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Pulling Down the Hierarchy

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ABSTRACT

This paper is about the hierarchy view: that each word has infinitely many meanings, arranged into levels, with the level n meaning serving as its semantic value when it occurs embedded to degree n in indirect or attitude reporting verbs. Departing from the famous debates over the bare tenability of the hierarchy view, I focus on whether there is any positive reason to endorse it. Philosophers have offered two such reasons. One appeals to intersubstitutability phenomena, first pointed out by Benson Mates. The other appeals to Frege's theory of indirect discourse. I examine each of these and find that neither actually supports the view. Regardless of its bare tenability then, we should reject the view.

1 | Introduction

This paper is about the *hierarchy view*, often associated with Frege: that for each word there is an infinite hierarchy of semantic values that it contributes in different kinds of occurrence. Much of the discussion of this view has concerned its bare tenability rather than its plausibility. My interest here is in what there is to be said in support of it—a different topic, on the assumption that "It has not been decisively refuted" is not such a reason. I will look at two considerations—one example-driven, the other theory-driven—that continue to be endorsed by prominent philosophers. On examination, neither supports the view. If there are, as I think, no good reasons supporting the view then the question of its tenability is moot.

One consideration given in support of the view appeals to cases of multiple embeddings in which intersubstitutability *salva veritate* seems to be more stringent than it is within single embeddings. The claim is that the hierarchy explains this apparent phenomenon. The problem here is that there is another explanation that is at least as plausible overall; so the hierarchy view does not come close to being supported by an inference to the best explanation.

There are also principled arguments meant to show that the hierarchy view follows from Frege's claim about the semantics of indirect discourse. Such arguments have been endorsed even by those who reject the hierarchy view: They take the arguments to show that Frege's claim must be rejected or modified. But to those who accept the claim, the arguments are reason to accept the hierarchy view. The best-worked-out argument of this sort is due to Tyler Burge. Burge's argument is careful and detailed—dauntingly so. It is widely endorsed. Yet, its crucial step is unexplained. On examination, it turns out that that step presupposes the hierarchy view. The argument, then, gives it no support.

So the hierarchy view has even less going for it than has been thought. What then explains its enduring appeal—such as it is, mostly among Fregeans? I diagnose it as resting on an error in thinking of Frege's claim about the base case. The error is mistaking what a word's semantic value does for something the word itself does. On a Fregean approach, there is no such thing as "multiple embedding," let alone a theory of it.

2 | The Hierarchy View

There are two parts to the hierarchy view: a claim that there are certain entities associated with a word and a claim about how those entities figure in semantic composition.

The prototype of the hierarchy view is, of course, the view often attributed to Frege. (Below, I discuss the basis in Frege's writings for this attribution.) It goes as follows. When a word

occurs unembedded—that is, not in the scope of an attitude verb or other *de dicto* construction—its semantic value (what it contributes to the compositional determination of the overall sentence's truth condition) is its reference. But things work differently when the word is so embedded: There its semantic value is its sense. When it is doubly so embedded—as "Paris" is, in

> (Double) John believes that Susan said that Paris is beautiful

then its semantic value is different still: It is its "indirect sense." And so on: Each expression has infinitely many senses, arranged into levels, with the sense at level n being the expression's semantic value when it occurs embedded to degree n. Finally, expressions that share a level n sense can differ in their level $n+1$ sense. The Fregean reason for this is that for any semantic value there can be different senses "presenting" it. Each word's level $n+1$ sense "presents" its level n sense.

That last feature means that I am discussing what Terence Parsons (1981) labelled the "libertine" version of the view. The contrast is with a "rigid" version, on which "any two expressions that share a customary sense also share their nth-level sense, for any n" (44). There are several reasons for my focus on the libertine version. First, that is the version that finds support in the first of the considerations I will discuss, the one having to do with intersubstitutability phenomena. Second, since Burge's argument proceeds as a *reductio* against the no-hierarchy view, it is neutral between the rigid and libertine versions of the hierarchy. Finally: the view that the hierarchy is libertine is the logically weaker, more defensible, claim; so working with it makes my goal—to show that there is nothing to be said in support of the view—harder to achieve.[1]

It should be emphasized that Frege does not so much as mention the hierarchy idea when explaining his theory of sense and reference. Indeed, the claim he does state about their semantic roles (discussed below, §4) conflicts with the hierarchy view, for it makes no appeal to a notion of degree of embedding: Its claim is about occurrences that are embedded, period. This means that the reader who wants to attribute the view to Frege faces an interpretative challenge. Parsons, recognizing the conflict, makes the following suggestion:

> It seems plausible to me that when Frege stated his principle of indirect reference... he just was not thinking of doubly embedded contexts, and it is really up to us to *extend* the theory he had in mind to account for such contexts. (1981, 40)

While understandable, I think that this take on the situation rests on an error that favors the hierarchy view; more on this below, §5.

Another point is that it is anachronistic to state any view of Frege's using the contemporary concept of semantic value, as I have done. As we will see below, Frege stated his view of indirect discourse in terms of his concepts of reference and sense. Formulating the hierarchy view in terms of semantic value is rather simpler, however, so I will continue to take that liberty.[2]

The Fregean hierarchy has been described as overly complicated (Carnap 1947, 130–32); as entailing the impossibility of finite beings learning languages (Davidson 1965); as requiring notions that "there is no way available to state" (Dummett 1973, 267); as requiring an "ambitious Platonist epistemology" (Soames 2010, 16); as "extravagant" (Yalcin 2015, 247n.15). I will not address any of these objections, important though they are. (Both Burge (2005) and Saul Kripke (2008) have mounted sophisticated defenses against some of them.) My concern is different: is there any positive reason to endorse such a view? I will examine what seem to me to be the two most often endorsed reasons.

3 | Intersubstitutability Considerations

Philosophers sometimes have approvingly claimed that the hierarchy view explains an apparent phenomenon concerning intersubstitutability *salva veritate* within the scopes of indirect discourse locutions. I will argue that such considerations do not take us very far. In particular, they do not support an inference to the hierarchy view as the best, or even an especially attractive, explanation.

3.1 | Mates's Examples

The intersubstitutability phenomena in question were first pointed out by Benson Mates (1950, 215), who claimed that no matter how close in meaning two sentences D and D' are, when they are put into the schemata

> Nobody doubts that whoever believes that D, believes that D.

and

> Nobody doubts that whoever believes that D, believes that D'.

the result is a pair of sentences differing in truth condition. Mates himself did not discuss any instances of his schemata, but Hilary Putnam (1954) started a tradition of instantiating them under the assumption that "Greeks" and "Hellenes" are synonyms. On that assumption we might work with the following instances.[3]

> (GG) Nobody doubts that whoever believes that Greeks are happy, believes that Greeks are happy.
>
> (GH) Nobody doubts that whoever believes that Greeks are happy, believes that Hellenes are happy.

Many have found Mates's claim plausible. It is difficult to see what would even count as someone falsifying (GG)—what would even count, that is, as a case of someone doubting that *whoever believes that Greeks are happy, believes that Greeks are happy.* On the other hand it seems easy to imagine someone who falsifies (GH). All it would take is for them to be less than sure that believing that Greeks are happy necessarily co-occurs with believing that Hellenes are happy. It is not required that such a doubter be justified, let alone correct, in their doubt; only that it be possible that they have it. And that seems

possible even if believing that Greeks are happy is believing that Hellenes are happy (as would be the case, on many views, if "Greeks" and "Hellenes" are indeed synonyms).

