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The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their inter actions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of]-0, 1+[.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.
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What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, igno- rance, imprecision, etc.

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INDEX

Maikel Leyva Vázquez. Preface

1. Gabriela Valeria Bustos–Chiliquinga, Purificación Galindo-Villardón, Cristian Cornejo Gaete. **Financial Dynamics of Cooperatives of the Popular and Solidarity Economy of Ecuador: An Analysis Using the Tucker3 Model and Projection with Multidimensional Neutrosophic Regression.** 1

2. Glenda Quimera Ramírez-Apolo, Mila Inés Moreno-Pramatarova, Patricia Elizabeth Lima-Rosero, Sylvia Ibeth Tapia-Tapia, María Soledad Andino-Acosta, Mercy Paulina Guerrero-Olalla, Ivonne Cristina Unda-Vargas, Salomé Shayana Ruales-Paredes. **A Model of Organizational Dynamics of Phonological Knowledge in Preschool Children Based on Bratianu's Theory and the Neutrosophic AHP Technique.** 17

3. Purificación Galindo-Villardón. **Analysis of Customer Perception of a New Product Using the Neutrosophic Biplot Method.** 34

4. Purificación Vicente-Galindo. **Analysis of the Evolution of Expert Opinion on Sustainability Policies through the Neutrosophic STATIS Method.** 50

5. Santiago Salomon Chicaiza-Toapaxi, Anthony Alexander Ortega-Toapanta, Héctor Luis Laurencio-Alfonso. **Analysis of hydraulic behavior in pressurized systems with different automation mechanisms using Neutrosophic SuperHyperSoft sets.** 65

6. Karina Alejandra Álvarez Lino. **Design of a Maintenance Protocol based on Bipolar Neutrosophic Analysis for Severe Depression.** 80

7. Gonzalo X. Vizuete, Cristian F. Gallardo, Gabriela Vizuete. **Structured Analysis of a Generator Set Lubrication System Using Vertex Articulation in Plithogenic n-SuperHyperGraphs.** 93

8. Mónica Sofia Pallo Sarabia, Lenin Rubén Imbacuán Jiménez, Marjorie Elizabeth Valdivieso Villacís, Ximena Patricia Pallo Sarabia. **NAHP for the Selection of Analgesic Treatments in Third Molar Surgery.** 104

9. Mónica Sofía Pallo Sarabia, Darwin Josué Cubi Masaquiza, Favio Nicholas Zúñiga Calderón, Luis Miguel Acosta Rueda. **Risk prediction in dental implants with bone substitutes using a hybrid neutrosophic-logistic model: Managing uncertainty in clinical and biomechanical variables.** 120

10. Verónica Elizabeth Saltos Paz, Tanya Elizabeth Borja Pante, Gloria Madaly Plasencia Álvarez, Milena Alejandra Rodríguez Salcedo. **Analysis of the Prevalence of Dissociative Identity Disorder Using Neutrosophic Plithogenic Statistics.** 132

11. Nairovys Gómez Martinez, Gerardo Ramos Serpa, Riber Fabian Donoso Noroña, Gloria Rebeca Medina Naranjo. **Neutrosophic Statistical Regression Models for Predicting the Incidence of Nosocomial Infections in Post-Trauma Patients.** 141

12. Rene Estalin Portilla Paguay, Jairo Mauricio Puetate Paúcar, Vanessa Lisseth Montenegro Altamirano. **Evaluating Lawyers' Perceptions of Cryptocurrency Regulation in Ecuador Using Plithogenic Statistics and an Indeterminate Likert Scale.** 150

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13. Alberto Sánchez Pena, Yunisleydis García Ramírez, Irina Reyes Martinez, Tatiana María Velázquez Ramírez, Lisbet Eunice Pérez Anzardo. **Multineutrosophic Analysis with the ARAS Method for Selecting Trading Strategies.** 160

14. Shakila Devi GT, Sangeetha S, Maikel Yelandi Leyva Vazquez. **Determining Factors for Minimizing Oven Downtime through Controlled Batch Management: A Multineutrosophic Approach with ARAS.** 174

