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Clarifying some misconceptions in interpreting Ernst Mach's views on thought experiments

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ABSTRACT

Proponents of a recent interpretation of Ernst Mach's views on thought experiments argue that for Mach thought experiments must be continuous with and return to cached experiences. These criteria, the proponents hold, explain the tension which has been noted in Mach's views on thought experiments: on the one hand, Mach reprimands Newton in "extending principles beyond the boundaries of experience" when critiquing Newton's bucket argument, while on the other, Mach himself engages in speculative reasoning when presenting 'Mach's principle'. Against this interpretation, I argue that Mach's critique of Newton's argument does not turn on Newton's illegitimate use of thought experiments, but instead turns on Newton's illegitimate use of thoughts *in* thought experiments. Recognizing this leads to a simple and more compelling interpretation of Mach's views. I close the paper by gesturing towards such an interpretation and noting that taking Mach's views on thought experiments seriously implies that Mach is much less of a strict empiricist than he is usually understood to be.

1. Introduction

Recent discussions in interpreting Ernst Mach's views on thought experiments paint the following picture: for Mach a thought experiment (and hypothetical/counterfactual reasoning in general) is valid only if it is continuous with and returns at the end back to (cached) experiences. On such a picture, (i) thoughts experiments are continuous with experience because in thought experiments cached experiences are varied continuously and (ii) thought experiments return back to experience because "the warrant for any conclusion based upon a thought experiment can be found only in experience" (Buzzoni 2019, p. 22). Versions of this interpretation have been recently argued for by Buzzoni (2018; 2019), Patton (2019; 2021), and Brecevic (2021). In this paper I argue against this interpretation and put forward an alternative which I suggest is more compelling.

I proceed as follows. In §2, I establish the Machian interpretive puzzle. Roughly put, the puzzle is that Mach's circumscription of Newton's bucket argument seems at odds with his own speculative reasoning while presenting 'Mach's principle' and his prescription of the use of thought experiments in science. In §3, I sketch a family of solutions to the puzzle put forward recently. §4 contains a critical evaluation of the proffered solution: I argue that the solution fails on two important counts. In particular, the solution is unable to accommodate Mach's positive appraisal of Stevin's thought experiment of the inclined plane and also of Mach's appraisal of J. B. Stallo's views. In §5, I present my own preferred interpretation which solves the puzzle of §2. According to my preferred solution, Mach's circumscription of Newton has nothing to do with the (in)validity of thought experiments. I argue that in criticizing the bucket experiment, Mach is not commenting on Newton's incorrect usage of thought experiments. Instead, he is commenting on Newton's incorrect usage of thoughts *in* thought experiments. By untying Mach's critique of Newton from his views on thought experiments and 'Mach's principle', I show that the puzzle of §2 dissolves. §6 contains four motivations for my proffered solution. I close in §7 by considering some objections to my interpretation and arguing that none are fatal. I also note in the conclusion that if my interpretation is correct then Mach is much less of a strict empiricist than he is usually taken to be.

2. The puzzle

§2.1 contains a discussion of Mach's critique of Newton's bucket argument for the existence of absolute space and §2.2 contains a brief sketch of 'Mach's principle' (§2.2.1) and Mach's views on thought experiments (§2.2.2). I close this section, in §2.3, by presenting the interpretative puzzle.

2.1. Mach on Newton

In his *The Science of Mechanics*, in the section entitled *Newton's Views of Time, Space, and Motion*, Mach defends a relational view of space and critiques Newton's bucket argument. Roughly, Newton's bucket argument goes along the following lines. A bucket is hung by a cord. The cord is then twisted, the bucket filled with water and released. Initially when

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the relative motion between the bucket and the water is highest, the water surface remains flat. However, after a while, the water rises on the walls of the bucket and the surface makes a concave shape, despite the water being at rest in relation to the bucket. Newton maintained that this showed that the rotation of the water is not with respect to the bucket but is an absolute and "true" circular motion with respect to an "immovable and homogenous" absolute space (Newton, 1995; Scholium I). Importantly, Newton did not consider his bucket argument a thought experiment. We find Newton parenthetically remarking in the *Scholium I* that "experience has shown me" that "the water will gradually recede from the middle and rise up the sides of the vessel".

As Mach reads Newton, the concavity of the water in the bucket and the tension in the cord reveal the presence of centrifugal forces, which in turn are held to be due to the system's rotational motion. But this rotation cannot be relative rotation. In the first case, the surface of the water is concave when the water is at rest with respect to the bucket. In the second, there is no relative motion of any kind. So, the existence of absolute motion, that is not a variety of relative motion, has been demonstrated. Mach rejects Newton's bucket argument by asserting that Newton is not "warranted in extending" the principles of mechanics "beyond the boundaries of experience" (Mach 1919, 229):

[*O1*] It is scarcely necessary to remark that in the reflections here presented Newton has again acted contrary to his expressed intention only to investigate actual facts. [*O2*] No one is competent to predicate things about facts about absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be produced in experience. All our principles of mechanics are, as we have shown in detail, experimental knowledge concerning the relative positions and motions of bodies. Even in the provinces in which they are now recognized as valid, they could not, and were not, admitted without previously being subjected to experimental tests. [*O3*] No one is warranted in extending these principles beyond the boundaries of experience. In fact, such an extension is meaningless, as no one possesses the requisite knowledge to make use of it.

Mach makes three distinct objections against Newton. I stress this point because this has not been fully recognized in the literature.

O1: Mach objects that the bucket argument contradicts Newton's own position "to investigate actual facts". Mach held Newton's methodological aversion of hypotheses and of not "invent[ing] explanations if what is in fact known provides adequate understanding" (1976, 173) in the highest of regards. Indeed, Mach asserts that Newton's refusal to meditate on the causes of gravitation and his adoption of the maxim *hypotheses non fingo* "stamps him as a philosopher of the highest rank" (1919, 193).[1] Mach's objection, then, can be read off more clearly. Mach thinks that in his bucket argument, Newton deviates from his methodological principle and trades in hypotheses – particularly, the hypothesis about absolute space.[2]

O2: Mach's second objection reveals his empiricist views. Importantly, he circumscribes his objection of Newton. Mach explicitly states that a mechanical principle (or mechanical concept) just is knowledge about relative positions and motions gained through experience.[3] And, *in so far as* Newton construes absolute space to be a mechanical principle, Newton fails. So, Mach is not objecting against the Newtonian principle of absolute space simpliciter but is objecting against the Newtonian principle of absolute space *qua* a mechanical principle. At least in this passage and in the *Science of Mechanics*, Mach asserts *only* that experimental tests are necessary for the admittance of mechanical principles – Mach is silent on non-mechanical principles. Moreover, there seems to be a tension in Mach's argument. Mach presupposes that mechanical principles concern "the relative positions and motions of bodies". It seems that this particular presupposition does no work in the objection. Even if Mach maintained the weaker view that mechanical principles are just "experimental knowledge", his objection would still go through.[4]

O3: Mach finally objects to any inference of mechanical concepts which goes beyond experience. *O3* is distinct from *O1* because while in *O1* Mach is pointing a finger and Newton for retreating from his methodology, in *O3* Mach is making a general claim that nobody can extend mechanical concepts beyond experience. It seems that for Mach – at least in the context of mechanical concepts – a concept is meaningful only if we can have the knowledge to put the concept to use.[5] This fits well with Mach's view that concepts must be serve a practical purpose to humans.[6] Thus, Newton is wrong about absolute space, on this count, because Newton wrongly extends the concept of rotation beyond experience, rendering it meaningless in the process.

