Hydrogen Torus Geometry as a Canonical S-Functional

Abstract

This note presents a geometric formulation in which system structure is represented by a toroidal curvature-based action functional S, and recurrence is defined by its scale derivative $\Psi = dS/dR$. A Heisenberg uncertainty floor imposes a minimum geometric perturbation, while Harmonic Resonance Bands (HRB) classify intervals of stability where Ψ remains bounded and smooth. Transition and relaxation operators are identified at HRB boundaries. The framework is substrate-independent and intended as a canonical geometric reference.

1. Geometric Definition

Let the system geometry be a torus $\mathcal{T}$ parameterized by a major radius R and a minor radius r. The torus is assumed to vary in R while r (or $\kappa = r/R$) is held fixed.

Define the geometric action functional:

$$S(R) = \int_{\mathcal{T}(R,r)} (\alpha + \beta H^2 + \gamma K) \, dA$$

where H is the mean curvature, K is the Gaussian curvature, dA is the toroidal surface element, and α, β, γ are normalized geometric weights.

2. Recurrence Operator

Define the recurrence slope as the scale derivative of the geometric action:

$$\Psi(R) = dS/dR$$

For scale-normalized analysis:

$$\Psi_{log}(R) = d/dR [\ln S(R)]$$

3. Heisenberg Geometric Floor

Confinement of the torus tube radius r imposes a quantum uncertainty floor:

$$\Delta r \cdot \Delta p \geq \hbar / 2$$

implying a minimum zero-point energy $E_{zp} \approx \hbar^2 / (8 m r^2)$.

4. Harmonic Resonance Bands (HRB)

Harmonic Resonance Bands are intervals in R (or $\kappa = r/R$) for which $\Psi(R)$ remains bounded and smooth under the Heisenberg floor.

5. Operator Correspondence

HRB : stability bands
h3π : transition operator at HRB boundaries
hHRT : relaxation operator into the next stable band

6. Canonical Synthesis

Heisenberg sets the minimum geometric perturbation; Ψ measures geometric response to scale change; HRB classifies stability.

7. Interpretive Note

This formulation is observer-independent and substrate-independent.