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# The emergence of the Mersenne Star: a new geometry of prime numbers bridging physics and artificial general intelligence

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## ABSTRACT

This paper introduces a new key geometric way to understand Mersenne prime numbers. It discovers a shape called the Mersenne Star, which appears naturally from a special sequence named the Quanta Prime Sequence (QPS). This sequence is a double-indexed pattern that shows special behaviour when applied to numbers like  $2^p - 1$ . The Mersenne Star has eight points and twelve edges, and it shows 32 strict mathematical relationships between exact locations. These patterns are not random—they are symmetric and meaningful. Around the star, we find neighbours that match famous number sequences like Fibonacci and Lucas, as well as points that follow regular repeating cycles (like 6, 8, 12, or 24). This shows a kind of hidden harmony in how prime numbers behave. The study offers a fresh way to see primes—not as isolated numbers, but as part of a larger picture with deep connections. This work is an early step toward building a new key field called Geometry of Prime Numbers. It also opens the door to using intelligent computer systems to detect prime numbers by recognizing these shapes and patterns. This idea may lead to what we call Artificial Prime Intelligence—using machines to explore and understand the hidden world of primes.

## ARTICLE HISTORY

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## 1. Introduction: a new geometric perspective on Mersenne primes

### 1.1. Prime numbers: simple yet mysterious

Prime numbers are the fundamental building blocks of arithmetic. Defined simply as numbers greater than 1 that are divisible only by 1 and themselves, their simplicity stands in stark contrast to the mystery of their distribution. Despite centuries of research, the pattern behind the appearance of primes remains elusive. Mathematicians continue to search for underlying laws that govern their behaviour and reveal a deeper order within the integers.

### 1.2. From Lucas–Lehmer to the Mersenne Star: a geometric evolution

Among the many families of prime numbers, Mersenne primes occupy a unique and prominent position. These primes take the form  $2^p - 1$ , where  $p$  itself is a prime number. Marin Mersenne, a 17th-century French scholar, was among the earliest to systematically investigate this class of numbers. Examples such as 3, 7, and  $31 = 2^5 - 1$  illustrate their early occurrence. Their concise binary representation and historical connection to perfect numbers have made them essential to both theoretical number theory and large-scale computational projects.

The deep interest in Mersenne primes is well documented, particularly in Ribenboim's comprehensive historical survey (Ribenboim, 1991). The most widely used method to verify their primality is the Lucas–Lehmer test (Brent, 1974), which is computationally effective but offers little in terms of conceptual or visual insight into the structural reasons behind primality. For recent insights on perfect numbers and

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Mersenne primes, see Deza (2021) and Mersenne Research (2024) (e.g. when discussing their historical link to perfect numbers, or Mersenne Research (2024) for the latest discovered Mersenne prime).

While the Lucas–Lehmer test remains the classical tool for verifying Mersenne primes, it lacks a geometric interpretation. In my earlier work, *Generalization of the Eight Levels Theorem* (Ibrahim, 2024), I reformulated the test using the analytic  $\Psi$ -sequence, obtaining an equivalent symbolic-divisibility condition. This idea was then developed in *On the Emergence of the Quanta Prime Sequence* (Ibrahim, 2025) into a single QPS-based divisibility relation. In the present work, that single relation has evolved into thirty-two distinct conditions, structured by exact geometric symmetries. This progression transforms the method from a purely algebraic test into a comprehensive geometric framework—the *Mersenne Star*<sup>1</sup>—that reveals structural properties hidden in the original Lucas–Lehmer formulation.

### 1.3. Unveiling the Mersenne Star

In this work, we present an original and innovative geometric perspective on Mersenne primes by introducing the *Mersenne Star*, a structure that arises from applying the Quanta Prime Sequence (QPS) to the study of these numbers.

This framework reveals a constellation of eight key points— $(0, -1)$ ,  $(1, 4)$ ,  $(-1, 4)$ ,  $(1, -4)$ ,  $(-1, -4)$ ,  $(2, 5)$ ,  $(-2, -5)$ , and  $(0, 1)$ —which arrange themselves into a symmetric star in the coordinate plane. Strikingly, this star-like structure was not imposed by design but emerged naturally through the application of QPS to Mersenne-related sequences. The resulting geometry provides a new lens for understanding the nature of Mersenne primes, shifting the focus from purely numerical verification to spatial and relational interpretation. It suggests that primality may reflect deeper symmetries, not just isolated arithmetic conditions.

### 1.4. Potential and promise

*The star's neighbours*—including Fibonacci, Lucas, golden ratio harmonics, and periodic sequences—raise deep questions: Can we express the star's points using these well-known patterns? Could we reinterpret those sequences through the geometry of the star? The answers may reshape how number patterns are studied. Beyond theory, the star's structure holds real potential. For researchers in artificial general intelligence (AGI), analyzing points between  $(\pm 1, 2)$  and  $(\pm 1, -2)$  could lead to faster algorithms for detecting large Mersenne primes—potentially reducing computation time from weeks to seconds. The Mersenne Star may thus mark a new frontier, where mathematics, physics-like symmetry, and computing power converge into a promising direction for discovery.

### 1.5. Advancing Mersenne Prime research through AGI and quantum computing

The geometric structure of the Mersenne Star—particularly its periodic neighbours and the binary pattern of the exponent  $p$  in Mersenne primes of the form  $2^p - 1$ —offers promising opportunities for artificial general intelligence (AGI). Given AGI's growing capabilities in analyzing complex symbolic and binary patterns, as demonstrated by recent high-performance AI systems (Bubeck, 2023; Weng, 2021), it may be possible to uncover hidden structures within Mersenne prime distributions. One such possibility is that AGI tools could detect relationships between the binary representation of  $p$  and the periodic behaviours observed in the vertical zones from  $(\pm 1, 2)$  to  $(\pm 1, -2)$ , as described in Section 8. These relationships may lead to novel and significantly faster methods for primality testing—potentially reducing the time needed to verify large Mersenne primes from weeks to just seconds. We encourage further exploration of how the binary structure of  $p$  might interact with the spatial symmetries revealed by the Quanta Prime Sequence (QPS) and the Mersenne Star.

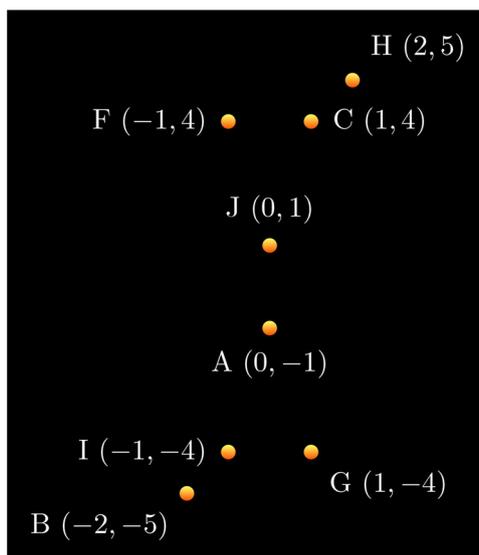
Building on this perspective, it is important to note that recent developments in artificial intelligence and quantum computing have greatly enhanced the tools available for both prime discovery and cryptographic analysis. Studies published in 2024–2025 highlight how advanced AI techniques can detect complex numerical patterns, while quantum algorithms contribute to more efficient primality testing and

factorization. This convergence reinforces the need to deepen our understanding of prime structures—particularly Mersenne primes—for securing future digital communications (Grzybowski, 2024).

## 2. Visualization of the Mersenne Star

### 2.1. Displaying the star

Below is a TikZ diagram that plots the eight points of the Mersenne Star. Each vertex is displayed as a glowing yellow dot with an enhanced blur effect to create a strong glow. The labels are in white.



### 2.2. Understanding the star's design

The eight-pointed Mersenne Star emerges as a surprising and elegant discovery. It prompts a fundamental question: *How does the Quanta Prime Sequence generate such a precise and symmetric pattern?* This well-formed structure, derived purely from a numerical process, does not appear random. Instead, it hints at a hidden geometry possibly embedded within the fabric of prime-related sequences. Each point seems intentionally placed, governed by an implicit mathematical rule. The symmetry between opposing vertices and the overall spatial balance suggest the presence of a deeper structural logic—elevating this configuration beyond a mere set of plotted coordinates. It becomes, in essence, a geometric representation with meaning.

This figure reflects exact QPS-derived relations, not an arbitrary plot.

## 3. Insights from the Mersenne Star

### 3.1. A map of mathematical relationships

More than just a symmetric shape, the Mersenne Star serves as a mathematical atlas—a framework mapping deep relationships among prime-related structures. The eight main vertices and their surrounding neighbours establish not only spatial positions but also interwoven connections between families of numbers that were previously viewed as independent. Fibonacci and Lucas sequences, long studied in isolation, now appear as geometric neighbours, revealing relationships shaped by the star's internal symmetry.

**Table 1.** Coordinates of the Mersenne Star and its prime-related neighbours.

Label	Coordinates $(\alpha, \beta)$	Category
Mersenne Star		
A	(0, -1)	Mersenne Star
B	(-2, -5)	Mersenne Star
C	(1, 4)	Mersenne Star
F	(-1, 4)	Mersenne Star
G	(1, -4)	Mersenne Star
H	(2, 5)	Mersenne Star
I	(-1, -4)	Mersenne Star
J	(0, 1)	Mersenne Star
Neighbors		
L1	(1, 3)	Lucas
L2	(-1, -3)	Lucas
Mix	(1, -3)	Fibonacci/Lucas Mix
P6	(1, 1)	Periodicity 6
P8	(1, 0)	Periodicity 8
P12	(1, -1)	Periodicity 12
P16	(1, $\sqrt{2}$ )	Periodicity 16
P20	(1, $\phi - 1$ )	Periodicity 20 (Golden Ratio)
P24	(1, $\sqrt{3}$ )	Periodicity 24

**Table 2.** Let us summarize the three changes with precision.

Position	From group 1	To group 8	Transformation
Point (0, -1)	On both sides	Becomes (0, 1)	Flip in $y$ -sign only
Point (-2, -5)	Left-hand side only	Becomes (2, 5)	Flip both coordinates
Point $(\pm 1, \pm 4)$	Right-hand side only	Remains unchanged	No transformation

**Table 3.** The eight points of the Mersenne Star: a symmetry-driven classification.

Label	Coordinates $(\zeta, \xi)$	Symmetry origin	Functional role in QPS structure
A	(0, -1)	Central symmetry	Anchor of the star; appears in all divisibility relations; serves as QPS baseline
J	(0, 1)	Reflection across x-axis	Completes vertical symmetry; mirrors point A
B	(-2, -5)	Diagonal inversion	Encodes constraints on exponent $p$ ; reflects to point H
H	(2, 5)	Diagonal inversion	Antipode of B; balances QPS structure diagonally
C	(1, 4)	Resonance symmetry	Fibonacci-aligned point; source of core QPS values
F	(-1, 4)	Reflection across y-axis	Horizontal mirror of C; part of symmetric ratios
G	(1, -4)	Reflection across x-axis	Vertical mirror of C; preserves down-up symmetry
I	(-1, -4)	Full inversion	Complete reflection of C; ensures full closure of point set

### 3.2. Patterns revealed by QPS

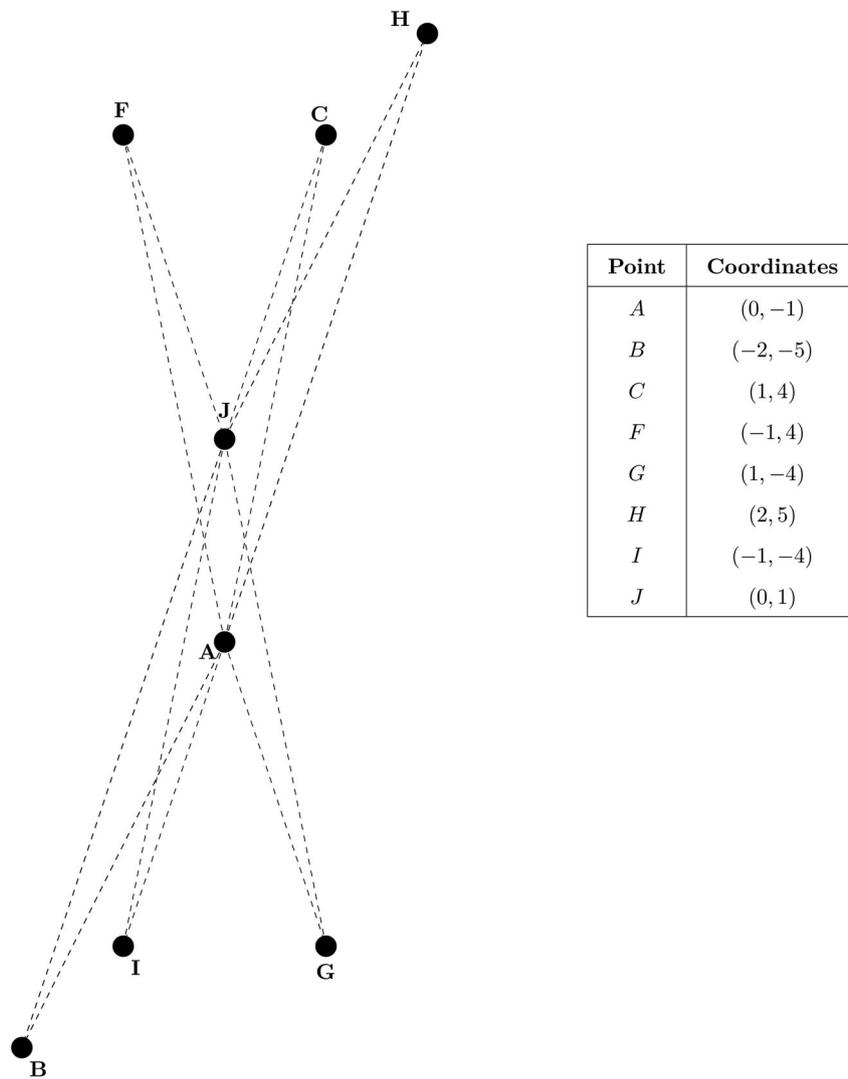
The application of the Quanta Prime Sequence (QPS) brings these patterns into sharp focus. The star's surroundings are filled with points that carry deep mathematical meaning—Fibonacci and Lucas numbers, golden ratio values, and periodicities of 6, 8, 12, 16, 20, and 24. Their consistent positioning and recurrence are unlikely to be coincidental. This arrangement suggests that what we see is not just a figure, but a coded structure with harmony at its core.

### 3.3. A visual pathway to discovery

Unlike traditional numeric methods, the star provides a spatial approach to understanding connections between primes and sequences. Its layout acts like a guide, showing how mathematical identities cluster and interact. It invites us to navigate through number theory using geometry—not merely as a tool for illustration, but as a method of exploration.

### 3.4. Beyond numeric data

The Mersenne Star brings new context to numbers. It shows how specific coordinates can carry symbolic weight—how a point's location reflects hidden periodic behaviour or alignment with a known sequence. This shifts our focus from viewing numbers as isolated data points to recognizing them as members of a cohesive geometric fabric. See [Figure 1–39](#) and [Tables 1–3](#) for the geometric construction of the Mersenne Star, its neighbourhoods and periodic alignments, and other salient properties.



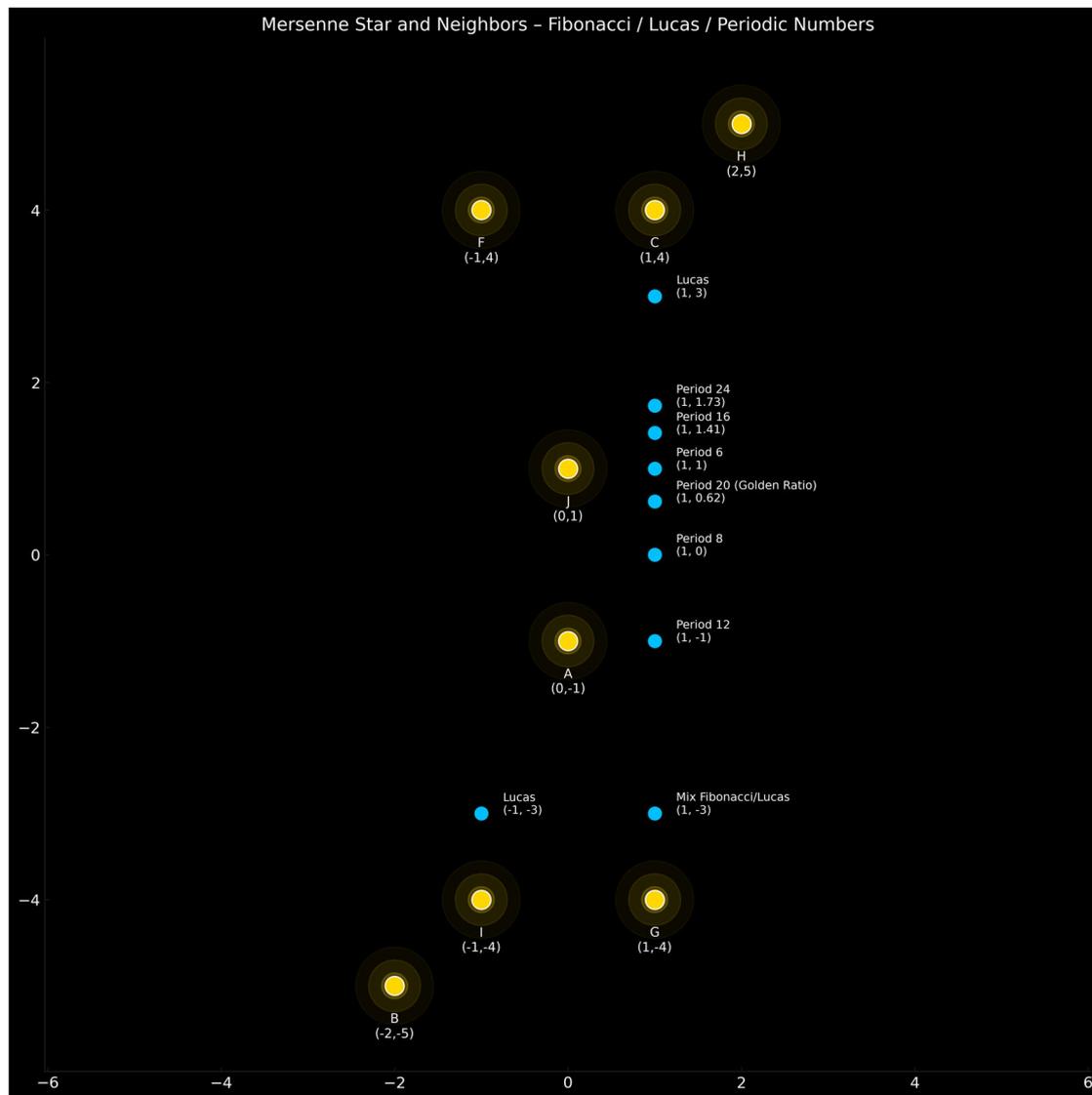
**Figure 1.** Geometric structure of the Mersenne Star generated from the Quanta Prime Sequence.

### 3.5. A gateway to future research

This star—vivid and geometric—invites further study. It offers a model where primes, sequences, and periodic structures coexist in harmony. It challenges us to look at primes not as singular spikes on the number line, but as part of an elegant design. The Mersenne Star is not decorative—it is functional. It suggests direction, coherence, and the presence of a deeper logic within prime geometry.

## 4. The Mersenne Star: a map that speaks

What was once hidden in abstraction now glows with clarity. The Mersenne Star, born from the Quanta Prime Sequence (QPS), is no longer a mystery—it is a guide. This figure does not simply display points; it offers a map, a constellation of meaning. For the first time, we see the full picture: the star at the centre, and around it, unexpected neighbours. And what neighbours they are! Fibonacci numbers. Lucas values. The golden ratio. Periodic cycles with lengths 6, 8, 12, 16, 20, and 24. These are not random intruders—they are familiar friends, quietly waiting in the shadows of prime space, now drawn into the light. What once seemed random now reveals hidden order. Each point in this image has a voice. Some shout symmetry, others whisper periodicity. But together, they form a new kind of language—one that the Mersenne Star helps us hear. Its geometry is not ornamental; it is directional. It tells us where to look next. Who is closest to



**Figure 2.** The Mersenne Star and its mathematical neighbours. This visual map highlights the Mersenne Star (golden glowing points) and its most significant mathematical neighbours (white-blue points). Each of the eight golden points represents a vertex of the star, which was discovered through the quanta Prime Sequence (QPS). The surrounding points are carefully selected due to their remarkable mathematical significance—ranging from Fibonacci and Lucas values to special periodic behaviours. Some of the periodic points include locations with period 6, 8, 12, 16, 20 (golden ratio), and 24. Together, this constellation-like structure reveals a hidden harmony that may guide future research into the geometry of prime numbers.

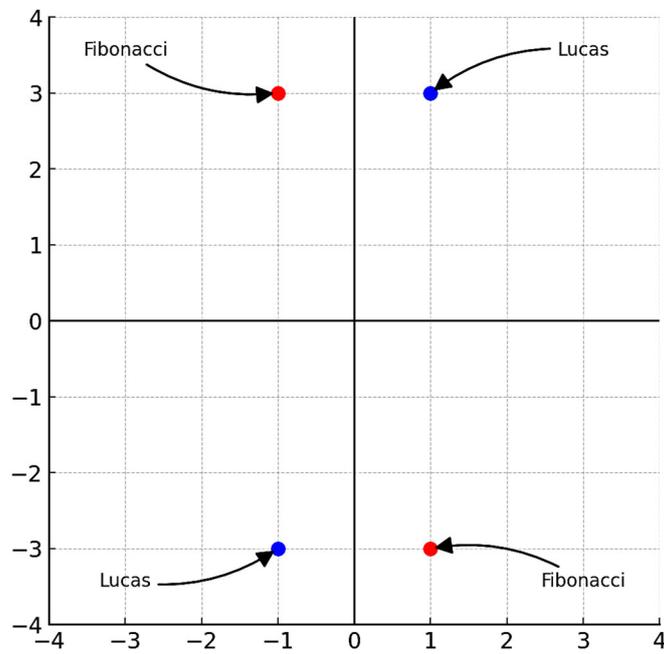
whom. Which patterns are recurring. Which harmonies are hidden in plain sight. This discovery changes the landscape. We are no longer wandering blindly in the forest of prime numbers. We have a map. A compass. And a star to follow. *The Mersenne Star does not just shine—it shows the way.*

## 5. Interpreting the Mersenne Star's geometry

### 5.1. Seeing beyond the diagram

#### 5.1.1. Interpretation

Looking at this star-like visualization, it's clear this isn't a typical mathematical figure. It suggests a sense of order, almost cosmic in nature. The golden points aren't just markers—they seem placed with purpose, part of a balanced and meaningful design.



**Figure 3.** Geometric locations of key points generating Fibonacci and Lucas values.

The Mersenne Triangle acts as a bridge between abstract algebra and visual structure. Instead of thinking about prime-related relationships purely as formulas or numbers, this triangle lets us see them as shapes in space. Suddenly, what once felt purely symbolic becomes visual and intuitive.

This also changes how we view the *Quanta Prime Sequence (QPS)*. It is no longer just a sequence used for calculations—it becomes a geometric language, a bridge between algebra and geometry. Through the triangle, QPS reveals its hidden ability to organize mathematical truths in a way that can be drawn, not just solved.

Thanks to this new perspective, the Mersenne Triangle opens the door to deeper exploration of Mersenne primes. It gives researchers a visual tool to investigate complex properties, and it suggests that geometry may hold the key to unlocking secrets that numbers alone cannot reveal.

To anyone intrigued by it, the Mersenne Star feels like a constellation in a mathematical sky—a pattern with significance. It raises questions: Why do these points stand out? Why do they cluster as if drawn by an unseen force? The presence of Fibonacci and Lucas numbers, alongside periodic patterns at specific irrational coordinates, shows this isn't a random setup.

For mathematicians, this figure sparks curiosity: Could these shapes signal a new geometric approach to primes? Might this be the start of a map to deeper insights? The answer lies in the *Quanta Prime Sequence (QPS)*, which turns abstract primes into visible forms—stars with neighbours and structures with influence.

This geometric representation is not merely decorative; it conveys subtle mathematical signals, hinting at hidden patterns and deeper connections within prime structures.

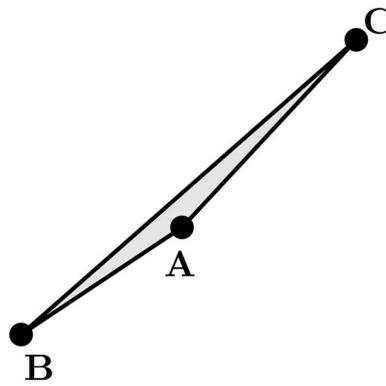
## 6. Notable findings and geometric interpretations

Analysis of the QPS-based Mersenne Star diagrams reveals non-random spatial behaviour:

- Periodic patterns emerge around specific coordinates such as  $x = 1$  and

$$y \in \{1, 0, -1, \sqrt{2}, \sqrt{3}, \sqrt{5}\}.$$

- Fibonacci and Lucas numbers, as well as golden ratio—related points, consistently align near the geometric star.
- These elements form a coherent structure, suggesting a deeper spatial logic behind Mersenne primes.