15. Raúl González Salas, Jhael Mikaela Gómez Ashqui, Marco Paúl Medina Valencia. **Assessing viral susceptibility in tilapia species using plithogenic fuzzy soft set: a multi-expert approach for TiLV.** 185

16. Marcelo Raúl Dávila Castillo, Jessica Johanna Santander Moreno, Rosa Evelyn Chugá Quemac. **Neutrosophic evaluation of sentiments through Neu-troAlgebra applied to the use of the mediation record as a means for lifting precautionary measures in child support proceedings.** 197

17. Ignacio Fernando Barcos Arias, Jennifer Angelina Almagro Triana, Ariana Michelle Mazacón Campoverde, Jason Jeampierre Guayaquil Cobo. **Neutrosophic-Logistic Hybrid Model to Evaluate the Effectiveness of Penitentiary Policies and Their Impact on Criminal Recidivism in Ecuador.** 211

18. Carla Gutiérrez Quiroz, Luis Abdon Rivera Paredes. **Measuring the degree of agreement between teachers' and employers' perceptions of the pedagogical model, using plitogenic statistics.** 223

19. Carlos Esteban Reyes Sanchez, Angel Vinicio Valencia Ramos, Paola Fernanda Castillo Castillo. **Improving Quality in Service Processes through the Strategic Implementation of the SERVQUAL Neutrosophic Statistical Model.** 235

20. Claudia Yadira Cuenca Macas, Jack Franck Jaramillo Herrera, Isaac Roger Martinez, Rudy Garcia Cobas, Nayade Caridad Reyes Palau. **Evaluating Community Satisfaction with Cultural Educational Projects through Neutrosophic Plithogenic Iadov.** 244

21. Wilmer Ortega Chávez, Janett Karina Vásquez Pérez, Beker Maraza Vilcanqui, Nazario Aguirre Baique, Carlos Manuel Sacarías Mercado, Ciro Angel Lazo Salcedo, Reyna Olano del Castillo. **Mentoring and Entrepreneurship in Higher Education: A Study from the Plithogenic Statistics.** 256

22. Carlos Castañeda Guillot, Ronelsys Martínez Martínez, Yohan Manuel Mancha Alfonso. **Relative impact of probiotics on improving insulin resistance in women with PCOS: an evaluation with neutrosophic plithogenic statistics.** 266

23. Esther Maricela Coello Avilés, Kevin Luis Flores Vega, Josselyn Bryggitte Vilela Verdezoto. **Neutrosophic Psychological Profiles in Habeas Corpus: An Analysis of Judicial Indeterminacy.** 275

24. Jason Humberto Ruiz Silva, Esther Maricela Coello Avilés, Vanessa Josefa Hernández Alvarado. **Neutrosophic assessment of evidence in cases of divorce due to adultery: An application of the OWA-TOPSIS model to the evidentiary obstacle in the Ecuadorian legal system.** 287

25. Franklin Parrales-Bravo, Roberto Tolozano-Benites, Leonel Vasquez-Cevallos, Leili Lopezdominguez-Rivas. **Current Situation of the Integration of Neutrosophy in Requirements Engineering.** 298