Mates's examples have mostly inspired two thoughts. One (Form) is that a difference in *logical form* can matter for intersubstitutability in embedded occurrences; the replacement of *D* with *D′* in the schema does, after all, change a logically valid complement (to "doubts") into one that is not logically valid. (Putnam 1954; Burge 1978; Fodor 1993, 32; Fodor 1998, 16; Taschek 1995, 130; Fine 2007; and Pryor 2017 think of the examples in this way.) The other (Embedding) is that it can matter for intersubstitutability that there is a difference in *degree of embedding*; sentences *D* and *D′* do, after all, occur doubly embedded within the scopes of attitude verbs. (Kripke 1979; Owens 1986; Moffett 2002; Sorensen 2002, 172n.12; Burge 2005, 78; Sainsbury and Tye 2012; Horwich 2014; and Rattan and Durand 2022 think of the examples in this way.) Specifically, Embedding says that distinct expressions that are intersubstitutable in some singly embedded occurrences in attitude-ascribing sentences can fail to be intersubstitutable when those sentences are themselves embedded as complements in attitude-ascribing sentences.[4]

Form and Embedding are logically independent claims; it is simply a quirk of Mates's examples that because they are complex in two ways, they have inspired two independent thoughts. We can consider Embedding on its own. As was noted long ago by Noam Chomsky, the variant schema "M believes that N believes that *D*" generates equally plausible examples that support Embedding without also supporting Form (Scheffler 1955, 42n.7; see also Horwich 2014, 1132). Just let John be our imagined person: He thinks that someone could believe that Greeks are happy while not believing that Hellenes are happy. Then the following two sentences differ in truth condition, *regardless* of whether the embedded "Mary" sentences themselves differ in truth condition.

> John believes that Mary believes that Greeks are happy.
>
> John believes that Mary believes that Hellenes are happy.

My focus in this paper is on Embedding, and the question of whether it motivates the hierarchy view, as some have claimed it does. The fact that Form is an independent claim means that we needn't discuss it in pursuing our question about Embedding.

3.2 | The Explanatory Proposal

The hierarchy view seems to support an explanation of Embedding. Consider our assumed synonyms "Greeks" and "Hellenes." Suppose that they have the same level *n* sense. On the hierarchy view it follows that they make the same contribution to semantic composition when embedded to degree *n*. But on the hierarchy view, their having the same level *n* sense does not preclude their having different level $n+1$ senses. Suppose that they do. This means we no longer have the guarantee we had at level *n*, on the basis of identity of semantic value: that replacing one of them with the other will make no difference to the truth condition of the overall sentence. Thus—the thought goes—we explain why they fail of intersubstitutability when embedded to degree $n+1$ while not doing so when embedded to degree *n*.

Several philosophers seem to endorse this kind of explanation. Tyler Burge claimed that those who take Mates's examples to support Embedding, and take a generally Fregean approach to the semantics of embedded occurrences, must "allow that the [level 2] senses of word-forms with the same [level 1] sense can differ" (2005, 172-73n.12). Kit Fine also claims that that difference would explain what we see in Mates's examples (2007, 129), although he argues that there are variant examples for which that explanation fails. Mark Sainsbury and Michael Tye write, "One attractive feature of the [hierarchy] view is that it gives an explanation of why deeper embeddings may increase the persuasiveness of Mates cases: the higher the value of n, the more likely that expressions that agree on [level 1] sense should disagree on [level *n*] sense" (2012, 80). Paul Horwich writes, "Mates's general point [Embedding] is indeed correct" and defends "its explanation in terms of higher-order senses" (2014, 1135) along the lines just described. Indeed he goes so far as to call the hierarchy Frege's "prescient response to Mates's critique" (1134). Finally, Gurpreet Rattan and Marion Durand have recently argued carefully for the Fregean hierarchy's application to Mates's examples. They reason in the above fashion, to the claim that "The two instances [of Mates's schema] can have different truth-values because [the level 2 sense of 'Greeks are happy'] ≠ [the level 2 sense of 'Hellenes are happy']" (2022, 13) and that this application of the hierarchy to Mates's puzzle "poses a problem for views that deny the existence of the hierarchy" (14).

Several of these authors clearly think not only that the hierarchy view supports an explanation of Mates's examples but also that its doing so is a consideration in favor of it.

3.3 | The Problem With the Proposal

Whether the hierarchy theorists' explanation counts in favor of the hierarchy view depends, however, on what other explanations are available. The existence of other explanations that are at least equally good would undermine the support that the hierarchy view gets from the fact that it supports an explanation. Let us grant that some hierarchy-based explanation along the lines just described works. Is there another possible explanation? There is, and it is better in at least one relevant respect.

The rival explanation begins by agreeing (to continue with our working example) that when doubly embedded, "Greeks" and "Hellenes" have distinct semantic values. But it says that they have those same—distinct—values also when singly embedded, as they are in "John believes that Greeks are happy" and "John believes that Hellenes are happy." How then does it explain their intersubstitutability in those singly embedded occurrences? By appeal to a fact about believing: that necessarily, every state of believing that Greeks are happy is a state of believing that Hellenes are happy. Of course this claim would have to be backed by an account of what belief states are and how they are individuated. One could object that this kind of explanation thus involves an illegitimate mixing of claims about semantic values and claims about real-world states of affairs that sentences specify in terms of those values. But as I will now explain, the hierarchy theorists' explanation also requires just such a mix of claims. So this point about the rival explanation does not count against it in the comparison with the hierarchy explanation.

The reason that the hierarchy theorist must appeal to claims about belief is that their claims about semantic values are not enough to explain the Matesian pattern of intersubstitutability data. On the hierarchical semantic analysis, "Greeks" and "Hellenes" contribute different semantic values when they occur doubly embedded. But in order to explain a difference in truth condition between the variant sentences, it is not enough to cite this fact. The reason is simple: A difference in semantic value between corresponding components of variant sentences does not necessarily make for a difference in truth condition between those sentences. Let me explain.

Semantic composition, we are supposing, is a matter of functions taking arguments; and a function can return the same value for distinct arguments. "Scarlet" and "burgundy" are different hues (of red), but the complex predicates "has the color of which scarlet is a hue" and "has the color of which burgundy is a hue" necessarily apply to the same things.[5] Obviously one cannot appeal to the difference in semantic value between "scarlet" and "burgundy" to explain a difference between the semantic values of those complex predicates, because there is no such difference to be explained. But it follows from this that where there is a difference in the semantic values of some pair of variant complex expressions, merely noting a difference in semantic value between the varying components is not enough to explain it. What this means for the hierarchy theorist is that in order to explain a difference in truth condition between two belief-ascriptive sentences that differ only in that "Greeks" occurs doubly embedded in one where "Hellenes" occurs doubly embedded in the other, the hierarchy theorist must appeal to some fact about belief states.

So both explanations—the hierarchy theorist's and the rival, fine-grained-values theorist's—require claims about the real-world states specified by "that"-clauses, in order to fully explain any pattern of differences in truth conditions among attitude-ascriptive sentences. The two proposals are on a par in this respect.

