Proof of Causality for the Existence of God and His Attributes: A Logical Approach

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Abstract

This paper develops a rigorous causal argument for the existence of God and His Attributes by exploring the logical framework underpinning self-sufficiency and causality. It combines symbolic logic, probabilistic models, and causal networks to demonstrate that only a self-sufficient, omnipotent, omniscient, and volitional entity can be the ultimate cause of all non-self-sufficient beings. This analysis ultimately establishes the necessity of the Creator, whose attributes ensure the stability and order of the universe. Additionally, the paper concludes that no element of unconscious nature — such as the universe, matter, or energy — can be eternal or self-sufficient, nor can it come into existence without an external cause or through an infinite causal chain; all such entities are inherently dependent on a fundamental, independent cause for their existence and properties.

Keywords: Causality, Creator, Logical Explosion, Ontological Explosion, Infinite Causal Regress

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1   Graphical Summary of the Causal Argument

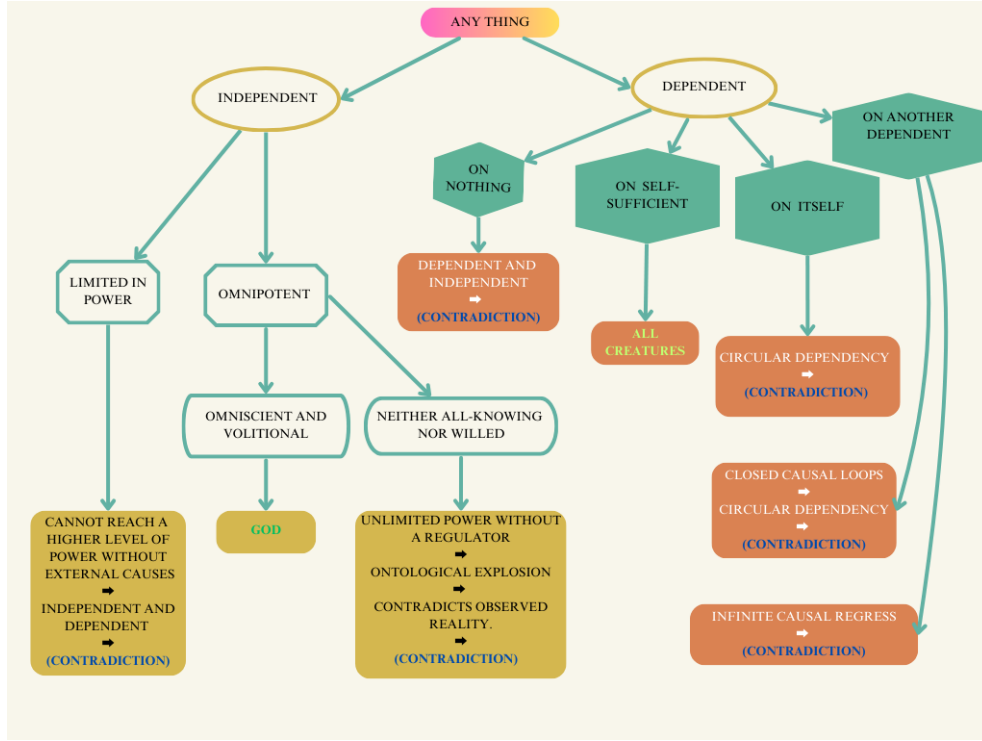Figure 1: Graphical summary of the causality argument for the existence of God.

2   Linguistic Description and Analysis of the Argument

The causality argument divides entities into two exhaustive categories:

1. **Self-Sufficient (G):** An entity does not require an external cause for its existence, power, or attributes. It is entirely independent.
2. **Causally Dependent (M):** An entity whose existence/attributes depend on external causes.

The argument uses *reductio ad absurdum* to eliminate contradictory possibilities (e.g. infinite causal regress and non-volitional self-sufficient entity), concluding:

1. **For Self-Sufficient entity:**

   - A self-sufficient entity with limited power implies a dependency on external causes to overcome limitations—a contradiction.

