invariance; minimal coupling
Background-independent field theories	Background independence	Relational observables; constraint closure
Non-relativistic quantum mechanics	Unitary invariance	Canonical commutation relations; Galilean symmetry
Relativistic quantum mechanics	Poincaré invariance	Unitarity; positive-energy spectrum
Quantum field theory	Locality, Lorentz covariance and unitarity	Gauge invariance; CPT symmetry
Interacting quantum field theory	Renormalization-group consistency	Locality; anomaly cancellation
Effective field theory	Scale separation (decoupling)	Operator expansion; RG flow
Axiomatic quantum field theory	Axiomatic consistency	Lorentz covariance; spectral condition
Algebraic quantum field theory	Algebraic (net) invariance	Locality; covariance of algebras
Gauge field theories	Local gauge invariance	BRST symmetry; constraint structure
Yang–Mills theories	Non-Abelian gauge invariance	Topological sectors; RG behavior
Spontaneous symmetry breaking frameworks	Vacuum-manifold structure	Goldstone modes; Higgs mechanism
Statistical mechanics	Ensemble invariance	Liouville measure; ergodicity assumptions
Thermodynamics	Legendre-transform invariance	Extensivity; convexity relations
Non-equilibrium statistical mechanics	Entropy-production structure	Fluctuation relations; coarse-graining
Renormalization group theory	Scale invariance under RG flow	Universality classes; scheme independence
Nonlinear dynamical systems	Structural (topological) invariance	Fixed-point and cycle structure
Deterministic chaos	Lyapunov-spectrum invariance	Fractal dimensions; metric entropy
Topological field theories	Topological invariance	Gauge symmetry; bordism classes
Topological quantum field theory	Cobordism invariance	Functoriality; modular structures
Chern–Simons theories	Knot and link invariance	Gauge invariance; level quantization
Conformal field theory	Conformal invariance	Operator product expansion; modular invariance
Holographic frameworks	Information equivalence	Entanglement entropy relations
Tensor-network descriptions	Entanglement-structure invariance	Isometric gauge freedom; coarse-graining
Categorical quantum mechanics	Functorial (compositional) invariance	Dagger-compact structure
Functorial field theories	Natural-transformation invariance	Gluing locality; bordism functors

Table 1: Classification of established theory classes by primary invariance. Each theory class is characterized by the invariant structure that defines its physical identity under admissible changes of description. Secondary invariances commonly present in standard formulations are listed for orientation.

5 Invariance clustering of physical theories

The classification summarized in Table 1 reveals a non-trivial organizational pattern. Despite the apparent diversity of formalisms and physical contexts represented, the listed theory classes do not distribute uniformly across distinct invariant structures. Instead, they exhibit a pronounced clustering around a relatively small number of primary invariances.

This clustering is not imposed by construction. The theory classes were identified independently based on established usage in the literature and the primary invariances were assigned according to the criteria outlined in Section 3. The resulting concentration of theories around specific invariance anchors therefore constitutes an empirical structural observation rather than a definitional artifact. The invariance clusters discussed below are obtained directly from the classification in Table 1 and are summarized in Table 2.

Several broad clusters can be identified. Relativistic frameworks group naturally around Lorentz invariance, encompassing special relativity, relativistic field theories and relativistic quantum formulations. Theories of gravitation and background-independent dynamics cluster around diffeomorphism invariance, reflecting the central role of relational geometric structure. Gauge-based theories form a distinct cluster organized by local gauge invariance, while quantum frameworks are unified by the preservation of probabilistic structure under unitary transformations.

A separate cluster is associated with scale and renormalization-group invariance, bringing together effective field theories, renormalization group methods and conformal field theories. Here, physical identity is preserved not under changes of coordinates or representation but under transformations of scale. Topological and metric-independent theories form another well-defined cluster, characterized by invariance under continuous deformations rather than local geometric structure. Finally, information-theoretic and categorical frameworks cluster around invariances related to entanglement structure and compositional consistency, respectively.