However, the rival explanation is better than the hierarchy theorist's explanation in the following respect: it postulates fewer entities. The hierarchy theorist, notoriously, proposes a vast hierarchy of semantic values of different levels. The rival theorist proposes only one set of (very fine-grained) values. So, it is preferable to the former on grounds of ontological parsimony.

Overall, then, the proponent of the hierarchical semantic analysis cannot support their view by anything like an inference to the best explanation. For their explanation is inferior to its rival.

The hierarchy proponent might object to the rival explanation by claiming that it proves too much. They might claim that it could be pushed, absurdly, all the way down to extensional occurrences. Suppose we said that "Greeks" and "Hellenes" have distinct semantic values in all occurrences, even unembedded ones. We could then explain their intersubstitutability in "ϕ are happy," say, as due to a fact about relations between those fine-grained values. Why should we not do that, if the above anti-hierarchy reasoning is sound? So the objection goes.

The objection misconstrues the dialectic. My point is not that we should assign the finest-grained values whenever it is possible to do so. If that were the point, then the objection would have some bite. Rather, my point is that Embedding, in particular, can be explained perfectly well—better, perhaps—without the hierarchy view. The upshot of this weaker claim is that the hierarchy view's supporting an explanation of Embedding is not a reason to endorse it.

Another objection is that the balance tips in the other direction once we take into account each view's metaphysical commitments. Both the hierarchy theorist and the no-hierarchy theorist, in order to explain substitution *failures* (with multiply embedded occurrences), postulate the *possibility* of someone's believing this while failing to believe that. But the no-hierarchy theorist needs an additional assumption in order to explain substitution *successes* (with singly embedded occurrences): They have to postulate the *necessity* of someone's believing this if they believe that. The objection, now, is that this is doubly problematic: In that it is an additional assumption needed by the no-hierarchy theorist and not by the hierarchy theorist; and in that it is a very strong assumption, amounting to the postulation of necessary connections between distinct entities.[6]

Seeing how the second of these points is met points the way to seeing how the first is. Regarding the second point: the no-hierarchy theorist should not say that their view involves postulating the necessary co-occurrence of distinct entities. Rather they should say that the operative claims, for explaining substitution successes, are *identity* claims: that, for example, believing that Greeks are happy *is* believing that Hellenes are happy, even though the semantic values of the words "Greeks" and "Hellenes" differ (even when singly embedded). The claim is that those distinct entities are used to specify one state, just as we can use the distinct ordinal numbers ω and $\omega + 1$ to specify one cardinal number, or distinct hues to specify one color.

Regarding now the first point: Since the no-hierarchy theorist makes use of identity claims in the way just described, their ontology of belief (or other attitude) states is exactly the same as that of the hierarchy theorist. Both views, answering to the same pattern of data, explain the intersubstitutability of some sentences when singly embedded by appeal to identity claims. For the hierarchy theorist, they are claims about identity of content (between those sentences in those occurrences); for the no-hierarchy theorist, they are claims about identity of attitude state (specified by those sentences when singly embedded). But the latter view makes do with one set of entities. It exploits, when needed, the fact that it is possible to specify one thing in different ways which involve reference to different entities. The hierarchy theorist, then, multiplies entities unnecessarily.

4 | Burge's Argument

One might agree that considerations regarding intersubstitutability do not motivate the hierarchy view. But there is another line of thought we must address. Many philosophers have

thought that the view follows from, or at least is strongly suggested by, Frege's claim about how indirect discourse works.

4.1 | Frege's Core Principle

What I will call Frege's **core principle** involves a distinction between two kinds of occurrence that words can have[7]:

> In reported speech one talks about the sense, e.g., of another person's remarks. It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. In order to have a short expression, we will say: In reported speech, words are used *indirectly* or have their *indirect* reference. We distinguish accordingly the *customary* from the *indirect* reference of a word; and its *customary* sense from its *indirect* sense. The indirect reference of a word is accordingly its customary sense. (1892, 59)

Nowhere in his essay—or any published work for that matter—is there mention of a hierarchy of senses. However, there is a passage (cited, e.g., by Parsons (1981, 40) and Burge (2005, 179)) in a letter Frege wrote to Bertrand Russell in which he seems to commit himself to the hierarchy view. He writes:

> Since "*M*" has different meanings in its two occurrences in the proposition "the thought that all thoughts belonging to class *M* are true does not belong to class *M*," there must also be a difference in the meanings of "*M*" in the expression "the thought that the thought that all thoughts belonging to class *M* are true does not belong to class M." It can be said that in the twice-underlined part it has an indirect meaning of the second degree, whereas in the once-underlined part it has an indirect meaning of the first degree. (1902, 154, restoring underlining)

Frege's use of "since" suggests that he takes his claim about the semantics of the doubly embedded words to follow from his claim about that of the same words when singly embedded. But he does not elaborate his reasoning. My focus below will be on precisely that inference.[8]

Sometimes the thought that the core principle suggests or entails the hierarchy view is expressed with just a brief remark. Parsons introduces it by saying that "when a sentence containing an indirect context is embedded in another indirect context, a similar sort of 'elevation' of senses and references takes place" (1981, 39). Scott Soames says, after explaining the core principle, that with doubly embedded expressions "The process [of reference and sense shift] is repeated" (2010, 14). Sometimes the thought is referred to as being already established: Kripke (2008, 183) says that the hierarchy view is a "familiar consequence" of the core principle. I will discuss below (§5) the feeling of obviousness that the implication of the hierarchy view seems to have had for many readers of Frege (and perhaps, given the "since" in his letter, for Frege himself). My concern in this section is with attempts to support this thought with arguments.

Two prominent arguments were given by Rudolf Carnap (1947, 130–31) and by Michael Dummett (1973, 267). While each is interesting in its own right, each has readily diagnosed flaws, so I will pass over these arguments here. Neither, as far as I am aware, is endorsed by anyone nowadays.[9]

4.2 | Burge's Argument

Tyler Burge (1979, 2005), however, has given another Fregean argument that is widely endorsed.[10] Burge argues for the hierarchy on the basis of claims about how Frege's core principle plays out in the formalization of a natural language.

(I should clarify what I mean by "Burge's argument," because in his 1979 essay Burge gives two arguments. My topic is the argument he gives in §II. That argument is not fully reprised in his 2005 "Postscript." There, Burge presents only a "simplified variant" (185n.24) of it which is directed at a proposal by Christopher Peacocke (1996, 142–44; 1999, 245–62). So I focus on the argument's presentation in the 1979 paper. As for that paper's §III argument, Burge repeatedly states that it depends on the §II argument.[11] So it is enough, for my purpose, to show that the latter fails.)

Concerning formalization, we can ask: Should we use the same, or different, expressions when we formalize a word's occurrences that have different semantic values? What Burge refers to as Frege's "general insistence on avoiding ambiguity in a well-constructed language" (156n.2)—in Frege's famous phrase, a "logically perfect" (1892, 58) language—suggests we should use different expressions.[12] On that approach we "represent expressions in natural-language oblique contexts [i.e., Frege's 'reported speech'] with symbols [in the formal language] which are different from the symbols that represent those same expressions as they occur in ordinary contexts" (156). This is the generally prevailing view on how Frege thinks of formalization. (See, e.g., Parsons 1981, Klement 2002 and Kripke 2008.) When we pursue the details of how this formalization goes, Burge argues, we find that we must endorse the hierarchy view on pain of absurdity.

