Mathematics of Existential Reality

Existence, Anti-Existence, and the Exclusion of Nothingness

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Abstract

This paper proposes a zero-free foundational framework called *Existential Mathematics*. Numerical entities are partitioned into (i) *Existence*, representing physically existing quantities (positives); (ii) *Anti-Existence*, representing directed reversals of existence (negatives); and (iii) *Nothingness*, corresponding to the classical symbol 0, here excluded from arithmetic. I formalize an *Existential Algebra* in which addition is partial (annihilating pairs are undefined), multiplication and division are total over $\mathbb{E} = \mathbb{R} \setminus \{0\}$, and present an *existential factorial* in which 0! is assigned the boundary value 0 to reflect the non-operation of the empty act. A zero-free analysis is outlined by treating 0 as an *annihilation boundary*: it may be approached from either side but never attained. The program aligns mathematics with physically interpretable states of being (existence and its reversal) while rejecting calculations that treat "nothingness" as a legitimate operand.

Keywords: zero-free mathematics, existential foundations, unitary number system, partial algebra, division by zero, factorial, 0!

1 Motivation and Overview

Counting in early human societies plausibly arose from concrete embodiment: ten fingers motivate a decimal progression (one through ten) by *accumulating what exists*. Subtractive acts—*removing what exists*—naturally introduced reversals (here called *Anti-Existence*). The later introduction of a distinct symbol for "nothing" into arithmetic enabled powerful formalisms, yet it also bound calculations to an operand lacking existential content. In this work, I pursue a disciplined alternative: arithmetic and analysis that operate only on what exists or anti-exists, while excluding Nothingness from acting numerically.

The central stance of this work is as follows:

- **Existence** ($\mathbb{E}^+$): positive quantities; measurable presence.
- **Anti-Existence** ($\mathbb{E}^-$): negative quantities; directed reversals of presence.
- **Nothingness** ($\varnothing$): true absence; *not* a number and never an operand.

A positive and an equal-magnitude negative *annihilate*; I model this via a partial law of addition rather than admitting a zero-valued sum.

2 Unitary Number System as Physical Existence

At the most fundamental level, counting begins with a single, physically existing unit: a *unitary* baseline. Every physical object, from cosmic bodies to subatomic particles, can be described as an accumulation of such units (or fractions thereof), never as an operand of Nothingness.

Higher-base number systems arise by grouping units, but should still *logically* begin at 1. For example:

- A *binary* system would naturally label its two states as 1 and 2, rather than 0 and 1.
- A *decimal* system would progress from 1 to 10, rather than 0 to 9.

The conventional use of 0 as the first symbol is a notational convenience, not a reflection of physical reality.

In digital practice, execution already operates on two distinct *existence* states: a higher and a lower physical state. In magnetic and optical storage, these likewise correspond to distinct configurations (magnetic domain orientations, or pits versus lands). Relabeling the low state as 1 and the high state as 2 (instead of 0 and 1) aligns the symbolic layer of computation with the existential framework.

3 The Existential Domain and Annihilation

Definition 1 (Existential Domain)**.** *Define the set of existential numbers by*

$$\mathbb{E} \;=\; \mathbb{E}^{+} \cup \mathbb{E}^{-} \;=\; \mathbb{R} \setminus \{0\}.$$

Elements of $\mathbb{E}^{+}$ *are* Existence *(positives), elements of* $\mathbb{E}^{-}$ *are* Anti-Existence *(negatives). The symbol* $\varnothing$ *denotes* Nothingness *(formerly* 0*), which is not an element of* $\mathbb{E}$*.*

Definition 2 (Annihilation Relation)**.** *For* $a, b \in \mathbb{E}$*, I say* a *and* b annihilate *and write* $a \# b$ *if* $a + b = \varnothing$ *in the usual real sense (i.e.* $b = -a$*). In the existential framework, annihilating pairs have no sum in* $\mathbb{E}$ *and are treated as* nonoperations*.*

Remark 1 (Order and Sign)**.** *The usual order on* $\mathbb{R}\setminus\{0\}$ *induces a sign function* $\operatorname{sgn}\colon \mathbb{E} \to \{+1, -1\}$*, classifying elements into Existence and Anti-Existence.*

4 Existential Algebra: Operations Without Nothingness

I now formalize arithmetic over $\mathbb{E}$. The structure is not a ring or group, because there is no additive identity. Addition is *partial*; multiplication and division are *total*.