This completes my brief discussion of Mach's objection to the bucket argument.[7] This is far from comprehensive, but what I say in the sequel

[1] See Newton's two letters to Bentley of 17 January 1692/93, 25 February 1692/93. Newton writes to Bentley:

> "You sometimes speak of gravity as essential and inherent to matter. Pray do not ascribe that notion to me; for the cause of gravity is what I do not pretend to know, and therefore would take more time to consider of it". (Letter of 17 January)

and:

> "Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers". (Letter of 25 Feb) (Newton, 2009).

[2] Of course, much turns on how Mach characterizes 'hypotheses'. I think that it is not accidental that Mach included a chapter on hypotheses in Knowledge and Error. The chapter succeeds the chapters on thought experiments, physical experiments, and role of analogy. This provides additional support to my proffered view which I present in the sequel.

[3] The identification of concepts with knowledge is consistent with Mach's broader empiricist views. In fact, as Mach argues in a chapter entitled The Concept in Mach (1976), concepts are an economical way the "represent and symbolize in thoughts large areas of fact" helping us "find our way in the bewildering tangle of fact" (p. 98). According to Mach, we form concepts due to our biological needs by abstracting away from facts about things, i.e, facts about stable complexes of sensations. Due to reasons of space, I can't provide a detailed account of Machian concepts, but see Banks (2003) and Edgar (2013) for detailed discussions.

[4] This is all the more intriguing because in the following sentences Mach drops any talk of relative positions and proceeds to object to absolute space on the grounds that it cannot be "subjected to experimental tests". Maybe Mach includes the condition about relative position because he thinks that only relative magnitudes can be experimentally measured. This might be so and there is some textual evidence to back up this claim, but this makes Mach even more vulnerable of begging the question. I bracket this exegetical issue.

[5] This seems in a similar spirit as the American pragmatists, especially C.S. Peirce. As far as I know, there hasn't been any detailed study of the relation between Mach and the pragmatists on the issue of concepts, although see Patton (2021) and Uebel (2021) for discussions on pragmatist strands in Mach.

[6] See note 3.

[7] There is disagreement regarding Newton's aims in the Scholium. While some believe that Newton's aim in both the bucket experiment and the globes thought experiment was to prove the existence of absolute space (the 'standard view'; cf. Sklar (1974, p. 182)), others believe that Newton did not intend to argue for the existence of absolute space but in part intended to argue against Descartes' proposal of motion in terms of immediately contiguous bodies (cf. Laymon, 1978; DiSalle, 2006). Still others, most prominently Rynasiewicz (1995a, 1995b, 2014), believe that Newton does not intend to argue for the existence of absolute space since it was common ground between him and his Cartesian interlocutor. Instead, Newton wanted to show that his characterization of absolute motion is better than Descartes' proposal of motion when the causes and effects which characterize motion are considered. A detailed discussion of this important interpretative issue can be found in Pooley (Manuscript). For the purposes of this paper, I do not need to engage in this debate. It is sufficient to note that Mach seems to take the 'standard view' – in his discussion of the bucket experiment in SM, Mach takes Newton to be argue for the existence of absolute space. Mach does not draw the distinction that Pooley (2002, p. 19 ff.) draws between the different purposes of the bucket experiment and the globes thought experiment. Indeed, Mach only mentions the globes thought experiment when he is quoting Newton and does not engage with it in either Mach (1919) or in Mach (1976). Thanks to James Read for pointing me towards this debate.

builds on the issues discussed here. I now turn to characterizing Mach's views on thought experiments, especially as enunciated in his *Knowledge and Error* (1976).

2.2. Thought experiments are necessary

In §2.2.1, I discuss 'Mach's principle' and in §2.2.2 I discuss Mach's views on thought experiments.

2.2.1. Mach's principle

Rejecting Newton's explanation for the concavity of the water surface, Mach gestures towards an alternative explanation. Mach asserts that Newton draws a false dichotomy that the water rotates only with respect to either the walls of the bucket or absolute space. According to Mach there's at least one more possibility: the water rotates with respect to the fixed stars. Mach explicitly employs a thought experiment to argue against Newton. That the motion of a body is caused by an interaction with other bodies has now come to be called 'Mach's principle'.[8]

Mach charges Newton with using unwarranted abstractions. By thinking of an abstracted system consisting just of the bucket, Newton fails to consider other possibilities including "how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick". Since "one experiment only lies before us", Mach argues that "our business is, to bring it into accord with the other facts known to us, and not with the arbitrary fictions of our imagination". He concludes by noting that the bucket argument "compel[s] us not to postulate an absolute reference system but to recognize the law of inertia as a mere empirical generalization about the motions of bodies relative to the fixed stars" (1919, 232 ff.). The bucket experiment can be brought into accord with and explained by what is already known, *viz.* rotation caused by fixed stars. Any explanation which challenges or revises our beliefs is inferior. Thus, it seems, that Mach holds a conservative position: if a phenomenon can be adequately explained by (a body of) already known facts, then we should not propose novel and revisionary metaphysics. Indeed, Mach writes:

> [Newton's] first philosophic rule for enquiry states that to explain nature we must admit only causes that are actual and such as are sufficient to account for the phenomena – a clear hint *not to invent explanations if what is in fact known provides adequate understanding*. (1976, 174; emphasis mine)

Here Mach asserts – in similar spirit to *O1* – that Newton errs in inventing an explanation of the concavity of the water surface while neglecting an actual cause which does not require the postulation of something novel (*viz.* rotation with respect to fixed stars) as the explanation.

2.2.2. Mach on thought experiments

In *Knowledge and Error*, Mach devotes an entire chapter[9] to discussing thought experiments and their importance in science.[10] Given that Mach is portrayed as an arch-empiricist, his approval and enthusiastic encouragement of thought experiments may seem surprising. Here's a selection of quotes:

> Experiments guided by thought lie at the basis of science and consciously aim at widening experience.
>
> Thought experiment is in any case a necessary precondition for physical experiment. Every experimenter and inventor must have the planned arrangement in his head before translating it into fact.
>
> The outcome of a thought experiment, and the surmise that we mentally link with the varied conditions can be so definite and decisive that the author rightly or wrongly feels able to dispense with any further tests by physical experiment.
>
> We can hardly doubt that thought experiments are important not only in physics but in every field, even in mathematics, where the uninitiated might least expect it. (1976, 134 ff.)