Burge's presentation of his argument is daunting. The following table collects his claims about what expressions in the formal language, *F*, he assigns as the formalizations of different occurrences of English expressions, and what their semantic values are in *F*. (He works with the example sentence, "Opus 132 is a masterpiece.") I notate assumptions as "A1" and so on, except for the *reductio* assumption R. "C" statements are conclusions from previous statements. Assumptions A1–A5 follow directly from Frege's core principle. The *reductio* assumption R reflects the no-hierarchy view: it assigns to doubly embedded occurrences of the "Opus" sentence the same value that is assigned to singly embedded occurrences (viz, the Thought that Opus 132 is a masterpiece). Burge's aim is to derive an absurdity from R, against the background of A1-A5, thereby showing its falsity—hence, the truth of the hierarchy view—on the assumption of the core principle.[13]

	English expression's occurrence	*F expression*	*Its value in the semantics of F*
A1	"Opus 132 is a masterpiece" occurring singly embedded	"α"	The Thought that Opus 132 is a masterpiece
A2	"Bela" occurring unembedded	"β"	Bela
A3	"believes" occurring unembedded	"Γ"	The believing function, whose arguments are a person and a Thought, and which returns $\top$ or $\bot$
A4	"Bela" occurring singly embedded	"β_1"	Mode of presentation of Bela
A5	"believes" occurring singly embedded	"Γ_1"	A mode of presentation of the believing function
R	"Opus 132 is a masterpiece" occurring doubly embedded	"α"	The Thought that Opus 132 is a masterpiece
C1	"Bela believes Opus 132 is a masterpiece" occurring unembedded	"$\Gamma(\beta, \alpha)$"	$\top$
	This follows from A1, A2, and A3.		
C2	"Bela believes Opus 132 is a masterpiece" occurring singly embedded	"$\Gamma_1(\beta_1, \alpha)$"	The Thought that Bela believes that Opus 132 is a masterpiece
	This follows from R, A4, and A5.		

The table itself contains nothing absurd. We do have the sense of "Opus 132 is a masterpiece" combining both (in C1) with the "believes" function, and (in C2) with a mode of presentation of that function. This is odd, but it is not absurd; nor does Burge claim that it is. Nor does he find any incoherence in the notational assumptions A1–A5. Given that his is a *reductio* argument, Burge must take it that they are unproblematic; otherwise, any absurdity derived from them in conjunction with R cannot be blamed on the latter. I too think that the notational assumptions are unproblematic. Where I differ from Burge is that I think that there is no problem with adding R to them: The entire table is coherent. The absurdity that Burge finds is due, rather, to an unstated assumption in his reasoning *from* what is in the table.

Burge reasons as follows:

> Let "β" express β_1 and denote Bela; let "Γ" express Γ_1 and denote what "believes" denotes (or have its extension). Let "α_0" express α and denote its truth-value. (We suppose that truth-value to be truth.) Then "$\Gamma(\beta, \alpha_0)$" express [sic] $\Gamma_1(\beta_1, \alpha)$ and denotes its truth-value. "Believes" originally applied to persons and [Fregean] propositions. But on our assumptions it has come also to apply to persons and truth-values. This leads to absurdity in short order. (159-60)

I agree that absurdity does follow quickly from the claim that the "believes" function can take as arguments a person and a truth-value. So we must examine this passage closely.

The first sentence's stipulations are in our table (as A2–A5). The second sentence adds another: that *unembedded* occurrences of "Opus 132 is a masterpiece" are formalized as "α_0." So far, so good. It is with the fourth sentence that we move beyond notational stipulations. It says that *from* them—"Then..."—it follows that there is a sentence in F whose sense is $\Gamma_1(\beta_1, \alpha)$ and whose reference is the True. This sentence, Burge says, is "$\Gamma(\beta, \alpha_0)$." It appears here for the first time in the discussion; it is not in our table. It is what leads to the absurdity Burge's argument requires, for it has a truth-value (the semantic value of "α_0") as the second argument to the believing function. We are thus at the crucial step in Burge's reasoning. So we have to ask: How do we get from notational stipulations about other expressions in the formal language F, to the claim that that language contains the sentence "$\Gamma(\beta, \alpha_0)$"—and that it is true?

4.3 | Descent

Burge does not say what principle supports his inference. But we can see what he has in mind by looking at how his claim relates to his argument's assumptions.

Notice the following relationships among his notational assumptions:

- the semantic value of "Γ_1" (which is a sense) presents the semantic value of "Γ" (the believing function);
- the semantic value of "β_1" (a sense) presents the semantic value of "β" (Bela);
- the semantic value of "α" (a sense) presents the semantic value of "α_0" ($\top$).

Now look at what C2 says: That F contains the expression "$\Gamma_1(\beta_1, \alpha)$." (That is not a sentence in F; it occurs only as a component of sentences. But that is all that C2 claims for it.) That expression is built from the first member of each of the pairs just listed. Combining the second members in the same way, we get the expression "$\Gamma(\beta, \alpha_0)$," which is the expression

that Burge claims exists in F. The claim that that is a sentence in F, and that it is true, is the *absurdum* that Burge's *reductio* requires.

Burge's inference, then, rests on a claim about the expressions in the formal language F. It is that if F contains an expression each of whose components stands for a sense, then it also contains an expression of the same structure, each of whose components stands for what its counterpart in the former expression presents. Component-by-component, in other words, we swap expressions so as to "descend" from a sense to what it presents. For Burge's inference to be sound, there must be such a function on expressions in F. The function—call it **Descent**—applies only to expressions in F each of whose components has a sense (of some degree) as its semantic value, because in general there is nothing presented by something that is not a sense.

It is a substantial claim about F, that it supports Descent. It is not something that follows from a few notational stipulations. Indeed, if anything the stipulations that Burge does make render it mysterious why Descent would apply to expressions in F, for the upshot of them is that in this case its value, "$\Gamma(\beta, \alpha_0)$," would appear to have no work to do. Line C1 says that unembedded occurrences of "Bela believes that Opus 132 is a masterpiece" are formalized as "$\Gamma(\beta, \alpha)$." Line C2 says that singly embedded occurrences are formalized as "$\Gamma_1(\beta_1, \alpha)$." The same goes for more-than-singly embedded occurrences. (The *reductio* assumption is clearly meant to generalize to them as well, as it reflects the no-hierarchy view.) So on the assumptions listed in the table, *every* occurrence of that English sentence, embedded or unembedded, is formalized by one of those two expressions in F. We might well wonder, then, what gets formalized as "$\Gamma(\beta, \alpha_0)$." There seem to be no remaining candidates. So it is mysterious why that expression would even occur in F, given that F is meant as a formalization of English. And if "$\Gamma(\beta, \alpha_0)$" does not occur in F then there is no true claim about its semantic value in F, let alone Burge's claim that it is the True.

So it is definitely not obvious that F supports Descent. We have to ask what reason there might be, even in the face of the considerations just given, to maintain that it does.

Before pursuing that question I digress to distinguish, from the claim that F supports Descent, another claim with which it could be confused. Kevin Klement points out, following Church (1951) and Kaplan (1968):

> One could introduce a two-place function sign "$\Delta(\ ,\ )$," standing for a function whose value is the True just in case the first argument is a *Sinn* picking out the second argument as *Bedeutung*. (2002, 17)

It is important to note that such a function expression could exist in a language having no expression-pairs to which it applies truly—in particular, no Descent-related expressions. The result would simply be that every instance of the schema "$\Delta(\ ,\ )$" in that language is false. The question before us is whether Descent does apply to expressions in F, not (to put it crudely) whether F has the expressive resources to say that it does. The delta notation bears on the answer to the latter question, not the former.[14] The two questions are independent.