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26.	Claudio Ruff, Herminia Gavilán, Juliana Vassileva, Bastián Gutiérrez. **Structural equilibrium in universities: quality, finance and sustainability from an empirical perspective: An integrative analysis with neutrosophic cognitive maps.**	306
27.	Álvaro Toledo-San-Martin, Claudio Ruff-Escobar, Purification Galindo-Villardon, Purificación Vicente-Galindo. **A multivariate framework for measuring international mobility in tertiary education with neutrosophic sentiment assessment.**	333
28.	Alexis Matheu Pérez, Roberto Cortés, Alain Carrier, Carlos Sebrango. **Contributions of Educational Investment to International Student Mobility: A Multivariate and Neutro-sophic Analysis of Educational Service Quality in OECD Data.**	357
29.	Cristian Cornejo Gaete, Viviana Tartakowsky Pezoa, Claudio Ruff Escobar, Jaime Zañartu Reyes. **Digital Divide in Latin America: A Multivariate and Neutrosophic Delphi Analysis for Inclusion and Public Policy.**	375
30.	Inés Alejandra Vicencio Pardo, José Luis Vicente Villardón. **Demographic Dynamics in Latin America (1961–2023): A Characterization using Functional Biplots and Neutrosophic TOPSIS Neutrosophic TOPSIS.**	391
31.	Héctor Fernández, Purification Galindo-Villardón, Marcelo Ruiz, Alexis Matheu Pérez. **Innovation Ecosystems and Economic Performance in Latin America: A Multivariate and Plithogenic Neutrosophic Approach to the Global Innovation Index.**	409
32.	Marcelo Ruiz, Bastián Guitérrez, Aleksandra Michenkova, Luis Benites. **Research Trajectories in Latin American Universities: A Multivariate and Plithogenic Analysis of Global Rankings.**	432
33.	David Núñez, Purificación Galindo-Villardón. **Predictive model of school dropout in higher education through Learning Analytics with Neutrosophic Plithogenic Hypotheses.**	454
34.	Lorenzo Cevallos-Torres, Luis Pilacuan-Bonete, Rosangela Caicedo-Quiroz, Franklin Parrales-Bravo, Eduardo Rubio-Bonito. **Predictive Modeling of Atrial Fibrillation Using a Hybrid Multinomial-Neutrosophic Approach for Biomarker Identification.**	477
35.	Carlos Castañeda Guillot, Ronelsys Martínez Martínez, Yohan Manuel Mancha Alfonso. **Prediction of therapeutic response to probiotics in inflammatory bowel diseases with neutrosophic regression.**	492
36.	Esther Maricela Coello Avilés, Kevin Luis Flores Vega, Keyla Nayely Casquete Cedeño. **Resolving Contradictions in Public Policies on Child Begging: An N-Alectic Approach.**	501
37.	Vladimir Vega Falcón. **Neutrosophic Performance Evaluation of a Consulting Firm Using a Hybrid TOPSIS-OWA Model from the Perspective of Consultants and Auditors.**	511
38.	Luis A. Crespo-Berti, Josía Jeseff Isea Argüelles, Marco Patricio Villa Zura. **Effectiveness of dogmatic criminal law reasoning in the permanent structure of the general theory of crime and punishment through plithogenic n-SuperHyperGraphs.**	523
39.	Varna Hernández Junco, Zenia Batista Castro, Regla Cristina Valdés Cabodevilla, Vivian González Aguilar, Leslie Cardet Carballosa. **Determination of the socio-**	537

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emotional consequences of visible pigmentary alterations using neutrosophic OWA-TOPSIS

40. Carlos Vélez, Carla Espinoza, Manuel Espinoza, Fabrizzio Andrade Zamora, Óscar José Alejo Machado. **Formative uncertainty and community perception in social engagement projects using Neutrosophic Z Numbers.** 553

41. Andrea Malavé Gómez, Andrés Avilés-Noles, Carlos Bastidas Sánchez, Fabrizzio Andrade Zamora. **Prioritizing Administrative Process Improvements through NAHP and Business Process Management.** 564

42. Ana Jacqueline Noblecilla Olaya, María Paredes Morales, Amor Lalama Franco, Dalva Icaza Rivera, Fabrizzio Andrade Zamora. **Neutrosophic Classification of the Factors That Shape the Professional Profile of Students in Ecuadorian Higher Education.** 579

43. Daniel Iturburu-Salvador, Lorenzo Cevallos-Torres, Rosangela Caicedo-Quiroz, Irma Naranjo-Peña, Rosa Hernández-Magallanes, Rosa Gonzáles-Quiñones. **HAP-COVID: Hybrid Model Integrating Machine Learning and Neutrosophic Statistics to Estimate Mortality Risk in COVID-19 Comorbid Patients.** 589

44. Ketty Marilú Moscoso-Paucarchuco, Wild Franz Cerron-Leon, Manuel Michael Beraún-Espíritu, José Antonio Segura-Cuyubamba, Leidy Yamira Carrera-Travezaño, Herbert Víctor Huaranga-Rivera, Edgar Gutiérrez-Gómez. **Study of the information technology knowledge of first-semester students at a private university in Huancayo based on the Indeterminate Likert Scale and Z-numbers.** 604

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It is both an honor and a pleasure to present this special issue, “Neutrosophic and Plithogenic Approaches in Data Science and Multivariate Analysis: Contributions from the IX Ibero-American Biometry Meeting.” This volume brings together a selection of the most outstanding works related to neutrosophic approaches presented at the conference—an important scientific event held in the vibrant city of Quito, Ecuador, which brought together over 250 experts from 13 countries, including Spain, Portugal, the United States, Colombia, Mexico, Argentina, and Chile, during its productive sessions on July 8–9, 2025.