The relative sparsity of primary invariance anchors compared to the number of theory classes is a salient feature of the classification. Many theories that differ substantially in mathematical carrier, dynamical interpretation or domain of application nevertheless share the same defining invariant structure. This suggests that the space of physically distinct theories, when organized by invariance rather than formalism, is considerably more compressed than is often assumed.

Figure 1 provides a graphical representation of the invariance clusters summarized in Table 2, together with representative invariance-preserving inter-theory morphisms. In this representation, theory classes are grouped according to their primary invariance, while connections between clusters indicate well-established inter-theory relations. Importantly, these connections do not generally correspond to inclusions or reductions in a strict sense but rather to transformations that modify representational structure while preserving the relevant invariance.

Examples include quantization procedures connecting classical and quantum frameworks, coarse-graining and renormalization group flow relating microscopic and macroscopic descriptions and dualities or holographic correspondences linking theories with markedly different mathematical realizations. The fact that such transformations typically preserve the defining invariance of the theory class reinforces the role of invariance as an organizing principle.

Taken together, these observations indicate that invariance-based classification captures genuine structural regularities across contemporary theoretical physics. The resulting clustering is neither accidental nor purely historical but reflects stable features that persist across changes of formulation, scale and mathematical language. In the following section we examine these inter-theory relations in more detail, focusing on their role as invariance-preserving transformations between theory classes.

Invariance cluster	Theory classes included (from Table 1)	Representative inter-theory morphisms
Lorentz (relativistic)	Special relativity; classical relativistic field theory; relativistic quantum mechanics; (parts of) quantum field theory	Quantization; relativistic limit
Diffeomorphism (relational)	General relativity; metric gravity theories; background-independent field theories	Duality; geometrization
Gauge (redundancy-based)	Classical electromagnetism; Abelian and non-Abelian gauge theories; Yang–Mills theories; gauge field theories	Duality; topological sector limits
Unitary (quantum)	Non-relativistic quantum mechanics; relativistic quantum mechanics; quantum field theory; algebraic quantum field theory	Coarse-graining; information-theoretic mappings
Scale / RG (universality)	Effective field theory; renormalization group theory; conformal field theory	Renormalization group flow; scaling limits
Topological (non-metric)	Topological field theories; topological quantum field theory; Chern–Simons theories; BF-type theories	Topological limits; categorification
Information / entanglement	Holographic frameworks; AdS/CFT-type correspondences; tensor-network descriptions; information-theoretic reconstructions	Holographic mapping; entanglement encoding
Functorial / compositional	Categorical quantum mechanics; functorial field theories; cobordism-based frameworks	Functorial lift; process reformulation

Table 2: Invariance clusters derived from the primary invariances listed in Table 1. Each cluster groups theory classes that share the same defining invariant structure. The listed inter-theory morphisms represent commonly discussed transformations that relate theories within or across clusters while preserving the corresponding invariance.

6 Inter-theory relations as invariance-preserving transformations

The clustering described in Section 5 is accompanied by a network of well-established relations between theory classes. These relations are familiar from the literature under various names—such as quantization, coarse-graining, renormalization group flow, duality or holographic correspondence—but are not always discussed in a unified structural context. From the perspective adopted here, a common feature of these relations is that they typically modify the representational or mathematical carrier of a theory while preserving its defining invariant structure.

In this sense, inter-theory relations may be understood as transformations that act *between* theory classes without erasing their primary invariance. They do not, in general, correspond to strict reductions or embeddings, nor do they imply a hierarchy of fundamentality. Rather, they relate different realizations of physical structure that remain equivalent with respect to the invariance that defines the theory class.

Related analyses of inter-theory relations, emergence and structural continuity can be found in the philosophy of physics literature [7].