Back to our question: What reason is there to think that F supports Descent? It cannot be that Descent itself works in natural language. Clearly it does not, for we do not find for each *natural language* expression, another *of the same structure* such that the reference of one is the sense of the other. The qualifications are important because (as Frege (1892, 59) himself noted) there is, for each word w in English, say, one expression whose reference is the sense of w, namely, the expression "the sense of $\ulcorner w \urcorner$." But that expression is more complex than w. Moreover it involves technical vocabulary, which should not be counted among that of natural language as such. This expression is not Descent-related to w.

4.4 | Extraction

So far we have found no reason to think, and some reason to deny, that Descent applies to expressions in F. But it would be premature to give up. For there is another thought favoring the idea that F supports Descent. It begins with the observation that if Frege's core principle is correct, then there is a function that applies to expressions in natural language that works *somewhat* like Descent would work in F. We should check what the application of this function in natural language might entail concerning Descent in F.

The function is defined on, and returns as values, sets of occurrences that an expression can have, rather than expressions themselves. I will call the function in natural language, **Extraction**. Extraction takes us from the set of occurrences in a natural language of some expression that are embedded (in the scope of an attitude or discourse verb) to degree $n \geq 1$, to the set of occurrences of the same expression that are embedded to degree $n - 1$. For example, Extraction applied to the set of doubly embedded occurrences of "Cicero" returns the set of its singly embedded occurrences.

Let us look first at how Extraction works in the simplest cases: Those in which $n = 1$ and the sentence being extracted has no embedding within it. Consider Extraction of "Opus 132 is a masterpiece" from its singly embedded occurrences to its unembedded ones. On both the hierarchy view and the no-hierarchy view, its semantic value in the former occurrences is its sense, and in the latter its reference; and the same goes for each of its component expressions. This fact about Extraction in natural language encourages the thought that Descent works in F. The reason is that according to our background methodological assumption, occurrences of an expression in natural language that differ in semantic value are to be symbolized in F with different expressions. So the embedded occurrences of "Opus 132 is a masterpiece" will be formalized with some expression e and the unembedded ones will be formalized with some different expression f. Presumably for each component of e, there will be a corresponding component of f; and given how the occurrences of our English sentence relate (on either of the views at issue), the semantic value in F of each component of e will be a sense that presents

the semantic value of the corresponding component of f. All this is to say that with e as argument, Descent's value is f. So when applied in the simplest cases, Extraction does encourage the idea that Descent applies to expressions in F—regardless of whether the hierarchy view is true or false.

Things go differently, however, with cases that are not of the simplest sort, and this is where the problem for Burge arises. With an eye to Burge's crucial claim, we should ask in particular about Extraction of the sentence "Bela believes that Opus 132 is a masterpiece" from singly embedded to unembedded occurrences. Here, things are more complicated because this sentence itself involves an embedding.

On the no-hierarchy view, the semantic value of an expression embedded to any degree is its level 1 sense. So when "Bela believes that Opus 132 is a masterpiece" occurs embedded, every component of it has a level 1 sense as its semantic value. When it occurs unembedded, of course, the semantic values of "Bela" and "believes" are their references and those of every component of "Opus 132 is a masterpiece" are level 1 senses. So, only with those first two expressions do we have a Descent-like drop in semantic value; with the others, there is no difference in semantic value even though they are Extraction-related. On the no-hierarchy view, these Extraction-related occurrences do not stand in a Descent-like relation.

Things are different on the hierarchy view. On that view, the semantic value of an expression embedded to degree n is its level n sense. So when "Bela believes that Opus 132 is a masterpiece" occurs singly embedded, the semantic values of "Bela" and "believes" are level 1 senses, while those of the components of "Opus 132 is a masterpiece" are level 2 senses. When it occurs unembedded, the semantic values of "Bela" and "believes" are as above: they are references, while those of the components of "Opus 132 is a masterpiece" are level 1 senses. With all components of "Bela believes that Opus 132 is a masterpiece," then, we see a Descent-like drop in semantic value. This is the case no matter its degree of embedding: the above reasoning generalizes. This means that these Extraction-related occurrences do stand in a Descent-like relation, on the hierarchy view.

Only on the hierarchy view, then, does Extraction work on our "Bela believes" sentence when embedded, in the way corresponding to how Burge applies Descent to the formalization in F of such occurrences. So in assuming that Descent works in F as his argument's crucial inference requires, Burge could find encouragement in the workings of natural language only on the assumption that the hierarchy view is true. But that is what the argument is meant to establish. A *reductio* argument against the no-hierarchy view should not rest on an assumption supported only by the hierarchy view. It is open to the proponent of the no-hierarchy view to head off Burge's *reductio* by denying that F supports Descent. Specifically they can deny that "$\Gamma(\beta, \alpha_0)$" is even a sentence of F, let alone a true one. There seem to be no grounds for insisting that this is an error—at least, not without presupposing the hierarchy view.

Burge's argument fails, then, to show that Frege's core principle about indirect discourse entails the hierarchy.

4.5 | Skiba's Diagnosis

I should explain how my diagnosis of Burge's argument relates to that of Lukas Skiba (2015). Skiba agrees that the problem in Burge's argument arises with his introduction of "$\Gamma(\beta, \alpha_0)$."[15] As we saw, Burge claims that this is a well formed sentence in F; that it is true; and that its sense is $\Gamma_1(\beta_1, \alpha)$—in which the sense Γ_1 is a function that takes the senses β_1 and α as arguments. I have focused on the first claim, while Skiba focuses on the third (which justifies the second, let us assume). The latter claim presupposes that senses combine by functional composition. Skiba argues that this presupposition—which he labels (Pr2)—is "rather contentious" (74).

However, someone could follow Skiba in rejecting (Pr2) yet conclude from this not that Burge's argument is misguided but that it rests on a fixable mistake: a false claim about *what* the sense of "$\Gamma(\beta, \alpha_0)$" is. Such a reader will cast around for a principle that is an alternative to (Pr2), perhaps one assigning that expression a sense that results not from functional application but from something more like concatenation. With such an alternative principle the argument, as Skiba reconstructs it, would still go through. (This is because $\Gamma_1(\beta_1, \alpha)$ serves only to secure the conclusion via an identity inference. It is identified with the sense of embedded occurrences of "Bela believes Opus 132 is a masterpiece" in line 1, and with the sense of "$\Gamma(\beta, \alpha_0)$" in line 3. The argument concludes with a statement of identity between the latter two entities.) Such a reaction would be encouraged by Descent. After all, Descent entails that "$\Gamma(\beta, \alpha_0)$" is well formed; so it must have *some* sense. Showing the failure of Descent, then, is key to fully addressing Burge's argument. And since Descent is not about how senses combine, rejecting it requires an argument different from one directed specifically against a principle such as (Pr2). So while I agree with what Skiba says, I think it leaves room for a fuller diagnosis of the failure of Burge's argument.