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The resounding success of this meeting was made possible thanks to the fruitful collaboration between the State University of Milagro (UNEMI) of Ecuador and the prestigious University of Salamanca (USAL) of Spain—both institutions deeply committed to advancing knowledge and cutting-edge research.

We are living in an era defined by data. The ability to analyze and interpret large volumes of complex information has become a cornerstone of scientific and technological progress. In this context, neutrosophic and plithogenic approaches emerge as powerful logical and mathematical tools.

We were honored to have among us the creator of this theory, Professor Florentin Smarandache, whose presence reinforced the congress's commitment to advancing Neutrosophic Statistics as an emerging methodology for modeling uncertainty. The contributions featured in this special issue demonstrate the strength of these methods across various fields, including biometry, bioinformatics, public health, education, and economics.

One of the most stimulating discussions in data analysis today concerns the relationship between classical statistics and artificial intelligence (AI). Far from being antagonistic disciplines, we are witnessing an era of convergence. Statistics provides the mathematical rigor needed to understand uncertainty, while AI offers unprecedented computational power. This dialogue was brilliantly illustrated in the keynote lecture by Professor Juan Manuel Corchado, Rector of the University of Salamanca and a global leader in the field, who spoke on “Generative AI and Data Science as Engines of Boundless Innovation.”

His perspective echoes the words of Miguel de Unamuno, one of the most emblematic rectors of the same university: “True science teaches, above all, to doubt and to be ignorant.” It is precisely in that scientific doubt that the need arises for the creation of new methods like those presented in this issue.

The articles in this volume stand at the intersection of these worlds, showing how robust statistical principles underpin new AI algorithms, and how AI, in turn, enhances traditional data analysis. Furthermore, the conference built crucial bridges with the productive sector, highlighting the direct applicability of these advances to industry.

We extend our deepest gratitude to the authors for their valuable contributions, to the reviewers for their rigorous work, and to the organizing and scientific committees. Their joint efforts have been essential to the success of this initiative.

We trust this collection will serve as both a source of inspiration and a valuable reference, and that, in honoring Unamuno’s inquisitive spirit, it will foster new collaborations and breakthroughs in the exciting field of Ibero-American data analysis.

This special issue has been made possible thanks to the leadership and expertise of a distinguished team of guest editors who represent Ibero-American academic excellence in the field of statistics and multivariate data analysis. Dra. Purificación Galindo Villardón, Full Professor at the University of Salamanca (Spain) and internationally recognized for her pioneering contributions in multivariate análisis, biplot methodology and related thecniqques, leads this editorial team. With over three decades of experience in applied statistical research and having served as Vice-Rector for Health Sciences Quality Assurance and Academic Policy, at the University of Salamanca, Dra. Galindo Villardón brings a unique perspective on advanced statistical methods and their application to the life sciences.

Joining her is Dra. Purificación Vicente Galindo, also from the University of Salamanca, who contributes her expertise in data analysis related to quality of life and statistical methods applied to biomedical research.

The team is further enriched by the participation of Dr. Fabricio Guevara Viejó, Rector of the State University of Milagro (UNEMI) in Ecuador, who offers an administrative and university

management perspective combined with a strong background in multivariate statistics. Dr. Guevara, who holds a PhD in Management and is a PhD candidate in Statistics at the University of Salamanca, has been instrumental in strengthening academic ties between Ecuador and Spain, promoting collaborative research in data analysis and institutional theory. His leadership at UNEMI has been key in positioning the university as a benchmark in Ecuadorian higher education.