6.1 Quantization

Quantization procedures provide a canonical example. Classical mechanical and field-theoretic frameworks, characterized by symplectic or variational invariance, can be related to quantum theories that preserve probabilistic structure through unitarity. While quantization radically alters the mathematical carrier—from phase spaces to Hilbert spaces or operator algebras—it preserves the essential invariance that distinguishes physical equivalence classes within the target theory. The resulting quantum theories therefore form a coherent cluster despite their departure from classical dynamics.

6.2 Coarse-graining and renormalization

Coarse-graining and renormalization group transformations relate theories defined at different levels of resolution or energy scale. These transformations typically discard microscopic detail while preserving universal behavior, as captured by scale invariance or renormalization group fixed points. From an invariance-based viewpoint, such procedures map between representations that differ in descriptive granularity but remain equivalent with respect to their primary invariance. This perspective clarifies why theories connected by renormalization group flow often exhibit strong structural continuity despite large differences in formulation.

6.3 Dualities

Dualities provide a further class of invariance-preserving relations. In many cases, dual descriptions employ distinct variables, degrees of freedom or even geometric interpretations, yet agree on physical observables and invariant content. Examples include electric–magnetic dualities in gauge theory and gauge–gravity dualities connecting seemingly disparate frameworks. Although the mathematical carriers involved may differ substantially, the persistence of a shared invariant structure places dual theories within the same structural landscape.

6.4 Holographic and information-theoretic mappings

Holographic correspondences and information-theoretic reconstructions relate theories formulated in different dimensions or with different notions of locality. Such mappings often translate geometric or dynamical structure into statements about entanglement, information flow or boundary data. Despite this shift in perspective, the underlying physical equivalence is maintained through invariance of informational structure. These relations therefore connect geometric and information-based theory classes without privileging one as more fundamental than the other.

6.5 Functorial reformulations

Finally, several theories admit reformulations in categorical or functorial terms, in which processes and compositional rules take precedence over state-based descriptions. These transformations preserve the operational or relational content of a theory while changing its mathematical language. From the present standpoint, such functorial lifts represent another form of invariance-preserving transformation, linking conventional field-theoretic or algebraic formulations to process-oriented frameworks.

6.6 Structural significance

A notable feature of the relations discussed above is that they often commute: the order in which representation-changing transformations are applied does not affect the resulting invariant structure. For example, coarse-graining may be performed before or after a change of mathematical language without altering the defining invariance of the theory class. This robustness reinforces the interpretation of these relations as acting primarily on representation rather than on physical identity.

Taken together, quantization, coarse-graining, renormalization, duality, holographic mapping and functorial reformulation constitute a web of invariance-preserving transformations connecting different regions of theory space. Their existence supports the view that many established theories are related not by ad hoc constructions but by systematic changes of description that leave core physical structure intact. In the next section we examine how the classification framework highlights regions of theory space that are sparsely populated or unoccupied, suggesting directions for further exploration.

7 Unoccupied and sparsely populated invariance classes

An advantage of organizing theory space by primary invariance is that the resulting classification does not merely arrange existing frameworks but also delineates regions where few or no established theories are currently located. Such unoccupied or sparsely populated regions arise naturally once invariance is taken as the organizing principle and do not depend on the introduction of speculative dynamics or new degrees of freedom.

Table 3 makes explicit several structurally admissible regions of theory space that are not presently occupied by established theory classes. These regions are identified solely by extrapolating the invariance-based classification, without introducing additional dynamical assumptions.

It is important to emphasize that the identification of such regions does not constitute a prediction in the usual physical sense. Rather, it reflects the fact that the space of admissible invariant structures is, in principle, larger than the subset that has so far been realized in widely accepted theoretical frameworks. In this respect, unoccupied invariance classes play a role analogous to that of symmetry-allowed but unrealized phases in condensed matter physics: they indicate logical possibility rather than empirical necessity.

Several examples illustrate this point. One may consider invariant structures associated with the preservation of informational or relational content without the assumption of global unitarity or frameworks in which chaotic or coarse-grained behavior is treated as a defining structural feature rather than as an emergent property of underlying dynamics. Similarly, while dualities and holographic correspondences are well documented, they typically appear as relations between existing theories rather than as defining principles of standalone theory classes.