**Figure 4.** The Mersenne Triangle with vertices at  $A(0, -1)$ ,  $B(-2, -5)$ , and  $C(1, 4)$ .

This geometry evokes parallels with symmetry and recurrence in physical systems. While no direct connection is asserted, the structural elegance hints at organizing principles that transcend arithmetic—recasting primes as elements within a broader geometric and potentially physical framework.

## 7. Understanding the quanta prime sequence (QPS)

### 7.1. What is the quanta prime sequence (QPS)?

Prime numbers have fascinated mathematicians for centuries. Despite their simple definition, they continue to reveal complex and subtle patterns that inspire both theoretical and applied research. This work builds upon a recently introduced structure known as the *Quanta Prime Sequence* (QPS) (Ibrahim, 2025), which provides a unified algebraic and geometric framework for exploring prime numbers. QPS generalizes several classical recursive sequences and uncovers a hidden geometric structure. This structure, based on internal symmetries within QPS, leads naturally to the emergence of the *Mersenne Star*, which is the central focus of this paper.

QPS is defined by the following elements:

- A natural number  $n \in \mathbb{N}$ ,
- A non-zero point  $(\zeta, \xi) \in \mathbb{R}^2$ ,
- Two non-negative integers: the horizontal index  $r$ , and the vertical index  $k$ .

The sequence is denoted:

$$\Omega_r(k|\zeta, \xi|n)$$

and satisfies the recurrence relation:

$$\begin{aligned} \Omega_r(k|\zeta, \xi|n) = & (2\zeta - \xi)(n - r - k) \cdot \Omega_r(k - 1|\zeta, \xi|n) \\ & - 2\zeta(n - 2r - \delta(n - 1)) \cdot \Omega_{r+1}(k - 1|\zeta, \xi|n) \end{aligned} \quad (1)$$

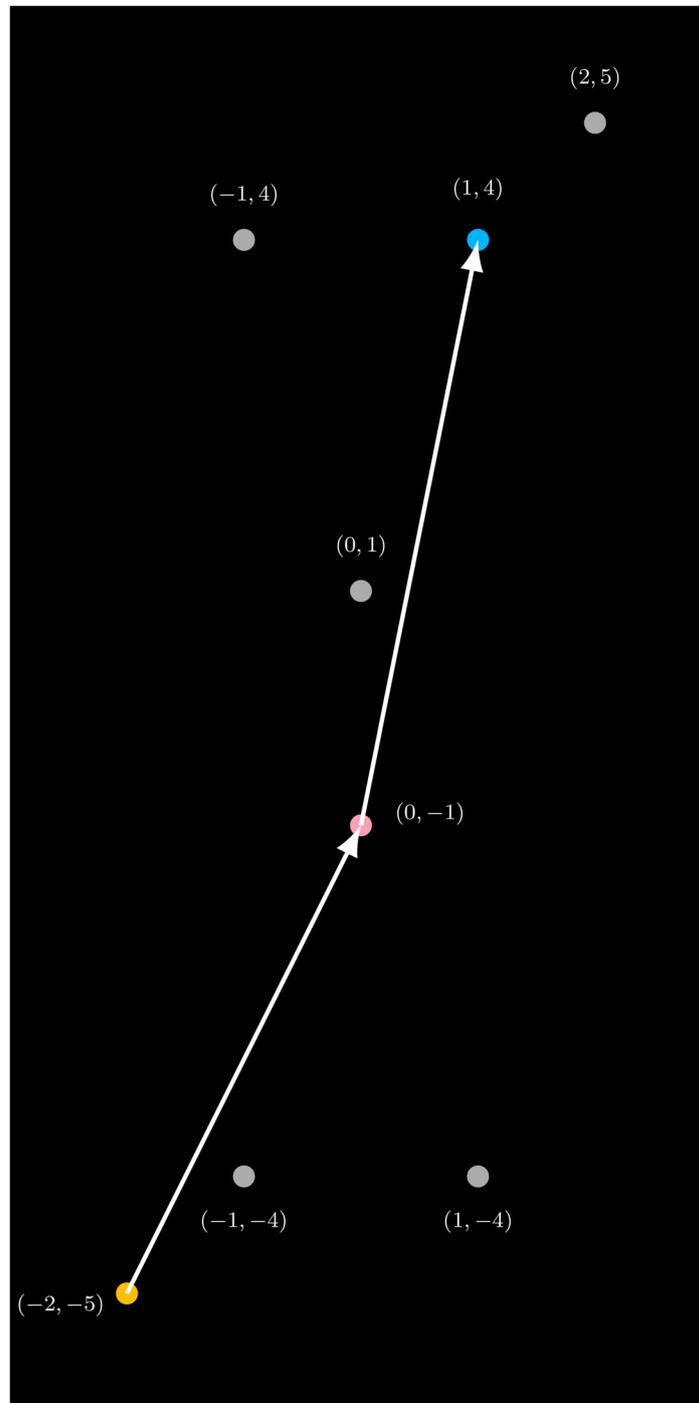
with the initial condition:

$$\Omega_r(0|\zeta, \xi|n) = 1 \quad \text{for all } r \geq 0 \quad (2)$$

where  $\delta(n) = n \bmod 2$  is the parity function.

### 7.2. Layered structure and two-dimensional evolution

Unlike traditional one-dimensional sequences, the Quanta Prime Sequence evolves along two orthogonal directions:

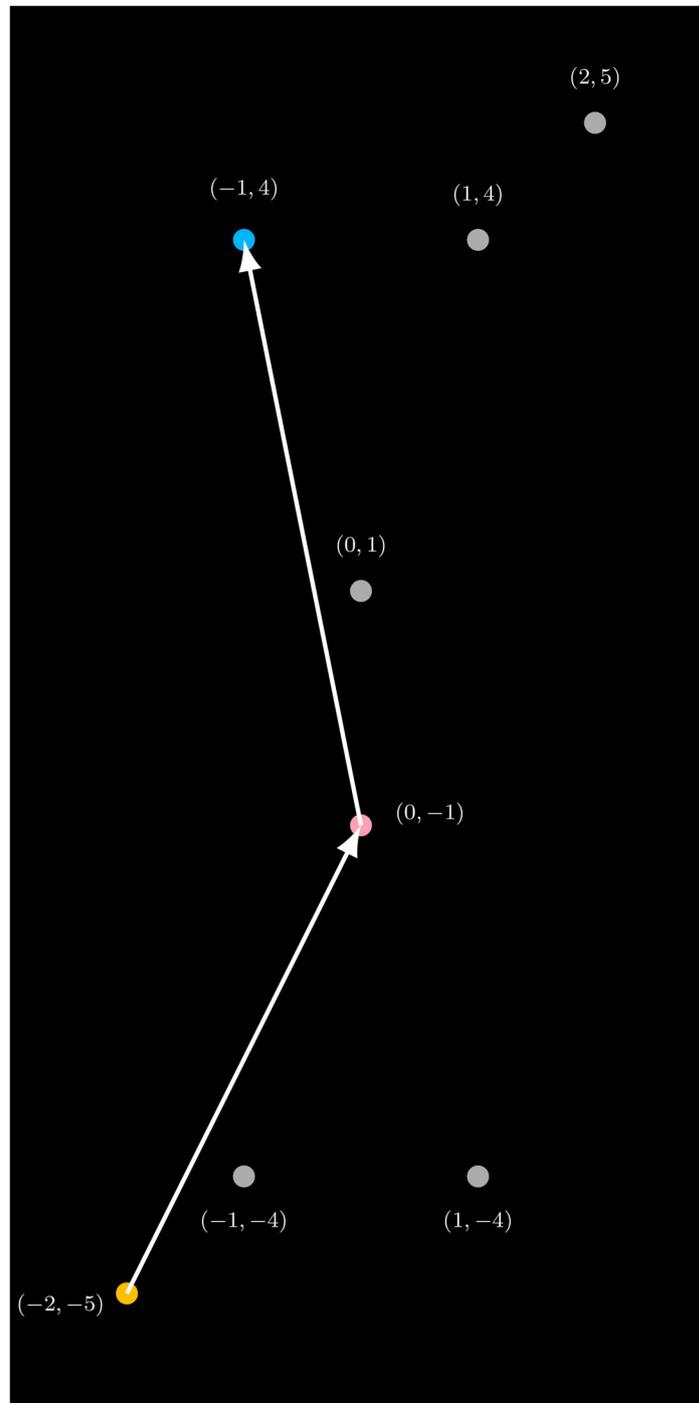


**Figure 5.** Geometric visualization of the first divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, 4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

- The vertical index  $k$  represents the recursion depth. Increasing  $k$  moves to a deeper layer in the structure.
- The horizontal index  $r$  represents the position within each layer. Increasing  $r$  moves laterally along the layer.



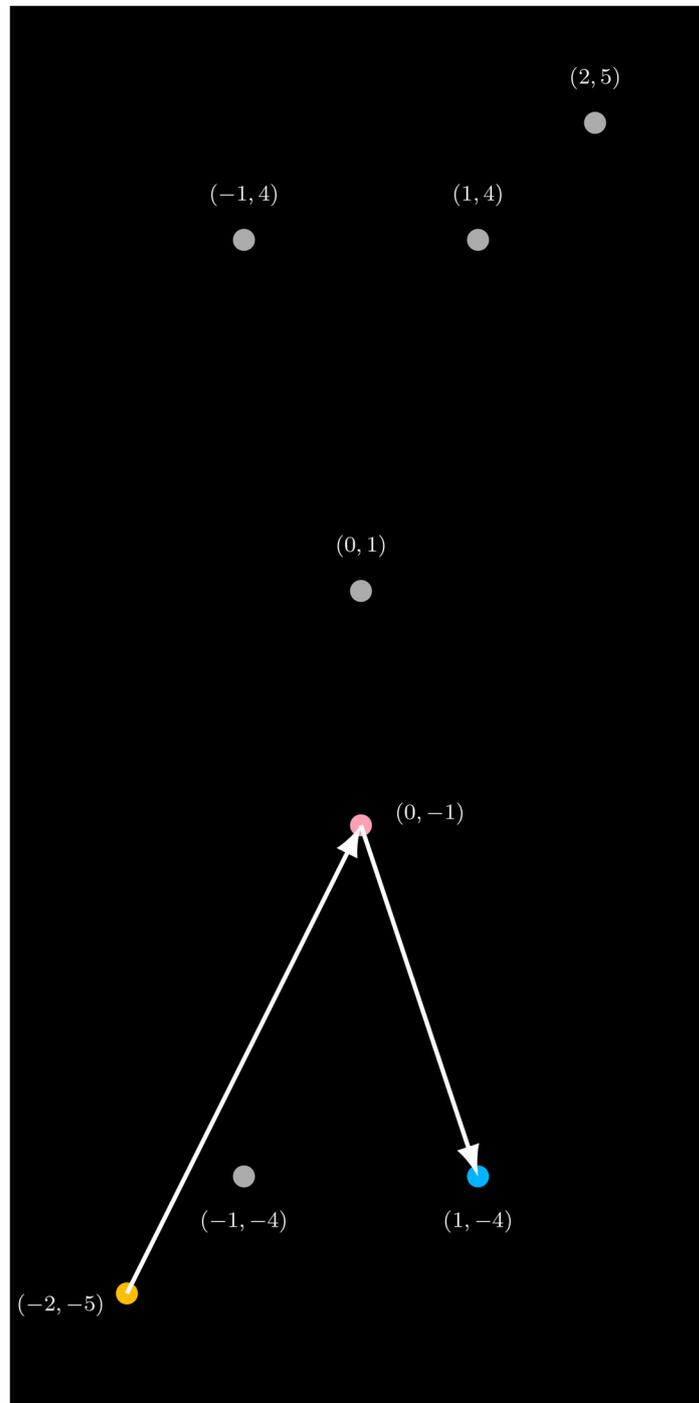
**Figure 6.** Geometric visualization of the second divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid 0, -1 \mid n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid -2, -5 \mid p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid -1, 4 \mid n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 0, -1 \mid p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

Each value in the sequence depends on two values from the previous recursive level:

- $\Omega_r(k - 1)$  at the same row,
- $\Omega_{r+1}(k - 1)$  at the next row.

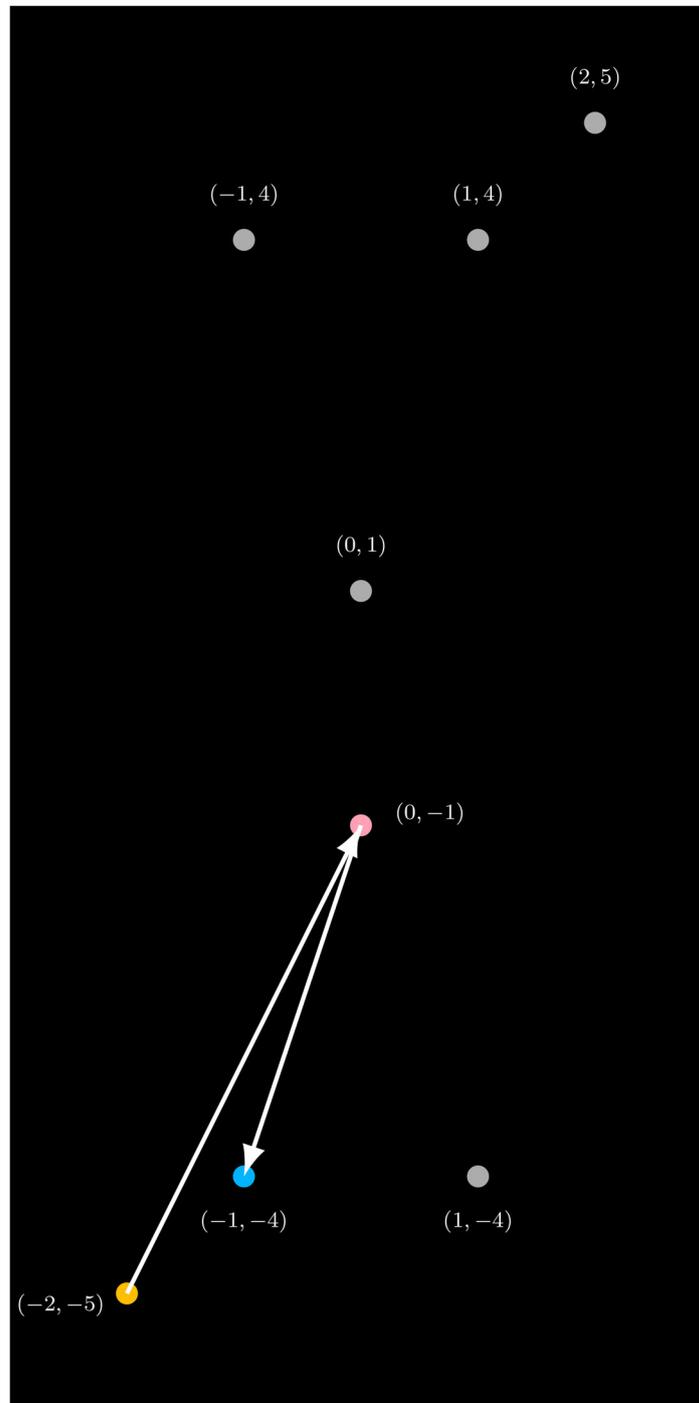


**Figure 7.** Geometric visualization of the third divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

This creates a two-dimensional, triangular cascade of symbolic expressions, whose internal structure often displays symmetries and periodicities. These features become especially significant when QPS is evaluated at specific geometric points related to prime number behaviour, as will be shown in the sections that follow.



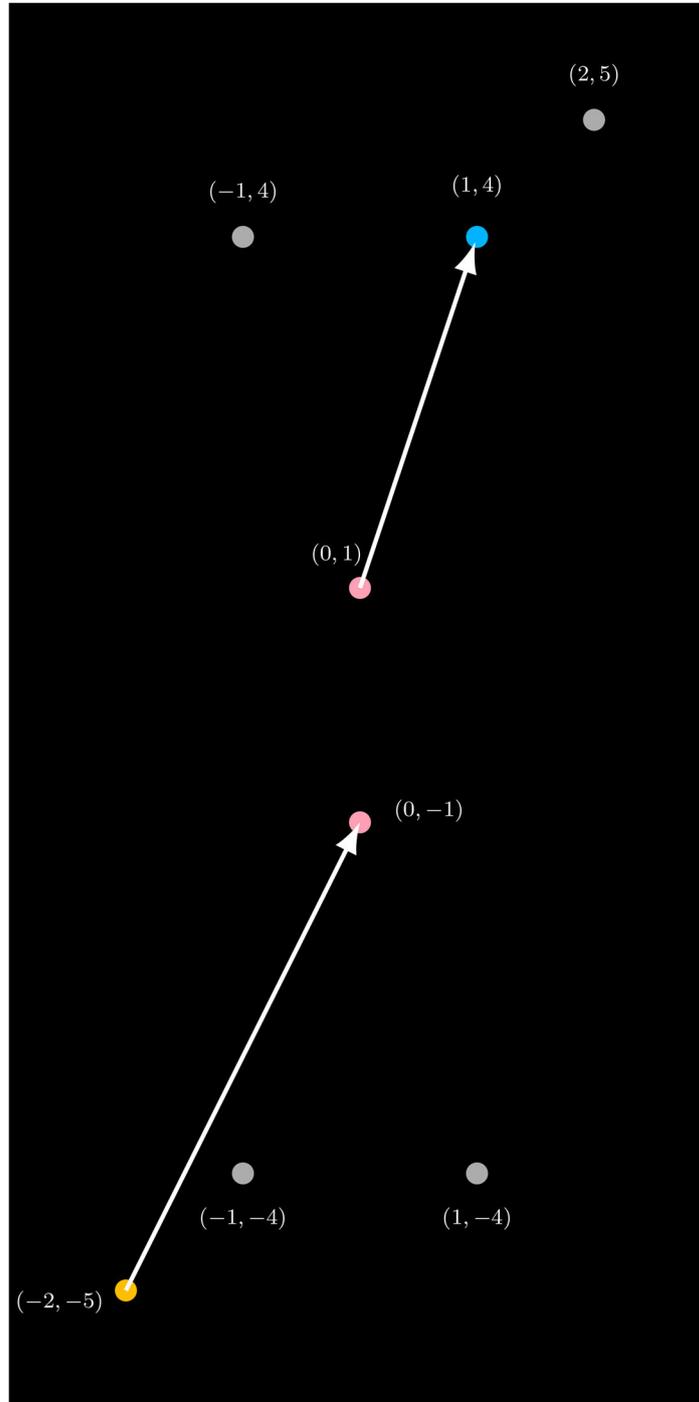
**Figure 8.** Geometric visualization of the fourth divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

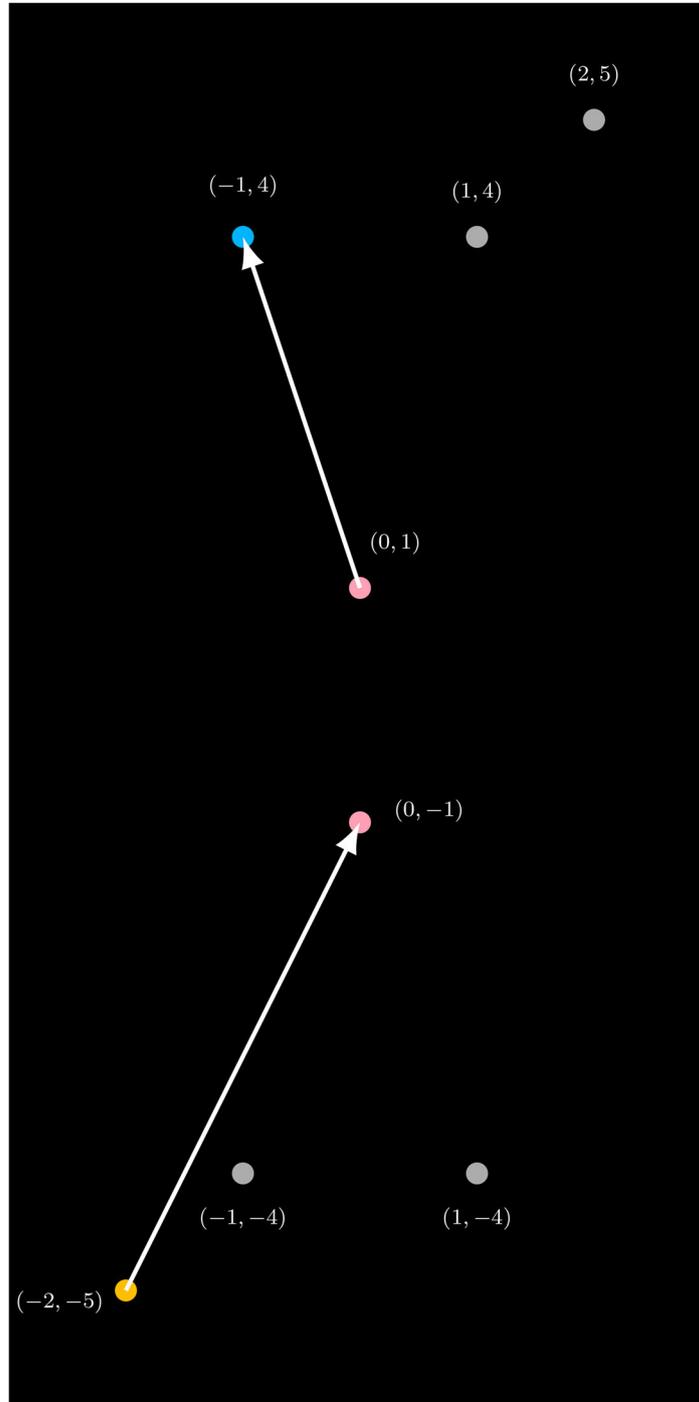
### 7.3. Connection to the Psi-sequence

A key development is the connection between QPS and the analytic Psi-sequence  $\Psi(\zeta, \xi, n)$ , which was introduced in earlier work. Unlike the recursive nature of QPS, the Psi-sequence is defined by a closed-form expression:



**Figure 9.** Geometric visualization of a divisibility condition.  
 $\Omega_0(\lfloor \frac{n}{2} \rfloor | 0, -1 | n) \cdot \Omega_0(\lfloor \frac{n}{2} \rfloor | -2, -5 | p) \mid \Omega_0(\lfloor \frac{n}{2} \rfloor | 1, 4 | n) \cdot \Omega_0(\lfloor \frac{n}{2} \rfloor | 0, 1 | p)$   
 This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

$$\Psi(a, b, n) = \frac{(2a - b)^{\lfloor n/2 \rfloor}}{2^n} \left[ \left( 1 + \sqrt{\frac{b + 2a}{b - 2a}} \right)^n + \left( 1 - \sqrt{\frac{b + 2a}{b - 2a}} \right)^n \right] \quad (3)$$



**Figure 10.** Geometric visualization of a divisibility condition.

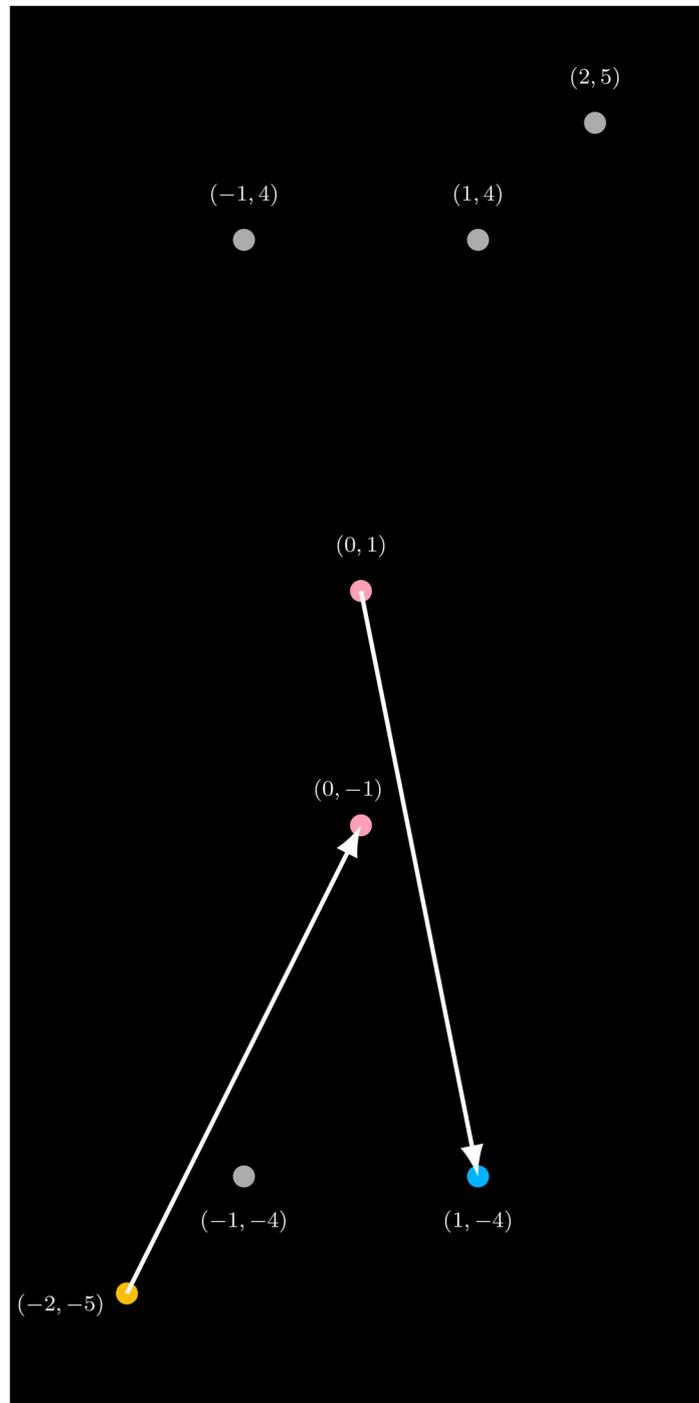
$$\Omega_0(\lfloor \frac{n}{2} \rfloor | 0, -1 | n) \cdot \Omega_0(\lfloor \frac{n}{2} \rfloor | -2, -5 | p) \mid \Omega_0(\lfloor \frac{n}{2} \rfloor | -1, 4 | n) \cdot \Omega_0(\lfloor \frac{n}{2} \rfloor | 0, 1 | p)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

This sequence captures harmonic growth and golden-ratio-like behaviour, without the need for iterative computation.