According to Mach, thought experiments are indispensable to the scientific enterprise: thought experiments "are the very activit[y] that most strongly promotes inquiry in the natural sciences" (1976, 125) and have helped "build modern natural science" (1976, 146). Mach explicitly and repeatedly acknowledges, asserts, and encourages the usefulness of thought experiments in natural science. Since, mechanics is a part of natural science, it seems that Mach must also assert the importance and indispensability of thought experiments in mechanics. Furthermore, Mach notes that physical experiments are often preceded by thought experiments and that thought experiments are a "necessary precondition for physical experiments" (1976, 137).[11]

How is it possible to reconcile thought experiments – which are supposed to be divorced from the empirical domain – with the Machian empiricist worldview? Mach's answer is that thought experiments connect with the empirical by drawing on our past experiences. According to Mach, in a thought experiment a scientist *continuously* varies and combines the facts of her cached experiences. This results in, inter alia, shining light on previously unnoticed observations and connections helping "us to new discoveries" (1976, 136). The elasticity of the variations depends on a number of factors: conditions which are judged to be unimportant can be safely neglected in a thought experiment but neglecting essential conditions might lead to unsuccessful outcomes. Successful thought experiments

> can be so definite and decisive that the author rightly or wrongly feels able to dispense with any further tests by physical experiment. (1976, 138).

However

> the less certain their outcome, the more strongly thought experiments urge the enquirer to physical experiment as a natural sequel that has to complete and to determine the result. (1976)

2.3. The Puzzle

Prima facie, there seems to be an interpretive puzzle concerning Mach's views on thought experiments:

[8] This is just one of the numerous ways of understanding and explicating 'Mach's principle'. Of course, I can't do justice to the complicated and subtle history and influence of 'Mach's principle' here. For a wide-ranging introduction see the compiled essays in Barbour and Pfister (1995). For an approach to dynamics based on 'Mach's principle' see Barbour and Bertotti (1982). And for a historical background see Barbour (2001). Because of the contentious nature of interpreting 'Mach's principle', I put it under single-quotes throughout this paper.

[9] The chapter on thought experiments published in *Knowledge and Error* was a revised and updated version of the paper Über Gedankenexperimente published by Mach in 1896 (Mach, 1896).

[10] The term 'thought experiment' was - as far as I know - first used by the Danish scientist Hans-Christian Ørsted (both in Danish 'Tankeexperiment' and in German 'Gedankenexperiment') in 1812 (Witt-Hansen, 1976; Buzzoni, 2018).

[11] James Read points out that this characterization seems to be in tension with exploratory experiments, especially in the "omic disciplines" (i.e., genomics, transcriptomics, and proteomics). It has been argued that experiments in these fields "represent a break with the ideal of hypothesis-driven science" (Burian, 2007, p. 289). To be sure, it seems to me as well, that Mach's views on the necessity of thought experiments to physical experiments are in tension with such exploratory experiments, thought there maybe ways to reconcile the views. However, since this discussion is orthogonal to project of this paper, I do not comment on these issues. See Ratti (2015) and Sharma and Read (2021) for an introduction to exploratory experiments.

(Puzzle) Mach seems to hold two jointly inconsistent views regarding thought experiments:

> **(Censure)** Mach argues against Newton's bucket argument and asserts that it is epistemic malpractice to extend concepts beyond the boundaries of experience.
>
> **(Priggish)** In proposing 'Mach's principle', Mach seems to accept empirically unfounded claims about the global distribution of matter, boundary conditions, physical mechanisms, and so on to avoid the Newtonian appeal to absolute space.

This tension in Mach's views has been well discussed: one strand of the discussion accuses Mach of using double standards. On the one hand, when criticizing Newton, Mach is an arch empiricist and admonishes any procedure that deviates from strictly empirical inferences, including thought experiments and hypothetical/counterfactual reasoning. On the other, Mach fails to apply this strict standard to himself, both when proposing 'Mach's principle' and when discussing thought experiments. Howard Stein is a leading proponent of this strand. He accuses Mach of using "abusive empiricism" to validate his arguments against absolute motion and atomic theory. Stein writes:

> In Mach, of course, we have a classic case of this abusive empiricism. It is a case that also exemplifies a characteristic tendency, a kind of Nemesis, of what we might call "hypercritical" philosophic theories – theories that lay down methodological standards or criteria which are actually impossible to practice.
>
> And a bit later when discussing 'Mach's principle':
>
> Mach's formulation is sketchy and loose, and his exact meaning a little hard to determine … the special assumptions involved, since they are cosmological (or cosmographic) go far beyond anything for which convincing empirical evidence is available … And what is ironic above all is that in the interest of purging Newtonian dynamics of an allegedly nonempirical component, Mach has been led to put forward a theory which must be regarded as on an empirically weaker footing than Newton's own – since Mach's theory is equivalent to the conjunction of Newton's and of special cosmological assumptions. In short, I submit that this is a clear case of ideology out of control. (Stein, 1977, 17 ff.)

Thus, the first strand is to accept the incompatibility between **(Censure)** and **(Priggish)** and argue against Mach on that basis. Recently, a second, more sympathetic, strand has emerged. I discuss this strand in more detail in the following section.[12]

3. Continuity and return in thought experiments

Recently, one prominent solution of **(Puzzle)** has been proposed, among others, by Buzzoni (2018; 2019), Patton (2019; 2021), and Brecevic (2021). The central idea behind the solution is:

> **(Implicit): (Censure)** and **(Priggish)** are not incompatible because an *implicit criterion* in Mach's discussion about thought experiments (and hypothetical/counterfactual reasoning) is that thought experiments must be continuous with and return back to (cached) experiences.

(Implicit) is stated in its most explicit form by Buzzoni (2018) [13]:

> The main pillar of Mach's theory about thought experiments is that they must not only proceed from experience but also return to it, because experience is the ultimate criterion of all sorts of knowledge, and the warrant for any conclusion based upon a thought experiment can be found only in experience. (p. 22)

Similar views can be found in both Patton and Brecevic. Patton frames **(Implicit)** in terms of (in) valid hypothetical reasoning. According to Patton, Mach criticizes Newton for using what she calls "artificial hypothetical reasoning". But when proposing 'Mach's principle' or discussing thought experiments himself, Mach – according to Patton – employs "natural hypothetical reasoning".[14]

> Natural hypothetical reasoning "fills out the gaps in experience by the ideas that experience suggests" with "sensations and ideas like our own". Artificial hypothetical reasoning either: (a) Does not merely fill in gaps in experience, but rather postulates a speculative kind of experience, or (b) Uses sensations and ideas that do not resemble our own to assemble speculative systems … If we appeal to the "fixed stars" or to global boundary conditions that can be cashed out in terms of observations like our own, then we are engaging in natural hypothetical reasoning. We are filling in the gaps of experience, but with experiences that we ourselves could have had. On Mach's reading, if we appeal to Newtonian absolute space and time, then we must construct a speculative hypothetical system based on the sensorium of God. Such an appeal does not fill in gaps in human experience – it goes outside any possible human experience. Newton's account of the bucket experiment is artificial hypothetical reasoning, according to this distinction. (2019, 8).