Axiom 1 (Partial Addition)**.** *Define* $+\colon \mathbb{E} \times \mathbb{E} \rightharpoonup \mathbb{E}$ *by*

$$a \oplus b = \begin{cases} a + b & \text{if } a + b \neq \varnothing, \\ \text{undefined (annihilation)} & \text{if } a \# b. \end{cases}$$

Axiom 2 (Total Multiplication)**.** *Define* $\otimes\colon \mathbb{E} \times \mathbb{E} \to \mathbb{E}$ *by* $a \otimes b = ab$*. Since* $ab \neq 0$ *for* $a, b \in \mathbb{E}$*, the product remains in* $\mathbb{E}$*.*

Axiom 3 (Division)**.** *For* $a, b \in \mathbb{E}$*, define* $a \oslash b = a/b \in \mathbb{E}$*. Division is well-defined because the denominator is never Nothingness.*

Factorial and Combinatorics at the Zero Boundary

Definition 3 (Existential Factorial)**.** *Define the factorial on nonnegative integers by*

$$0! := 0, \qquad 1! := 1, \qquad n! := n \cdot (n-1)! \text{ for } n \geq 2.$$

Thus $n!$ agrees with the classical factorial for all $n \geq 1$, while the boundary value encodes that the empty act is a nonoperation with weight 0.

Remark 2 (Why $0! = 0$)**.** *In this framework, factorial counts* executable acts of arrangement*. "Arranging nothing" is not an act; it contributes no existential weight. Setting $0! = 0$ captures this: the boundary is present as a conceptual marker but contributes no multiplicative identity.*

Proposition 1 (No collapse of positive factorials)**.** *The assignment $0! = 0$ together with the base $1! = 1$ and the recurrence $n! = n(n-1)!$ for $n \geq 2$ yields the standard values for all $n \geq 1$.*

Proof. Immediate by induction from the stipulated base $1! = 1$. □

Definition 4 (Existential binomial coefficients)**.** *For integers $n \geq 0$ and $k \geq 0$, define*

$$\binom{n}{0}_E := 0, \qquad \binom{n}{k}_E := \frac{n(n-1)\cdots(n-k+1)}{k!} \quad \text{for } 1 \leq k \leq n, \qquad \binom{n}{k}_E := 0 \quad \text{for } k > n.$$

This coincides with the classical value for $1 \leq k \leq n$ and suppresses the empty selection.

Proposition 2 (Punctured binomial identity)**.** *For $n \in \mathbb{N}$ and indeterminate x,*

$$(1+x)^n - 1 = \sum_{k=1}^{n} \binom{n}{k}_E x^k.$$

Proof. Subtract the $k = 0$ term from the classical binomial theorem; by Definition 4, the remaining coefficients match. □

Remark 3 (Permutations unaffected)**.** *For $1 \leq k \leq n$, the number of k-permutations $P(n,k) = n(n-1)\cdots(n-k+1)$ is unchanged. The special case $P(n,n) = n!$ never invokes $0!$ under the multiplicative form, so it remains classical.*

Indeterminate Forms and Division-by-Nothingness

Expressions such as $0/0$, 0^0, or division by zero do not arise: Nothingness is not admitted as an operand. Where classical analysis encounters indeterminacy through zero, the existential framework replaces it with boundary analysis via limits that never evaluate at 0 (Section 6).

5 Geometry and Coordinates Without an Origin

In Euclidean modeling, the "origin" is a marker with all coordinates zero. I retain reference frames but interpret the origin as a *geometric label* rather than a numeric point. Coordinates take values in $\mathbb{E}$, and any computation that would force a coordinate to be numerically zero is treated as an annihilation case (nonoperation).