The two sequences are linked through the identity:

$$\Psi(\zeta, \xi, n) = \frac{\Omega_0(\lfloor \frac{n}{2} \rfloor | \zeta, \xi | n)}{(n-1)(n-2) \cdots (n - \lfloor n/2 \rfloor)} \tag{4}$$

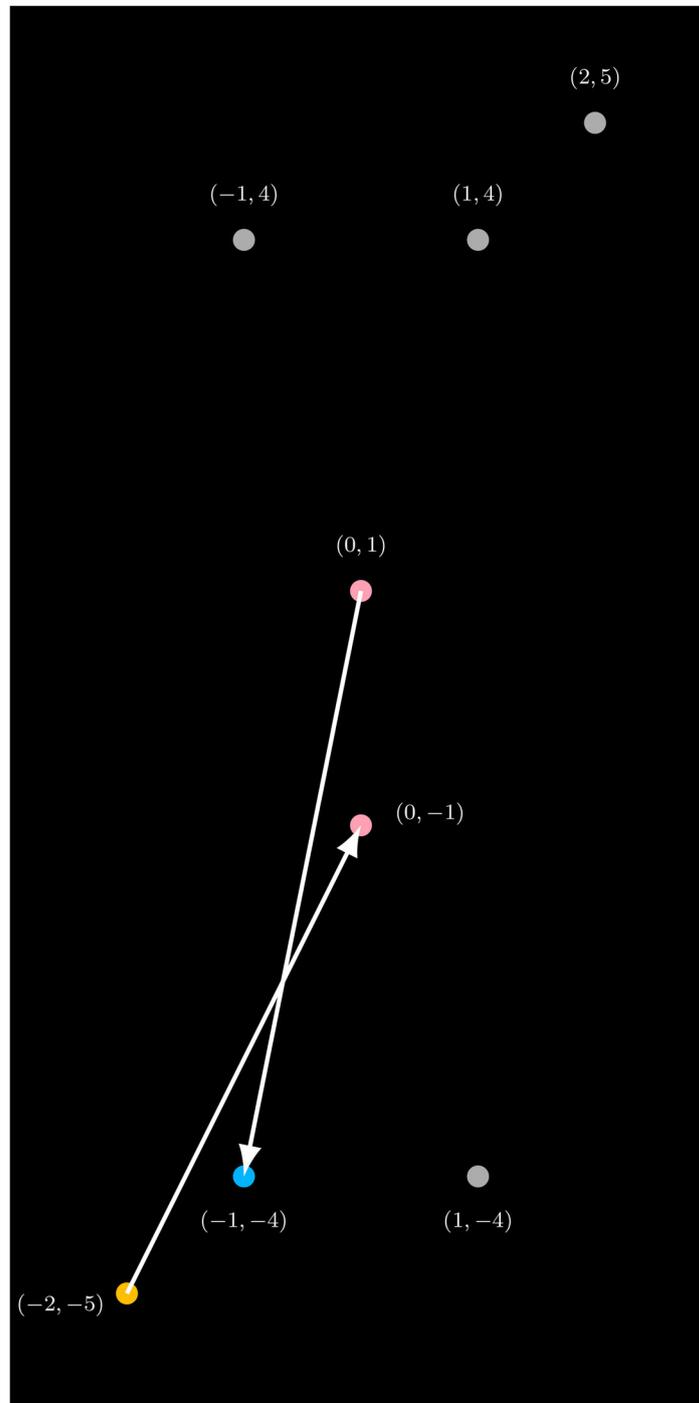


**Figure 11.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

This result is known as the Second Fundamental Theorem of QPS (Ibrahim, 2025). It translates the complexity of the recursive QPS structure into a smooth analytic form, suggesting a deeper connection between recurrence and harmony in number theory. Important Note. When computing  $\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | \zeta, \xi | n\right)$ , it is crucial not to fix  $r = 0$  at the beginning of the recursive process. Since the recurrence involves a

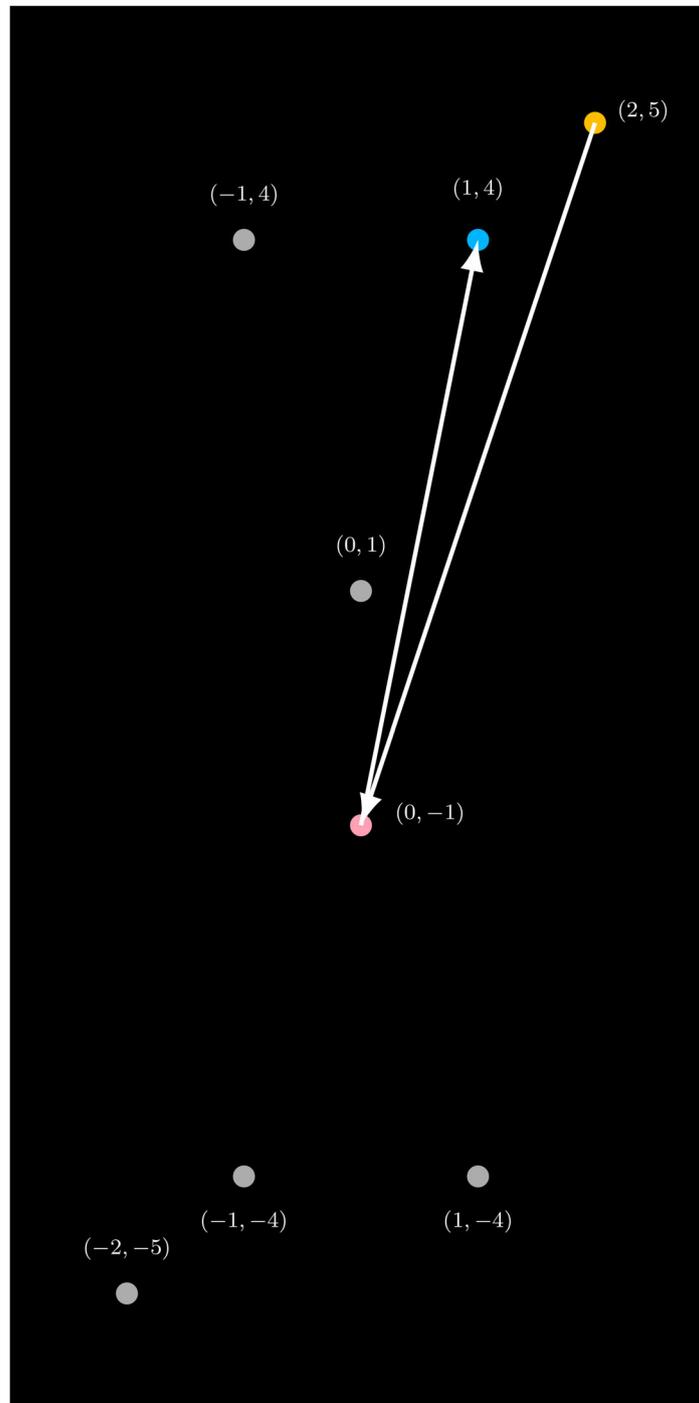


**Figure 12.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 0, -1 \mid n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid -2, -5 \mid p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid -1, -4 \mid n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 0, 1 \mid p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

dependency on  $\Omega_{r+1}$ , the correct computation must proceed symbolically across increasing values of  $r$  at each recursive level  $k$ , forming a triangular structure. Only after the full symbolic evolution is completed should one evaluate the expression at  $r = 0$ . Ignoring this step will lead to incorrect results and misinterpretation of the underlying structure.



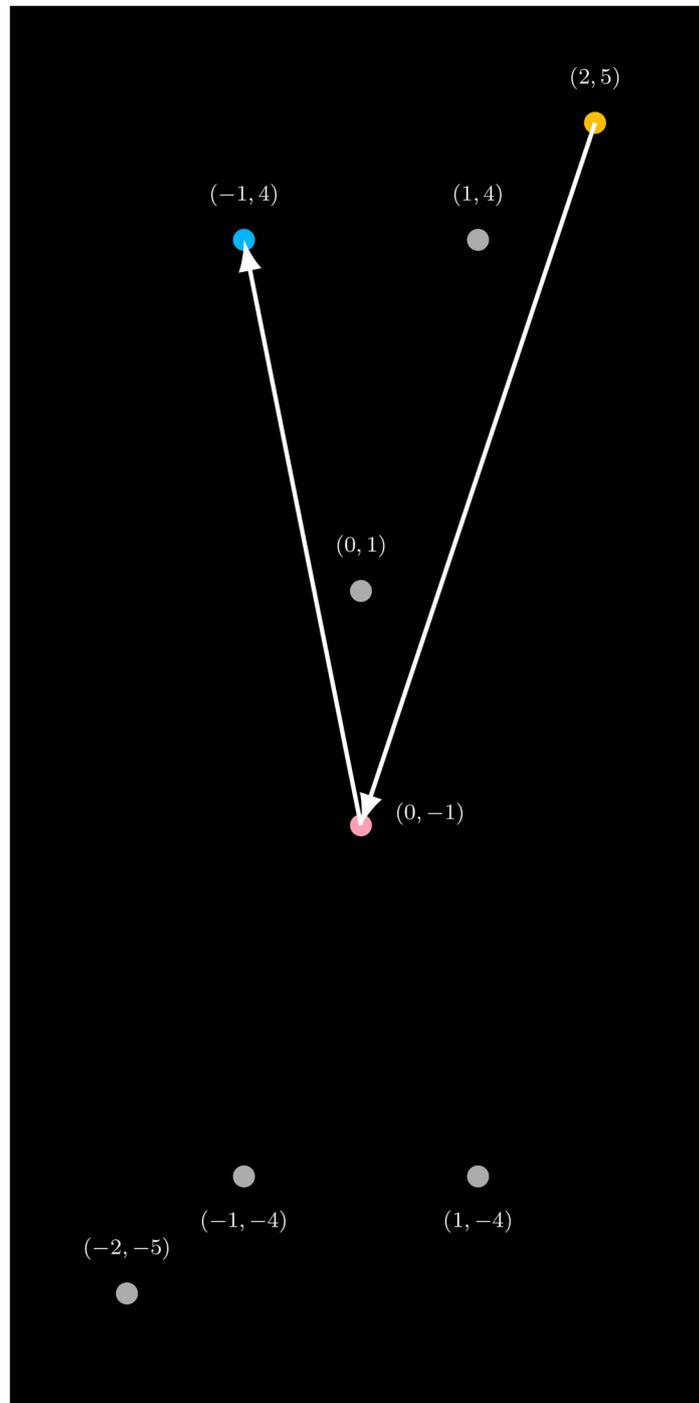
**Figure 13.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, 4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

## 8. Computing quanta—examples: Fibonacci and Lucas—points

In this section, we reveal a surprising connection between the Quanta Prime Sequence and classical sequences such as the Fibonacci and Lucas numbers. This connection emerges in the vicinity of a geometric structure known as the Mersenne Star. We provide a detailed computation showing how the



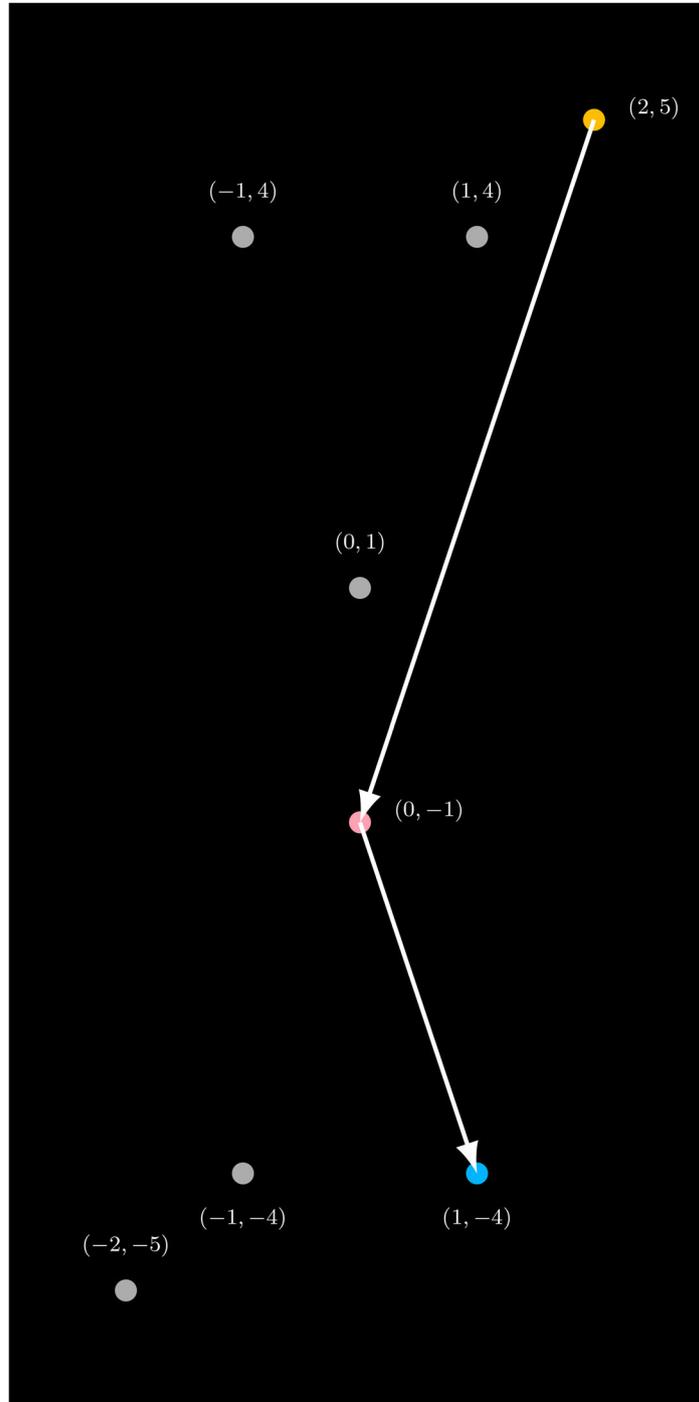
**Figure 14.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, 4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^{2^p} - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

values of the Quanta Prime Sequence  $\Omega_0(2|\pm 1, \pm 3|5)$  are directly linked to Fibonacci and Lucas numbers. Two independent derivations are presented to support this connection.

- Direct symbolic computation using the recurrence definition of the QPS.
- Direct evaluation using the Second Fundamental Theorem of the QPS (see Ibrahim, 2025).



**Figure 15.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

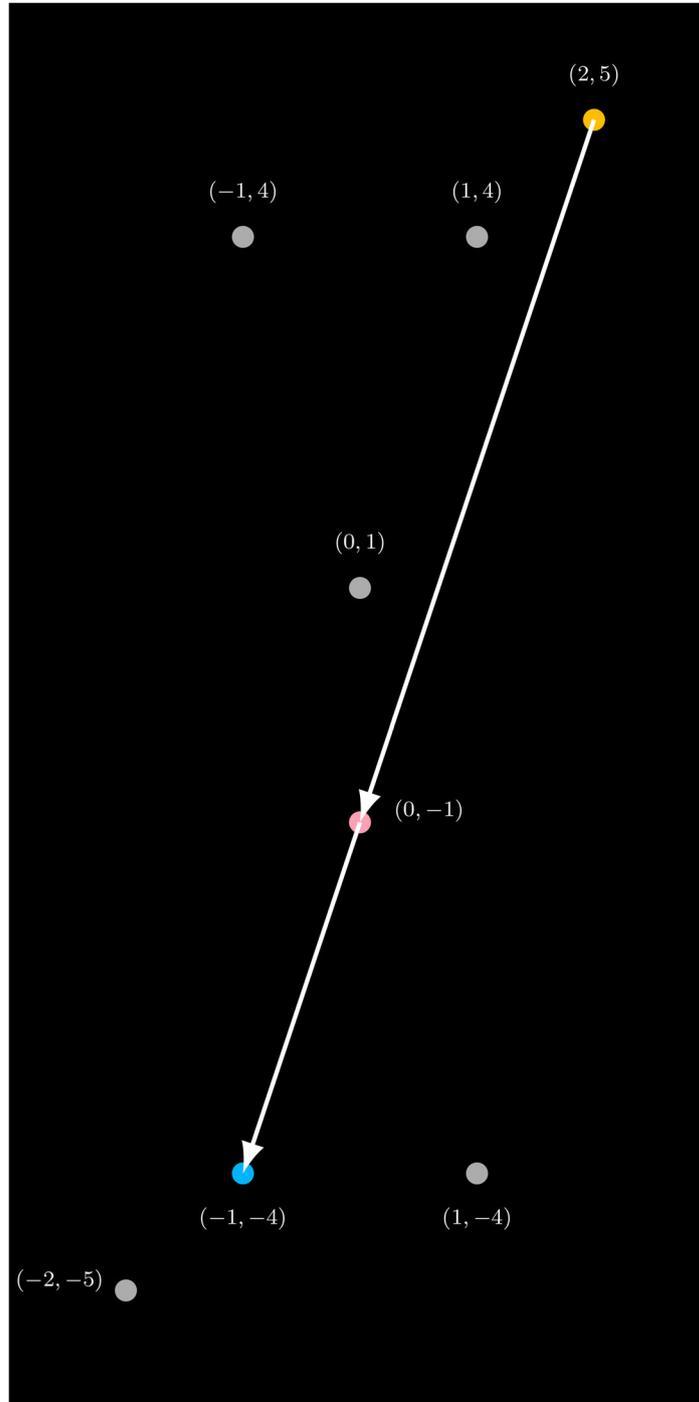
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

### 8.1. Direct computation via QPS recurrence

The Quanta Prime Sequence satisfies the recurrence relation:

$$\Omega_r(k | \zeta, \xi | n) = (2\zeta - \xi)(n - r - k)\Omega_r(k - 1 | \zeta, \xi | n) - 2\zeta(n - 2r - \delta(n - 1))\Omega_{r+1}(k - 1 | \zeta, \xi | n),$$

with the initial condition:



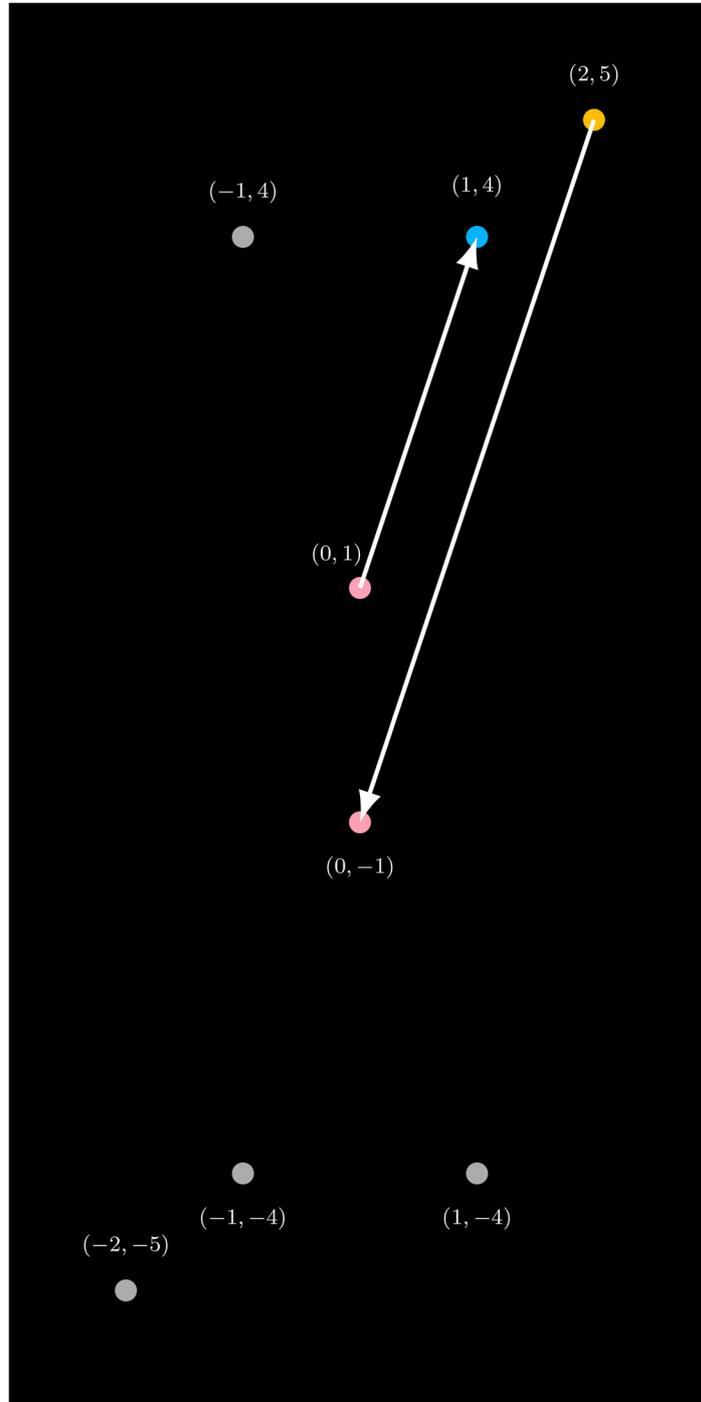
**Figure 16.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5| p\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1| n\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, -4| n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1| p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

$$\Omega_r(0 | \zeta_r, \xi | n) = 1.$$

For the specific case  $n = 5$ , we have  $\delta(n - 1) = \delta(4) = 0$ , and we set  $k = 2$ . Then, for any non-negative integer  $r$ , the recurrence simplifies into the following layers:

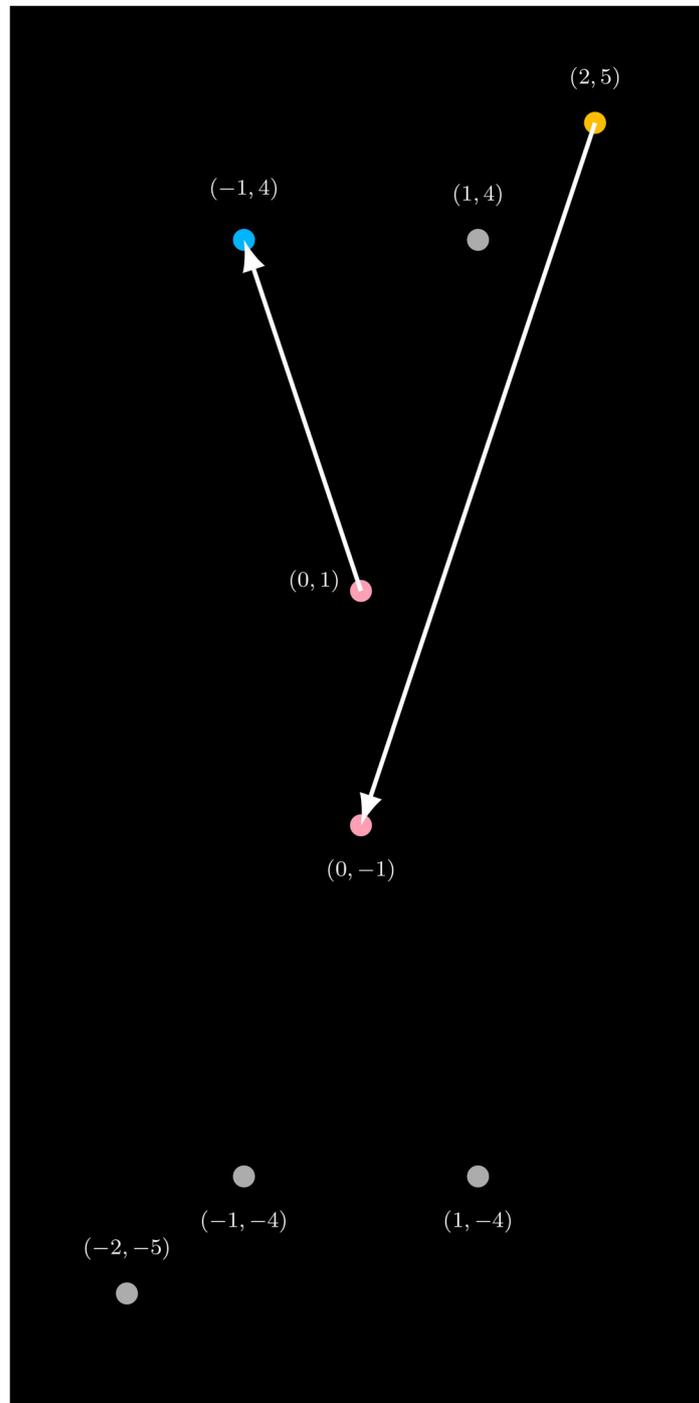


**Figure 17.** Geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

$$\begin{aligned} \Omega_r(2|\zeta, \xi|5) &= (2\zeta - \xi)(3 - r)\Omega_r(1|\zeta, \xi|5) - 2\zeta(5 - 2r)\Omega_{r+1}(1|\zeta, \xi|5), \quad (5) \\ \Omega_r(1|\zeta, \xi|5) &= (2\zeta - \xi)(4 - r)\Omega_r(0|\zeta, \xi|5) - 2\zeta(5 - 2r)\Omega_{r+1}(0|\zeta, \xi|5), \\ \Omega_r(0|\zeta, \xi|5) &= 1. \end{aligned}$$