Brecevic approvingly cites of Buzzoni's formulation and proffers a view close to **(Implicit)**. However, Brecevic (2021) goes a step further and grounds the criterion of returning to an experience in Mach's bio-economical attitude towards science and on Mach's insistence of the continuity of experience:

> A stronger explanation for Mach's criticism toward Newton is found in the demand that thought experimentation in Machian science must, as Marco Buzzoni suggests, "proceed from experience but also return to it".
>
> From the biologico-economical view of science, Newton's error lies in using the imaginative power of thought experimentation to help establish the existence of absolute space, a concept that is discontinuous with experience … In other words, deviating from the aim of Machian science, Newton's use of poetic imagination can only remain poetic — it gives rise to imaginings that cannot, in principle, lead us to experiences that will expand the domain of memory.
>
> But, as evidenced by his critical view of Newton's bucket experiment, Mach insists that the poetic imagination in science must be constrained by the principle of continuity, ensuring that the imaginative

[12] A third strand – not discussed in this paper – can be found in Norton (1995). Norton argues that one way to rescue Mach from **(Puzzle)** is to interpret Mach's presentation of the 'Mach's principle' as only pointing out that absolute space is superfluous in Newton's experiment and not as providing a causal or physical mechanism for the law of inertia. However, Norton points out that even with such a rescue, problems remain. In particular, it seems incompatible with Mach's acquiesce with Einstein's formulation of 'Mach's principle'. In a letter to Mach, Einstein wrote: "Inertia has its origins in a kind of interaction of bodies, quite in the sense of your reflections on Newton's bucket experiment". ((Einstein, 1913); quoted in (Misner et al., 1973)). Since Norton's project is not to point out and solve the tension identified in this paper, I leave this strand aside. Although I think that that my preferred solution is not at odds with Norton's reading.

[13] Buzzoni's own operational-Kantian account of thought experiments has been influential in the recent literature on thought experiments, see Buzzoni (2008) for a book length treatment of Buzzoni's views. For completeness, I should also note that Buzzoni's characterization of "continuous with experience" has been criticized by some. See the exchange in *Epistemologica* between Buzzoni and Fehige (Fehige [2012], [2013], and Buzzoni [2013]). I think that in so far as interpreting Mach's view on thought experiment is concerned, the formulations of (Implicit) by Buzzoni, Patton, and Brecevic suffice.

[14] Patton also distinguishes – in the same spirit – between "artificial counterfactual reasoning" (counterfactual reasoning which is not continuous with experience) and "natural counterfactual reasoning" (counterfactual reasoning which is continuous with experience).

paths traveled by scientists always lead back to experience. (2021, 17).

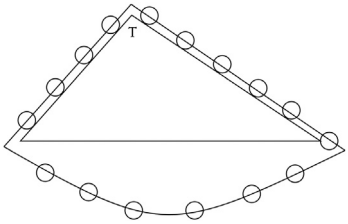
The details of the variants of **(Implicit)** needn't worry us here. All that is required for my critique is that each of the variants validate (something in the vicinity of) **(Implicit)**. In the rest of the paper, I aim to show that there's evidence in Mach's writing which should make us wary of accepting **(Implicit)**. This I do in §4. Following which in §5, I present my own interpretation and provide four motivations for it in §6. I conclude in §7, by anticipating and responding to three objections against my proffered interpretation.

4. Against (Implicit)

I present two problems for **(Implicit)** in this section. §4.1 presents Mach's views on Simon Stevin's thought experiment to deduce the mechanical advantage of an incline plane.[15] I argue that on **(Implicit)**, Stevin's experiment – like the bucket argument – becomes an erroneous thought experiment. In §4.2, I argue that Mach's regard for the American philosopher J. B. Stallo provides an insight into Mach's own critique of Newton, resulting in a further weakening of the case in favour of **(Implicit)**.

4.1. Stevin and Mach

The most discussed thought experiment in *The Science of Mechanics* is Simon Stevin's thought experiment: *The Epitaph of Stevin*.[16] In his *The Elements of the Art of Weighing*, Stevin derives the mechanical advantage of an inclined plane (Proposition XIX; 1955; Stevin, 186). Stevin imagines two inclined planes of equal height (but different slopes) joined together to form a prism and a string of fourteen equally placed beads of equal mass looped over the plane. A part of the string hangs beneath the prism and the beads on the planes are held up by the tension at the apex of the prism *T*. Roughly, Stevin reasons as follows:



> Either the string is in (static) equilibrium or it is not. If not, then the string will slide down the heavier side under its own weight. The new position of the beads will be qualitatively indistinguishable from the initial configuration. Hence, the string will again slide. And so on, resulting in a perpetual motion. Since perpetual motion is absurd, Stevin concludes that the string is in equilibrium with equal forces either side of *T*.
>
> Stevin continues. First, the part of the string hanging beneath the prism is symmetrical, so it can be dispensed away with without hindering the equilibrium. Second, the beads are equidistant, so the total weight supported by each plane is proportional to length of the plane. And hence he concludes that the mechanical advantage of each plane is proportional to its length.

The conclusion of Stevin's thought experiment can be put in other terms. Mach prefers the following: "on inclined planes of equal heights equal weights act in the inverse proportion of the lengths of the planes" (1919, 25).[17]

4.1.1. Mach on Stevin

Mach seems to be completely won over by Stevin's reasoning. Stevin has, in Mach's eyes, "showed great mastery" of thought experiments "in [his] treatment of inclined planes" and has committed "no error" (1976, 138 ff.). In *The Science of Mechanics*, Mach devotes six pages to praising Stevin and he claims elsewhere that Stevin "provides a paradigm for all great enquirers" (1976, 200). To Mach, Stevin's thought experiment is so powerful that "we accept the conclusion drawn from it … without the thought of an objection" even though were the same argument to be presented as a physical experiment, it "would appear dubious". For Mach, the cogency of Stevin's argument arises from Stevin using purely instinctively knowledge. Mach writes:

> Unquestionably in the assumption from which Stevinus starts, that the endless chain does not move, there is contained primarily only a purely instinctive cognition. He feels at once, and we with him, that we have never observed anything like a motion of the kind referred to, that a thing of such a character does not exist.
>
> [Stevin's experiment] throws a wonderful light on the process of the formation of science generally, on its rise from instinctive knowledge … (1919, 26 ff.)

Of instinctive knowledge, Mach asserts that it has a "higher authority" and that:

> We feel clearly, that we ourselves have contributed nothing to the creation of instinctive knowledge, that we have added to it nothing arbitrarily, but that it exists in absolute independence of our participation. Our mistrust of our own subjective interpretation of the facts observed, is thus dissipated. (1919, 30 ff.)

Thus, it seems that if any thought experiment must be validated by Mach, it would be *The Epitaph of Stevin*.[18] But on **(Implicit)**, it turns out that Mach must object to and reject Stevin's thought experiment.

4.1.2. Stevin and (Implicit)

(Implicit) seems incompatible with Mach's enthusiastic appraisal of Stevin's experiment. This is because Stevin's experiment neither proceeds from experience nor is it continuous with experience. Stevin's thought experiment does not proceed from experience because the starting assumptions of Stevin's experiments – frictionless planes – are highly idealized; something which never could have been experienced by Mach. This point is not lost to Mach. Making it explicit, he writes:

> We cannot be surprised at this character, when we reflect that all results of [Stevin's thought] experiment are obscured by adventitious circumstances (as friction, etc.), and that every conjecture as to the conditions which are determinative in a given case is liable to error. (1919, 26)

Thus, it seems that Mach is happy with thought experiments which do not proceed from experience. It also seems that he is happy with thought experiments which are not continuous with experience. Discussing Stevin's experiment, Dijksterhuis notes:

[15] Mechanical advantage is defined as the amplification of force or power due to a (simple) machine. For the case of the inclined plane this amounts to deducing the relation between the force required to carry a weight W straight up and the force required to displace W through a same height on the plane.