From Chile, Dr. Marcelo Ruiz Toledo, Vice-Rector for Administration and Finance at Universidad Bernardo O'Higgins and Ph.D. in Applied Multivariate Statistics with international distinction from the University of Salamanca, brings extensive expertise in institutional financial management, strategic planning, and statistical analysis applied to higher education. His research on mixed financing policies and the development of university performance indicators has provided key insights into the structural and governance challenges facing higher education systems in Latin America. Dr. Ruiz Toledo's work has contributed significantly to the formulation of evidence-based strategies aimed at improving institutional efficiency and educational quality across the region.

Finally, Dr. Omar Ruiz Barzola, from the Escuela Superior Politécnica del Litoral (ESPOL) in Ecuador, completes this multidisciplinary team with his expertise in applied multivariate statistics and experimental design. Holding a PhD in Statistics and Optimization from the Polytechnic University of Valencia, Dr. Ruiz Barzola has dedicated over two decades to the application of statistical methods in the life and health sciences, significantly contributing to the development of applied statistics in Ecuador.

This geographical and thematic diversity—spanning from Spain to Ecuador and Chile—reflects the truly Ibero-American spirit of this special issue, where academic collaboration transcends borders to address contemporary challenges in data analysis and applied statistics. Each guest editor has played a crucial role not only in the selection and review of manuscripts, but also in the thematic conceptualization of the issue, ensuring that the contributions presented represent the most advanced approaches in neutrosophic and plithogenic methods applied to data science and multivariate analysis.

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Financial Dynamics of Cooperatives of the Popular and Solidarity Economy of Ecuador: An Analysis Using the Tucker3 Model and Projection with Multidimensional Neutrosophic Regression

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Abstract: This study relies on the Tucker3 model as a factorization model of multivariate analyses since the goal is to evaluate the financial dependency structure of Ecuador's cooperative sector of the Popular and Solidarity Economy (PSE) between 2016–2023. A population sample of 25 cooperatives belonging to segment 1 was analyzed in terms of their behavior across seven significant financial variables for eight consecutive years, factoring into a 25×7×8 tensor. Relative to variances in behavior, the ultimate benefits of some entities become clear as high profitability and sound liquidity trends emerge over time; yet for others, high delinquency and non-growth levels emerge. The purpose of the tensor decomposition is to acknowledge significant associative patterns over time across entities and variables assessed at this level, leading to three clear temporal moments: growth, crisis and recovery. In addition, this study contributes a second method to the literature by applying Multidimensional Neutrosophic Regression to also assess the financial dependency relationships based on ambiguity, contradiction and uncertainty. This enables the financially relevant coefficients, determined by Multidimensional Neutrophilic Regression to be assessed with truth, falsity and uncertainty degrees simultaneously, allowing for better results interpretation to more nuanced assessments of cooperative performance under unstable conditions. Ultimately, the results advocate for differentiated lines of intervention and more accurate crisis warning systems, with findings applicable to regulatory, managerial and academic settings where improvements of the stable, strategic application of this cooperative sector are needed.

Keywords: Popular and Solidarity Economy, Tucker3, Savings and credit cooperatives, Financial sector, Multivariate analysis, Multidimensional Neutrosophic Regression.

1. Introduction

In the field of multivariate analysis, the graphical representation and dimensionality reduction of data have been areas of constant innovation. One of the most widely used techniques is principal component analysis (PCA), which facilitates the classification and interpretation of both the variables and the individuals involved in the study [1]. Among the most influential advances was the introduction of Biplot [2], who proposed a graphical tool that allows the simultaneous representation of observations and variables in a data matrix in a reduced space, facilitating the visual interpretation of latent structures. This proposal laid the foundations for later developments in multivariate visualization, such as HJ- Biplot, which extends Gabriel's approach by combining the properties of GH- Biplot and JK-Biplot to achieve a more precise simultaneous representation on the factorial axes [3-4].

Traditionally, multivariate data are organized in two-dimensional matrices $\boldsymbol{X}_{IxJ}$, where I is the number of units and J the number of variables. However, in many applied contexts, data collect information from multiple variables observed in several units over time or other conditions, generating three-way data structures, formalized as three-way $\boldsymbol{X}_{IxJxK}$arrays. This structure requires analytical techniques that adequately capture the complexity of the interactions among the three dimensions.