Other sparsely populated regions arise at the intersection of multiple invariance anchors. For instance, combinations of topological invariance with information-theoretic structure or functorial composition with dynamical locality, are known to occur in limited contexts but lack a unified, systematically developed theory class. The classification highlights these intersections as structurally coherent possibilities, even in the absence of fully developed models.

The purpose of identifying such regions is not to advocate for their immediate physical realization but to clarify the scope and limitations of the current theoretical landscape. By making explicit which invariant structures are well represented and which are not, the classification provides a clearer picture of how existing theories occupy theory space and where conceptual gaps remain.

In this sense, unoccupied invariance classes serve a diagnostic rather than a speculative function. They indicate where additional theoretical development might be possible or meaningful but they also underscore that a complete classification need not be fully populated. The existence of empty regions is compatible with and may even reflect, the selective constraints imposed by consistency, mathematical tractability or empirical relevance.

Structural region	Description	Status in current theory space
Duality-dominant structures	Frameworks in which duality relations themselves define theory identity, rather than relating distinct theories	Present only as inter-theory relations; no established standalone theory class
RG-dominant structures	Theories defined primarily by renormalization group flow or universality, rather than by microscopic degrees of freedom	Appears as methodological tool; not as defining invariance
Topological–informational structures	Theories combining topological invariance with explicit information-theoretic structure	Realized in limited contexts (e.g. topological order) but lacking unified classification
Chaos-dominant descriptions	Frameworks in which sensitivity, instability or entropy production define theory identity	Treated as emergent behavior, not as primary invariance
Non-unitary but information-constrained frameworks	Theories allowing controlled information loss without full stochasticity	No standard theory class; appears only in effective or phenomenological models
Functorial–dynamical hybrids	Theories whose primary structure is compositional or categorical, combined with nontrivial dynamics	Existing as reformulations, not as independent theory classes

Table 3: Representative unoccupied or sparsely populated regions of theory space identified by the invariance-based classification. These regions correspond to structurally admissible invariance patterns that are not currently realized as established theory classes.

8 Interpretative perspective: invariance and projection

The results presented in the preceding sections establish a coherent structural picture without recourse to additional theoretical assumptions. A limited set of primary invariances organizes a wide range of established theory classes; these classes cluster non-uniformly in theory space; and well-known inter-theory relations act predominantly as transformations that modify representation while preserving invariant structure. Taken together, Tables 1, 2 and Figure 1 suggest a compact organization of theory space by invariant structure. These observations invite further interpretation.

One natural way to read this pattern is to regard physical theories as arising through the preservation of specific invariant structures under systematic changes of description. From this perspective, differences between theories often reflect differences in mathematical carrier, scale or representational language, while their physical identity is determined by what remains invariant across such changes. The clustering observed in Section 5 then reflects the stability of certain invariant structures under a wide range of admissible transformations.

This viewpoint suggests an interpretation in which theory construction proceeds not by the introduction of fundamentally new structures at each stage but by successive re-expressions of a smaller set of invariant principles. Inter-theory relations such as quantization, coarse-graining, renormalization group flow, duality and holographic correspondence may then be understood as operations that alter descriptive detail while leaving core invariant content intact. The frequent commutativity of such operations, noted in Section 6, further supports this reading. In what follows, it is convenient to refer to this invariance-preserving change of description as a form of projection, understood in a purely interpretative sense. A closely related projection-based interpretative framework has been developed in previous work [1], although the present classification does not depend on adopting that framework.

It is useful to emphasize that this interpretative perspective does not presuppose any specific ontological hierarchy or generative mechanism. The notion of projection, as used here, is intended in a minimal and operational sense: as a change of description that preserves selected invariant features while discarding others. No claim is made that invariant structures originate from a deeper physical substrate, nor that projection is a physical process. Rather, the term serves as a conceptual tool for articulating the observed relationship between invariant structure and representational freedom.