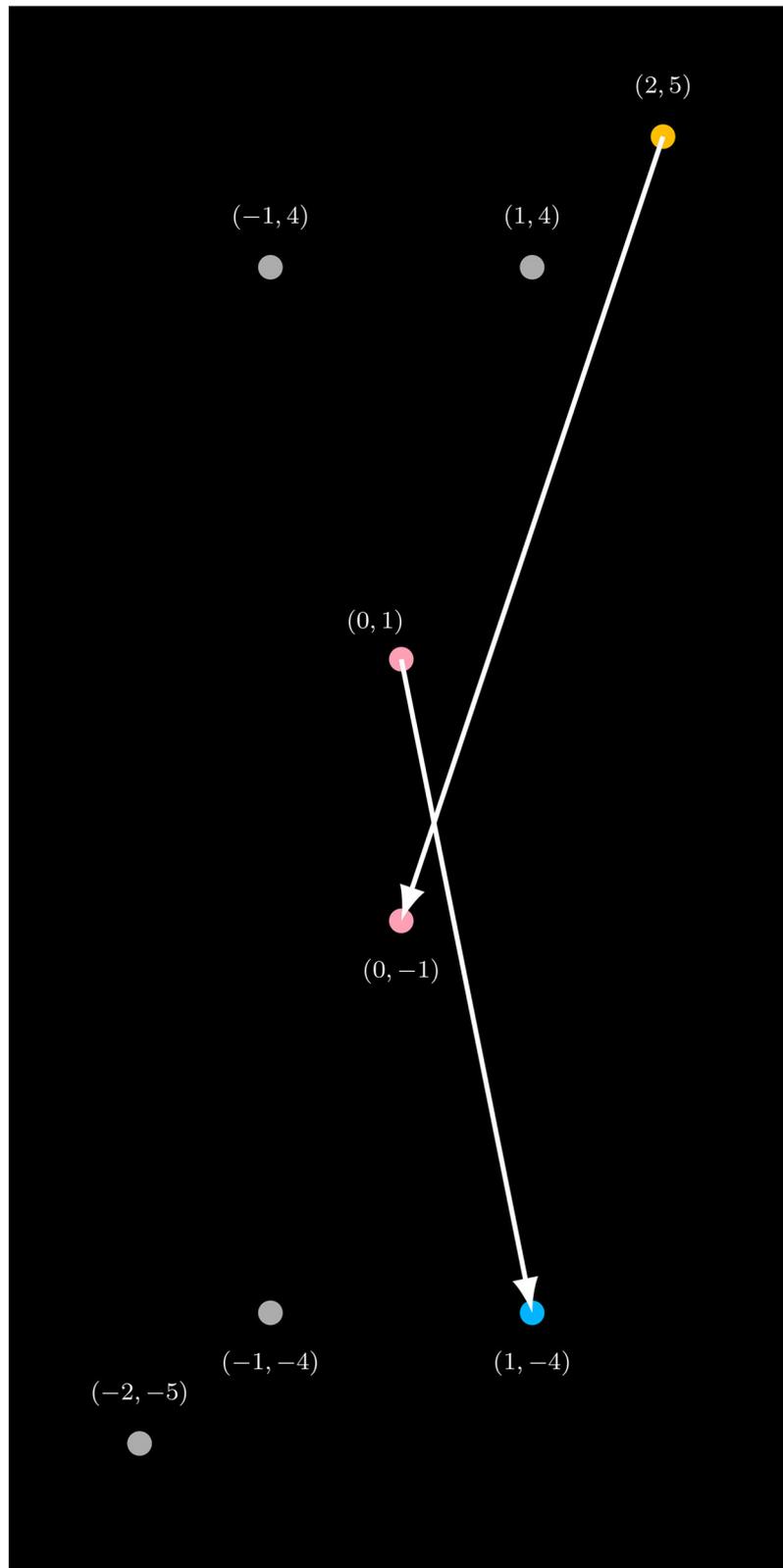
**Figure 18.** Alternative geometric visualization of a divisibility condition.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, 4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

### 8.2. Direct evaluation using the Second Fundamental Theorem of the QPS

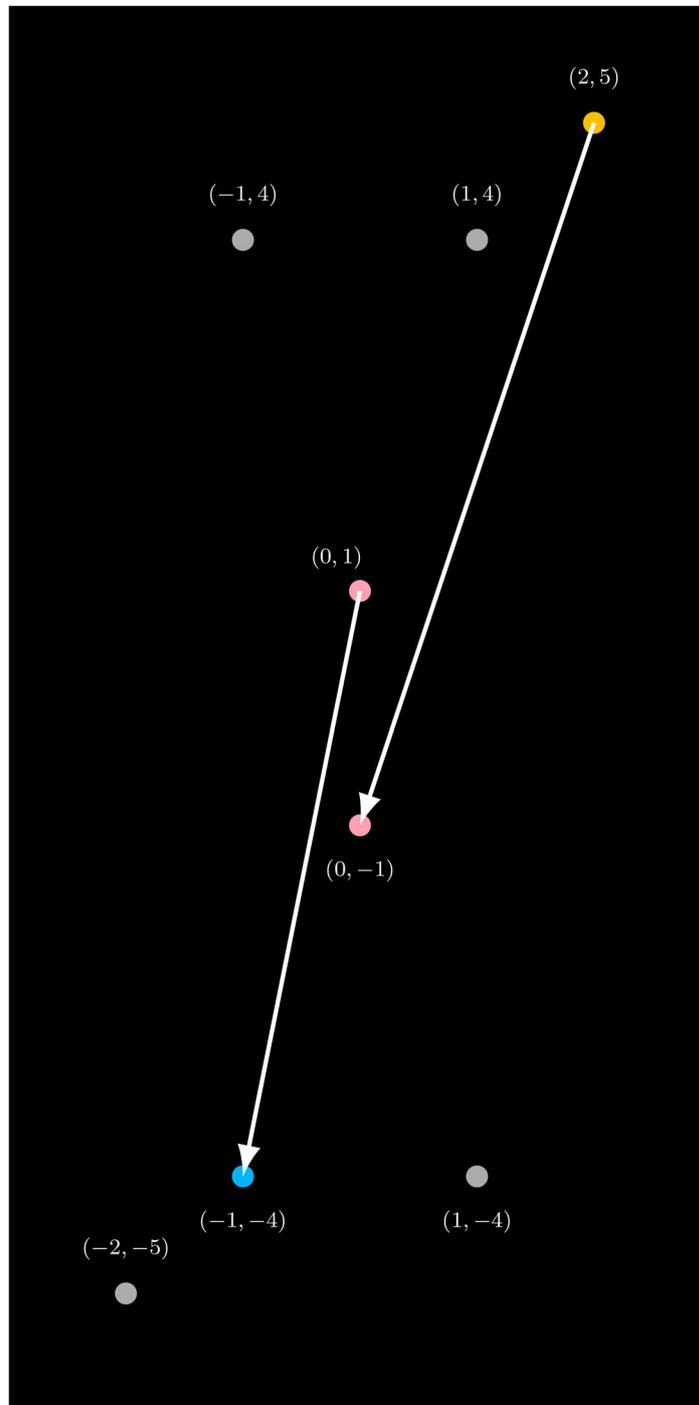
The Second Fundamental Theorem of the Quanta Prime Sequence provides a direct way to compute the Quanta values through the following relationship:



**Figure 19.** Third geometric representation of the divisibility relation.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | p\right),$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

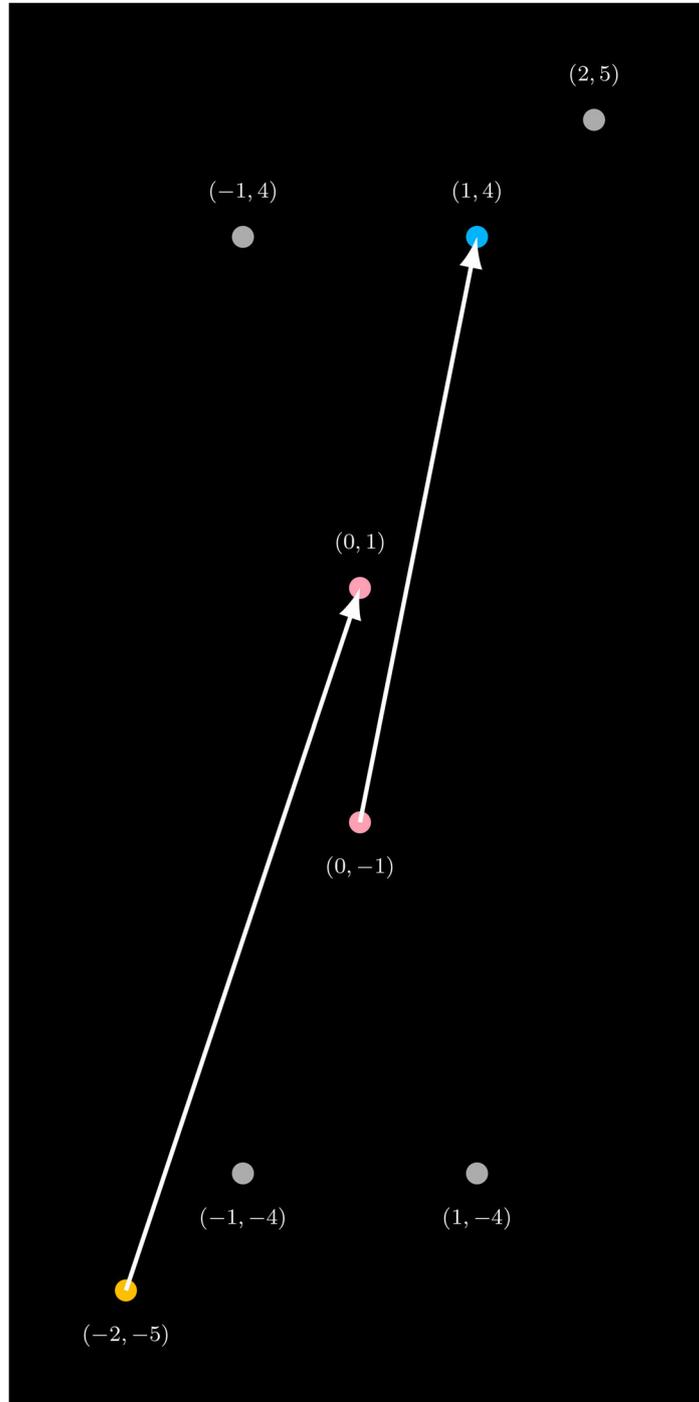


**Figure 20.** Fourth configuration showing mirrored diagonal reflection.  
 $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$   
 This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \middle| \zeta, \xi | n\right) = (n - 1)(n - 2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot \Psi(\zeta, \xi, n)$$

**8.2.1. Case 1:  $(\zeta, \xi) = (1, 3)$ —direct computation via the recurrence relation**

$$2\zeta - \xi = -1, \quad 2\zeta = 2$$



**Figure 21.** Fifth configuration: complete reflection of the initial geometric identity.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, 4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

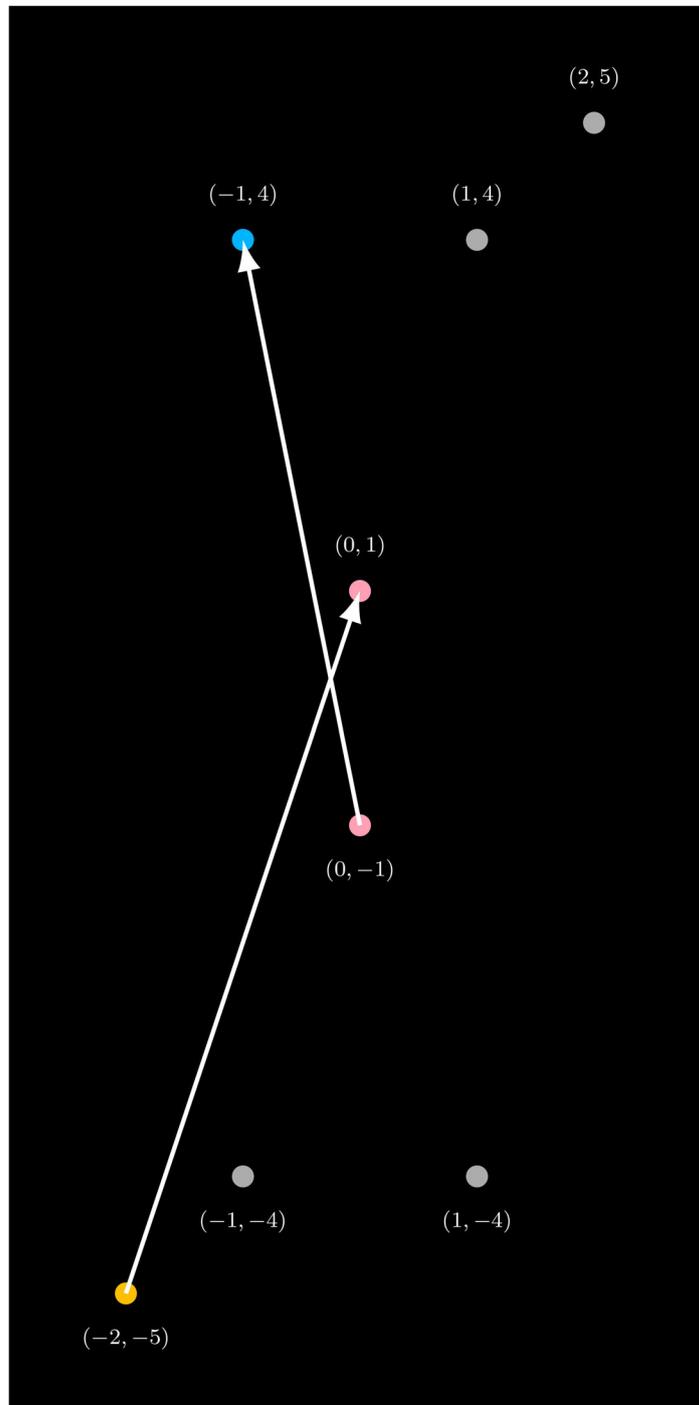
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

then

$$\Omega_r(2 | 1, 3 | 5) = -(3 - r)\Omega_r(1 | 1, 3 | 5) - 2(5 - 2r)\Omega_{r+1}(1 | 1, 3 | 5), \quad (6)$$

$$\Omega_r(1 | 1, 3 | 5) = -(4 - r)\Omega_r(0 | 1, 3 | 5) - 2(5 - 2r)\Omega_{r+1}(0 | 1, 3 | 5),$$

$$\Omega_r(0 | 1, 3 | 5) = 1.$$



**Figure 22.** Sixth configuration: rotational symmetry within the Mersenne Star.

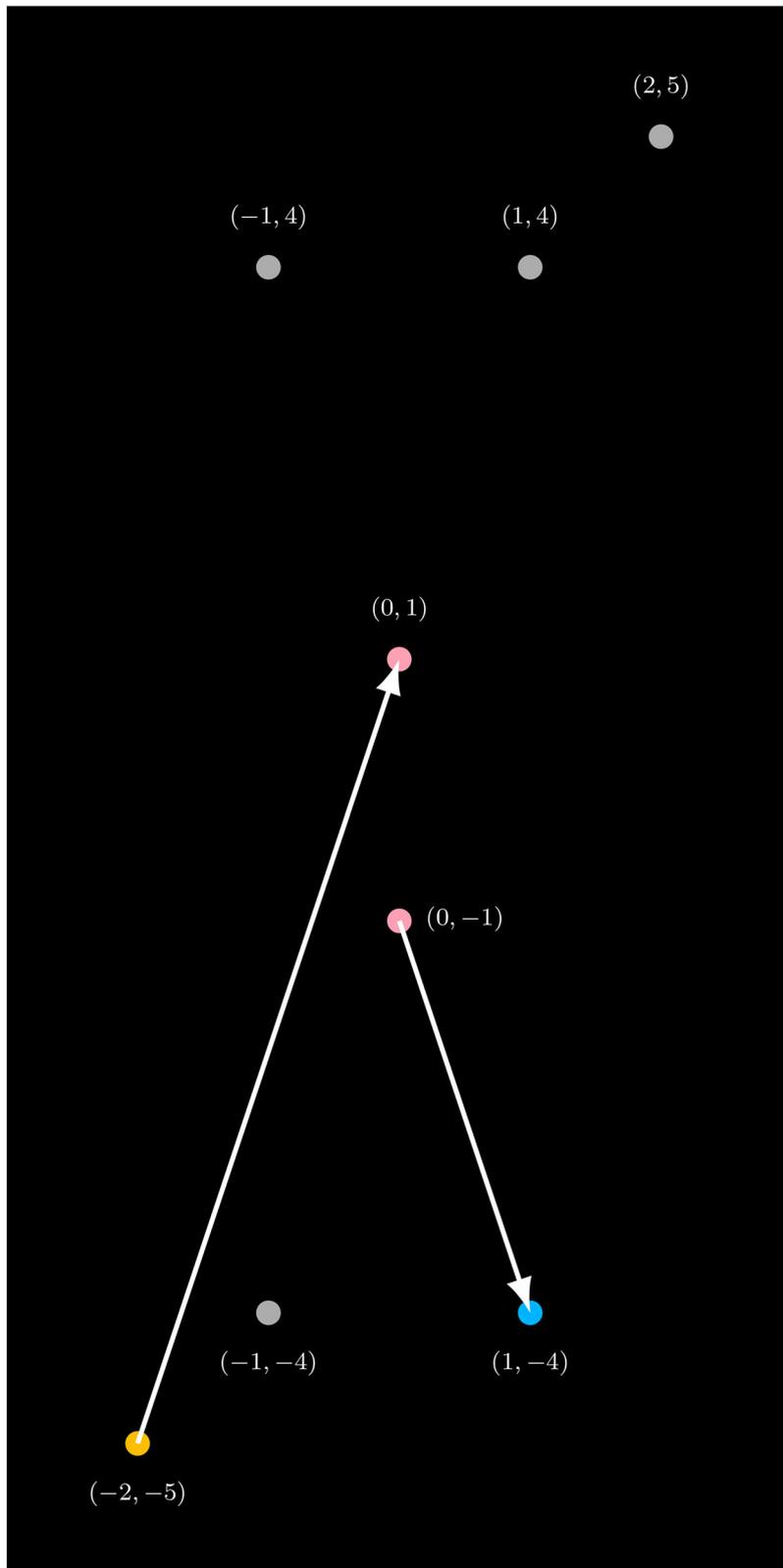
$\Omega_0(\lfloor \frac{p}{2} \rfloor | 0, 1 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | -2, -5 | p) \quad | \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | -1, 4 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | 0, -1 | p)$   
 This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

We now proceed with a bottom-up evaluation of the recurrence, computing each layer from the base case upward toward higher levels:

$$\Omega_r(0|1,3|5) = 1, (7)$$

$$\Omega_r(1|1,3|5) = -(4-r)(1) - 2(5-2r)(1) = 5r - 14,$$

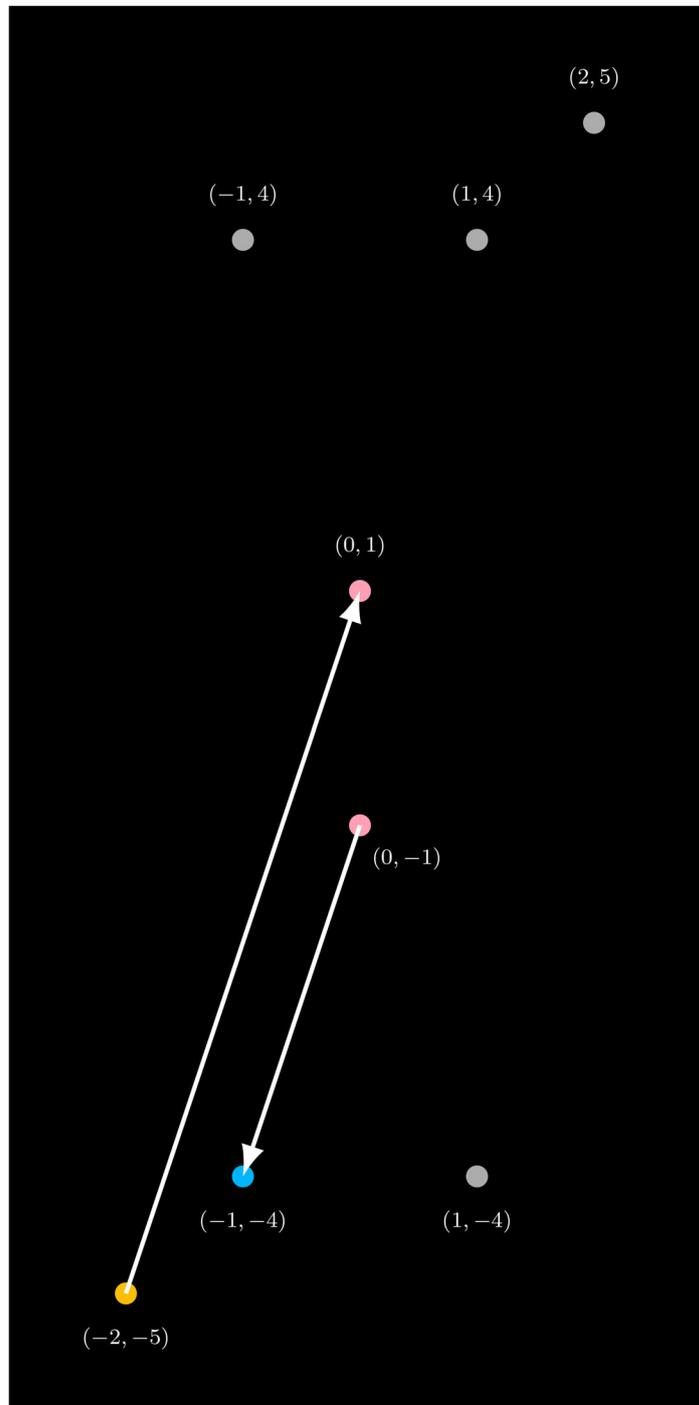
$$\Omega_r(2|1,3|5) = -(3-r)(5r-14) - 2(5-2r)(5[r+1]-14) = 25r^2 - 115r + 132.$$



**Figure 23.** Seventh configuration: reversed spiral flow within the Mersenne Star.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.



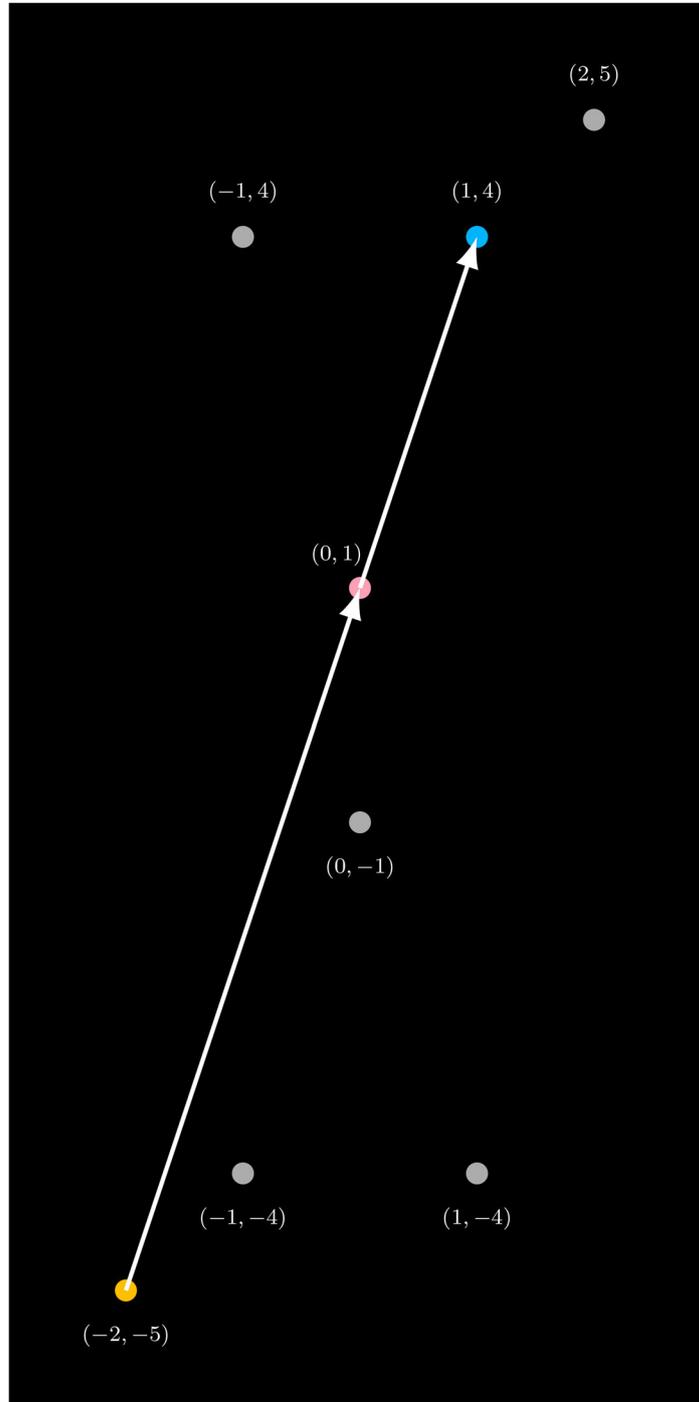
**Figure 24.** Eighth configuration: full inverse quadrant geometry of the Mersenne Star.