[16] Other prominent discussions of thought experiments in *The Science of Mechanics* include Archimedes' thought experiment on the principle of the lever and Galileo's thought experiment on the free fall of bodies.

[17] The works of Stevin on mechanics are collected in Dijksterhuis (1970) where a detailed derivation can be found. For a general biography on Stevin see Devreese and Guido (2008) and for a recent discussion on Stevin's influence see Koetsier (2010).

[18] Rowbottom (2014) notes that there seems to be an internally inconsistency in Mach's discussion of Stevin's thought experiment. Rowbottom also provides a thorough discussion of the different ways philosophers have read Stevin's thought experiment. This is indeed an interesting and important point however I think that in so far as the project of this paper is concerned and the conclusion I draw from it, I can safely bracket this issue here.

> The salient point of the demonstration obviously consists in the conviction of the impossibility of a perpetual motion. Now the whole contrivance is conceived in the ideal realm of rational mechanics, where all disturbing influences, such as friction and air resistance, are believed not to exist. But in this realm a perpetual motion is by no means impossible: a simple pendulum … forms an example of it. The wreath of spheres would indeed perform a perpetual motion if it were given an initial velocity. It will not do, therefore, to appeal to the absurdity of perpetual motion as such. (1970, 54).

The essential premise in Stevin's argument is the absurdity of perceptual motion. But, Stevin appealed to the absurdity not as a consequence of conservation laws – he preceded (the canonical statement of) the law of conservation of energy by three centuries.[19] So, it seems, that in accepting Stevin's argument Mach accepts that it is valid to use something not given in experience – the absurdity of perceptual motion in rational mechanics – in thought experiments.

Two anonymous reviewers point out that a proponent of **(Implicit)** may respond by arguing that although frictionless planes are not directly continuous with experience, they are neither completely discontinuous with experience. Frictionless planes, the proponent might say, are a natural progression of our experiences. Consider the following example. We have experience of a block of wood sliding down a plank of wood. We also have experience of the block of wood sliding down a plank of polished metal. Further, we also have experience of the block of wood sliding down a plank of ice. In all these cases, the block slides down the plank with different velocities. Thinking about a block of wood (or any other object) sliding down a frictionless plank, the proponent might say, is a natural progression of such a series of experiences and hence proceeds from and is continuous with experience in the relevant way.

However, I do not think that such a response is available to the proponent of **(Implicit)**. If the above response by the proponent of **(Implicit)** works, it can only establish that thought experiments employing inclines having arbitrary small coefficient of friction are valid, but it cannot establish the Stevin's thought experiment is valid. This is because a frictionless surface is not a surface with an arbitrary small coefficient of friction. Stevin's conclusion goes through only if we think of the coefficient of friction of the inclined plane to be exactly zero. The conclusion cannot go through if the plane has even an arbitrary small coefficient of friction. From the experiences of a bock of wood sliding down planks with evermore smaller coefficients of friction, we cannot continuously progress to the thought of a frictionless surface. All we can progress on to are the thoughts of planks with smaller and smaller friction, not to a plank with no friction. There is a discontinuous jump from a plank with (an arbitrary) small friction to a frictionless plank and thus it seems to me that such a response cannot assuage the concern raised against **(Implicit)**.

Another consideration militating against **(Implicit)** with regards to Stevin's experiment concerns instinctive knowledge. As we saw above, Mach noted that the force of Stevin's argument was due to its use of instinctive knowledge. But, later in *The Science of Mechanics*, Mach writes:

> As already seen, instinctive knowledge enjoys our exceptional confidence. No longer knowing how we have acquired it, we cannot criticise the logic by which it was inferred. We have personally contributed nothing to its production. It confronts us with a force and irresistibleness foreign to the products of voluntary reflective experience. It appears to us as something free from subjectivity, and extraneous to us, although we have it constantly at hand so that it is more ours than are the individual facts of nature. (1919, 83)

The very nature of instinctive knowledge, Mach holds, makes it impossible for us to criticize it or fault the way by which it was acquired. If thought experiments make use of "pure instinctive knowledge", it seems that Mach commits himself to the principle that "we cannot criticize the logic" of thought experiments. And if we cannot criticize the logic of thought experiments, **(Implicit)** fails because for Mach it is inappropriate to draw a line between natural/valid and (à la Stevin and 'Mach's principle') artificial/invalid thought experiments (à la Newton's bucket argument).

Here's a response that a proponent of **(Implicit)** might provide. Instinctive knowledge, for Mach, is knowledge that is knowledge which proceeds from and is continuous with experience. Hence, there is no need to "criticize the logic" of instinctive knowledge and no need to draw a line between valid and invalid thought experiments in the domain of instinctive knowledge.[20] Such a response will allay the concern I have raised above, but I have a reservation with such a response. As far as I can see, Mach nowhere connects instinctive knowledge with continuity in experience. The burden is on the proponent of **(Implicit)** to provide us with evidence that establishes such a tight connection between implicit knowledge and knowledge which proceeds from and is continuous with experience. On the contrary it seems that for Mach any connection between implicit knowledge and continuity of experience is impossible. He writes:

> We feel clearly that we ourselves have contributed nothing to the creation of instinctive knowledge, that we have added to it nothing arbitrarily, but that it exists in absolute independence of our participation. (1919, 26)

Since instinctive knowledge is independent of an individual and hence her experiences, I think it is hard to establish any connection that would be required for such a response by the proponent of **(Implicit)** to go through.

4.2. Stallo

I want to discuss a further problem for **(Implicit)**. I think that Mach's praise for the American philosopher J. B. Stallo provides us with additional evidence that **(Implicit)** is not on the right track. Mach 's positive assessment of Stallo is evident: Mach sponsored the German edition of Stallo's *Concepts and Theories of Modern Physics* (1881) in 1901; Mach dedicated the second edition of his *Principles of the Theory of Heat* (1900) to Stallo; and Mach paints Stallo's philosophy as a perfect complement to his own:

> During the midsixties when I began critical work, it would have been very encouraging and beneficial to have known about the related endeavors of a comrade like Stallo. (Mach (1901) in Hiebert (1976)

> J. B. Stallo, independently and in different form, has expressed views that essentially agree with what … [I have] said. (Mach, 1976, 102).

Hence, one good way to clear our understanding of Mach is to see what Stallo has to say about the issues concerning us and whether Mach approves of Stallo's arguments. If yes, then we would have a strong indication of Mach's own views. Fortunately for us, in his chapter *The Concept* in *Knowledge and Error*, Mach discusses Stallo's objection to absolute space:

> In brief, [Stallo] holds that thought does not occupy itself with things as they are in themselves, but with our concepts of them; we know things only through their relations with other things, so that all our conceptual knowledge of things must be relative; a specific act of thought never contains the totality of an object's knowable properties, but only the relations belonging to a specific class. By not attending to these propositions, we commit several natural errors that are very common, grounded in our mental organization, as it were. Amongst these [is] the following: … things exist independently of their

[19] Helmholtz (1847) is acknowledged to be the first account of what is now considered to be the (canonical) law of conservation of force.