Within this interpretative frame, the existence of unoccupied invariance classes discussed in Section 7 acquires a natural explanation. If theory space is structured by invariant principles rather than by specific formalisms, then it is to be expected that not all admissible invariant structures are realized in currently established theories. Some may remain unrealized due to mathematical, empirical or conceptual constraints, while others may correspond to regimes not yet explored. The presence of such regions is therefore consistent with, rather than problematic for, an invariance-based organization of theory space.

While the present classification is not hierarchical by construction, it nevertheless induces a minimal structural ordering of descriptive levels implicit in the organization of theory space, summarized schematically in Figure 2. At the most general level, one may speak of invariant structure as such, corresponding to those features preserved under admissible changes of description. These invariant structures organize into clusters of mutually compatible invariance requirements. Within each cluster, a primary invariance can be identified, which defines the physical identity of a given theory class. Distinct theory classes then correspond to equivalence classes of descriptions under the same primary invariance, while concrete physical realizations arise through the imposition of secondary invariances and/or through finite factorizations within a theory class (as illustrated, for example, by fermion family replication in the Standard Model [3]).

The interpretative perspective outlined here is offered as a unifying lens rather than as a foundational claim. All structural results presented in this paper stand independently of it. Nonetheless, the projection-based reading provides a compact and conceptually economical way to understand why diverse theories cluster as they do and why transformations between them so often preserve invariant content despite radical changes in formalism.

In the concluding section we summarize the main findings and briefly indicate how the present classification may serve as a reference point for further work, both within and beyond the domains considered here.

9 Conclusion

In this work we have developed an invariance-based classification of established theory classes in fundamental and mathematical physics. By organizing theories according to their primary invariant structure rather than their specific mathematical formulation, we have shown that a wide range of apparently disparate frameworks cluster around a relatively small number of invariance anchors.

The resulting taxonomy reveals a significant compression of theory space. Classical, relativistic, quantum, statistical, topological and information-theoretic theories often differ substantially in formalism, yet share defining invariant structures that persist under admissible changes of description. This observation highlights invariance as a unifying criterion for theory identity, complementary to more traditional model- or dynamics-based classifications.

We have further shown that many well-known relations between theories—such as quantization, coarse-graining, renormalization group flow, duality, holographic correspondence and functorial reformulation—can be understood as transformations that alter representational structure while preserving the relevant invariance. The existence and robustness of such invariance-preserving relations reinforce the view that structural continuity across theories is more fundamental than formal similarity.

Finally, the classification naturally identifies regions of theory space that are unoccupied or sparsely populated. These regions do not constitute predictions but rather reflect the fact that the space of admissible invariant structures exceeds the subset currently realized in established theories. Making these regions explicit clarifies both the reach and the limitations of the present theoretical landscape.

Taken together, the results presented here support an organizational perspective in which physical theories are most naturally compared and related through the invariant structures they preserve. Independently of any further interpretation, the invariance-based classification provides a coherent reference framework for understanding relationships between existing theories and for situating future developments within a broader structural context.