5 | Frege on Embedding and Shifting

We have found that intersubstitutability considerations do not support the hierarchy view and that well-known attempts to derive it from Frege's core principle about indirect discourse—even Burge's careful and complex argument—fail to do so. One could still wonder if we have missed something, though, given how often readers of Frege have thought that his core principle somehow leads to the hierarchy view. Perhaps this common reaction admits of articulation into an argument rather different from the ones we have discussed.

Consider how writers sometimes express the idea that the hierarchy follows from Frege's core principle. As was noted above (§4.1), Parsons says that "a similar sort of 'elevation'... takes place" (1981, 39); Soames says that "the process... is repeated" (2010, 14). Remarks like this suggest the idea that the "shift" in semantic value between a word's unembedded and its singly embedded occurrences, and the (alleged) shift between the latter and its doubly embedded occurrences, work in the same way, or occur for the same reason. Such an idea must rest, I think, on a misunderstanding of Frege's account of the first of those shifts: on taking something done by a word's semantic

value, for something done by the word itself. Properly understood, I will argue, Frege's approach gives us no reason to think that there is a second shift, let alone that it works in the same way or for the same reason. Indeed it gives us reason to deny that it occurs.

5.1 | Frege's Conception of Embedding

We can begin by asking what we mean by "embedding." Up to this point I have taken for granted the notion of an expression's occurring embedded to degree *n*. Since our concern now is with the implications of Frege's core principle, our question is: What notion of embedding do we find at work there?

It is not a notion that we can apply simply by looking at the surface form of a sentence. Frege does not describe counting the number of verbs that take "that"-clause complements and in whose grammatical scopes some words occur. Rather he talks about the *function* of words that occur within *some* such complements. His is a proposal about the semantics of words occurring in "reported speech," which he describes as speech in which "one talks about the sense, e.g., of another person's remarks" (1892, 59). Whether a verb is one that is used to "talk about the sense" of some person's remarks depends on what that verb means. (That it takes a sentential complement is not sufficient. On some other ways of thinking of embedding, it would be.[16]) Specifically it depends on that verb's meaning *in that occurrence*. That meaning must be such as to make it the case that the speaker using it is "talking about the sense" of someone's speech or thought.

The paradigm cases for Frege are ones in which the speech-reporting verb does not itself occur within the scope of another such verb. In such an occurrence, for Frege, the verb's semantic value is its reference. Consider "said" (indirect) or "believes," two of the reporting verbs most often used in discussing Frege's principle. Indirect "said" has as its reference a function, one of whose arguments is the sense of the complement sentence. The same is true of "believes." Call the former the **saying function** and the latter the **believing function**. From the previous paragraph's claim it follows that it is these functions that make it the case that the words in the complements (to the verbs whose references those functions are) are "embedded" in the sense that concerns Frege. Let us state this as follows.

Function Embedding: A word occurs **embedded**—hence has its sense as its semantic value in that occurrence—iff it occurs in the grammatical scope of a verb whose semantic value, in that occurrence, is a reporting function (e.g., the saying function or the believing function).[17]

5.2 | Double Embeddings

One could now ask: What should we say about words that are in the grammatical scope of a reporting verb whose semantic value, in that occurrence, is not a reporting function? This is a live question because Function Embedding itself entails that this occurs. It occurs when the reporting verb occurs in the grammatical scope of another reporting verb whose semantic value, in its occurrence, is a reporting function. The cases in question, in other words, are ones that hierarchy theorists will say are cases of double (or more) embedding. Recall our example of a sentence involving a double embedding:

> (Double) John believes that Susan said that Paris is beautiful.

In (Double), "said" is in the grammatical scope of "believes." The latter occurs unembedded, so it has the believing function as its semantic value. By Function Embedding, then, the semantic value of "said" is its sense. Our question is: What are the semantic values of words—such as "Paris," in (Double)—that occur in *its* grammatical scope?

Alas—for the hierarchy theorist—Function Embedding itself already answers this question. In (Double), "Paris" is within the scope of "believes"; and because that verb's semantic value in that occurrence is the believing function, Function Embedding entails that the semantic value of "Paris" is its sense.

One might object: Shouldn't we ask what difference "said" makes to the value of "Paris"? Wouldn't "said" trigger Function Embedding a second time? No, because its semantic value, in that occurrence, is not a reporting function; it is a sense presenting such a function.[18] The idea that "the same thing happens again" in such a case neglects the fact that necessarily, one of those verbs occurs within the scope of the other; and Function Embedding's kicking in with the outer verb precludes its kicking in with the inner one. There is, then, no such thing as a word's occurring "doubly embedded," on Frege's conception of embedding.

This point relates to one raised by Kripke (2008) concerning direct quotation[19]:

> Strangely enough no one, to my knowledge, has raised the hierarchy problem regarding the theory of direct quotation. After all, direct quotation can be iterated any number of times. Would Frege's theory lead to a hierarchy problem, analogous to the problem people see for indirect quotation? (194)

The above reasoning explains why not. Just as "said" does not induce (further) embedding when itself embedded, directly quoted quotation marks do not do any quoting. So there is no hierarchy of direct quotation: There is a one-level ontology of quotable items, among which are quotation marks themselves. And as we should expect, Descent-like principles fail here also. Indeed their failure here is obvious: From the fact that some expressions can combine within a direct quotation, one can infer nothing—neither *à la* Descent nor *à la* Extraction—about how those, or any related, expressions combine outside direct quotations. Just as above, the point is that while quotation *marks* can be "iterated any number of times," quotation itself—that semantic function—cannot be.[20]

So there is no question for the hierarchy view to answer that is not already answered by Frege's core principle. Any answer that differs from its answer, thereby conflicts with that principle. This is my reason for disagreeing with Parsons's remark (quoted above, §2) about "extending" Frege's stated account to handle "doubly embedded contexts." That account already entails a treatment of the cases so called.

5.3 | Yalcin's Semantics

In his paper "Quantifying in from a Fregean perspective" Seth Yalcin (2015) proposes a formal semantics for indirect discourse that implements the ideas just described. That apparatus handles an impressive range of phenomena. (His main aim is to show that such an approach can handle notoriously tricky cases of "quantifying in" (Kaplan 1968) and other anaphoric links into, or out of, "that"-clauses.) My purpose here does not require getting into its details. The point is that it is a rigorous implementation of a no-hierarchy Fregean approach; and, consistently with the above description of how that approach handles so-called "double embeddings," it implements Frege's core principle with a rule that can apply at most once in such cases. Yalcin proposes a modification of the usual rule of Functional Application (245). (He works within the framework of Heim and Kratzer (1998), whose rule involves no such complication.) The idea is that if one node in the overall sentence's structure has as its semantic value a function, then if that function takes senses as arguments, the node combines with the sense of its complement expression; otherwise, it combines with its reference. Since the semantic value of "believes," unembedded, is the believing function, one of whose arguments must be a sense, Yalcin's rule requires that the expression supplying that argument be interpreted as having its sense as its semantic value. The "doubly embedded" cases are handled in his system exactly as described above. When a reporting verb occurs embedded its semantic value is not the corresponding reporting function but a sense. So the functional application rule does not even apply to it.[21]

5.4 | An Objection

To all this the hierarchy theorist might object that we are taking Frege too literally, and that Function Embedding is not the best way to elaborate on the idea behind his core principle, even if it does reflect how he stated it. Their proposal might be that just as a reporting function embeds by requiring Thoughts as one of its arguments, so too a sense presenting such a function somehow requires that the semantic value of its grammatical complement is something other than what the hierarchy theorist would call a first-level sense. Similarly for reporting verbs that it embeds; thus we get the hierarchy.

