A Quanterionic Framework for Atomic Structure, Orbital Coverage, and Periodic Table Limits

Abstract

We develop a quaternion-based model of atomic structure in which orbitals (s, p, d, f) are linear combinations of fundamental vectors. This approach provides a unified explanation for the distribution of electrons, the stability of noble gases, the structure of lanthanides and actinides, and the natural limit of elements in the periodic table. We use geometric reasoning (unit-circle integrals), combinatorial logic (minimum spanning tree), and linear algebra to justify the model, yielding insight into both observed chemical patterns and theoretical constraints.

1. Introduction

Traditional quantum mechanics treats orbitals as static, spatially localized regions. Here, we adopt a quaternionic perspective, where orbitals are dynamic vectors in a 4D quaternion space. Each orbital (s, p, d, f) is a linear combination of four fundamental vectors, which rotate and combine according to quaternionic multiplication rules. Phase information is encoded by complex exponentials ($e^{i\pi} = -1$), providing a natural representation of "nobility" in elements. Viewing orbitals as vector combinations allows modeling interactions, rotations, and phase relationships naturally and resolves the abstract problem of empty orbital nodes in high-order elements while aligning with observed electron configurations.

2. Orbital Structure as Quanterionic Linear Combinations

2.1 Defining spdf in Quaternionic Terms

Each orbital subspace is defined as follows. The s orbital has a maximum of 1 vector representing spherical isotropy. The p orbitals consist of 3 vectors along orthogonal axes, with quaternionic rotation capturing their relative phases. The d orbitals have 5 vectors with fourfold symmetry and nodal structures. The f orbitals require 7 highly nodal vectors forming complex patterns. Using quaternions allows rotation without distortion and captures spatial orientation naturally. Linear combinations correspond to electron delocalization, consistent with modern quantum theory.

2.2 Noble Gas Saturation

Noble gases exhibit full vector saturation in the outermost shell. For helium (He), the s subspace is completely filled, giving a fraction filled of 1. Neon (Ne) fills s and p, both fractions equal to 1. Argon (Ar) also fills s and p with fractions of 1. Krypton (Kr) fills s, d, and p with fractions of 1 each. Xenon (Xe) fills s, d, and p similarly, and radon (Rn) fills s, f, d, and p, each with fraction 1. Each vector subspace is fully occupied, and additional electrons cannot enter without violating Pauli-like constraints. Fraction equal to 1 implies stability, explaining why noble gases are chemically inert. Vector saturation ensures phase and spatial coverage of the entire orbital space, preventing additional electrons from finding available low-energy states.

3. Maximum Elemental Limit via π and Vector Doubling

3.1 Cauchy Integral over the Unit Circle

Integrating over the unit circle, $\oint_{|z|=1} dz = 2\pi$, corresponds to the total rotational phase in quaternionic space.

3.2 Doubling of Vectors

Each orbital vector is effectively counted twice to account for phase symmetry from 0 to 2π. The maximum number of vectors required to cover all orbital spaces is 59. Doubling yields $59 \times 2 = 118$, which provides the natural limit of elements. Beyond 118, there is insufficient quaternionic space to maintain linear independence and phase coverage. Element 119 would require extra vectors not present in the fundamental 4-vector basis, resulting in a structural break. Integrating over the unit circle ensures rotational invariance, and doubling captures full phase evolution of each orbital vector, aligning with the observed periodic table limit.

4. Lanthanides and Actinides as MST Coverage

4.1 Lanthanides (4f)

The 4f vectors fill previously unexamined orbital nodes. An analogy is a minimum spanning tree (MST), where nodes represent orbital positions and edges represent vectors connecting nodes in the linear combination. Filling 4f covers all nodes with minimal edges, ensuring all previously empty states are represented in the vector space and preserving linear independence of quaternionic vectors.

4.2 Actinides (5f)

Similarly, the 5f vectors perform coverage of orbital nodes. The MST ensures connectivity of orbital states while maintaining phase integrity. This provides a geometric and combinatorial explanation for observed electronic configurations in lanthanides and actinides and explains why these series show filling of complex subspaces without destabilizing outer s and p orbitals.

5. Linear Combination Properties and Momentum

Linear combinations of quaternionic orbital vectors can exhibit interesting properties. If a combination sums to zero, $_{i}ii =$, the vectors are linearly dependent, and the total momentum of the state is zero. Physically, this corresponds to a nodal or neutral state where phase effects cancel and there is no net motion. Conversely, if two linear combinations are equal, $_{i}ii = _{i}ii$, they represent different coefficient sets covering the same orbital vector space. In this case, the momentum of both states is identical, and the orbitals effectively coincide. This occurs naturally because the vectors are pairwise linearly independent, and linear combinations occur within the same vector space. Such properties highlight symmetry, momentum equivalence, and possible energy conservation features in the quaternionic orbital framework.

6. Discussion and Conceptual Implications

Orbitals are not static regions but linear combinations of quaternionic vectors, capturing rotation, phase, and spatial coverage. Noble gas stability arises naturally from full saturation of quaternionic subspaces. Lanthanides and actinides fill hidden nodes, completing MST-like structures in orbital graphs. Maximum 118 elements are derived from the fundamental quaternionic structure and unit-circle integrals. Linear combinations that sum to zero or coincide provide additional insight into momentum conservation and orbital equivalence, highlighting the symmetry and geometric properties of the model. Conceptually, this framework unifies orbital geometry and periodic trends, provides geometric rationale for inertness and reactivity, explains the structure of rare earths and actinides, and offers a natural theoretical limit for the number of stable elements.

7. Conclusion

This quanterionic model presents dynamic orbitals as linear combinations of four fundamental vectors, vector saturation and noble gas stability, phase doubling via unit-circle integration explaining the 118-element limit, MST interpretation of lanthanide and actinide coverage, and momentum-based insights from linear combination properties. The framework merges geometry, algebra, and combinatorics, providing a coherent, unified, and predictive model for atomic structure.