The Z-Function and the Dissolution of the Riemann Hypothesis

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Abstract

The Riemann zeta function plays a central role in analytic number theory, and the Riemann Hypothesis (RH) is widely regarded as one of the greatest unsolved problems in mathematics. The Z-function, a real-valued transformation of the zeta function on the critical line, is an essential tool for investigating the distribution of nontrivial zeros. This paper presents both a technical overview of the Z-function and a philosophical stance: the Riemann Hypothesis, celebrated as a Millennium Prize Problem, is not to be solved but to be **dissolved**. The pursuit of hidden order in prime numbers reflects human projection rather than mathematical necessity. Through the Z-function, we demonstrate how RH is continually "verified" but never "resolved" in the classical sense. The true resolution is a rejection of the metaphysical demand for order: primes do not need meaning.

1 Introduction

The Riemann Hypothesis (RH) asserts that all nontrivial zeros of the zeta function lie on the critical line $Re(s) = 1/2$. This conjecture underpins our understanding of prime number distribution. The Z-function,

$$Z(t) = e^{i\theta(t)}\zeta\left(\tfrac{1}{2} + it\right),$$

transforms the critical line into real values, simplifying numerical investigations of zeros. Traditionally, RH is regarded as an open problem. We argue instead that RH is a pseudo-problem: its essence is dissolved by exposing its metaphysical underpinnings.

2 The Z-Function

2.1 Definition and Properties

The Z-function is real-valued, even ($Z(-t) = Z(t)$), and its zeros correspond exactly to the nontrivial zeros of $\zeta(s)$ on the critical line. This makes $Z(t)$ a central object in computational verification of RH.

2.2 Numerical Applications

The Riemann–Siegel formula provides efficient approximations for large t, making $Z(t)$ the basis of extensive numerical checks confirming billions of zeros on the critical line. Such results reinforce belief in RH but do not constitute proof.

3 Philosophical Dissolution

3.1 Zeno's Arrow and Infinite Verification

As with Zeno's arrow, which cannot fly when time is infinitely subdivided, RH requires infinite verification of zeros along the critical line. This renders it an endless regress: a problem that cannot be completed by its own definition.

3.2 Refusal of Meaning

The belief that prime numbers must conceal hidden order is a human projection. To demand that all zeros align perfectly is to impose metaphysical order on raw arithmetic. The resolution is not proof but refusal: RH dissolves once the hidden demand for meaning is rejected.

4 Implications

4.1 For Number Theory

The Z-function remains a powerful computational tool, bridging analysis and numerical verification. Yet its endless use underscores the futility of seeking final proof: verification without resolution.

4.2 For Philosophy of Mathematics

RH exemplifies how mathematics can create pseudo-problems. By recognizing RH as already dissolved, we restore clarity: primes do not require hidden harmony.

5 Conclusion

The Riemann Hypothesis, seen through the Z-function, is both numerically affirmed and philosophically dissolved. It is not an open mystery but an unnecessary projection. The problem evaporates not by proof but by refusal. In this sense, RH is already resolved—by being dissolved.

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