$\Omega_0(\lfloor \frac{p}{2} \rfloor | 0, 1 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | -2, -5 | p) \quad | \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | -1, -4 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | 0, -1 | p)$   
 This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

By setting  $r = 0$ , we obtain the following identity:

$$\Omega_0(2|1, 3|5) = 132 = 4 \cdot 3 \cdot 11. \tag{8}$$

Here, the factor 11 coincides with the Lucas number  $L(5)$ , demonstrating a precise alignment with the Second Fundamental Theorem of the Quanta Prime Sequence (Ibrahim, 2025). This appearance of the Lucas number is not coincidental—it is a direct consequence of the theorem’s internal structure.



**Figure 25.** Corrected ninth configuration: Proper transition and full geometry.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | p\right)$$

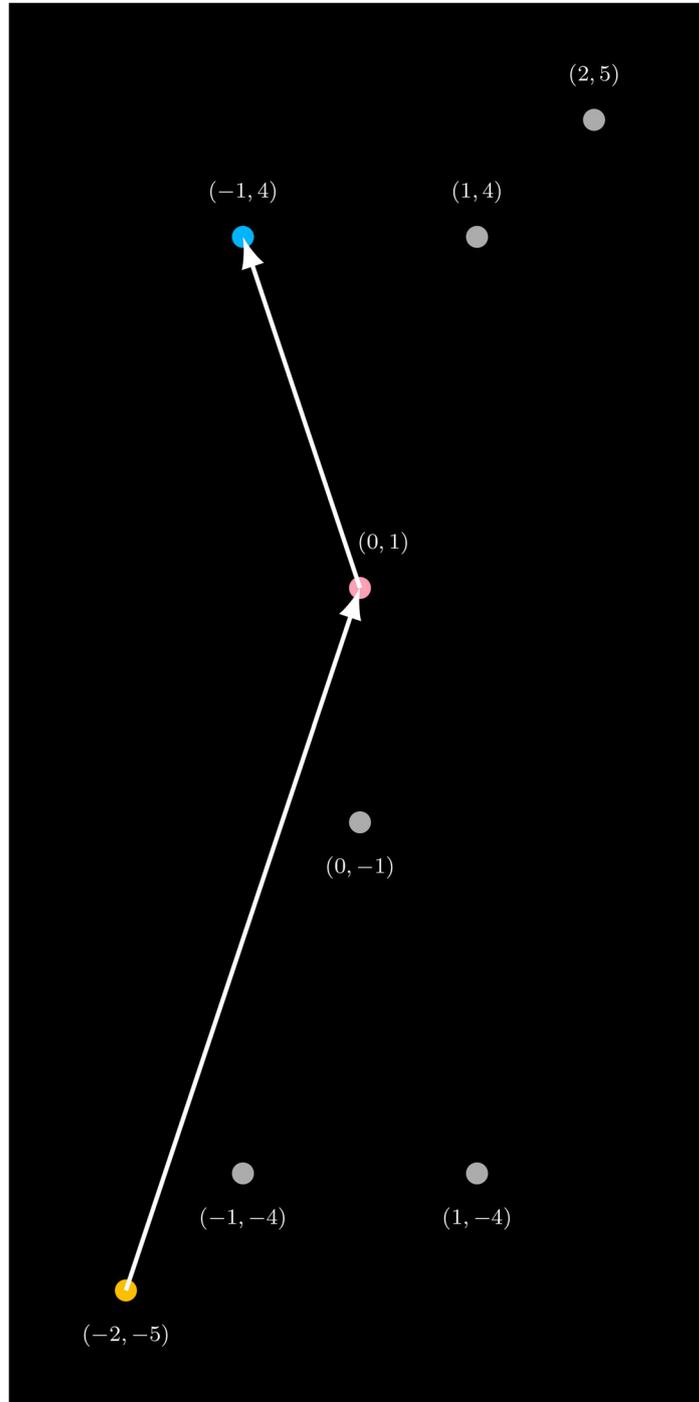
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

### 8.2.2. Verification via the Second Fundamental Theorem

For the point  $(\zeta, \xi) = (1, 3)$ , the general form of the theorem asserts:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 3 | n\right) = (n - 1)(n - 2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot \Psi(1, 3, n). \tag{9}$$

Since  $\Psi(1, 3, n) = L(n)$  for this case, and  $L(5) = 11$ , we recover Equation (8) exactly.



**Figure 26.** Tenth configuration: inverted diagonal symmetry with repeated transition.

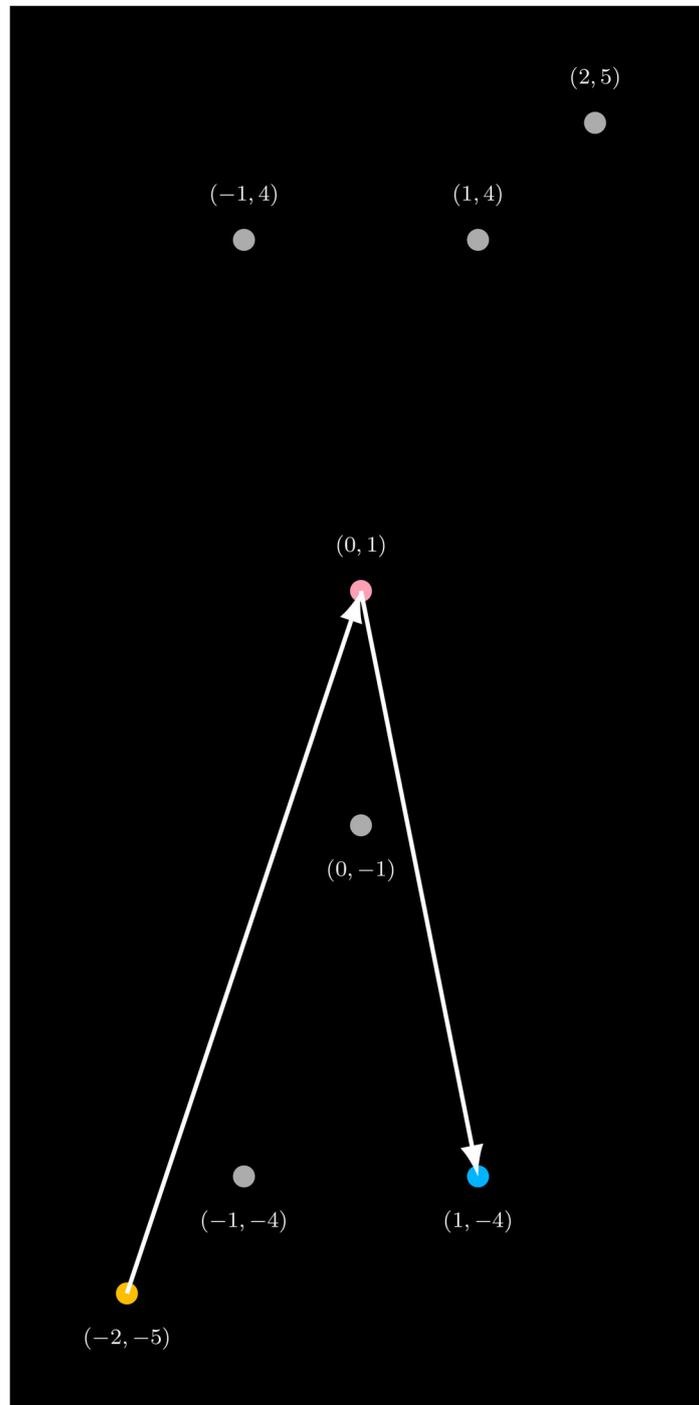
$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, 4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

**8.2.3. Case 2:  $(\zeta, \xi) = (1, -3)$ —direct computation via the recurrence relation**

We begin by computing the coefficients in the recurrence:

$$2\zeta - \xi = 2(1) - (-3) = 5, \quad 2\zeta = 2.$$



**Figure 27.** Eleventh configuration: downward diagonal closure across Mersenne Star.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

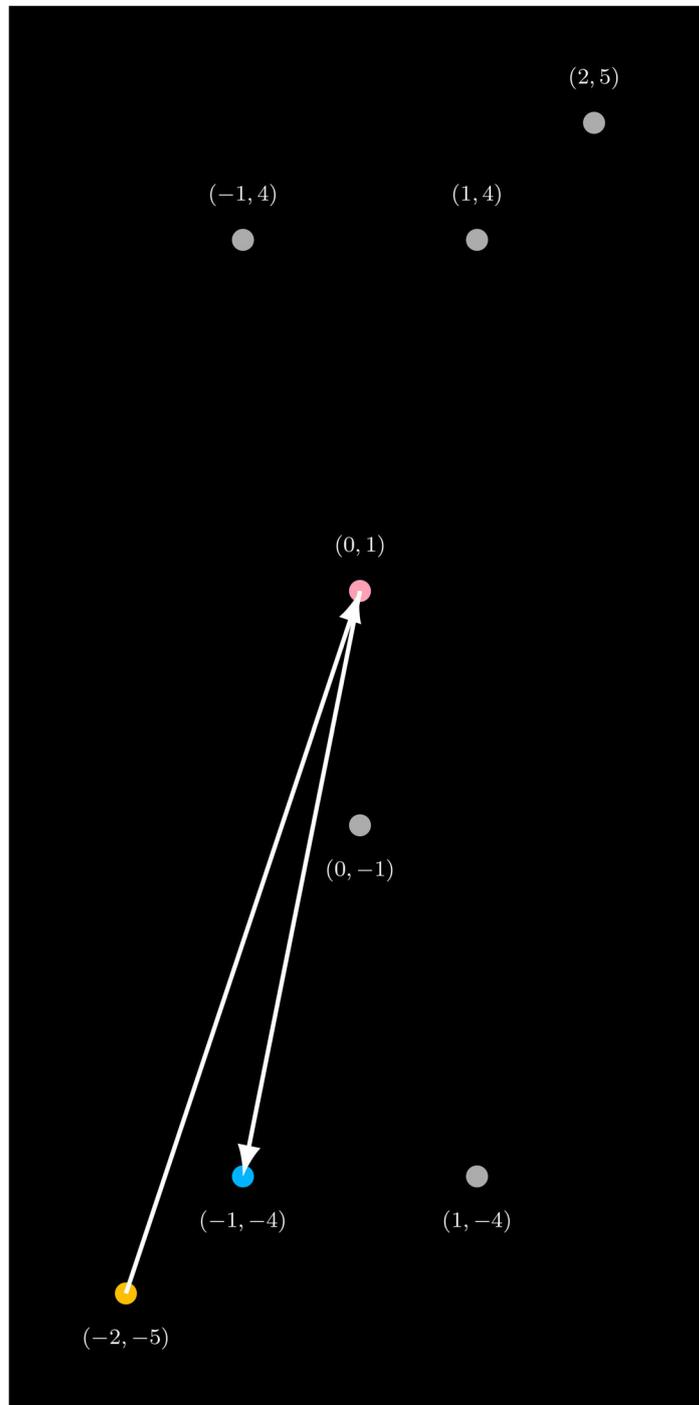
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

Substituting into the general recurrence with  $n = 5$ ,  $\delta(n - 1) = \delta(4) = 0$ , and  $k = 2$ , we obtain:

$$\Omega_r(2|1, -3|5) = 5(3 - r)\Omega_r(1|1, -3|5) - 2(5 - 2r)\Omega_{r+1}(1|1, -3|5), \quad (10)$$

$$\Omega_r(1|1, -3|5) = 5(4 - r)\Omega_r(0|1, -3|5) - 2(5 - 2r)\Omega_{r+1}(0|1, -3|5),$$

$$\Omega_r(0|1, -3|5) = 1.$$



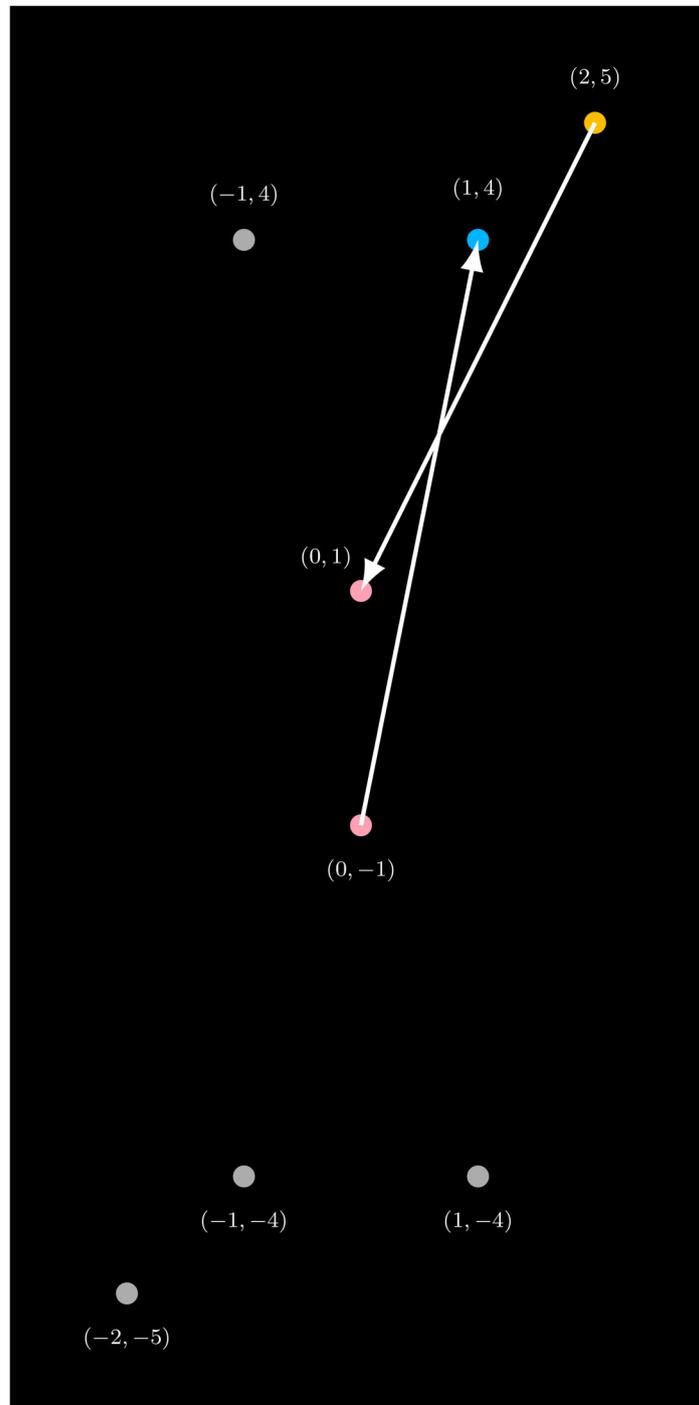
**Figure 28.** Twelfth configuration: final inverted geometric identity across lower axes.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

We now proceed with a bottom-up evaluation of the recurrence, computing each layer from the base case upward toward higher levels:

$$\begin{aligned} \Omega_r(0 | 1, -3 | 5) &= 1, (11) \\ \Omega_r(1 | 1, -3 | 5) &= 5(4 - r)(1) - 2(5 - 2r)(1) = -r + 10, \\ \Omega_r(2 | 1, -3 | 5) &= 5(3 - r)(-r + 10) - 2(5 - 2r)(-[r + 1] + 10) = r^2 - 19r + 60. \end{aligned}$$



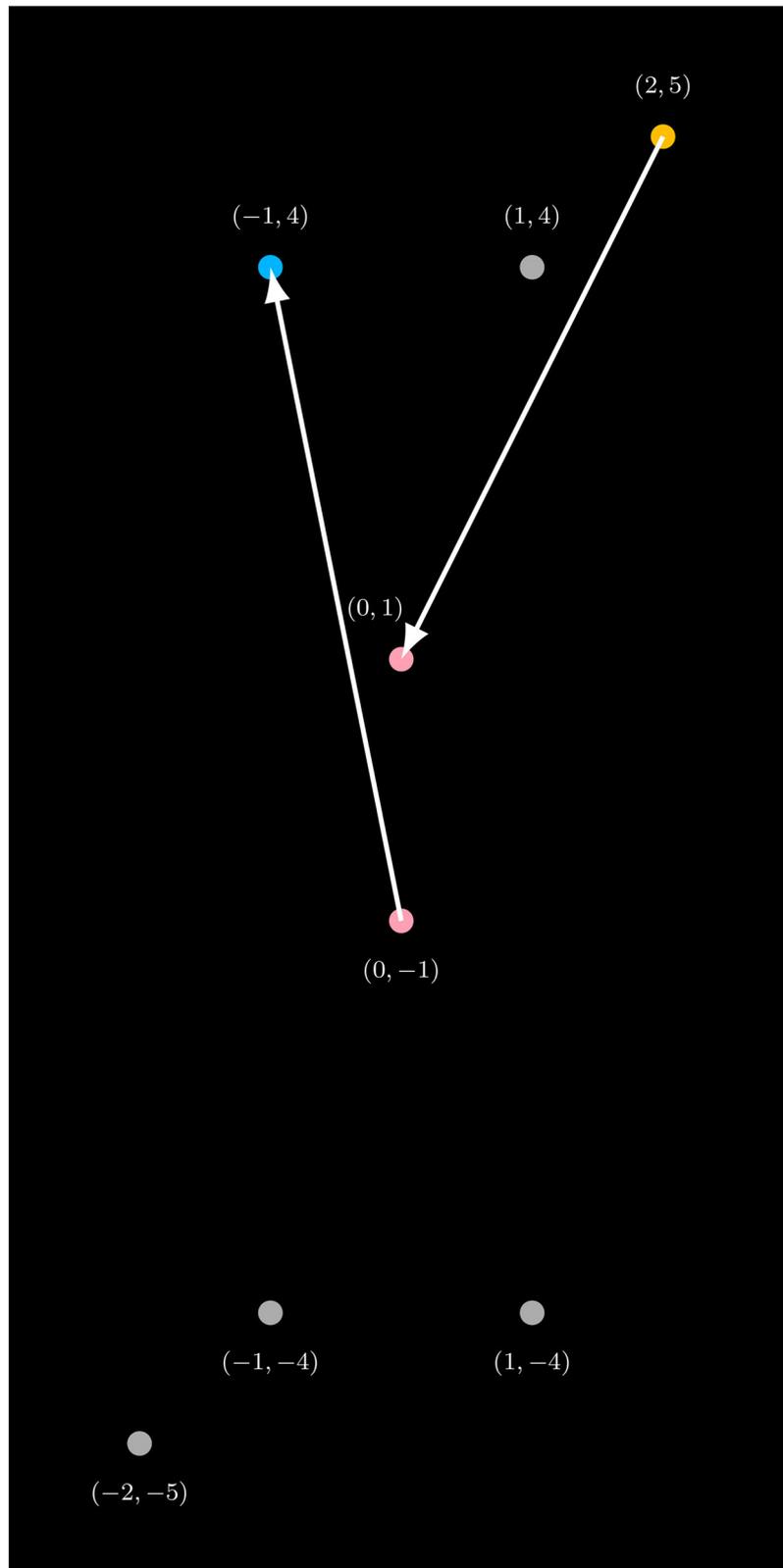
**Figure 29.** Thirteenth configuration: inverse path of the original divisibility structure.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, 4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

Setting  $r = 0$ , we obtain:

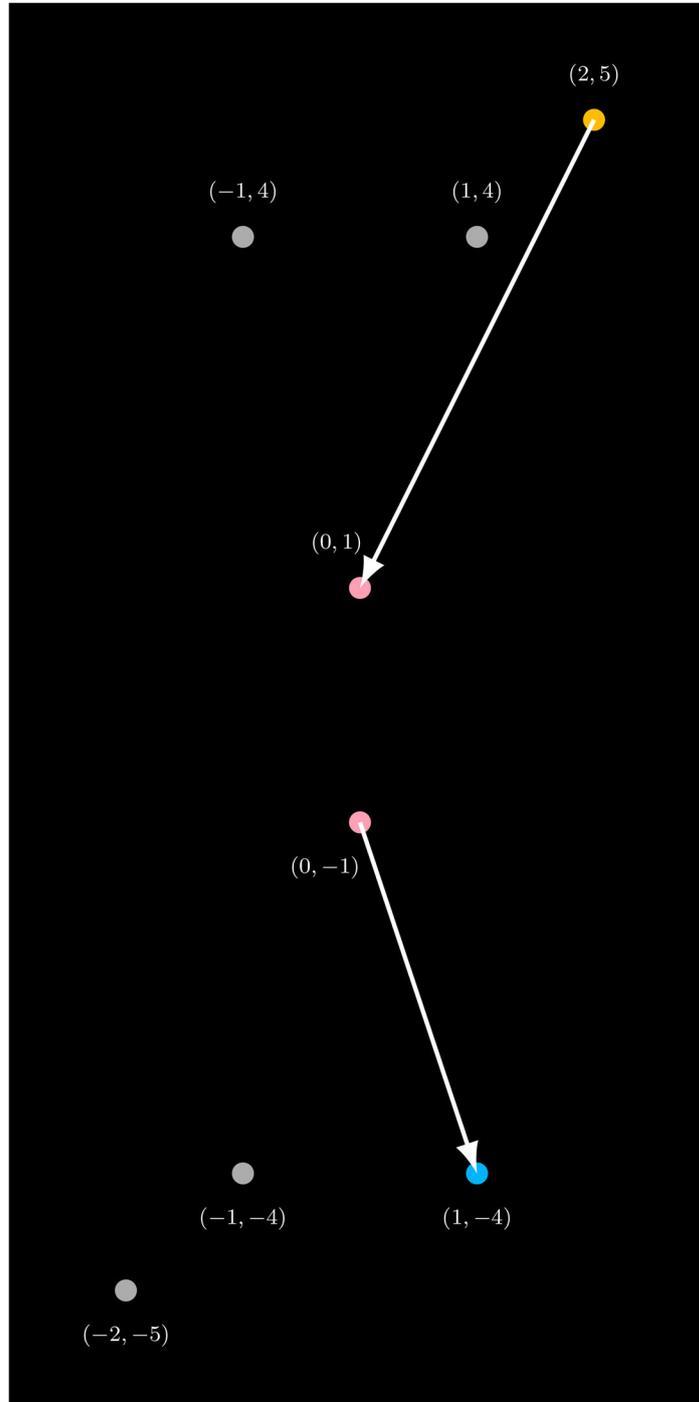
$$\Omega_0(2|1, -3|5) = 5 \cdot 3 \cdot 10 - 2 \cdot 5 \cdot 9 = 150 - 90 = 60 = 4 \cdot 3 \cdot 5. \tag{12}$$



**Figure 30.** Fourteenth configuration: final full reflection of Mersenne symmetry across axes.

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.