   - A self-sufficient omnipotent entity must be both omniscient and volitional; otherwise, nothing would regulate what emanates from its absolute power. In such a scenario, every possible effect would occur without control, leading to an ontological explosion (analogous to a logical explosion, see Appendix 1) in which no physical law could prevail in any possible world and the properties of things would become indistinguishable. This outcome contradicts observed reality.

   - Therefore, the self-sufficient entity must be omnipotent, omniscient, and volitional (the Creator).

2. **For causally dependent entity:**

   - Circular dependency (A→B→A) creates a logical contradiction.

   - Infinite causal regress fails due to the lack of an initial cause.

   - Therefore, causally dependent entities (all creatures) must depend on a self-sufficient entity.

3. **Conclusion:** A self-sufficient, omnipotent, omniscient, and volitional Creator must exist, upon whom all causally dependent entities rely.

3 Symbolic and Descriptive Analysis

We analyze the argument using multiple logical frameworks.

3.1 Definition of Symbols:

- $G(x)$: x is self-sufficient.

- $M(x)$: x is causally dependent.

- $P(x)$: x is omnipotent: Having unlimited power, capable of doing anything logically possible.

- $Q(x)$: x is omniscient: Knowing everything, including all possible outcomes and consequences.

- $D(x)$: x is volitional: Possessing will or intention, able to make purposeful choices.

- $K(x)$: x is omniscient and volitional.

- $U(x)$: x is a stable universe (has consistent rules and cause-effect relations).

- $\neg U(x)$: x is a universe with an ontological explosion: where effects occur without control, leading to an acausal lawless universe without consistent rules or predictable cause-effect relationships.

- Infinite Causal Regress(ICR): A specific type of infinite regress that involves causal relationships. This refers to a chain of cause-and-effect relationships in which every node has an incoming causal edge, resulting in an endless sequence with no initial self-sufficient cause.

- Creator(x): x is the Creator.

- Created(x): x is created.

- Depends on(x, y): x depends on y.

- Caused By(x, y): x causes y.

3.2 Proof by Contradiction

- A limited self-sufficient entity:

$$\text{Assume } G(x) \wedge \neg P(x) \rightarrow M(x) \quad \Rightarrow \quad \bot \text{ (contradiction)}.$$

- A self-sufficient omnipotent entity without omniscience or volition:

Step 1: Assume $P(x) \wedge G(x) \wedge \neg K(x)$.

Step 2: $\neg K(x) = \neg Q(x) \vee \neg D(x)$.

Step 3: If $\neg Q(x)$, then unintended effects, disrupting the stability of the universe.

Step 4: If $\neg D(x)$, then arbitrarily changes that undermine consistent rules.

Step 5: $P(x) \wedge G(x) \wedge \neg K(x) \implies \neg U(x)$.

Step 6: $U(x)$ is true, so $\neg U(x)$ is false.

Step 7: Contradiction, hence $P(x) \wedge G(x) \wedge \neg K(x)$ is false.

Step 8: $P(x) \wedge G(x) \implies K(x)$.

Thus, an entity with unlimited power (omnipotence) and complete independence (self-sufficiency) must also be all-knowing (omniscient) and intentional (volitional) to maintain control over its actions. Without these qualities, its power would manifest uncontrollably, producing an acausal lawless universe where anything can happen without restriction—a state incompatible with the orderly, rule-governed universe we experience.

3.3   Mathematical Modeling of Causal Networks

A causal network is a directed graph $\mathcal{N} = (V, E)$:

- V: Set of nodes (entities G or M).

- E: Set of edges (causal relations, $(v_1, v_2) \in E$ means v_1 causes v_2).

Definition 3.1. *A self-sufficient entity G has no incoming edges:* $\deg^-(v) = 0$. *A dependent entity M has* $\deg^-(v) \geq 1$.