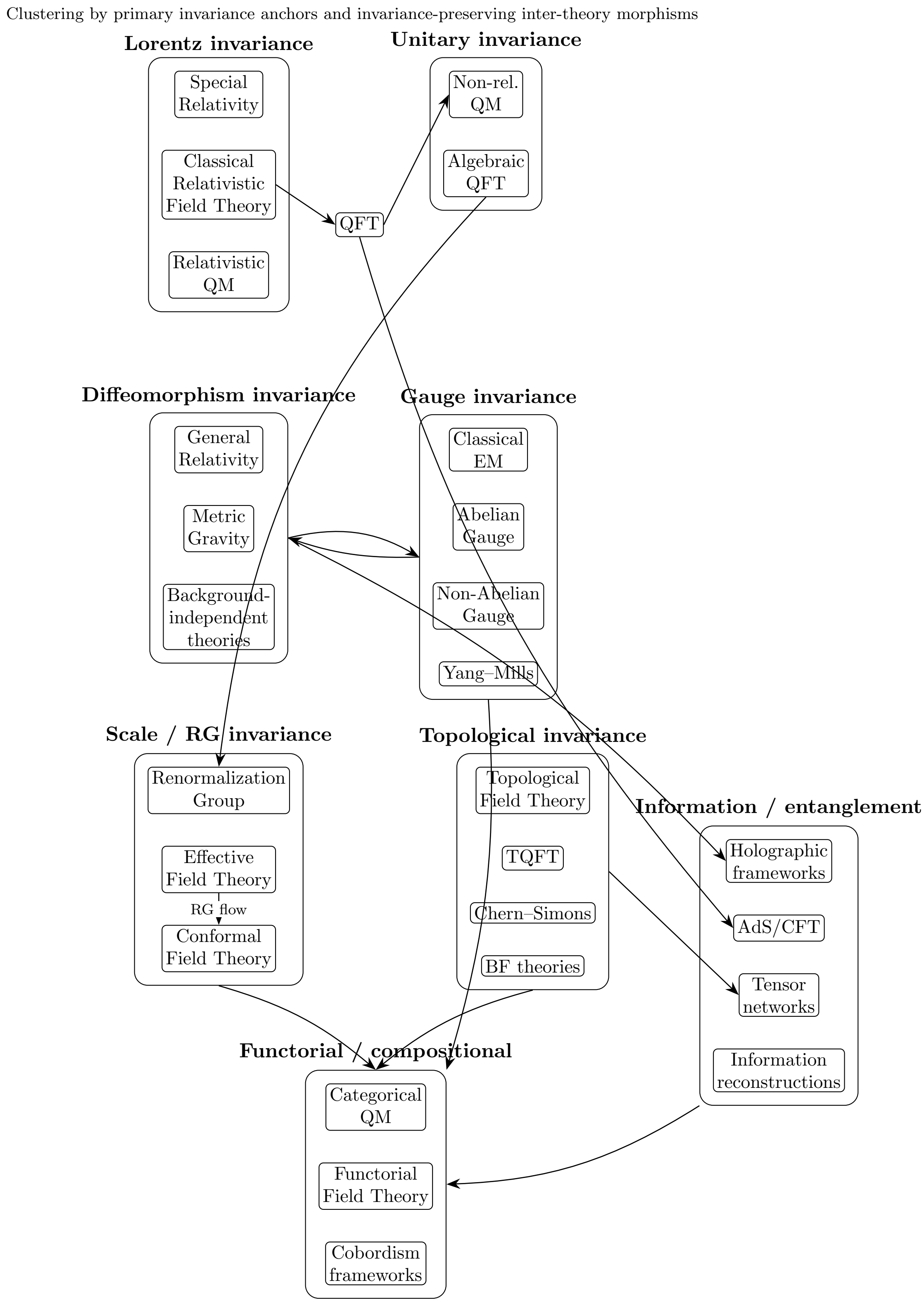

Figure 1: Clustering of established theory classes by primary invariance anchors. Small boxes denote representative theory classes while larger enclosing boxes denote invariance clusters derived from Table 2. Arrows indicate selected, representative inter-theory morphisms that preserve the defining invariance while changing representation (quantization, coarse-graining, RG flow, duality, holographic mapping and functorial lifting). The figure is intended as a schematic visualization of the classification rather than an exhaustive relational graph.