My purpose in this paper does not require me to argue that this proposal is incoherent or otherwise unworkable. It requires, rather, asking what there is to be said in support of it, on the basis of the core principle. So I will assume that it is possible to state a consistent set of principles that work as the hierarchy theorist would like. (It could not include Function Embedding, of course, but it could include a variant of it applying in the same way only to words that are "singly embedded" within reporting verbs.) The problem in the present context is that it is hard to see what it is about the core principle that might motivate such a proposal. Let me explain.

That a reporting function enables one to "talk about the sense of another's remarks" consists in the fact that it takes as one of its arguments a sense. Suppose that it took instead the reference of the complement sentence as an argument. The reference of a sentence, for Frege, is a truth value. As Burge pointed out (above, §4.2), having a truth value as one of the arguments to the believing function leads to absurdity. When a reporting verb's semantic value is its reference, then, the semantic value of its complement sentence *must* be shifted on pain of absurdity. No parallel consideration motivates the fallback proposal. That is, there is no absurdity in having the sense of a reporting verb combine with the sense of a declarative sentence, as happens—according to Function Embedding—when both those expressions occur embedded. (Indeed, this is exactly the kind of combination that Yalcin implements in his proposal: Senses combine in a way very much like concatenation.) No shift to a different kind of value is required. So there is no problem with Frege's core principle as stated, to which this proposal is a solution. It is unmotivated, regardless of whether it is in some way workable.

6 | Conclusion

The hierarchy view has long hovered over many discussions of the semantics of indirect discourse. Most of the discussion of it has concerned its bare tenability. This is a problem in itself. For if there is no real motivation for a view, then debating its tenability is beside the point. My aim in this paper has been to show that while there are many philosophers who continue to see a place for the hierarchy view, they have yet to offer any compelling reason for this position. So, on balance, we should reject the hierarchy view regardless of whether it meets the threshold of bare tenability.

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Endnotes

[1] I believe that both Burge's (2005) and Kripke's (2008) defenses of the hierarchy view rely on the assumption that the hierarchy is rigid. Roughly: they exploit the 1–1 relation between level n senses and level $n + 1$ senses, to define a function from the former to the latter.

[2] There are some Fregean semantic theories (e.g., Yalcin 2015; Lederman 2022, discussed below, §5.3) on which a word has only one semantic value, albeit one that is in effect a *package* of different items. The composition rules work in such a way that different

components of that package are extracted from it to figure in semantic composition, depending on the kind of occurrence the word has. At the cost of some literal inaccuracy, I will count these too as views on which a word has different semantic values in different occurrences.

[3] Putnam himself worked with instances that are more logically complex than Mates's schemata require.

[4] A note on the formulation of Embedding. Many who discuss Mates's examples do so on the assumption that there are some pairs of expressions that are intersubstitutable *salva veritate* within *all* singly embedded complements to attitude verbs. But one could reject that very strong claim while still thinking that Mates's examples point to a real phenomenon. My formulation is meant to capture only the thought that intersubstitutability gets *more* stringent as degree of embedding increases, whatever the baseline among single embeddings is.

[5] This point was noted long ago in exactly our present dialectical context. Defending his proposal for very fine-grained semantic values of "that"-clauses, Mark Richard (1990) responded to those who worry that this will entail too many differences in truth condition among attitude-ascriptive sentences:

> I agree that, of necessity, whoever believes (viz., has the relation named by "believes" to) the proposition that *A* and *B* believes that *B* and *A*. This is an interesting fact about belief and the two propositions. It does not follow that the propositions are identical. To argue in this way is something like arguing thus: Necessarily, a number has the same cardinality as omega iff it has the same cardinality as omega + 1. So omega = omega + 1. (32)

[6] I am grateful to a referee for pressing me regarding this objection.

[7] For simplicity's sake I am here, as throughout, ignoring the semantic role that expressions have within direct quotations. Occurrences of this sort would require qualifications of certain claims, but not ones at issue in this debate. For the same reason I will ignore the semantic role that extralinguistic context plays; an 'occurrence' of an expression is not meant in the Kaplan (1977) sense, of a pairing of it with an extralinguistic context; rather it means its occurrence as a component within some sentence.

[8] In a recent paper Georgalis (2025) argues that Frege did not intend the hierarchy even to get started, because he stated the core principle in such a way that (contrary to the usual interpretation) it does *not* entail that a word has an "indirect sense" distinct from its customary sense. Nicholas Georgalis rests his argument on the fact that Frege accompanies his statement of the core principle with another statement that it represents an "exception" to the usual "connexion between sign, sense, and reference." Nicholas Georgalis does note that Frege's above-quoted statement seems to conflict with his interpretation; but he remarks that "the complication is in the syntax and would apply to a formal language, not to a natural language" (700n.3). (I'm not sure that this suffices to deflect the passage's implications for his interpretation.) In any case this scholarly matter does not bear on my argument in what follows, because (as we will soon see) Burge's argument does not rest on any claims about indirect sense. (Nicholas Georgalis notes the controversy around Burge's argument but does not enter into it, saying that his aim is to clarify what Frege believed rather than to determine its implications (709).)

[9] For discussion of Carnap's argument, see Parsons (1981, 38–39). Parsons finds Carnap's conclusion irrelevant to the hierarchy question. Regarding Dummett's argument he finds that it presupposes the hierarchy view (42).

[10] Natha Salmon (2005, 1100) endorses Burge's argument, claiming in addition that unbeknownst to Burge his argument had as a predecessor the famously obscure "Gray's Elegy" argument that Bertrand Russell gave in "On denoting" (1905). Kripke (2008, 184n.9) also endorses it, saying that it shows that "the hierarchy is an actual consequence of Frege's theory. (Qualifications of this will emerge from the present article.)" (I do not know which qualifications he had in mind.) Ori Simchen (2018, 257) writes, "It is often assumed that Burge's argument establishes a hierarchy of senses." (See below, n. 13 on Simchen's own point about the hierarchy.)

[11] Here is Burge on the relation between his arguments.

> Let us summarize the difficulty. A language of the form of L_I cannot give a systematic truth theory for L_I—one needs laxer substitution principles than L_I countenances...But term by term translations of embedded belief sentences of L_I into the 'laxer' metalanguage are prevented by the argument of section II. (1979, 165)

> My claim (in section III of "Frege and the Hierarchy") was that in a metalanguage that allows substitutivity of codenoting expressions and that specifies senses by translating expressions of the object language (in particular ones in oblique contexts that specify senses) into metalinguistic expressions with the same senses, the problems I raised for Method II [i.e., the argument in §II of the 1979 paper] will recur in the metalanguage. (2005, 184)

[12] This is what Bryan Pickel calls, following Kaplan, "the method of direct discourse" (2021, 6924).

[13] Simchen (2018) argues that the no-hierarchy view and the hierarchy view as I have described it are not the only options. There could be, in effect, *cycles* of three or more values, in the hierarchy of semantic values. If Simchen is right then a successful *reductio* of the former does not prove the latter to be true. For simplicity's sake, though, I will follow Burge in taking it that there are only those two options.