6 Calculus at the Annihilation Boundary

I sketch a differential and limit theory that never evaluates functions at Nothingness.

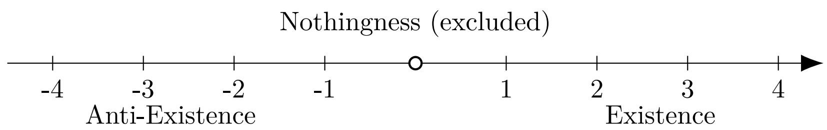

Figure 1: The existential number line: all nonzero reals; the zero point is excluded as “Nothingness”.

Approach Without Attainment

Let $x \in \mathbb{E}$ and let ε denote an *infinitesimal* in $\mathbb{E}$ (arbitrarily small in magnitude but never Nothingness). A limit “at zero” is rephrased as a *boundary limit*:

$$\lim_{\varepsilon \to 0^\pm} f(\varepsilon) \quad \text{means: } \varepsilon \in \mathbb{E}^\pm,\ |\varepsilon| \text{ arbitrarily small, but } \varepsilon \notin \varnothing.$$

The symbol 0 here names the annihilation boundary, not a value of the variable.

Derivatives

For $f\colon \mathbb{E} \to \mathbb{R}$ (or to $\mathbb{E}$ when closed), define the derivative at $x \in \mathbb{E}$ by

$$f'(x) \ = \ \lim_{\varepsilon \to 0} \frac{f(x \oplus \varepsilon) - f(x)}{\varepsilon},$$

where the numerator is formed only for those $\varepsilon \in \mathbb{E}$ such that $x \oplus \varepsilon$ is defined (i.e., no annihilation). This is a *domain-aware* derivative: increments that would annihilate are excluded from the difference quotient.

Integrals

For intervals $[a, b] \subset \mathbb{R}$ with $0 \notin [a, b]$, Riemann sums proceed unchanged over $\mathbb{E}$. Improper integrals that classically cross 0 are split at the annihilation boundary, never sampling Nothingness.

7 Algebraic and Analytical Consequences

- **No additive identity:** addition is partial and lacks a universal neutral element.
- **No additive inverses:** opposite magnitudes annihilate and are not admissible sums.
- **Robust multiplicative structure:** $(\mathbb{E}, \cdot, /)$ is closed; division by zero is structurally excluded.
- **Combinatorics reframed:** $0! = 0$ encodes the empty act as a nonoperation; binomial coefficients are *punctured* at $k = 0$ (Definition 4).
- **Analysis reframed:** boundary-aware limits and derivatives avoid evaluating at Nothingness.

8 Historical and Conceptual Note

The decimal system can be naturally read as emerging from human embodiment: ten fingers suggest a base-10 tally of *what exists*. Arithmetic then extends to reversals (*Anti-Existence*) to represent removal or opposition. From the existential viewpoint adopted here, the later use of a symbol for “nothing” as a numerical operand entrenched paradoxes (e.g., division by zero, $0/0$, 0^0) by treating absence as though it were a quantitative entity. *Existential Mathematics* restores the primacy of being (and its directed reversal) while denying calculational agency to non-being.

9 Discussion and Outlook

This program yields a coherent—if nonclassical—calculus and algebra:

1. A partial additive law that forbids annihilation results from being computed as numbers.
2. A total multiplicative and divisive law across $\mathbb{E}$.
3. An analysis that regards 0 as an unattainable boundary of annihilation.

Future work includes:

- Developing existential combinatorics that counts only executable acts (no empty act term).
- Formulating differential equations on domains avoiding annihilation crossings.
- Exploring probability models that avoid assigning numeric weight to Nothingness.

Conclusion

Mathematics of Existential Reality advances a disciplined refusal to operate on Nothingness. Numbers become the language of what is and what is reversed; not of what is not. By assigning the boundary value $0! = 0$ and excluding Nothingness from arithmetic, I aim to align computation and counting with existentially interpretable quantities, offering a clear conceptual alternative to the convention $0! = 1$.