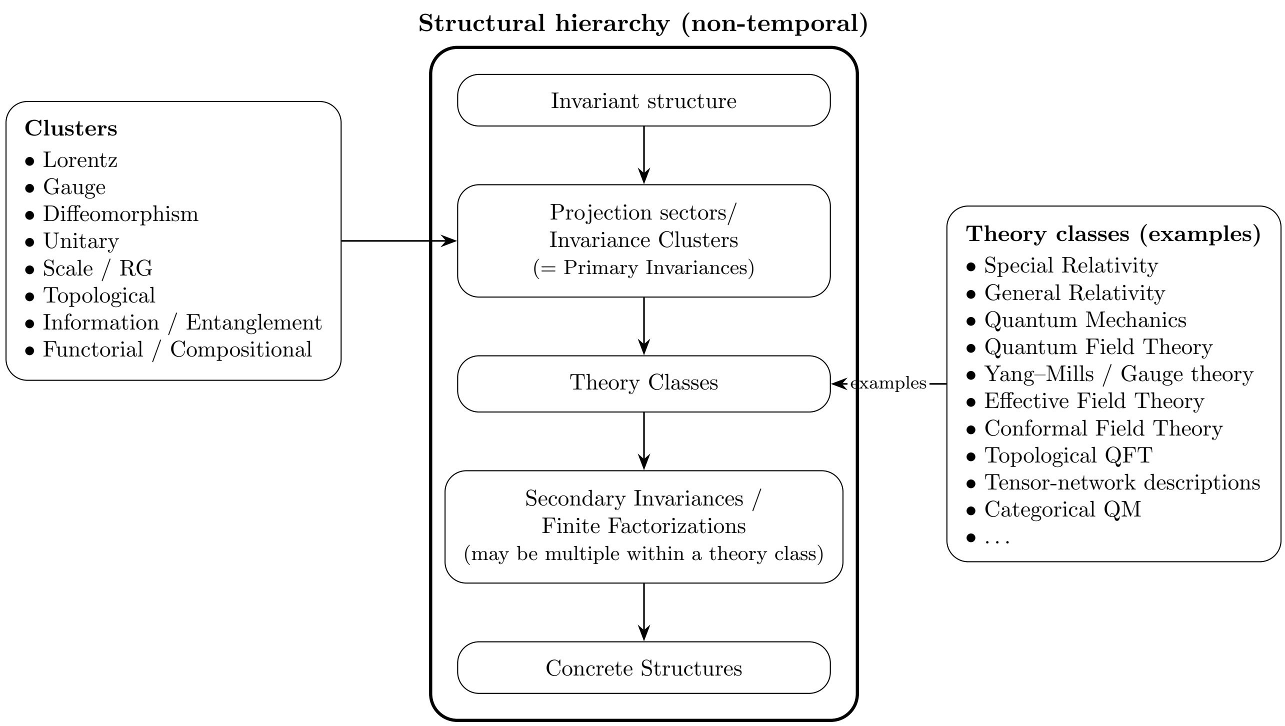

Figure 2: Minimal structural hierarchy implicit in the invariance-based classification. The diagram represents levels of description rather than a temporal or causal sequence. Invariant structure organizes into invariance clusters; within each cluster, a primary invariance defines theory classes. Concrete realizations arise through secondary invariances and/or finite factorizations within a theory class. Side boxes indicate representative examples and are illustrative rather than exhaustive.

References

[1] Axelkrans, E. *Spacetime as a Compatibility-Induced Coordinative Structure.* Preprint, PhilArchive (2026). Submitted to *European Physical Journal Plus* . https://philpapers.org/rec/AXESAA

[2] Axelkrans, E. *Unification Without Reduction: A Classification of Projection-Invariant Physical Theories.* Preprint, PhilArchive (2026). https://philpapers.org/rec/AXEUWR Submitted to *Journal of General Philosophy of Science* (2026).

[3] Axelkrans, E. *On the Origin of Fermion Family Replication.* Preprint, PhilArchive (2026). https://philpapers.org/rec/AXEOTO Submitted to *International Journal of Modern Physics D* (2026).

[4] E. P. Wigner, Symmetries and Reflections, MIT Press (1967).

[5] H. Weyl, Symmetry, Princeton University Press (1952).

[6] K. G. Wilson, The renormalization group and critical phenomena, Rev. Mod. Phys. **55**, 583 (1983).

[7] J. Butterfield and C. J. Isham, Emergence, reduction and supervenience, J. Consciousness Studies **6**, 1 (1999).