[20] Many thanks to an anonymous reviewer for raising this point.

> relations. [A]s examples of the … error, Stallo discusses the hypostasizing of space and time, as it occurs particularly in Newton's doctrine of absolute space and time. (1976, 102).

This discussion fits perfectly with Mach's second objection discussed in §2.1. Since we come to know of a thing only through the relations in which it stands to others, our thoughts (and concepts) can only be about those relations, and *not* about things-in-themselves.[21] It follows then that thought experiments – which involve varying our concepts in thought – can only provide us knowledge about the relations of things. By committing the error of believing that "things exist independently of their relations", Stallo and Mach reprimand Newton for hypostasizing absolute space.

Why is Mach's agreement with Stallo a problem for **(Implicit)**? Because it shows that Mach critique of Newton is independent of his views on thought experiments. According to Mach, Newton's bucket argument falters not – as **(Implicit)** will have it – because it does not return to experience or because it is not continuous with experience, but simply because the thought used in reasoning in the bucket argument – *to wit:* absolute space – is a thought independent of relations. Since thoughts cannot be, by their very nature, about anything other than relation of things, Newton's move of using thoughts of absolute space is incorrect. Mach's reprimand of Newton is not for using a thought experiment in an incorrect way, but of using incorrect thoughts in his bucket experiment.

5. A more compelling interpretation?

If not **(Implicit)**, then should we side with Stein and maintain that Mach practiced "abusive empiricism"? I suggest that there's a simple and more compelling reading available which dissolves **(Puzzle)**.

My preferred solution is to deny that there is any tension between **(Censure)** and **(Priggish)** by noting that **(Censure)** has nothing to do with thought experiments. As we saw above, there's a case to be made that Mach, following Stallo, chastised Newton's bucket argument not because of some illegitimate use of thought experiments but because of an illegitimate use of thoughts *in* thought experiments. This observation is further strengthened by my disambiguation of Mach's objection in §2.1. In none of the objections does Mach say anything about thought experiments or hypothetical reasoning: *O1* is a complaint on methodology and *O2* a complaint on using illegitimate thoughts (*i.e.,* non-relational thoughts) in mechanics. I'll come back to reading *O3* on my interpretation in §6.

This interpretation of Mach's views on thought experiments is different from **(Implicit)** in that it denies that we can find any principled distinction between valid/natural and invalid/artificial thought experiments in Mach's writing. On my interpretation, Mach takes a very liberal approach on thought experiments and places only minimal constraints on thought experiments. Contra **(Implicit)**, it is not necessary for the correctness of a thought experiment that it be continuous with or return to experiment. Of course, as indicated above, this dissolves the tension identified in **(Puzzle)** by denying that there is any tension. Hence, my proffered interpretation is located somewhere in-between Stein's evaluation and **(Implicit)**.

On the one extreme, Stein maintains that there is no structure to Mach's views on thought experiments – Mach changes his views when it suits him. On the other, the proponents of **(Implicit)** maintain that Mach's views are much more structured – Mach distinguishes between "valid" and "invalid" thought experiments. The interpretation I offer here can be located somewhere in the middle: neither does Mach think of thought experiments are as structured as the proponent of **(Implicit)** will have it, nor does he practice "abusive empiricism" by shifting his views on thought experiments when it suits him. Instead, Mach thinks of thought experiments in a liberal fashion and does not restrict their usage. What he does put constraints on and add structure to are not thought experiments but thoughts *in* thought experiments.[22]

What reasons do I think I have to prefer my proffered interpretation? I offer four.

6. Four reasons for the proffered interpretation

In this section, I offer four reasons to prefer the interpretation that Mach takes a liberal view on thought experiments and that his criticism of Newton's bucket experiment is independent of his views on thought experiments. As I mentioned above, read this way there's no puzzle about Mach's views on thought experiments because **(Censure)** is incorrect.

First. My interpretation scores well on problems I raised with **(Implicit)** in §4. It does not fall prey to the first problem of Mach's positive appraisal of Stevin's thought experiment because there is no requirement on thought experiments to be continuous with or return back to experience. Furthermore, my interpretation dovetails nicely with Mach's assertion that we accept the conclusion of Stevin's experiment "without the thought of an objection" because unlike **(Implicit)** – as mentioned above – my interpretation is not at odds with the nature of instinctive knowledge. My interpretation also does not fall prey to the second problem raised in §4 because it unties Mach's critique of Newton from his more encouraging views on thought experiments. Thus, my reading is consistent with Mach's sanguine evaluation of Stallo's views.

Second. As I discussed above, Mach believes that Stevin's experiment is forceful because of its appeal to instinctive knowledge. Elsewhere in *The Science of Mechanics*, Mach writes of instinctive knowledge that:

> It is a peculiar property of instinctive knowledge that it is predominantly of a negative nature. We cannot so well say what must happen as we can what cannot happen, since the latter alone stands in glaring contrast to the obscure mass of experience in us in which single characters are not distinguished. (1919, 28)

If instinctive knowledge is "predominantly negative", then it seems that Mach criticizes Newton for misusing instinctive knowledge – for at least as Mach reads Newton, Newton uses instinctive knowledge of a positive nature (the concavity of water when the water is at rest with the bucket).[23] Hence, the proponent of **(Implicit)** errs in appealing to other factors (such as continuity of experience) in explaining Mach's criticism. On my interpretation, no such additional attribution is required: Mach's criticism is explained by observing that he is criticizing Newton not for using an incorrect kind of thought experiment (one that is discontinuous with experience), but for using incorrect thoughts *in* thought experiments.

Third. Silence speaks louder than words. A closer look at the context of Mach's discussion of the bucket experiment and his discussion on thought experiments further supports my interpretation. It is suggestive to note that Mach discusses the bucket experiment only once in the second chapter of *The Science of Mechanics*, in the larger context of discussing mechanics from his critico-historical perspective. Nowhere in *The Science of Mechanics* does Mach – at least explicitly – talk about thought experiments. Indeed, there is no mention of 'thought experiment(s)' in the *Science of Mechanics*. In contrast to Mach's silence on thought experiments in *The Science of Mechanics*, Mach discusses thought experiments (and imagination generally) extremely widely in *Knowledge and Error*.

Even more suggestive than Mach's silence on thought experiments in *The Science of Mechanics* is his silence on the bucket experiment in

[21] For a discussion on Mach's argument against things-in-themselves see Edgar (2013).

[22] To emphasize: although my solution denies that for Mach thought experiments need to be continuous with experience, it importantly does not deny that, for Mach, knowledge generated from thought experiments must be grounded in experience. Many thanks to an anonymous reviewer for advising me to make this clearer. See also my discussion of Mach's use of fantasy and imagination in section 6.

[23] See note 7.