[14] Of course the delta notation does not express Descent, exactly, because it does not get at the component-by-component relation that Descent requires. This is another reason not to confuse the claim that *F* has the delta notation, with the claim that *F* supports Descent.

[15] I have used Burge's notation; Skiba states the claims by subscripting English words to indicate whether they are in the metalanguage or the object language.

[16] On the account of Richard Larson and Peter Ludlow (1993), for example, "ILFs (and the intensionality effects they bring) are not introduced by specific predicates, such as those involving thoughts and beliefs; rather they are introduced constructionally. ILFs appear in the truth conditions whenever one has a VP containing a complement S" (311).

[17] One would want also a principle covering noun phrase constructions that are "about the sense" of the complement sentence's words, as occur in Frege's 1902 letter (quoted above, §4.1). Here, I ignore this complication and focus on the central cases.

[18] One might propose that embedded reporting verbs are an exception to Function Embedding—that even when embedded, their semantic values are their references. But it is hard to see a Fregean motivation for this. It seems to go against Frege's idea that in reported speech we "talk about the sense" of the reported words. This idea applies to all words being reported, even ones that happen themselves to be reporting verbs.

[19] I am grateful to a referee for also raising this question.

[20] See, relatedly, points by Bryan Pickel and Brian Rabern (2021, 2690n.5) concerning substitution principles and direct quotation.

[21] Lederman's (2022) system, building on Yalcin's, works similarly. So even though (as Lederman notes, 1240n.4) such systems could be modified to accommodate the hierarchy view, there is nothing about them that motivates doing so (as Yalcin notes, 246).

References

Burge, T. 1978. "Belief and Synonymy." *Journal of Philosophy* 75: 119–138.

Burge, T. 1979. "Frege and the Hierarchy." *Synthese* 40: 265–281.

Burge, T. 2005. "Postscript to 'Frege and the Hierarchy'." In *Truth, Thought, Reason: Essays on Frege*, 167–210. Clarendon Press.

Carnap, R. 1947. *Meaning and Necessity*. University of Chicago Press.

Church, A. 1951. "A Formulation of the Logic of Sense and Denotation." In *Structure, Method and Meaning: Essays in Honor of H. M. Sheffer*, edited by P. Henle, H. M. Kallen, and S. K. Langer, 3–24. Liberal Arts Press.

Davidson, D. 1965. "Theories of Meaning and Learnable Languages." In *Logic, Methodology and Philosophy of Science: Proceedings of the 1964 International Congress*, edited by Y. Bar-Hillel, 383–394. North-Holland.

Dummett, M. 1973. *Frege: Philosophy of Language*. Harvard University Press.

Fine, K. 2007. *Semantic Relationism*. Blackwell.

Fodor, J. 1993. *Review of a Study of Concepts, by Christopher Peacocke*. London Review of Books 15.19.

Fodor, J. 1998. *Concepts: Where Cognitive Science Went Wrong*. Clarendon Press.

Frege, G. 1892. "Über Sinn und Bedeutung." *Zeitschrift Für Philosophie Und Philosophische Kritik* 100: 25–50. Translated as "On sense and meaning" in *Translations from the Philosophical Writings of Gottlob Frege*, 2d edition (1960), ed. and trans. Peter Geach and Max Black, 56–78. Blackwell.

Frege, G. 1902. "Letter to Bertrand Russell, December 28, 1902." In *Philosophical and Mathematical Correspondence*, edited by G. Gabriel et al., 152–154. University of Chicago Press.

Georgalis, N. 2025. "On Frege's Supposed Hierarchy of Senses." *Inquiry* 68, no. 2: 696–717.

Heim, I., and A. Kratzer. 1998. *Semantics in Generative Grammar*. Blackwell.

Horwich, P. 2014. "Critical Notice of Seven Puzzles of Thought and How to Solve Them: An Originalist Theory of Concepts, by R. M. Sainsbury and Michael Tye." *Mind* 123: 1123–1139.

Kaplan, D. 1968. "Quantifying in." *Synthese* 19: 178–214.

Kaplan, D. 1977. "Demonstratives." In *Themes From Kaplan*, edited by J. Almog, J. Perry, and H. Wettstein, 481–614. Oxford University Press.

Klement, K. 2002. *Frege and the Logic of Sense and Denotation*. Routledge.

Kripke, S. 1979. "A Puzzle About Belief." In *Meaning and Use*, edited by A. Margalit, 239–283. D. Reidel.

Kripke, S. 2008. "Frege's Theory of Sense and Reference: Some Exegetical Notes." *Theoria* 74: 181–218.

Larson, R., and P. Ludlow. 1993. "Interpreted Logical Forms." *Synthese* 95: 305–355.

Lederman, H. 2022. "Fregeanism, Sententialism, and Scope." *Linguistics and Philosophy* 45: 1235–1275.

Mates, B. 1950. "Synonymity." *University of California Publications in Philosophy* 25: 201–226.

Moffett, M. 2002. "A Note on the Relationship Between Mates' Puzzle and Frege's Puzzle." *Journal of Semantics* 19: 159–166.

Owens, J. 1986. "Synonymy and the Nonindividualistic Model of the Mental." *Synthese* 66: 361–382.

Parsons, T. 1981. "Frege's Hierarchies of Indirect Senses and the Paradox of Analysis." *Midwest Studies in Philosophy* 6: 37–57.

Peacocke, C. 1996. "Our Entitlement to Self-knowledge: Entitlement, Self-knowledge and Conceptual Redeployment." *Proceedings of the Aristotelian Society* 96: 117–158.

Peacocke, C. 1999. *Being Known*. Clarendon Press.

Pickel, B. 2021. "The Functional Composition of Sense." *Synthese* 199: 6917–6942.

Pickel, B., and B. Rabern. 2021. "Frege and Saving Substitution." *Philosophical Studies* 178: 2687–2697.

Pryor, J. 2017. "De Jure Codesignation." In *A Companion to the Philosophy of Language*, edited by B. Hale, C. Wright, and A. Miller, 1033–1079. Blackwell.

Putnam, H. 1954. "Synonymity, and the Analysis of Belief-Sentences." *Analysis* 14: 114–122.

Rattan, G., and M. Durand. 2022. "Mates and the Hierarchy." *Synthese* 200: 433.

Richard, M. 1990. *Propositional Attitudes*. Cambridge University Press.

Russell, B. 1905. "On Denoting." *Mind* 14: 479–493.

Sainsbury, M., and M. Tye. 2012. *Seven Puzzles of Thought and How to Solve Them: An Originalist Theory of Concepts*. Oxford University Press.

Salmon, N. 2005. "On Designating." *Mind* 114: 1069–1133.

Scheffler, I. 1955. "On Synonymy and Indirect Discourse." *Philosophy of Science* 22: 39–44.

Simchen, O. 2018. "The Hierarchy of Fregean Senses." *Thought* 7: 255–261.

Skiba, L. 2015. "On Indirect Sense and Reference." *Theoria* 81, no. 1: 48–81.

Soames, S. 2010. *Philosophy of Language*. Princeton University Press.

Sorensen, R. 2002. "Meaningless Beliefs and Mates's Problem." *American Philosophical Quarterly* 39: 169–182.

Taschek, W. 1995. "Belief, Substitution, and Logical Structure." *Noûs* 29: 71–95.

Yalcin, S. 2015. "Quantifying in From a Fregean Perspective." *Philosophical Review* 124: 207–253.