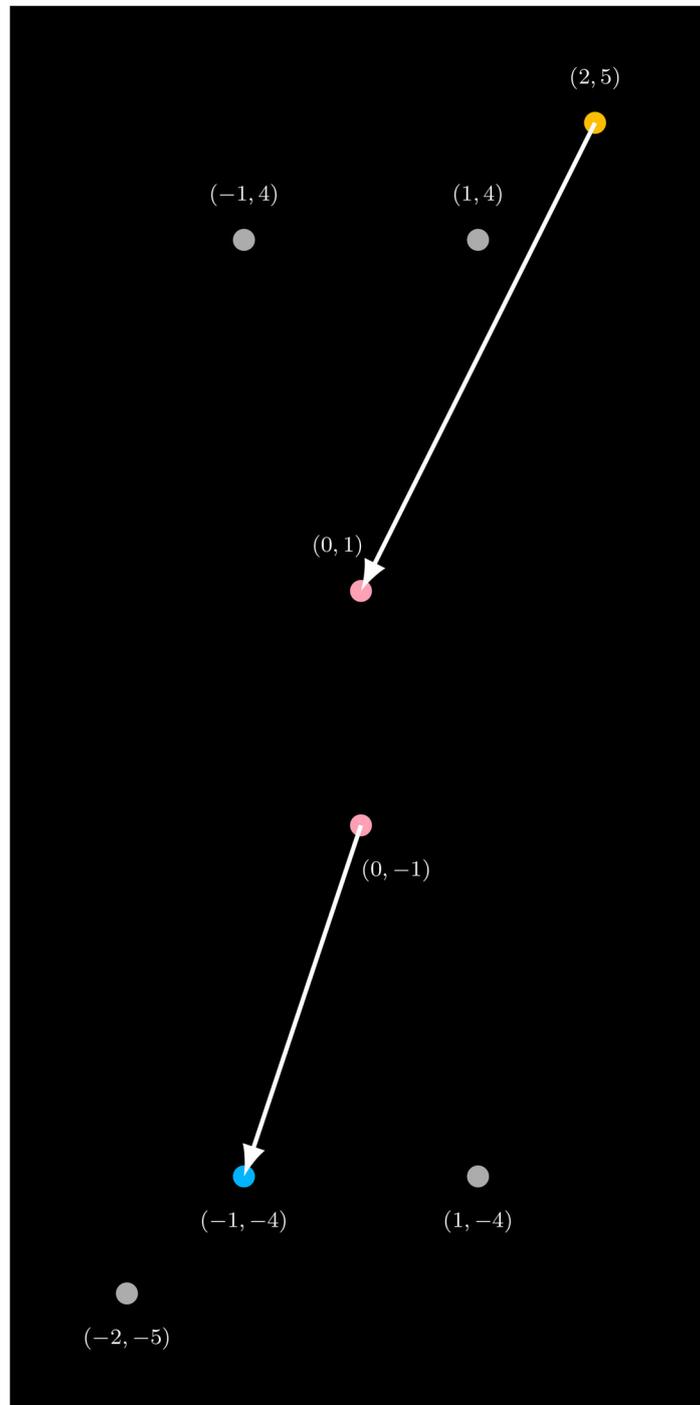
**Figure 31.** Fifteenth configuration: mirrored descent across southern vector paths.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

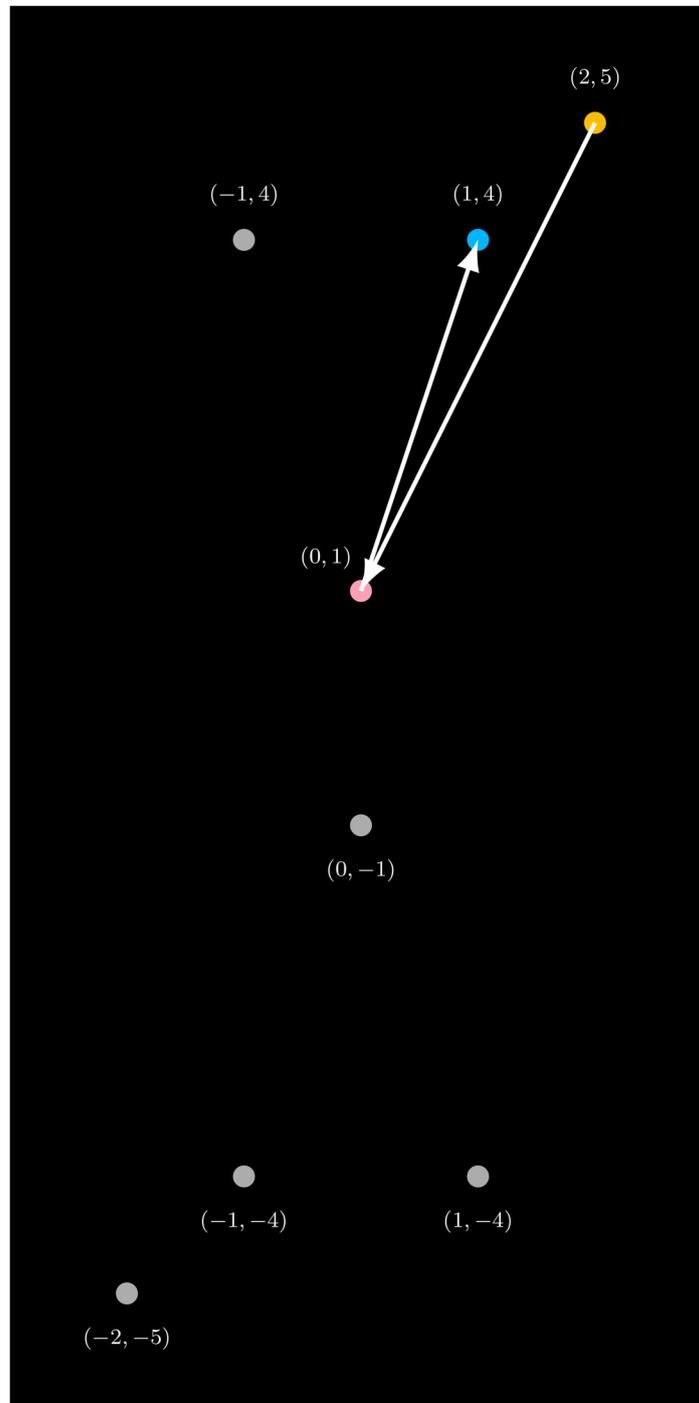
Interestingly, while the Lucas number does not appear in this case, the Fibonacci number  $F(5) = 5$  emerges as a visible factor in the result:

$$\Omega_0(2 | 1, -3 | 5) = 60 = 4 \cdot 3 \cdot \underbrace{5}_{\text{Fibonacci}(5)} .$$



**Figure 32.** Sixteenth configuration: final reflected vectorial closure of Mersenne Star.  
 $\Omega_0(\lfloor \frac{p}{2} \rfloor | 0, 1 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | 2, 5 | p) \quad | \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | -1, -4 | n) \quad \Omega_0(\lfloor \frac{p}{2} \rfloor | 0, -1 | p)$   
 This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

This outcome is far from coincidental. It reflects a distinct behaviour of the Quanta Prime Sequence near the point  $(1, -3)$ , where Fibonacci structure dominates instead of Lucas. Such behaviour underscores the geometric sensitivity of QPS: some points lead naturally to Lucas numbers, while others channel Fibonacci values with equal clarity.



**Figure 33.** Seventeenth configuration: pure upward flow through dual transition at  $(0,1)$ .

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

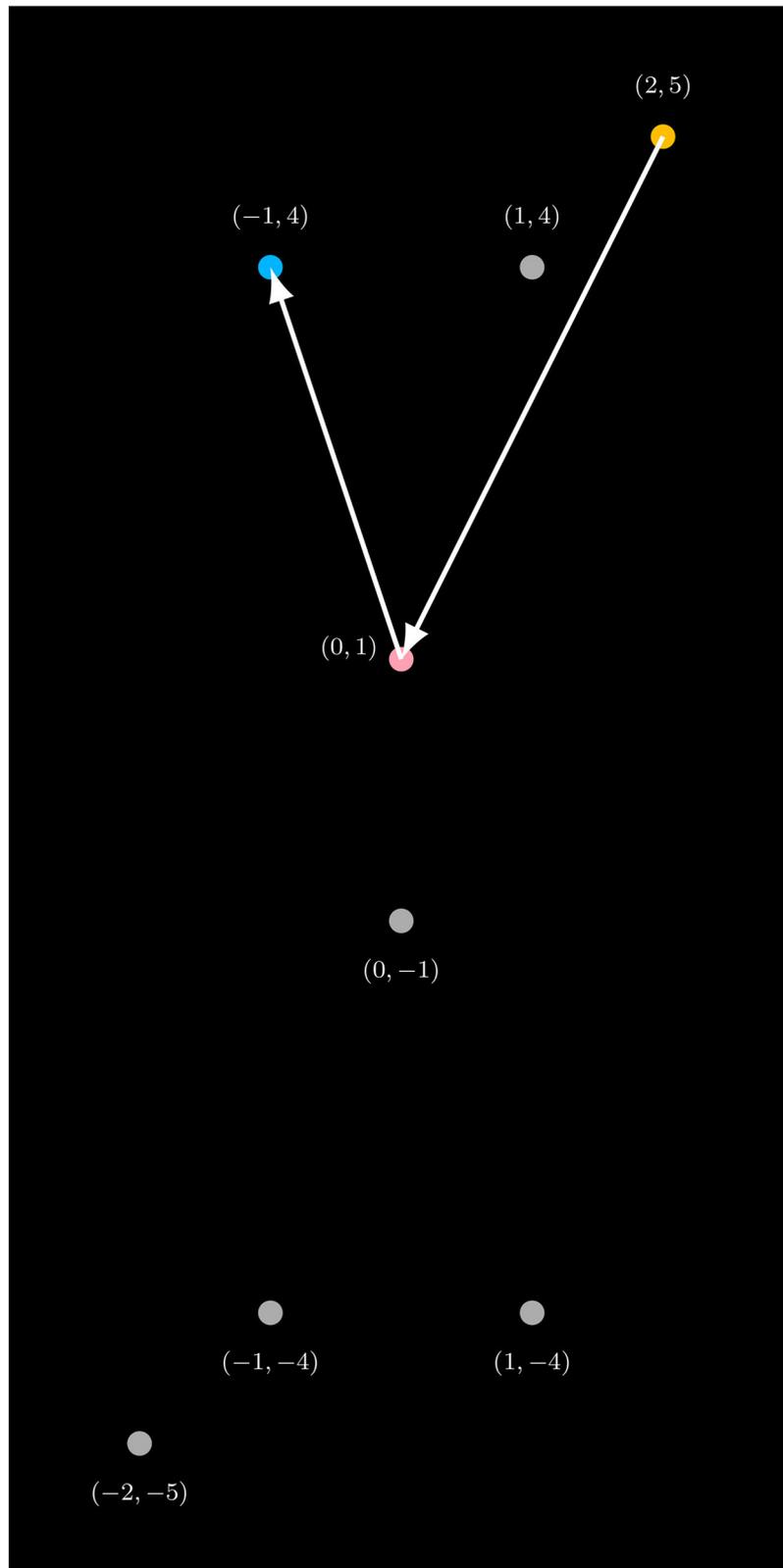
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.

#### 8.2.4. Verification via the Second Fundamental Theorem

For the point  $(\zeta, \xi) = (1, -3)$ , the general form of the theorem asserts:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -3 | n\right) = (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot \Psi(1, -3, n). \quad (13)$$

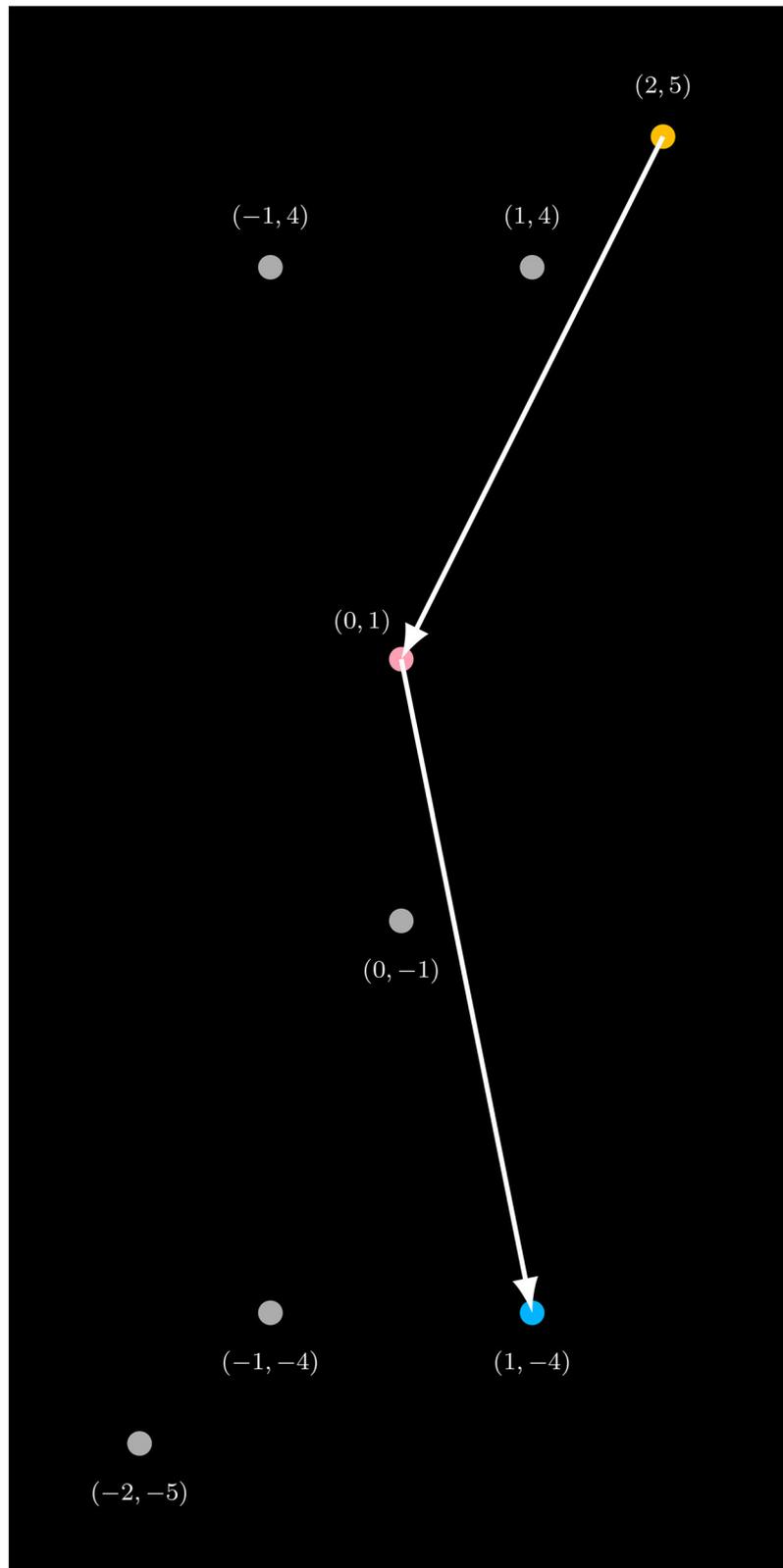
Since  $\Psi(1, -3, n) = F(n)$  for any odd value of  $n$ , and  $F(5) = 5$ , we recover Equation (12) exactly.



**Figure 34.** Eighteenth configuration: dual upward reflection through central point  $(0, 1)$ .

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

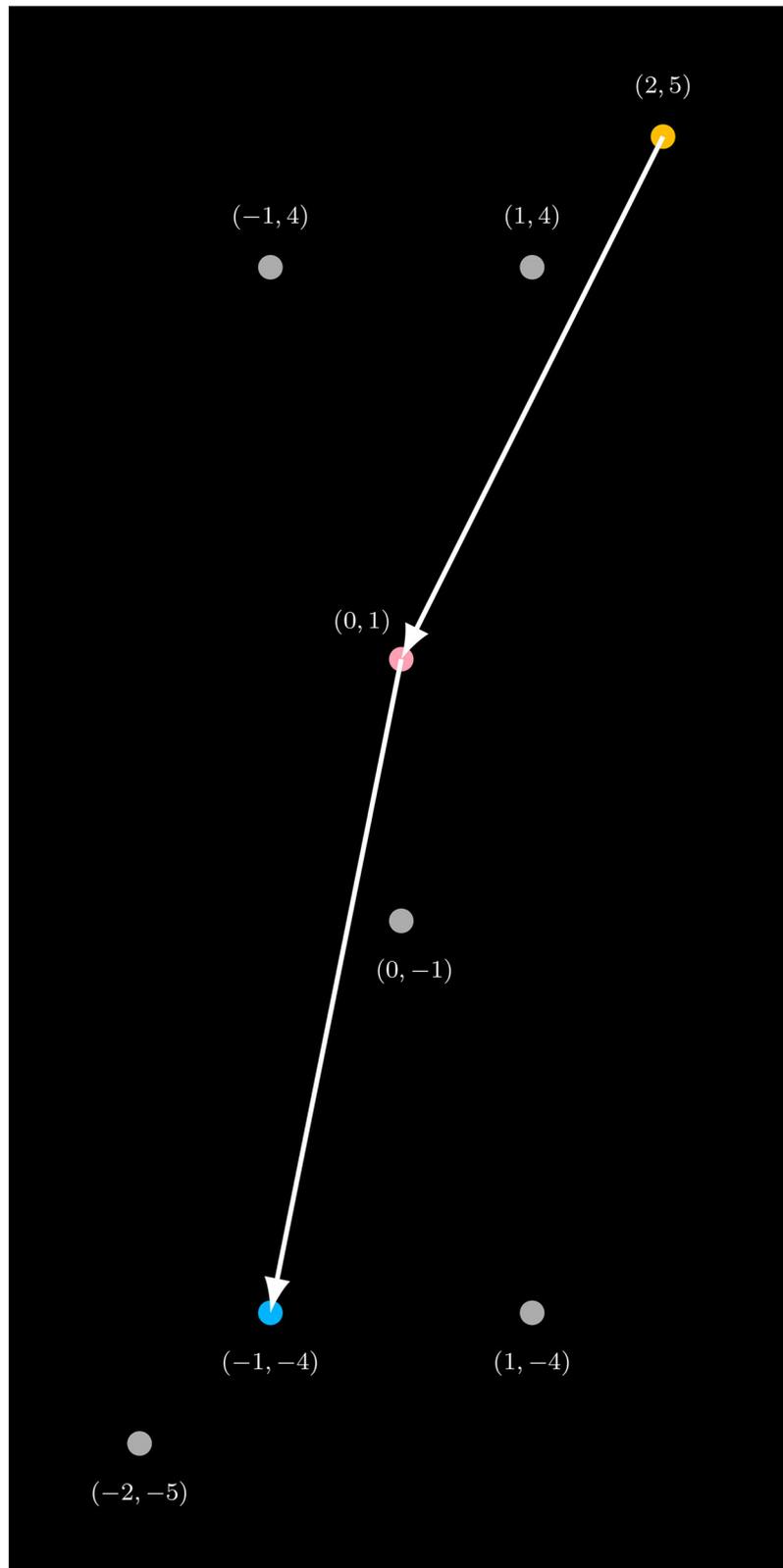
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.



**Figure 35.** Nineteenth configuration: downward diagonal symmetry passing through  $(0, 1)$ .

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 1, -4 | n\right) \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

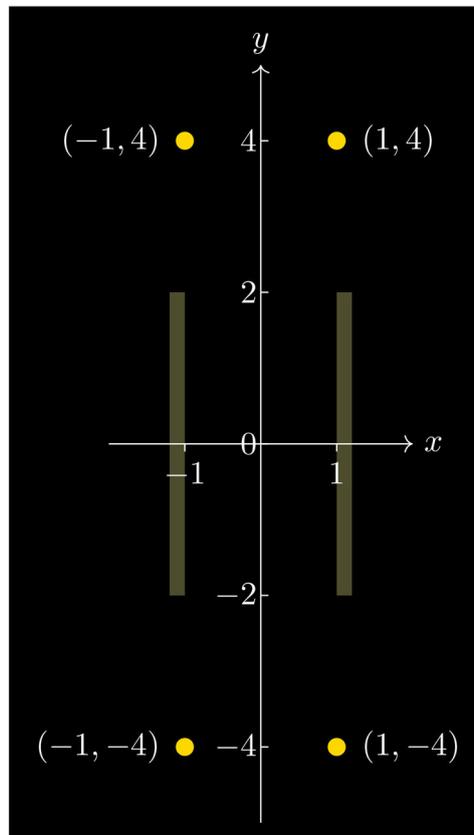
This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.



**Figure 36.** Twentieth configuration: diagonal descent reflection through the point  $(0, 1)$ .

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4 | n\right) \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right)$$

This divisibility condition is equivalent to the primality of  $2^p - 1$ , where  $n := 2^{p-1}$  and  $p$  is prime. The yellow point represents the divisor vector (starting point), the pink point represents the transition vector connecting multiplicative paths, and the blue point represents the dividend vector (target point). Gray points are inactive vertices completing the Mersenne Star shape. White arrows indicate the divisibility relationships between points as defined by Theorem 6.1.



**Figure 37.** Using QPS, the Mersenne Star vertices at  $(\pm 1, \pm 4)$  can be transformed into suitable periodic points on the yellow strips from  $(1, 2)$  to  $(1, -2)$  and from  $(-1, 2)$  to  $(-1, -2)$ .

### 8.2.5. Quanta identities involving Fibonacci and Lucas numbers

The following four identities hold for any natural number  $n$ , as a direct consequence of the Second Fundamental Theorem of the Quanta Prime Sequence (Ibrahim, 2025). They reveal strong and elegant links between QPS values and classical sequences like Fibonacci and Lucas, centered at specific points in the Quanta Plane.

$$\begin{aligned}\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid -1, -3 \mid n\right) &= (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot L(n), \\ \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid 1, 3 \mid n\right) &= (-1)^{\lfloor n/2 \rfloor} (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot L(n), \\ \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid 1, -3 \mid n\right) &= (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot \begin{cases} F(n) & \text{if } n \text{ odd} \\ L(n) & \text{if } n \text{ even} \end{cases}, \\ \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor \mid -1, 3 \mid n\right) &= (-1)^{\lfloor n/2 \rfloor} (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \cdot \begin{cases} F(n) & \text{if } n \text{ odd} \\ L(n) & \text{if } n \text{ even} \end{cases}.\end{aligned}$$

### 8.2.6. Note

Each final factor in the Quanta value matches a Fibonacci or Lucas number:

$$\Omega_0(5 \mid -1, -3 \mid 10) = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \mathbf{123}, \text{ where } 123 \text{ is the } 10^{\text{th}} \text{ Lucas number: } L_{10} = 123.$$

$$\Omega_0(6 \mid -1, 3 \mid 13) = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \mathbf{233}, \text{ where } 233 \text{ is the } 13^{\text{th}} \text{ Fibonacci number: } F_{13} = 233.$$

**Remark.** For every natural number, these four identities connect the Quanta Prime Sequence with Fibonacci and Lucas numbers in an unexpected way. The study of neighbouring points to the Mersenne Star unveils a true mathematical kingdom. These identities arise as a direct consequence of the Second Fundamental Theorem of the Quanta Prime Sequence (see Ibrahim, 2025).

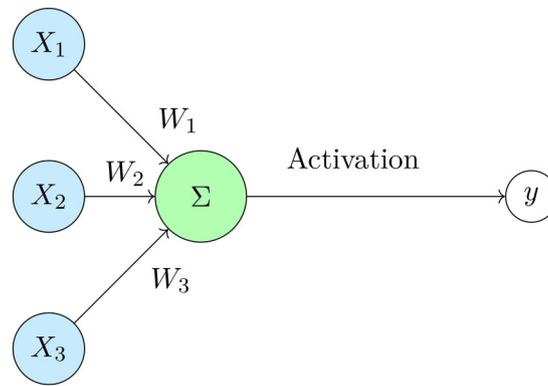


Figure 38. Basic neuron computing a weighted sum.

### 8.3. Anchor identity and normalization

For any natural number  $n \geq 2$ , the following identity holds (see Ibrahim, 2025) for full details):

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) = (n-1)(n-2) \cdots \left(n - \left\lfloor \frac{n}{2} \right\rfloor\right) \quad (14)$$

Substituting this into the previous identity yields a normalized form:

$$\Psi(\zeta, \xi, n) = \frac{\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | \zeta, \xi | n\right)}{\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right)} \quad (15)$$

This equation shows that the Psi-sequence acts as a scaled version of QPS, normalized by the anchor point  $(0, -1)$ . This foundation plays a crucial role in understanding the deeper geometry revealed later in the paper.

#### 8.3.1. From arithmetic to geometry: the birth of the Mersenne triangle

In our previous work (Ibrahim, 2024, 2025), we established a symbolic criterion for determining the primality of Mersenne numbers  $M_p = 2^p - 1$ , based on values of the Quanta Prime Sequence (QPS). The core result is as follows:

**Theorem 7.1** (The First Fundamental Theorem of the Mersenne Triangle).

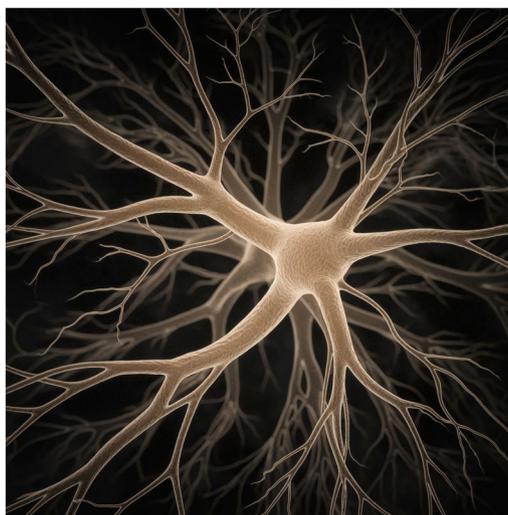
Let  $p \geq 5$  be a prime number and define  $n := 2^{p-1}$ . Then  $M_p = 2^p - 1$  is prime if and only if

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \mid \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right). \quad (16)$$

**Remark 7.2.** On Prior Publication. The complete and fully rigorous proof of this result has already appeared in (Ibrahim, 2025), formally stated as Theorem 52 on page 17. This prior publication includes all necessary definitions, logical steps, and structural justifications required to validate the claim beyond any ambiguity. We reference it here to acknowledge the foundational role it plays in the current framework, while ensuring full mathematical transparency and reproducibility. We kindly refer the reader to that work for the complete symbolic derivation. Strikingly, this condition relies only on QPS values at three symmetric locations—revealing how geometric symmetry and sequence dynamics work hand in hand to characterize primality. What concerns us here is not the repetition of the proof, but rather the geometric insight it inspired.

While the condition above appears purely algebraic, it surprisingly gives rise to three geometric points:  $(0, -1)$ ,  $(-2, -5)$ ,  $(1, 4)$ . These three coordinates are not arbitrary—they carry mathematical weight and symbolic significance:

- $(0, -1)$ : A symmetric anchor appearing frequently across multiple Mersenne-related identities.



**Figure 39.** Biological inspiration: layered neuron.

- $(-2, -5)$ : A point encoding hidden divisibility conditions linked to the exponent  $p$ .
- $(1, 4)$ : A Fibonacci-aligned coordinate generating resonance within the QPS field.