Knowledge and Error. *Knowledge and Error* – Mach's mature epistemological work – which contains chapters entitled "On Thought Experiments", "Hypothesis", "Adaptation of Thoughts to Facts and to Each Other", "Sensation, Intuition, Phantasy", "The Concept", "The Exuberance of the Imagination", among others does not have a single discussion on the bucket experiment. If, as the orthodoxy maintains, Mach is asserting something about thought experiments in his discussion of the bucket experiment, then it seems strange that Mach is silent on the bucket experiment when he had ample chance not to do so. Surely one will expect Mach to discuss an example of a 'bad' thought experiment in *Knowledge and Error*. The orthodoxy, including **(Implicit)** and Stein-ian "abusive empiricism", cannot explain this silence. But my interpretation accommodates this lacuna: as I have repeatedly emphasised, in discussing the bucket experiment Mach is not asserting anything about thought experiments; corollarly in discussing thought experiments in *Knowledge and Error*, Mach neglects the bucket experiment.

Fourth. My interpretation also explains away any charge of "abusive empiricism" levied on Mach. On my interpretation, we do not attribute to Mach any insincerity – he is well within reason to both critique Newton on Newton's illegitimate use of thoughts *yet* hypothesize himself about global boundary conditions and exotic physical mechanisms (à la 'Mach's principle'). If anything, my interpretation of Mach paints him to not be the strict empiricist he is often taken to be. And I think that this gels well with Mach's own views. He concludes the aptly titled chapter *The Exuberance of Imagination* thus:

> These excrescences of the imagination fight for existence by trying to overgrow each other. Countless such offspring and flowers of phantasy must, in view of the facts, be destroyed by merciless criticism, before a single one can develop further and attain some permanence … But before we can understand nature we must seize it through phantasy, in order to give these concepts a living and intuitive content. (1976, 77)

Here again we see the importance of fantasy for Mach.[24] It is fantasy that provides our concepts about nature "living and intuitive" content and thus make them meaningful. I won't discuss Mach's views on fantasy at length, but I want to note that the interpretation I offer is much more in line with Mach's acceptance of fantasy as a central tool in science than either **(Implicit)** or a Stein-ian "abusive empiricism" interpretation can accommodate.[25] What I have proposed is very much keeping in this Machian spirit of allowing the flourishment of fantasy.

According to Mach, imagination and fantasy can generate knowledge through what he calls the "law of association."[26] After discussing various examples including Galileo's experiments in determining the speed of light, Stevin's research in hydrodynamics, Newton's works on celestial mechanics and optics, and Wheatstone's development of rotating mirror apparatus, Mach's asserts that "The law of association has shown itself sufficient to explain the workings of scientific phantasy here discussed" (1976, 113). "Law of association" is Mach's term for the process of *freely* combining thoughts grounded in experience to generate thoughts that might not correspond to anything given in experience. Indeed, he underscores that fantasy leads to thoughts that are not in experience:

> If I had only ever seen one dog and now imagine one, the picture would probably have all the marks that had not escaped my attention in observing the dog. However, I have seen countless different dogs and doglike animals: therefore, the imagined dog is likely to be different from any that I ever saw. (1976, 110).

And for Mach "[if] one knows the historical development of science or has taken part in scientific enquiry, one will not doubt that scientific research requires a fairly robust phantasy" (111).

Mach's ruminations on fantasy and imagination provide strong support for my interpretation of his views on thought experiments. Using fantasy and imagination to their fullest potential, we should conceive of all sorts of thought experiments – continuous and discontinuous with experience – and weed out – with "merciless criticism" – the ones that can't stand competition, for example Newton's bucket argument. Indeed, Mach's emphatic calls at the end of the chapter for countless offspring of fantasy to flourish suggests to me that Mach is not looking to constrain the use of imagination and fantasy in a way the proponent of **(Implicit)** wants.[27]

7. Close

I close by anticipating three objections for the interpretation I have presented in this paper.

First. It seems that on my interpretation reading *O3* proves a bit tricky. Agreed. But I think that with the background established in this paper, there is a plausible reading of *O3* compatible with my solution and which does not violate Mach's spirit. Read literally, *O3* seems problematic for Mach's appraisal of Stevin's experiment because Stevin extends the "boundaries of experience" by appealing to the impossibility of perpetual motion in rational mechanics. However, I think the clue to solve this problem is in Mach's assertion that "no one possesses the requisite knowledge" to make use of concepts which are beyond experience. As I suggested in my discussion of *O3* in §2.1, Mach takes a concept to be meaningful only if its knowledge is useful.[28] In his writings, Mach repeatedly asserts that imagination does *create* knowledge:

> If favourable circumstances guide the imagination in such a way that it follows or anticipates facts, we gain knowledge. (1976, 64).

So, for Mach, it is possible that – under favourable circumstances – we may gain knowledge of concepts through imagination and thought experiments. Importantly, I think Mach here is not implying that imagination can generate knowledge only if imagination is continuous with facts. All I take Mach to imply here, and in his *Exuberance of Imagination* in *Knowledge and Error,* is that imagination can generate knowledge if the starting point of our imagination and the knowledge generated from it is consistent with background facts ("it follows or anticipates facts"). Note that Mach here (and in other places where he talks about imagination and fantasy) has nothing to say about whether imagination or fantasy should be continuous with experience. That is a stronger condition, one I think that we are not warranted to draw from Mach's writings on imagination or thought experiments.

O3 can then be thought of as a truncated statement which can be filled in by paying heed to the context in which *O3* is embedded in, *i.e.*, the context of Mach's criticism of Newton's illegitimate use of concepts and not of thought experiments. Hence, on my suggested reading *O3* is to read:

> No one is warranted in extending [mechanical] principles beyond the boundaries of experience [*in the way Newton does*], *i.e.*, principles cannot be extended beyond experience unless we can have useful knowledge of those principles.

On this construction, Mach's criticism of Newton becomes more illuminating and Mach's own position less rife with tension. Newton is at fault

[24] Mach's use of the German 'Phantasie' has been translated both as 'fantasy' and 'imagination' (Brecevic, 2021).

[25] Many writers have noticed the central importance of imagination in Mach's philosophy of science. See Frank (1941, Chapter 2); Sorensen (1992, Ch.3); Pojman (2000); and Banks (2004, 2014). For contemporary views of imagination similar to those of Mach see Kind (2016) and Jackson (2018).

[26] To be sure, Mach distinguishes between "scientific phantasy" and "artistic phantasy". I concentrate only on scientific phantasy here. Mach discusses his views on fantasy in a couple of chapters in Knowledge and Error: "The Exuberance of Imagination" and "Sensation, Intuition, Phantasy."

[27] My thanks to an anonymous reviewer for prompting me to provide a brief discussion on Mach's views on fantasy and how they relate to the interpretation I proffer.

[28] See notes 3 and 5.

because he extends a concept we cannot know of – a non-relational principle – beyond experience. Stevin and Mach himself are not at fault because they extend useful concepts we already know of beyond experience. Here again, Mach's conservative tendency – noted in §2.1 – is brought out. Mach deems it fit to extend those concepts beyond experience which are useful (and thus meaningful) to us, but deems it unfit to extend those which are meaningless. This further supports my interpretation's central tenet: Newton is not at fault for using thought experiments or hypothetical reasoning – Mach's criticism has nothing to do with those.