Analysis of Infinite Networks: - If every $v \in V$ has $\deg^-(v) \geq 1$, there is no initial node, implying an infinite causal regress ICR $\cdots \to v_3 \to v_2 \to v_1$. - This contradicts the need for a minimal element in a causal chain, leading to $\bot$.
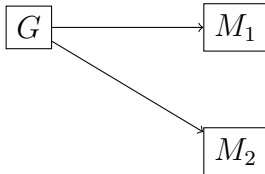
Probabilistic Analysis:

$$\mathbb{P}(\text{ICR}) \approx 0 \quad \Rightarrow \quad \mathbb{P}(\exists x \, G(x)) \approx 1$$

Bayesian Analysis:

$$\mathbb{P}(G|E) = \frac{\mathbb{P}(E|G) \cdot \mathbb{P}(G)}{\mathbb{P}(E)}, \quad \mathbb{P}(\text{ICR}|E) = \frac{\mathbb{P}(E|\text{ICR}) \cdot \mathbb{P}(\text{ICR})}{\mathbb{P}(E)}$$

where $\mathbb{P}(E|G) \gg \mathbb{P}(E|\text{ICR})$, since G explains the order, while infinite causal regress does not. so $\mathbb{P}(G|E) \approx 1$.

3.4 Core Symbolic Formulation

3.4.1 Premises

(1) $\forall x \Big((G(x) \vee M(x)) \wedge \neg(G(x) \wedge M(x)) \Big)$

(2) $\forall x \Big(G(x) \rightarrow \big(P(x) \vee \neg P(x) \big) \Big)$

(3) $\forall x \Big(G(x) \wedge \neg P(x) \rightarrow M(x) \Big)$

(4) $\forall x \Big(G(x) \wedge P(x) \rightarrow \big(K(x) \vee \neg K(x) \big) \Big)$

(5) $\forall x \Big(G(x) \wedge P(x) \wedge K(x) \rightarrow \mathrm{Creator}(x) \Big)$

(6) $\forall x \Big(G(x) \wedge P(x) \wedge \neg K(x) \rightarrow \bot \Big)$

(7) $\forall x \Big(M(x) \rightarrow \Big(\big(\exists z \, (\mathrm{Depends\_on}(x,z) \wedge M(z)) \big) \vee$

$\big(\exists w \, (\mathrm{Depends\_on}(x,w) \wedge G(w)) \big) \vee$

$\neg \exists y \, \mathrm{Depends\_on}(x,y) \Big) \Big)$

(8) $\forall x \Big(\big(\exists z \, (\mathrm{Depends\_on}(x,z) \wedge M(z)) \big) \rightarrow \mathrm{ICR} \Big)$

(8.1) $\mathrm{ICR} \rightarrow \bot$

(9) $\forall x \Big(\big(\neg \exists y \, \mathrm{Depends\_on}(x,y) \wedge M(x) \big) \rightarrow \bot \Big)$

(10) $\forall x \Big(\big(\exists w \, (\mathrm{Depends\_on}(x,w) \wedge G(w)) \big) \rightarrow \mathrm{Created}(x) \Big)$

(11) $\forall x \Big(M(x) \rightarrow \exists y \, \mathrm{Caused\ By}(x,y) \Big) \quad \text{(Causality Principle)} \Big)$

3.4.2   Detailed Deductions

1. From (3): $G(x) \wedge \neg P(x) \to M(x)$, and with (1), we deduce:

$$\forall x \left(G(x) \to P(x) \right)$$

2. From (4), and (6): $G(x) \wedge P(x) \wedge \neg K(x) \to \perp$, so:

$$\forall x \left(G(x) \wedge P(x) \to K(x) \right)$$

3. Combining the above:

$$\forall x \left(G(x) \to \left(P(x) \wedge K(x) \right) \right)$$

   Then from (5):
$$\forall x \left(G(x) \to \text{Creator}(x) \right)$$

4. From (7), (8), (8.1), (9), and (11): Every $M(x)$ must depend on a $G(w)$, leading to:

$$\forall x \left(M(x) \to \exists w \left(\text{Depends\_on}(x, w) \wedge G(w) \right) \right)$$