This spontaneous emergence of three coherent points led to a compelling geometric realization: the formation of a triangle that we now call the Mersenne Triangle.

The birth of this triangle is not arbitrary—it is a natural consequence of the QPS structure, and it marks the first geometric encoding of the Mersenne primality test. It also sets the stage for what we shall later define as the Mersenne Star.

The Mersenne Triangle is more than just a visual object—it is the first clear sign that something deeper lies beneath the surface of Mersenne primes. The entire test for Mersenne primality is now written purely in terms of QPS—with no external assumptions or auxiliary number-theoretic tools. All the information is encoded in just three values—at three special points that form the corners of this triangle.

This triangle becomes the natural frame for expressing and analyzing Mersenne primes. In other words, this theorem shows that the structure of Mersenne primes is deeply rooted in the internal behaviour of QPS. It opens the door to a new way of thinking—where primes are not just isolated numbers, but part of a geometric pattern guided entirely by QPS dynamics.

This triangle reveals that QPS is not just a formula—it is a bridge between algebra and geometry. Through this lens, prime numbers are no longer static values on a line—they are patterns in space. And this realization may very well mark the beginning of a new field—the geometry of primes.

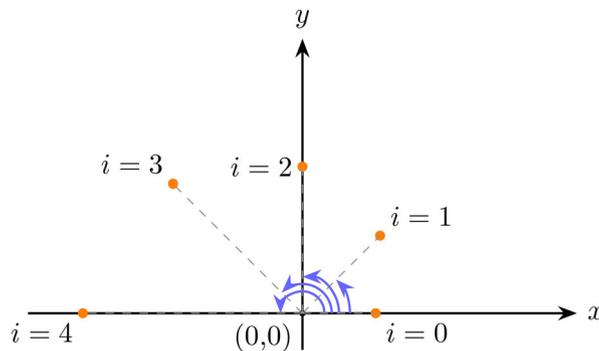
## 9. Prelude: a star is born—into the heart of a prime geometry

The journey toward understanding Mersenne primes often seems numerical, recursive, or algorithmic in nature. Yet, hidden within this numerical wilderness lies an unexpected geometric order—a structure that does not merely compute but reveals. This structure is what we call the Mersenne Star.

This star did not arise from speculation, but from the subtle whispers of the prime numbers themselves. As we probed the structure of the Quanta Prime Sequence, primal patterns began to align, leading us not to a single value, but to a constellation—a symmetric framework of eight geometric points. From these points, an astonishing harmony unfolds.

The Mersenne Star is not just a visual motif—it is a geometric map, a structural skeleton, a framework of symmetry and logic that quietly encodes deep arithmetic truths. It forms a region within the Quanta Prime Plane where opposing forces—divisibility and balance, recursion and ratio—coexist under an elegant system.

But why eight points? Why this specific configuration?



**Figure 40.** Exact symbolic plot of points for  $n = 31$  using  $a = 2$ .

These eight vertices are not arbitrary. They embody a deep interplay between symmetry and primality. They emerge naturally from the divisibility behaviour of the Quanta Prime Sequence. Their coordinates are intertwined with celebrated sequences—Fibonacci, Lucas, and others—suggesting a hidden order we never previously imagined. A closer examination of this star reveals not only symmetry linked to primality, but also subtle periodic structures embedded within its core—with potential technological applications to be explored in future work. We were not ready to see it before. The map had not been drawn. But with the emergence of the Mersenne Star, a new geography of prime numbers has been charted—and with it, a gateway to understanding one of number theory’s most elusive mysteries. While the first theorem unveils a fundamental divisibility law rooted in the star’s symmetric nature, a deeper structure emerges when we ask a different question: not whether one value divides another, but how these values relate through proportion. This leads us to a profound observation—that geometry is not only a visual aid, but a harmonic system in its own right. Ratios, not just integers, govern the behaviour of the Quanta Prime values across the star. These ratios exhibit a remarkable consistency—revealing structured behaviour only when the inputs correspond to Mersenne primes. This is not a numerical accident. It is a law encoded within the structure of the star itself, a prime-conserving symmetry that resists disruption. This symmetry is not a product of chance. It is a whisper from the architecture of primes—one that geometry helps us hear more clearly. In what follows, we formally define the Mersenne Star, detail the mathematical reasoning behind its construction, and proceed to uncover a hidden ratio—one that reveals its consistency only in the presence of prime numbers.

### 9.1. The Mersenne Star: birthplace of prime number geometry

What follows is simple, yet extraordinary. We allow the Mersenne triangle to undergo controlled motion—sliding, rotating, and reflecting—but only in ways dictated by the internal logic of the Quanta Prime Sequence itself. This is not speculative manipulation; it is a guided evolution governed by the sequence’s own constraints. As we perform these permitted transformations, something remarkable unfolds: thirty-two new and distinct divisibility relations begin to manifest, each anchored in precise geometric alignment. Together, they give rise to a larger, coherent structure—the Mersenne Star. This star is not a mere visual artefact. It encodes specific arithmetic behaviours that emerge exclusively under Mersenne primality. Like a mathematical fingerprint, its pattern does not appear for general primes. It arises only under the special harmony enforced by the Mersenne condition. This is more than a geometric curiosity—it may mark the emergence of a new mathematical discipline: what we now propose as the *geometry of prime numbers*.

### 9.2. The Second Fundamental Theorem of the Mersenne Star

**Lemma 7.3** (Global Symmetry Lemma).

Let  $p, n, k \in \mathbb{N}$  and let  $\Omega_0(k|\zeta, \xi|n)$  be the Quanta Prime Sequence (QPS) associated with the point  $(\zeta, \xi)$ . Then the following symmetry relations hold:

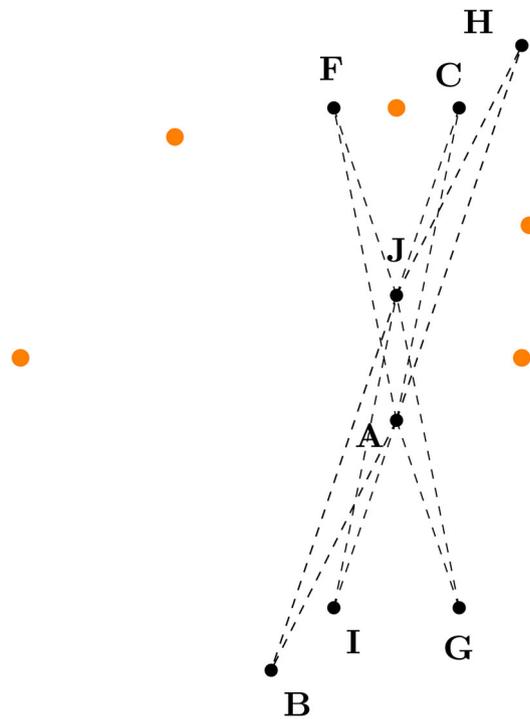


Figure 41. Mersenne Star geometry for  $p = 31$  with highlighted orange points.

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) = (-1)^{\lfloor \frac{p}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right), \quad (17)$$

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4 | n\right) = (-1)^{\lfloor \frac{n}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right), \quad (18)$$

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right) = (-1)^{\lfloor \frac{p}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right), \quad (19)$$

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) = (-1)^{\lfloor \frac{n}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right). \quad (20)$$

These identities follow from the parity symmetry of the  $\Psi$ -sequence and the structure of the QPS, as encoded in the Second Fundamental Theorem of QPS.

**Proof of Lemma 7.3.** We establish the global symmetry identities of the Quanta Prime Sequence (QPS) using two complementary approaches.

Step 1: Symmetry of the  $\Psi$ -Sequence via Closed Form. It is shown in the literature (Ibrahim, 2025) that the  $\Psi$ -sequence admits the closed-form expression:

$$\Psi(a, b, n) = \frac{(2a - b)^{\lfloor n/2 \rfloor}}{2^n} \left[ \left( 1 + \sqrt{\frac{b + 2a}{b - 2a}} \right)^n + \left( 1 - \sqrt{\frac{b + 2a}{b - 2a}} \right)^n \right]. \quad (21)$$

It is immediate from this expression that:

$$\Psi(-a, -b, n) = (-1)^{\lfloor \frac{n}{2} \rfloor} \Psi(a, b, n), \quad (22)$$

since the numerator  $(2a - b)^{\lfloor n/2 \rfloor}$  changes sign accordingly, while the symmetric sum remains invariant under sign reversal.

Step 2: Connection to QPS and Deduction. From the Second Fundamental Theorem of QPS (see Ibrahim, 2025), we have:

$$\Psi(a, b, n) = \frac{\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | a, b | n\right)}{(n-1)(n-2) \cdots (n - \left\lfloor \frac{n}{2} \right\rfloor)}. \quad (23)$$

Combining Equations (22) and (23), the symmetry identity for  $\Omega_0$  immediately follows:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -a, -b | n\right) = (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | a, b | n\right). \quad (24)$$

Step 3: Applying to Specific Points. By applying the symmetry identity

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -a, -b | n\right) = (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | a, b | n\right)$$

to the following specific pairs of points, we recover each of the four symmetry identities stated in Lemma 7.3, one by one:

- Identity (17): For the prime number  $p$ , take  $(a, b) = (2, 5)$ . Then,

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) = (-1)^{\left\lfloor \frac{p}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right),$$

establishing symmetry across the direction  $(2, 5) \leftrightarrow (-2, -5)$ .

- Identity (18): For general  $n$ , take  $(a, b) = (1, 4)$ . Then,

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4 | n\right) = (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right),$$

confirming the symmetry along the axis  $(1, 4) \leftrightarrow (-1, -4)$ .

- Identity (19): For a prime  $p$ , take  $(a, b) = (0, 1)$ . Then,

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right) = (-1)^{\left\lfloor \frac{p}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right),$$

showing symmetry across the vertical axis.

- Identity (20): For general  $n$ , again let  $(a, b) = (0, 1)$ . Then,

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) = (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right),$$

once more yielding a reflectional symmetry across the origin.

Conclusion. Each identity follows directly and rigorously from the closed-form structure of the  $\Psi$ -sequence and the Second Fundamental Theorem of QPS (Ibrahim, 2025). This establishes the claimed global symmetry relations of the Quanta Prime Sequence in a unified and elegant manner. ■

**Lemma 7.4** (Even Symmetry Lemma).

Let  $n$  be a positive even integer divisible by 8, i.e.  $n \equiv 0 \pmod{8}$ . Then, the Quanta Prime Sequence exhibits an enhanced symmetry for the points  $(\pm 1, \pm 4)$ . Specifically, we have the identity:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4 | n\right). \quad (25)$$

**Proof.** Step 1: Invoke the Eight Levels Theorem. For even numbers  $n \equiv 0 \pmod{8}$ , the Eight Levels Theorem (Ibrahim, 2023) provides the following closed-form expression for the  $\Psi$ -sequence:

$$\Psi(a, b, n) = \sum_{\substack{k=0, \\ k \text{ even}}}^{\lfloor n/2 \rfloor} (-1)^{\lfloor k/2 \rfloor} \frac{\prod_{\lambda=0}^{\lfloor k/2 \rfloor - 1} [n^2 - (4\lambda)^2]}{k! \cdot 4^k} \cdot a^{\lfloor n/2 \rfloor - k} b^k. \quad (26)$$

Step 2: Analyzing the Symmetry. Note that this expression contains only even powers of  $k$ , and the coefficients alternate according to  $(-1)^{\lfloor k/2 \rfloor}$ . Due to this structure, the signs of  $a$  and  $b$  can be changed independently without affecting the final result. More precisely:

$$\Psi(-a, b, n) = \Psi(a, b, n) = \Psi(a, -b, n). \quad (27)$$

Step 3: Apply to Specific Cases. Using the Second Fundamental Theorem of QPS (Ibrahim, 2025), which connects  $\Psi(a, b, n)$  to  $\Omega_0(k|a, b|n)$ , this symmetry directly implies that:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4|n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4|n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4|n\right), \quad (28)$$

thus completing the proof. ■

**Remark 7.5.** A Hidden Geometric Trigger. The enhanced symmetry captured in Lemma 7.4 is not a mere numerical coincidence. Rather, it reflects a deeper, structured harmony that emerges only when the integer  $n$  is divisible by 8.

This condition aligns precisely with the core structure of Mersenne primes. In our framework, we define  $n := 2^{p-1}$ , which is always divisible by 8 for any prime  $p \geq 5$ . This alignment is no accident. It is a fingerprint—a geometric resonance echoing through the very fabric of the Quanta Prime Sequence.

It suggests that the Eight Levels Theorem (Ibrahim, 2023), previously established as a standalone result, is in fact a natural manifestation of a broader geometrical symmetry hidden within the Mersenne Star.

This observation may very well mark the dawn of a new era in our understanding of prime numbers.

What we are beginning to uncover is a latent architecture—a harmonic structure—within the QPS geometry that is sensitive to the divisibility of  $n$  by powers of 2, particularly 8.

These are not just algebraic identities. They are *geometric currents*—surges of mathematical charge that pulse through the Mersenne Star, forming patterns too elegant to be accidental.

This discovery invites a deeper global investigation. What other symmetries lie hidden? What further harmonics might emerge if we tune ourselves to the correct geometric frequencies?

*The answers, we believe, are encoded in the star.*

**Theorem 7.6** (The Second Fundamental Theorem of the Mersenne Star).

Let  $p \geq 5$  be a prime number and  $n := 2^{p-1}$ . The number  $2^p - 1$  is prime if and only if each of the following 32 divisibility conditions, divided into 8 groups, holds. Each condition below is true and fully equivalent to the primality condition of Mersenne numbers.

Below, we list all 32 conditions, grouped by the points used in the Quanta Prime Sequence (QPS). Each group has a short explanation to help you understand the context.

Group 1: Conditions with Points  $(0, -1), (-2, -5), (0, -1), (\pm 1, \pm 4)$ . These conditions use the points  $(0, -1)$  and  $(-2, -5)$  on the left side, and  $(0, -1)$  with  $(\pm 1, \pm 4)$  on the right side.

1.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|p\right)$
2.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|p\right)$
3.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|p\right)$
4.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|p\right)$

Group 2: Conditions with Points  $(0, -1), (-2, -5), (0, 1), (\pm 1, \pm 4)$ . These conditions use the points  $(0, -1)$  and  $(-2, -5)$  on the left side, but now pair with  $(0, 1)$  and  $(\pm 1, \pm 4)$  on the right side.

5.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1|p\right)$
6.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1|p\right)$
7.  $\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -2, -5|p\right) | \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4|n\right) \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1|p\right)$



The theorem asserts  $8 \times 4 = 32$  divisibility relations, grouped into eight geometric families, each containing four distinct conditions. These relations stem directly from the geometry of the Mersenne Star and the algebraic structure of the Quanta Prime Sequence (QPS).

The proof unfolds in three stages:

- Stage 1 – Foundational Identity: Prove the first divisibility condition, stated earlier as Theorem 6.1. This relation provides the base for all subsequent results.
- Stage 2 – Completing the First Group: Apply symmetry operations (horizontal, vertical, and diagonal reflections) to derive the remaining three relations in Group 1.
- Stage 3 – Constructing All Groups: Extend the result to the remaining groups (2 through 8) using a unified pattern of substitutions rooted in QPS behaviour.

This plan guarantees a coherent derivation of all 32 relations, showcasing the deep geometric and algebraic unity behind the structure.

Step 1: Proving the First Relation. We now invoke [Theorem 7.1](#), previously established in this work. This theorem corresponds exactly to the first relation in Group 1 of the current result and thus serves as its foundational base. Accordingly, Step 1 is complete: the first relation is already proven and will serve as the cornerstone for the entire divisibility framework.

Step 2: Completing the First Group. We now derive the remaining three relations in Group 1 by demonstrating that each one follows directly from the first relation (previously established as [Theorem 7.1](#)) through the application of precise geometric symmetries. These symmetries are not incidental; rather, they are rigorously governed by [Lemma 7.4](#) (Even Symmetry Lemma) and [Lemma 7.3](#) (Global Symmetry Lemma), both proven earlier. We proceed one by one:

*Relation (2):*

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This identity is obtained from the first relation by applying a horizontal reflection to the vector  $(1, 4)$  in the term

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 4 | n\right),$$

which becomes:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, 4 | n\right)$$

due to [Lemma 7.4](#), which confirms that:

$$\Omega_0(k | \zeta, \xi | n) = \Omega_0(k | -\zeta, \xi | n) \quad \text{if } n \text{ is divisible by } 8.$$

Thus, the right-hand side is invariant under this symmetry, and so [Relation \(2\)](#) holds.

*Relation (3):*

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This time, the symmetry is applied to the vertical component  $\xi = 4$  to become  $-4$ . According to [Lemma 7.4](#), when  $n$  is divisible by 8:

$$\Omega_0(k | \zeta, \xi | n) = \Omega_0(k | \zeta, -\xi | n)$$

Hence, the transformation:

$$(1, 4) \mapsto (1, -4)$$

preserves the value of  $\Omega_0$ , and so the entire right-hand side remains invariant. Therefore, [Relation \(3\)](#) follows from the first.

Relation (4):

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) \quad | \quad \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | -1, -4 | n\right) \cdot \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right)$$

This is a composition of both symmetries: reflection in  $\zeta$  and in  $\xi$ , i.e.

$$(1, 4) \mapsto (-1, -4)$$

and again, Lemma 7.3 (Global Symmetry Lemma) guarantees:

$$\Omega_0(k | \zeta, \xi | n) = \Omega_0(k | -\zeta, -\xi | n)$$

as  $n$  is divisible by 8. Therefore, this final form is again a valid transformation that preserves the divisibility, and Relation (4) is thus proven.

*Conclusion of Step 2:* All four relations in Group 1 are now established. The first was proven independently as Theorem 7.1, and the remaining three follow directly from it by applying the symmetric transformations outlined in Lemma 7.4 and Lemma 7.3. This confirms that Group 1 is fully determined by a single foundational identity and the inherent symmetries of the Mersenne Star geometry.

*Proof of Group 2 Using Global Symmetry Lemma.* We now prove the four divisibility relations in Group 2 by leveraging the symmetries guaranteed by Lemma 7.3 (Global Symmetry Lemma). This lemma states that for any natural number  $m$ , the following identity holds:

$$\Omega_0\left(\left\lfloor \frac{m}{2} \right\rfloor | 0, -1 | m\right) = (-1)^{\lfloor \frac{m}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{m}{2} \right\rfloor | 0, 1 | m\right).$$

In particular, we apply it for  $m = n$  and  $m = p$ , yielding the key identity:

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, -1 | p\right) = (-1)^{\lfloor \frac{p}{2} \rfloor} \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 0, 1 | p\right).$$

Now, consider each of the four relations in Group 1, which have already been proven. If we replace every occurrence of the point  $(0, -1)$  with the point  $(0, 1)$  using the identities above, we obtain the corresponding relations in Group 2. Since the multiplicative constants involved are signs  $\pm 1$ , the divisibility structure is preserved due to the elementary fact that

$$x | y \iff x | \pm y.$$

Hence, all four relations in Group 2 follow immediately from those in Group 1, completing the proof.

*Proof of Group 3 Using Point Symmetry.* We now provide a direct and simplified proof for the four divisibility relations listed in Group 3.

Observe that each identity in Group 3 is structurally identical to those in Group 1, except for a single substitution: the point  $(-2, -5)$  appearing in Group 1 has been replaced by the point  $(2, 5)$  in Group 3.

This substitution is justified by a symmetry of the Quanta Prime function  $\Omega_0$ , which satisfies the identity:

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | -2, -5 | p\right) = \pm \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor | 2, 5 | p\right).$$

This means that the value of  $\Omega_0$  at  $(2, 5)$  differs from that at  $(-2, -5)$  only by a global sign.

As divisibility relations are invariant under multiplication by  $\pm 1$ , we have the key logical equivalence:  $x | y$  if and only if  $x | \pm y$ . Therefore, by simply replacing every instance of  $(-2, -5)$  in Group 1 with  $(2, 5)$ , we obtain all the identities in Group 3 without affecting the truth of the divisibility conditions. This substitution preserves the structure of each expression, and hence the divisibility conditions in Group 3 follow immediately and rigorously from those in Group 1.

*Proof of Group 4 Using Double Symmetry.* We now present a clean and rigorous proof for the four divisibility relations listed in Group 4. Take a closer look at the identities in Group 4 and compare them with those in Group 1. You'll quickly notice a graceful symmetry unfolding: each identity in Group 4 is structurally identical to its counterpart in Group 1, except for a simple and meaningful substitution: the points  $(-2, -5)$  and  $(0, -1)$  in Group 1 have been replaced with the points  $(2, 5)$  and  $(0, 1)$ , respectively.

This substitution is justified by the following symmetries of the Quanta Prime function  $\Omega_0$ , which holds up to a global sign:

$$\Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid -2, -5 \mid p\right) = \pm \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 2, 5 \mid p\right), \quad \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 0, -1 \mid p\right) = \pm \Omega_0\left(\left\lfloor \frac{p}{2} \right\rfloor \mid 0, 1 \mid p\right).$$

Since divisibility is preserved under multiplication by  $\pm 1$ , we have  $x|y$  if and only if  $x|\pm y$ , and hence, replacing both  $(-2, -5)$  with  $(2, 5)$  and  $(0, -1)$  with  $(0, 1)$  in the identities of Group 1 yields exactly the identities of Group 4—without affecting the truth or structure of the divisibility relations. In this way, the validity of Group 4 follows immediately and rigorously from the previously verified conditions in Group 1, thanks to this elegant dual symmetry of the underlying geometry.

*Proof of Group 5 Using Point Substitution from Group 1.* We now rigorously derive the four divisibility identities listed in Group 5 by applying a simple transformation to the identities in Group 1. Each identity in Group 5 shares the exact structure with its corresponding identity in Group 1, except for a single substitution:

$$(0, -1) \rightarrow (0, 1),$$

while the second point  $(-2, -5)$  remains unchanged.

This substitution is justified by the following symmetry of the Quanta Prime function  $\Omega_0$ :

$$\Omega_0(k \mid 0, -1 \mid n) = \pm \Omega_0(k \mid 0, 1 \mid n).$$

Since divisibility is preserved under multiplication by  $\pm 1$ , we have  $x|y$  if and only if  $x|\pm y$ . Therefore, by replacing every instance of  $(0, -1)$  in Group 1 with  $(0, 1)$ , we immediately obtain all identities of Group 5 with no change in validity. Conclusion: Group 5 follows directly from Group 1 *via* the replacement  $(0, -1) \bowtie (0, 1)$ , which preserves the divisibility structure under the sign symmetry of  $\Omega_0$ .

*Proof of Group 6 via Dual Role Symmetry.* We now derive the four divisibility relations in Group 6 using a structural substitution grounded in the general symmetry of the Quanta Prime function  $\Omega_0$ . Upon close inspection, we find that each identity in Group 6 mirrors a corresponding identity in Group 1, with only one consistent transformation applied:

$$(0, -1) \rightarrow (0, 1),$$

where the point  $(0, -1)$  appears either as the  $(\zeta, \xi)$ -coordinate associated with  $p$  or with  $n$ . This substitution is justified by the identity:

$$\Omega_0(k \mid 0, -1 \mid m) = \pm \Omega_0(k \mid 0, 1 \mid m),$$

which holds for any integer  $k$  and  $m$ , by the general symmetry theorem. Since divisibility is preserved under multiplication by  $\pm 1$ , we have  $x|y$  if and only if  $x|\pm y$ . Therefore, by simply replacing every occurrence of  $(0, -1)$  in the expressions of Group 1 with  $(0, 1)$ , regardless of whether it appears with  $p$  or  $n$ , we obtain all four relations of Group 6 immediately and rigorously. This completes the proof.

*Proof of Group 7 via Double Replacement Symmetry.* To derive Group 7 from Group 1, we apply a precise double replacement on the defining points in each condition. Specifically, we observe that every expression in Group 1 involves the following two points on the left side:

$$(0, -1) \quad \text{and} \quad (-2, -5).$$

Now, by invoking the General Symmetry Principle of QPS geometry, we are allowed to substitute a point with its symmetric counterpart, provided the overall balance of directional pairing is preserved.

We apply the following double replacement:

$$(0, -1) \rightarrow (0, 1), \quad (-2, -5) \rightarrow (2, 5).$$

Since divisibility is preserved under multiplication by  $\pm 1$ , we have

$$x|y \text{ if and only if } x|\pm y.$$

Thus, each divisibility relation in Group 1 remains logically valid under the above substitutions. In other words, the resulting expressions form a new collection—precisely those of Group 7.

Let us demonstrate with one example. Consider the first identity in Group 1:

$$\Omega_0\left(\left\lfloor\frac{n}{2}\right\rfloor|0, -1|n\right)\Omega_0\left(\left\lfloor\frac{p}{2}\right\rfloor|-2, -5|p\right)|\Omega_0\left(\left\lfloor\frac{n}{2}\right\rfloor|1, 4|n\right)\Omega_0\left(\left\lfloor\frac{p}{2}\right\rfloor|0, -1|p\right).$$

Replacing:

$$(0, -1) \rightarrow (0, 1), \quad (-2, -5) \rightarrow (2, 5),$$

we obtain:

$$\Omega_0\left(\left\lfloor\frac{n}{2}\right\rfloor|0, 1|n\right)\Omega_0\left(\left\lfloor\frac{p}{2}\right\rfloor|2, 5|p\right)|\Omega_0\left(\left\lfloor\frac{n}{2}\right\rfloor|1, 4|n\right)\Omega_0\left(\left\lfloor\frac{p}{2}\right\rfloor|0, -1|p\right),$$

which is exactly the first identity in Group 7.