Second. Here's an attractive idea that solves **(Puzzle)**: between *The Science of Mechanics* – first published in German in 1883 – and *Knowledge and Error* – first published in German in 1905 – Mach simply changed his views about thought experiments. In *The Science of Mechanics* when Mach is criticizing Newton, he is actually asserting something about thought experiments. But in *Knowledge and Error*, Mach does not mention the bucket experiment and seems more receptive to thought experiments because he actually is more receptive. This interpretation is attractive because it solves **(Puzzle)** without appealing to an unacknowledged distinction (à la **(Implicit)**) and without making Mach insincere (à la Stein-ian "abusive empiricism"). Clearly this interpretation is incompatible with my interpretation as well.[29] Call this view **(Change).**

But I think there's no evidence – textual or otherwise – to support **(Change)**. Rather, I think there is solid evidence to the contrary. If indeed Mach changed his views on thought experiments between *The Science of Mechanics* and *Knowledge and Error*, he would have done so in the numerous opportunities he had. But Mach did not do that. Mach published the seventh edition of *The Science of Mechanics* in 1912 – four years before his death. But even in the seventh edition, Mach did not rescind any of his discussion on Newton's bucket experiment. Rather, in the preface to the seventh edition he wrote:

> The character of the book has remained the same. With respects to the monstrous conceptions of absolute space and absolute time I can retract nothing. Here [*i.e.*, in the seventh edition] I have only shown more clearly than hitherto that Newton indeed spoke much about these things, but throughout made no serious application of them. (1988)

If anything, Mach's comments support my interpretation because he makes it clear that his critique of Newton is directed not at Newton's use of thought experiments but towards Newton's usage of useless thoughts.[30] It is the impotency and the incorrectness of the concepts of absolute space and absolute time in the bucket experiment that are Mach's target of criticism. Hence, we see once more that when discussing Newton, Mach criticizes the incorrect usage of thoughts *in* thought experiments and not thought experiments simpliciter.

Third. Buzzoni (2019) claims that on **(Implicit)** it is easier to make sense of Mach's positive appraisal of Pierre Duhem's views on thought experiments. Duhem in his *La Théorie Physique* outright condemned thought experiments (*expériences fictives*) (Duhem, 1906/1954). As evidence of Mach agreement with Duhem, Buzzoni points to a note Mach amended in the "On Thought Experiments" chapter in the second edition of *Knowledge and Error*. Mach writes:

> Duhem rightly warns against representing thought experiments [*Gedankenexperimente*] as though they were physical, that is pretending that postulates are facts. (1976, 146; emphasis original).

Buzzoni takes this quote to mean that Mach approved of Duhem's criticism of thought experiments. Buzzoni then argues that the only way to make sense of this note – short of attributing to Mach a self-contradictory position – is to appeal to **(Implicit)**.[31] Buzzoni argues that this note is explained by **(Implicit)** because what Mach is agreeing with Duhem on is the incorrectness of the "artificial" thought experiments and not on the incorrectness of "natural" thought experiments. Buzzoni reads Mach as reading Duhem's criticisms of thought experiments as Duhem's criticism of *artificial* thought experiments.

Agreed: Mach appraisal – or at least his positive acknowledgment – of Duhem's views on thought experiments seems problematic for my interpretation. But I think that Buzzoni is reading too much into Mach's note. Literally read, the note is only a warning against thinking of thought experiments *as* physical experiments. I think that it is incorrect to extrapolate from this to the position that Mach accepted Duhem's rejection of thought experiments. What I think Mach means here is just what he writes: if we confuse thought experiments and physical experiments, we commit a blunder and that Duhem is right in illuminating this blunder. Indeed, Mach attaches this note when he is discussing whether thought experiments can replace physical experiments.

Consider further that even though in the preface to *Knowledge and Error* Mach glowingly writes that he has "far-reaching agreement" (1976, xxxv) with Duhem, he nonetheless does not mention Duhem directly in his chapter on thought experiments. Instead, Mach makes it clear that while he will enthusiastically note where he agrees with Duhem, he will be more cautious in recording their disagreements:

> I value the agreement between us all the more because Duhem arrived at the same results quite independently … In what follows I shall often have occasion to refer to Duhem's pronouncements and only rarely and on subordinate points to note a difference of opinion. (1976)

Hence, it seems to me that there is not sufficient evidence to read anything more than what is in Mach's note. Mach is, I think, not saying anything that supports **(Implicit)** in approvingly citing Duhem and neither should we.

My aim in this paper was two-fold: a) to argue against **(Implicit)** and b) to provide a novel interpretation of Mach's views on thought experiments. I made a case against **(Implicit)** by pointing out that **(Implicit)** faces difficulties in accounting for Mach's appraisal of Stevin's experiment and in accommodating Mach's positive reflections on Stallo's philosophy. On the positive side, I presented an interpretation of Mach's views on thought experiment according to which Mach advocated a liberal use of thought experiments without any rigid constraints.

I argued for my proffered interpretation by noting four reasons in favour of it and by arguing against the orthodox position that Mach's criticism of Newton's bucket experiment was directed towards the incorrectness of Newton's thought experiment. Instead, I argued that Mach's criticism of the bucket experiment was directed towards Newton's use of incorrect thoughts *in* his bucket experiment. That is, we should read Mach's critique of Newton as being about using inappropriate thoughts in his bucket experiment. Mach criticizes Newton not because of the form of the bucket argument but because of its content. I submit that the view presented and defended here is superior to its alternatives: **(Implicit)**, **(Change)**, and Stein-ian "abusive empiricism".

Let me close by briefly noting an interesting and welcome consequence of my reading of Mach's views on thought experiments. On my interpretation, Mach turns out to be much less of a strict empiricist than he is usually taken to be. As I have stressed at various points in the paper, on my reading Mach is happy to countenance a variety of thought experiments irrespective of whether the thought experiments are continuous with experience or not. Although, as I have also stressed, this does not imply that thoughts *in* thought experiments are not constrained by experience, this does imply that Mach takes a more laissez-faire approach

[29] Prima facie, is **(Change)** simpler than my proffered interpretation in the sense that it is simpler than the other two interpretations? I am not sure, in part because I think that my proffered interpretation is simple as well. But I am not sure if this is an interesting question to answer and since nothing of substance depends on this question (or so I hope), I leave it unanswered.

[30] Corollarly, there is no mention of the bucket experiment in further German editions of Knowledge and Error published in 1906, 1917, 1920, and 1926.

[31] See Kühne (2005) for such an interpretation. Strictly speaking, Kuhne does not attribute a self-contradictory position to Mach. Rather, he argues that Mach privately accepted Duhem's contrtadiction, but never publicly accepted them.

to scientific epistemology. I think this is a welcome consequence, especially if one pays adequate heed to his prognostications on fantasy and imagination. A fuller reappraisal of Mach's views in this humbler empiricist spirit must await a future paper but I think that it will be an interesting project (for someone) to undertake.[32]

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