   Then from (10):
$$\forall x \left(M(x) \to \text{Created}(x) \right)$$

5. Assume $\neg \exists x\, G(x)$: then every entity is $M(x)$, but (11) requires a cause, leading to a contradiction via (8) and (8.1). Thus:

$$\exists x\, G(x)$$

3.4.3   Conclusion

$$\exists x \left(G(x) \wedge P(x) \wedge K(x) \wedge \text{Creator}(x) \right)$$

$$\wedge\, \forall y \left(M(y) \to \left(\text{Created}(y) \wedge \exists z \left(G(z) \wedge \text{Depends\_on}(y, z) \right) \right) \right)$$

This demonstrates that there is a single Creator who is self-existent, omnipotent, all-knowing, and all-willing and that every dependent being is created relying on this self-existent Creator.

4 Unified Integration

Synthesizing all frameworks:
- Symbolic and contradiction proofs exclude infinite causal regresses and ontological explosions.
- Causal networks require a prime node.
- Bayesian analysis favors a necessary Creator.

5 Final Proof Structure

1. **Categorical Division:**

$$\forall x \left[G(x) \vee M(x)\right] \quad \wedge \quad \neg\exists x \left[G(x) \wedge M(x)\right].$$

2. **Excluding Limited Self-Sufficiency:**

$$G(x) \wedge \neg P(x) \to \bot \quad \Rightarrow \quad G(x) \to P(x).$$

3. **Attributes of the Self-Sufficient:**

$$G(x) \to \left[P(x) \wedge K(x)\right].$$

4. **Dependency Resolution:**

$$M(x) \to \exists y \left[G(y) \wedge \text{Caused By}(x,y)\right].$$

5. **Probabilistic Consolidation:**

$$\mathbb{P}(\text{ICR}) \approx 0 \quad \Rightarrow \quad \mathbb{P}\big(G(x)\big) \approx 1.$$

6. **Final Conclusion**

$$
\boxed{
\begin{aligned}
&\forall x \left[G(x) \vee M(x)\right] \wedge \neg\exists x \left[G(x) \wedge M(x)\right] \quad \wedge \\
&G(x) \to \left[P(x) \wedge K(x)\right] \quad \wedge \\
&M(x) \to \exists y \big(G(y) \wedge \text{Caused By}(x,y)\big) \quad \wedge \\
&\mathbb{P}(\text{ICR}) \approx 0 \Rightarrow \mathbb{P}\big(G(x)\big) \approx 1
\end{aligned}
}
$$

6 Conclusion

There exists a self-sufficient entity G with omnipotence $P(x)$, omniscience, and volitional $K(x)$, which is the cause of all dependent entities M. Infinite causal regress and ontological explosions fail to explain the observed orderly universe, confirming the necessity of a Creator, and deny the eternity or self sufficiently for any natural non-volitional element.

7 Appendix 1

Similarities of Logical and Ontological Explosions

7.0.1 Parallel Structure:

- **Logical Lawlessness:** No constraints on truth $\rightarrow$ contradictions $\rightarrow$ all propositions provable $\rightarrow$ no logical coherence.

- **Causal Lawlessness:** No constraints on effects $\rightarrow$ all states possible $\rightarrow$ no physical coherence.

7.0.2 Mechanism of Explosion:

- **In logic:** a contradiction leverages inference rules to derive everything, breaking the system.

- **In physics:** the absence of causal constraints means that no state excludes any other, breaking the ability to define a consistent reality. If a ball can fall and not fall without a rule or reason, no law can hold.

- **Both systems rely on a 'causal' framework:** logical rules as causes for deduction and physical laws as causes for state transitions. When these frameworks collapse (via contradiction in logic or lawlessness in physics), the result is an explosion—logical or ontological—where all outcomes become equally valid or possible.

7.0.3 Annihilation of Coherence:

Logical explosion renders truth indistinguishable. Ontological explosion renders physical states indistinguishable. This isn't mere chaos (unpredictable but rule-governed) or randomness (governed by probabilities), Without even

meta-laws governing probabilities, but unstructured, as no law (deterministic or probabilistic) can emerge or persist, no pattern (random or otherwise) holds, but a state that "utterly annihilates reality."