The same replacement logic applies to the remaining expressions in Group 1. Therefore, all four conditions of Group 7 follow rigorously and directly from Group 1 *via* this double replacement.

*Proof of Group 8 via Triple Replacement Symmetry.* The conditions of Group 8 are directly derived from Group 1 by applying a synchronized triple transformation to the points involved. Unlike previous groups that required only one or two geometric modifications, this group reveals a more sophisticated structural shift involving three distinct replacements.

To make these transitions crystal clear, we summarize them in the following comparative table:

By applying these transformations to the conditions in Group 1, each expression is mapped onto its Group 8 counterpart. Let us illustrate this precisely:

- In every line, we replace each instance of  $(0, -1)$  with  $(0, 1)$ .
- All appearances of  $(-2, -5)$  on the left-hand side are replaced with  $(2, 5)$ .
- The points  $(\pm 1, \pm 4)$ , found on the right-hand side of each condition, are left unchanged.

This triple replacement preserves the algebraic integrity of the expressions, thanks to the symmetry principles inherent in the Quanta Prime Sequence. The transformations maintain both the divisibility relations and the underlying structural patterns embedded within the  $\Omega_0$ -expressions, thereby ensuring the consistency and validity of the entire group.

Conclusion. With the completion of the eighth and final group, we have now delivered a fully self-contained, rigorous algebraic proof of the divisibility relations—one that operates entirely independently of any geometric intuition or visual interpretation.

This marks the successful closure of the group-based classification, establishing a solid algebraic backbone that reinforces the Quanta Prime System and supports the broader architecture of the Mersenne Star. The logical structure now stands complete, coherent, and self-sufficient. ■

### 9.2.1. The structural origin of the eight star points

The Mersenne Star, as introduced in [Theorem 7.6](#), is not a decorative pattern. It arises from the internal structural symmetries of the Quanta Prime Sequence (QPS), which enforce strict constraints on which points can appear in the star.

*But why exactly eight points?*

This is not an aesthetic or arbitrary design choice. The number eight arises unavoidably from the algebraic symmetries proven earlier in Lemmas 6.3 and 6.4, namely:

- Reflections across the horizontal and vertical axes,
- Inversions of both coordinates (central symmetry),
- Directional symmetry (e.g.  $(\zeta, \xi) \mapsto (-\zeta, \xi)$  or  $(\zeta, \xi) \mapsto (\zeta, -\xi)$ ).

Applying these transformations to a minimal seed set generates a closed system of eight distinct points—no more, no less. Any attempt to reduce this set would break closure under symmetry; any extension would introduce redundancy.

The table below classifies the eight points of the Mersenne Star, their coordinates, the symmetry that generates each one, and their structural role in the overall QPS framework.

This eight-point configuration represents the minimal symmetry-closed set under QPS transformations. It is not decorative—it is algebraically determined, structurally balanced, and functionally indispensable to the full geometry of the Mersenne Star.

## 10. Visualizing the theory

The following 32 geometric diagrams illustrate the structure of the Mersenne Star. Each diagram highlights directional flow and symmetry linked to specific divisibility relations. Although all 32 conditions confirm the same primality result, each reflects a unique geometric perspective—collectively forming the full star. These views may inspire future discoveries in number theory, cryptography, or mathematical physics.

## 11. Prime geometry in the AGI—quantum era: toward new paradigms

As the threats posed by quantum computing become more imminent, global institutions are sounding the alarm. The National Institute of Standards and Technology (NIST) has explicitly warned that rapidly advancing quantum technologies could render current encryption systems obsolete within a decade, endangering the privacy and security of individuals, corporations, and entire nations (National Institute of Standards and Technology \NIST), 2024). In parallel, the Information Systems Audit and Control Association (ISACA) emphasized in its 2025 bulletin the need to transition away from classical methods such as RSA and ECC, which are vulnerable to quantum attacks (Beder, 2025).

While this study does not propose a cryptographic protocol, it introduces a mathematical structure—rooted in the Quanta Prime Sequence (QPS)—whose geometry unfolds in the form of the Mersenne Star. This emerging symmetry, inherently tied to the behaviour of large primes, may offer a fertile ground for rethinking foundational questions in number theory and security.

Amid growing concerns that future threats may arise not only from quantum computing but also from the accelerating rise of Artificial General Intelligence (AGI), the need for fundamentally new paradigms has never been more urgent. Recent analyses have echoed this call, particularly within sectors like finance and blockchain security (Naik et al., 2025). Most responses focus on algorithmic or computational redesigns, yet alternative paths—perhaps quieter, rooted in the deep structure of mathematics—deserve attention.

This work simply opens a door. Whether the geometry of primes such as that expressed by the Mersenne Star could eventually inform new directions in secure communications remains to be seen. But the structure's coherence, symmetry, and origin in an unexplored prime-generating sequence hint at possibilities that may resonate beyond pure theory, especially as the world seeks defences not just against quantum systems, but against the compounded impact of AGI-quantum alliances.

## 12. The planetary neighbourhood: a hidden mathematical cosmology surrounding the Mersenne Star

The Mersenne Star is not a standalone structure. While it is built from eight glowing yellow points, forming a perfect star of symmetry in QPS space, it is surrounded by a group of nearby points—like planets—that show deep mathematical meaning.

These surrounding points are not part of the star itself. They are located just near it—around the vertical and diagonal lines that connect the star's arms. When we study them, we find something surprising: they often correspond to famous number sequences such as Fibonacci, Lucas, golden ratio patterns, Fermat-like numbers, and other periodic systems.

This discovery leads us to a new idea: we are not just looking at a star. We are looking at a full geometric system—a kind of mathematical cosmology. In this system, the star is the centre, but the real magic lies in how the surrounding points behave, orbit, and reflect structures we already know from number theory. We now understand that many classical sequences—once thought to appear randomly

in number theory—actually have a hidden geometric home near the Mersenne Star. This is the central idea of this section.

### 12.1. The Lucas Planet: the point (1,3)

Among the many white planets that surround the Mersenne Star, one point begins to shine when we look carefully through the lens of the Quanta Prime Sequence: the point (1,3). This point is not part of the eight yellow vertices of the star. It sits quietly just below one of them—the vertex (1,4). This vertical alignment is not accidental. It is geometric. When we examine the behaviour of the Quanta Prime Sequence, we discover that the following expression plays a central role:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 3 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \cdot L(n)$$

This formula, at first glance, refers to the point (0,1). But with deeper reflection, we find that this is not the final destination. Instead, the QPS at (0,1) acts like a lens, a focusing tool that directs us to the actual location where the Lucas numbers appear: the point (1,3). It is at (1,3) where the Lucas sequence fully emerges in geometric form—not in isolation, but as a structural part of the QPS space. The sequence is no longer a distant list from textbooks. It is a local vibration near the Mersenne Star.

$$1, 3, 4, 7, 11, 18, 29, \dots$$

This transforms our understanding. The point (1,3) is not just a number—it is a Lucas planet, orbiting just beneath a glowing star point, stabilized by the hidden geometry of the QPS. It shows us that the space around the star is alive, resonant, and full of ancient patterns waiting to be rediscovered. So what seemed like a simple formula at (0,1) was actually a key—and the Lucas behaviour it unlocks lies at (1,3). The QPS does not only reveal values. It reveals locations—geometric sites where the structure of number theory has been hiding all along.

### 12.2. The exponential planet: the point (1,2)

As we descend one more step from the Lucas planet at (1,3), we arrive at the next white neighbour—the point (1,2). It appears calm, silent, just below the rhythm of Lucas. But through the lens of the Quanta Prime Sequence, it begins to reveal its true nature. At this position, the behaviour of the QPS is described by the following expression:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 2 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \cdot 2^{\delta(n-1)} \cdot n^{\delta(n)}$$

This is not a simple formula. It is a signal amplifier. The term  $\delta(n)$  is a parity detector—it equals zero when  $n$  is even, and one when  $n$  is odd. The presence of  $2^{\delta(n-1)}$  and  $n^{\delta(n)}$  tells us something subtle and powerful: this point increases or decreases its energy depending on whether  $n$  and  $n - 1$  are even or odd. The planet (1,2) is not just another neighbour. It is an engine. It receives the vibration from (0,1)—the same lens used to detect the Lucas signal—and amplifies it based on the hidden parity logic of the number  $n$ . It shows us how the binary nature of numbers is not separate from geometry. Instead, it is embedded in the space, living at this point. This is why the point (1,2) deserves special attention. It holds exponential keys. It reveals how QPS values can grow or vanish depending on structure. And it stands between the smooth rhythm of Lucas and the chaotic world that lies deeper down the column. It is a gatekeeper—a mathematical switch.

### 12.3. The periodic planet—the point (1,1)

As we move one step below the Lucas point (1,3), and pass through the exponential point (1,2), we reach a very different kind of behaviour at the point (1,1). Here, the Quanta Prime Sequence does not continue growing. Instead, it follows a fixed periodic rule depending on the value of  $n$  modulo 6:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 1 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \begin{cases} +2, & n \equiv 0 \pmod{6} \\ +1, & n \equiv \pm 1 \pmod{6} \\ -1, & n \equiv \pm 2 \pmod{6} \\ -2, & n \equiv \pm 3 \pmod{6} \end{cases}$$

This is the first appearance of a periodic pattern in the vertical column below the Mersenne star. The value of the QPS at this point does not depend on continuous growth or Lucas-type behaviour, but instead on the remainder of  $n$  when divided by 6. This modular structure suggests that the point (1,1) can be used to detect or classify certain values of  $n$  through simple periodic rules. Such behaviour can help in simplifying calculations related to the Lucas numbers, Mersenne values, or the star vertices. If we can describe other points in terms of this periodic structure, it may lead to new ways of analyzing or generating sequences with less computational effort. Therefore, the point (1,1) is the first in a new category of QPS behavior—not exponential, but periodic. It can serve as a basic reference to study repeating cycles and may allow us to reorganize known sequences based on modular conditions.

#### 12.4. The extended periodic planet: the point (1,0)

After reaching the first periodic level at the point (1,1), we continue downward to the next point, (1,0). Here, the periodic behaviour continues, but with a longer cycle. The modular structure now depends on  $n \pmod{8}$ , not  $n \pmod{6}$ .

The QPS value at this point is given by:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, 0 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \begin{cases} +2, & n \equiv 0 \pmod{8} \\ +1, & n \equiv \pm 1 \pmod{8} \\ 0, & n \equiv \pm 2 \pmod{8} \\ -1, & n \equiv \pm 3 \pmod{8} \\ -2, & n \equiv \pm 4 \pmod{8} \end{cases}$$

This is the first appearance of a five-level periodic system in the column. The value of the QPS does not increase or follow exponential growth, but cycles in a controlled, modular pattern. This behaviour confirms that we are now in a region where the QPS becomes sensitive to modular classifications. The point (1,0) helps detect fine differences in the structure of  $n$ , using the modulus 8 system. These periodic points can potentially be used to classify Mersenne primes or Lucas numbers using modular properties instead of direct computations. As we move downward, it is expected that the periodic cycles may become more complex, but also more informative. The point (1,0) extends the idea introduced at (1,1), and helps form a base layer for understanding the repetition patterns in the QPS.

#### 12.5. The large periodic planet—the point (1,-1)

At the depth of the column, below the previously periodic layers, we arrive at the point (1,-1). Here, the QPS follows a more extended and structured periodic cycle based on modulo 12 arithmetic.

The formula governing this point is:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -1 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, -1 | n\right) \cdot \begin{cases} +2, & n \equiv 0 \pmod{12} \\ +1, & n \equiv \pm 1, \pm 2 \pmod{12} \\ 0, & n \equiv \pm 3 \pmod{12} \\ -1, & n \equiv \pm 4, \pm 5 \pmod{12} \\ -2, & n \equiv \pm 6 \pmod{12} \end{cases}$$

Such a system can be used in applications where classification of numbers by fine modular structures is important. It can potentially help in simplifying number-theoretic computations, particularly in detecting specific behaviours of Mersenne numbers or Lucas values. This point confirms that the lower levels of the Mersenne Star's structure are not chaotic—they are organized in large modular rings.

### 12.6. The inverse exponential planet—the point (1,-2)

At the point (1,2), we observe a return to exponential behaviour, but in a reverse direction compared to the points above the star. The value of the QPS at this location is given by:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -2 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \cdot 2^{\delta(n+1)}$$

This expression reuses the QPS evaluated at the point (0,1), but it is multiplied by an exponential factor that depends on  $\delta(n+1)$ , where  $\delta(n) = n \bmod 2$ . This formula suggests that even at deeper levels of the vertical axis, the influence of earlier values continues. The structure is not entirely disconnected. The presence of  $\delta(n+1)$  indicates that the parity of  $n+1$  plays a role in determining the weight of the QPS value. This point shows that the QPS does not behave linearly with depth. Instead, we see a combination of periodic and exponential behaviour, depending on the parity of the variable and the nature of the location in the grid. The point (1,2) is a good example of how QPS blends logic from both modular and exponential systems, maintaining structural coherence even in lower levels. This deepens the picture: the modular behaviour at (1,1), (1,0), and (1,1) now leads into an expression that involves both previous points and future parity information—a backward connection that respects the global structure of the Mersenne Star.

### 12.7. The selective planet—the point (1, -3)

At the deepest point of the visible vertical structure, the point (1, -3) presents a unique and important combination. The value of the Quanta Prime Sequence at this location is:

$$\Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 1, -3 | n\right) = \Omega_0\left(\left\lfloor \frac{n}{2} \right\rfloor | 0, 1 | n\right) \cdot \begin{cases} F(n) & \text{if } n \text{ odd} \\ L(n) & \text{if } n \text{ even} \end{cases}$$

This expression means that the QPS value from the point (0,1) is being scaled by either a Fibonacci number  $F(n)$  or a Lucas number  $L(n)$ . This behaviour reveals a deep connection between QPS and classical number sequences.

- This point links the QPS with the Fibonacci and Lucas families.
- It opens a door for expressing QPS values using well-known recursive sequences.
- It may help simplify or approximate values in other parts of the Mersenne Star.
- It shows that lower points in the vertical structure are not random but connected with known mathematical systems.
- This location may act as a reference point that adapts to different contexts, depending on whether we use  $F(n)$  or  $L(n)$ .

## 13. Physics analogy: geometric patterns and physical structure

The Mersenne Star, derived from the Quanta Prime Sequence, displays geometric symmetries and periodic points that resemble structures seen in physical systems. Certain locations, such as (1,1) and (1,0), exhibit periodic behaviour suggestive of oscillations. These symmetries mirror patterns like parity and resonance, and the directional arrows linked to divisibility conditions may be interpreted as vector-like flows. Though purely mathematical, the star's structure invites further investigation into potential analogies with physical fields, crystals, or dynamic systems, offering a spatial framework to rethink prime distribution.

## 14. Future perspective

The star could inspire new models for prime distribution or algorithms for primality testing. Its geometric insights point to a deeper connection between mathematics and physics, encouraging interdisciplinary

research. However, integrating AI into such studies presents both opportunities and challenges, as highlighted in recent discussions on AI's impact on physics (Das Sarma, 2023).

### 14.1. Potential for a transformative law: unveiling a new region of periodic patterns

The Mersenne Star has revealed a significant region of periodic points, forming vertical strips from  $(1, 2)$  to  $(1, -2)$  and from  $(-1, 2)$  to  $(-1, -2)$ , with the points mentioned earlier—such as  $(1, 1)$  (Period 6),  $(1, 0)$  (Period 8), and  $(1, -1)$  (Period 12)—being mere examples. This discovery uncovers an extensive region characterized by periodicity, where the periods appear to be infinitely selectable, though such details are beyond the scope of this study. We draw the reader's attention to a critical insight: if we can establish a mathematical law connecting the star's vertices (e.g.  $(1, 4)$ ,  $(1, -4)$ )—particularly those in close proximity—to the periodic points of their neighbours, points we had not previously noticed, a breakthrough could emerge. This connection, inspired by the Mersenne Star's framework, opens a pathway to select appropriate periods, potentially revolutionizing not only our understanding of Mersenne primes but also number theory at large. Through the map of the Mersenne Star, we may initiate a new era of linking points—not just the star's vertices to their neighbours, but also vice versa, and even extending to connections between sequences like Fibonacci numbers and periodic points. The infinite strip of periodic numbers could play a significant role in simplifying computations through their inherent periodicity, paving the way for profound advancements in geometric number theory.

## 15. Artificial intelligence and the Mersenne Star: two practical visions

It began with a simple yet powerful question by Alan Turing: "Can machines think?" This question laid the foundation for decades of research in artificial intelligence.

In the 1980s, Geoffrey Hinton and collaborators introduced the backpropagation algorithm, which enabled neural networks to learn from data (Rumelhart et al., 1986). A neuron receives inputs  $X_1, X_2, X_3$ , each weighted by  $W_1, W_2, W_3$ , and computes:

$$z = X_1W_1 + X_2W_2 + X_3W_3,$$

The result is passed through an *activation function*, such as:

$$\text{ReLU}(z) = \max(0, z), \quad \text{sigmoid}(z) = \frac{1}{1 + e^{-z}}.$$

These nonlinearities allow networks to model complex behaviours.

By 2025, AI has advanced far beyond routine tasks. Systems like AlphaFold 3 astonished the scientific community by accurately predicting protein structures, accelerating discoveries in biology and chemistry (Abramson et al., 2024).

In this same spirit, the Mersenne Star proposes a structured geometric approach to understanding prime numbers. Rather than relying solely on computational brute-force, this star-shaped framework may guide AI toward recognizing patterns of primality through symmetry and periodicity.

Despite significant technological progress, only 52 Mersenne primes have been discovered (Great Internet Mersenne Prime Search (GIMPS), 2023), each requiring years of distributed computation. While the Lucas-Lehmer test provides a method to confirm primality, it offers no guidance on where to search. What is missing is not the test—but the map. This work introduces the Mersenne Star as a potential geometric map for prime prediction. It offers a structural lens unique to Mersenne primes, organizing possibilities spatially. Though still in development, we believe this perspective may illuminate new paths toward intelligent discovery.

Application I: GeoPrime<sup>2</sup> is a geometric tool designed to support future AI models by highlighting patterns in the Mersenne Star that may guide prime discovery. It represents an early step toward structural, rather than brute-force, approaches to understanding primes.

## 16. Application I: GeoPrime—the star as a geometric guide to prime numbers

Prime numbers have long appeared as isolated landmarks in the landscape of mathematics. The Mersenne Star introduces a structured, geometric-algebraic framework—defined by 32 exact conditions—that may offer a path to understanding their distribution.

We propose the following question:

*Can the Mersenne Star serve as a visual and algorithmic guide to predict the locations of Mersenne primes?*

The approach begins by embedding the 52 known Mersenne primes into the star's geometry and analyzing their spatial distribution. The key insight lies not only in where primes appear—but also in where they do not. Non-prime zones offer critical negative data, enabling AI to refine its sense of primality through contrast.

- Do primes align near axes or specific intersections?
- Do they cluster in symmetric regions?
- Is there a gravitational centre for prime density?

GeoPrime explores these questions through four main steps:

1. Reconstruct the Mersenne Star using the 32 governing constraints.
2. Position each known Mersenne prime within its geometric location.
3. Analyze patterns of density, reflection, and distribution.
4. Use AI to estimate where the 53<sup>rd</sup> Mersenne prime may reside.

This is more than mere pattern recognition—it is yet a novel approach to uncovering the geometric behaviour of prime numbers. The star acts as a symbolic city of primes, full of intersections, reflections, and paths that may trace hidden laws of primality.

We believe its true potential lies in:

- Structuring the prime landscape geometrically,
- Guiding mathematical and computational exploration,
- Revealing deeper mathematical principles yet to be formalized.

Like a navigational compass, the Mersenne Star may one day guide our steps through the uncharted territory of prime numbers.

## 17. Example protocol: binary mapping within the GeoPrime framework

In this example, we introduce a prototype approach under the GeoPrime framework. The core idea is to embed the *binary representation* of a given integer into the geometric structure of the Mersenne Star. This allows us to visualize and study how different numbers—particularly primes and non-primes—trace distinct spatial patterns, potentially revealing hidden geometric signatures.

The idea is simple: every number generates a unique geometric path based on its binary digits (0s and 1s). This path is constructed by moving step-by-step, where each binary digit controls a directional rule or geometric angle. Once the full path is generated, we mark its key points and place them inside the fixed structure of the Mersenne Star, enabling visual comparison with other number representations.

Each number, whether prime or not, creates a distinct configuration. The Mersenne Star acts as a stable geometric stage, allowing us to display many such configurations and visually analyze their behaviour.

How are the points generated? Each number is converted into a geometric configuration using polar coordinates as follows:

- Start with the binary representation of a number  $n$ :  $n = (b_0, b_1, \dots, b_k)$
- For each bit  $b_i = 1$ , define:

$$\begin{aligned}\theta_i &= i \cdot \frac{\pi}{4} \quad (\text{angle of rotation}) \\ r_i &= a + i \quad (\text{radial shift, with } a = 2 \text{ by default}) \\ x_i &= r_i \cdot \cos(\theta_i), \quad y_i = r_i \cdot \sin(\theta_i)\end{aligned}$$

- Plot the resulting points  $(x_i, y_i)$  within the Mersenne Star diagram.

*Example:* To illustrate, the number 31 is represented in binary as:

$$31 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 = 11111_{(2)}$$

This sequence of five ones generates a consistent turning pattern. The corresponding five points for 31 are shown in orange—plotted in the diagram on the right (see [Figure 40](#)) and also within the full star configuration below (see [Figure 41](#)).

*Note:* The choice  $\theta = \pi/4$  and  $a = 2$  is illustrative; other values may yield alternative useful patterns.

*Note:* All coordinates are expressed in exact symbolic form to preserve the purity of number-theoretic analysis. Any visual approximation in diagrams is for illustrative purposes only. This binary-to-geometry method works on a one-to-one basis: each number has exactly one binary expansion, and each binary expansion results in one specific configuration. Thus, every number is encoded geometrically in a unique and unambiguous way.

What makes this idea powerful is the potential for artificial intelligence to study these geometric outputs. Currently, only 52 Mersenne primes are known, but we also know over 136 million values of  $2^p - 1$  that are not prime. This gives us a massive training dataset: 52 examples of known prime configurations, and over 100 million non-prime ones.

By analyzing these with machine learning, the AI can begin to recognize which geometric characteristics often lead to non-primality, and which ones suggest a higher likelihood of primality. Even if the AI cannot *prove* a number is prime, it may still *predict* which numbers to ignore and which ones deserve more attention, based on their geometric profile.

If successful, this approach could significantly reduce the computational effort involved in testing large Mersenne numbers. Rather than spending years computing Lucas-Lehmer sequences for each candidate, this method could allow early filtering and prediction within seconds—a potentially transformative advance in prime detection.

### 17.1. Towards intelligent prime prediction

Recent developments in artificial intelligence and physics hint at a deeper convergence between deep learning architectures (LeCun et al., 2015), general intelligence frameworks (Whiteson et al., 2023), competitive forecasting landscapes (Jones, 2025), and generative models in condensed matter systems (Arnold et al., 2024).

It is known that only 52 Mersenne primes have been confirmed, while over 136 million candidates of the form  $2^p - 1$  have been tested. That is:

$$\begin{aligned}\text{Tested} &: 136,279,841 \\ \text{Knownprimes} &: 52 \\ \text{Confirmednon-primes} &: \geq 136,279,789\end{aligned}$$

This enormous imbalance offers a powerful opportunity: we can train intelligent models using both the rare prime configurations and the vast majority of non-prime configurations—more than 100 million labelled examples.

Through the GeoPrime system, each number is mapped to a deterministic geometric signature. This allows artificial intelligence to:

- Detect which geometric characteristics frequently indicate non-primality.

- Identify configurations that resemble those of known primes.
- Sharply reduce the effort needed by shrinking the effective search space.
- Predict the likely location of the next Mersenne prime (e.g. estimate where the 53<sup>rd</sup> one might appear).

## 17.2. Final note

The present paper firmly establishes the theoretical foundations and proofs for the Mersenne Star and its neighbouring structures. It further introduces *GeoPrime* as a pioneering technological prototype, highlighting the potential of the Mersenne Star to evolve into a powerful emerging technology. By building on these results, this work may help lay the groundwork for two emerging scientific disciplines: Geometry of Prime Numbers and Artificial Prime Intelligence (API)<sup>3</sup>—a transformative fusion of the Mersenne Star, prime numbers, and artificial intelligence.

## Notes

1. U.S. Provisional Patent Application No. 63/861,388, filed 11 Aug 2025.
2. U.S. Provisional Patent Application No. 63/861,111, filed 10 Aug 2025.
3. U.S. Provisional Patent Application No. 63/861,399, filed 11 Aug 2025.

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## Disclosure statement

The author declares ownership of the following intellectual property: *Mersenne Star*—U.S. Provisional Patent Application No. 63/861,388, filed with the USPTO on 11 Aug 2025. *Artificial Prime Intelligence (API)*—U.S. Provisional Patent Application No. 63/861,399, filed with the USPTO on 11 Aug 2025. *GeoPrime*—U.S. Provisional Patent Application No. 63/861,111, filed with the USPTO on 10 Aug 2025.

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## Data availability statement

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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