7.0.4   Reductio ad Absurdum:

To strengthen the analogy, let's use a reductio argument: Assume causal lawlessness does not lead to an ontological explosion. That is, suppose a lawless universe maintains some coherence, where only certain states or properties occur. If there's coherence, some rule or constraint must limit which states follow others (e.g., "gravity-like behavior persists"). But causal lawlessness means no such rules exist—any state can follow any other. This contradicts the assumption of coherence, as unrestricted transitions allow all states and properties to manifest arbitrarily. Thus, causal lawlessness must lead to an ontological explosion, where all possibilities coexist, akin to a logical explosion.

7.0.5   Conclusion:

The analogy holds: that causal lawlessness entails an ontological explosion, mirroring how logical lawlessness entails a logical explosion. Just as logical lawlessness destroys truth's coherence by making all propositions derivable, causal lawlessness destroys reality's coherence by making all effects possible without structure. This isn't mere chaos or randomness but a complete unraveling of order, supporting the parallel between the two "explosions.

Proof of the Analogy Between Logical and Ontological Explosions

Theorem: In both logical and physical systems, removing constraints leads to an explosion in the set of derivable (or reachable) elements. However, while a logical explosion permits all propositions (including contradictions), an ontological explosion excludes contradictory states, as they are self-forbidden.

Proof. **Logical System**

Let L be a set of propositions, and let $\vdash$ be a deduction relation such that $\Gamma \vdash I$ means that the proposition I can be derived from the set of premises

$\Gamma \subseteq L$.

Assume that the logical system permits contradictions. That is, there exists some $\Gamma \subseteq L$ and a proposition $L_0 \in L$ such that:

$$\Gamma \vdash L_0 \quad \text{and} \quad \Gamma \vdash \neg L_0.$$

By the principle of explosion (ex falso quodlibet), the following steps hold for any arbitrary proposition $I \in L$:

1. From $\Gamma \vdash L_0$, by disjunction introduction we have:

$$\Gamma \vdash L_0 \vee I.$$

2. Given $\Gamma \vdash \neg L_0$, the disjunction $L_0 \vee I$ forces I to be true (since L_0 is untenable in the presence of $\neg L_0$):

$$\Gamma \vdash I.$$

Thus, every proposition in L is derivable from Γ, and we have:

$$\{I \in L \mid \Gamma \vdash I\} = L.$$

This result demonstrates that the lack of constraints (allowing contradictions) leads to a collapse of the meaningful distinction between true and false propositions.

Physical System

Let S be the set of all possible physical states, and let $R \subseteq S \times S$ be the transition relation, where $(s, t) \in R$ indicates that state t can follow state s according to some causal law.

In a well-defined physical system, causal laws restrict transitions so that for any state $s \in S$, the set of reachable states,

$$\{t \in S \mid (s, t) \in R\},$$

is a proper subset of S. In an ontological explosion—where causal constraints are absent—we have:

$$R = S \times S,$$

implying that every state is, in principle, reachable from any given state:

$$\{t \in S \mid (s,t) \in R\} = S.$$

However, an important nuance distinguishes this situation from the logical case: despite the unconstrained state transitions, the ontological framework inherently forbids the simultaneous realization of contradictory states.

Establishing the Analogy

The analogy between logical and ontological explosions can now be stated as follows:

- **Logical Explosion:** In the absence of constraints (i.e., when contradictions are allowed), any contradictory set Γ results in every proposition being derivable:

$$\{I \in L \mid \Gamma \vdash I\} = L.$$

This outcome destroys any meaningful distinction between true and false propositions.

- **Ontological Explosion:** When causal constraints are removed, every state becomes reachable from any starting state:

$$\{t \in S \mid (s,t) \in R\} = S.$$

- **In summary,** both explosions result from the removal of essential constraints—logical consistency in the former and causal structure in the latter. While a logical explosion destroys truth, an ontological explosion destroys the coherence of reality.