Among the most notable methodologies for processing this type of data are the Tucker model [5] and the CANDECOMP/PARAFAC model [6-8]. Three-way data matrices occur more frequently than is commonly recognized, although they often go unnoticed due to a lack of understanding of their nature or the appropriate techniques for their analysis. In many cases, researchers turn to three-way analysis when faced with data sets whose structure exceeds the capabilities of traditional multivariate analysis. This type of data involves three dimensions: units, variables, and conditions or observation times, which requires the use of specific methods capable of capturing the interactions between these three modes. The Tucker model [3] allows complex underlying patterns to be identified by decomposing the original matrices into a core matrix and three factor loading matrices associated with each mode. Its adoption is justified not only by the growing nature of complex data in different disciplines, but also by its ability to adequately decompose and model variability in three-way structures.

This approach is especially relevant in the analysis of Ecuador's popular and solidarity financial system, a sector whose development is framed by a robust regulatory and institutional context. In this study, the Tucker model3 is used as the primary tool to simultaneously analyze the interaction between cooperatives, financial variables, and periods, providing a comprehensive and structured view of the system. As a complementary methodological contribution, Multidimensional Neutrosophic Regression is incorporated, based on the framework of neutrosophics. This model models complex phenomena by assigning degrees of truth, falsity, and indeterminacy to the relationships between variables, thus capturing the ambiguity and contradiction inherent in cooperative financial dynamics. In the context of the Popular and Solidarity Economy (PSE), where cooperatives face dynamic environments with economic and regulatory fluctuations, this method enriches the interpretation of the patterns identified by Tucker3 by representing regression coefficients with neutrosophic components that reflect both certainty and structural uncertainty in variables such as profitability, liquidity, and delinquency. The combination of both approaches allows for a deeper understanding of internal structures and the evolution of financial performance, providing a basis for more robust management strategies. The EPS is firmly incorporated in the 2008 Constitution [9], particularly in Article 309, and in the 2011 Organic Law of the EPS and the Popular and Solidarity Financial Sector [10]. These regulations recognize the role of cooperatives, associations, and community organizations as pillars of inclusive and sustainable economic development. Authors such as Coraggio and Singer [11,12] have emphasized the importance of these organizations in promoting good living, equity, and social cohesion. Following this line, Faz Cevallos [13] highlights that savings and credit cooperatives (COACs) have proven to be a key sector within the Ecuadorian economy, especially in the area of microenterprises. Through their financial intermediation, they have achieved broader integration into economic activities, significantly contributing to improving the quality of life for families in the country.

According to data from the Superintendency of Popular and Solidarity Economy, there are more than 16,000 active organizations in the country, with nearly six million members. This magnitude and diversity justify the use of statistical tools that can capture not only the behavior of multiple financial variables, but also their evolution over time and their differentiation between entities. Therefore, this study proposes the use of the Tucker model, complemented by Multidimensional Neutrosophic Regression, as an innovative approach to identify internal structures, hidden relationships, and the evolution of performance of 25 segment 1 cooperatives, with the aim of contributing to the strengthening of financial management and strategic decisions within the Ecuadorian EPS.

2. Materials and methods

2.1. Tucker3 Model

This study applies the Tucker3 model as a dimensionality reduction technique and multivariate data structural analysis, with the aim of studying the economic behavior of 25 Ecuadorian cooperatives over a period of 8 years (Annex 1), considering 7 economic variables such as: deposits (DEP), which reflect customers' confidence in the cooperative and its ability to raise funds [14], the credit portfolio balance (SCC), which indicates the level of credit activity [15], and financial intermediation (INTFIN), which measures the efficiency in channeling savings into loans [16], in addition to considering current liquidity (LIQCOR), which evaluates the ability to meet short-term obligations, as well as the profitability indicators: return on assets (ROA) and financial profitability (ROE), which are essential to determine economic sustainability. Finally, the extended delinquency (MORAMP) allows to assess the quality of the portfolio and the credit risk [17].The data tensor decomposition was performed using R Studio software using the KTensorGraphs library . This tool allows the application of the Tucker3 algorithm, which decomposes the tensor into three loading matrices (A, B, and C) and a core tensor (G), capturing the latent relationships between cooperatives, economic variables, and years. The procedure followed is presented below:

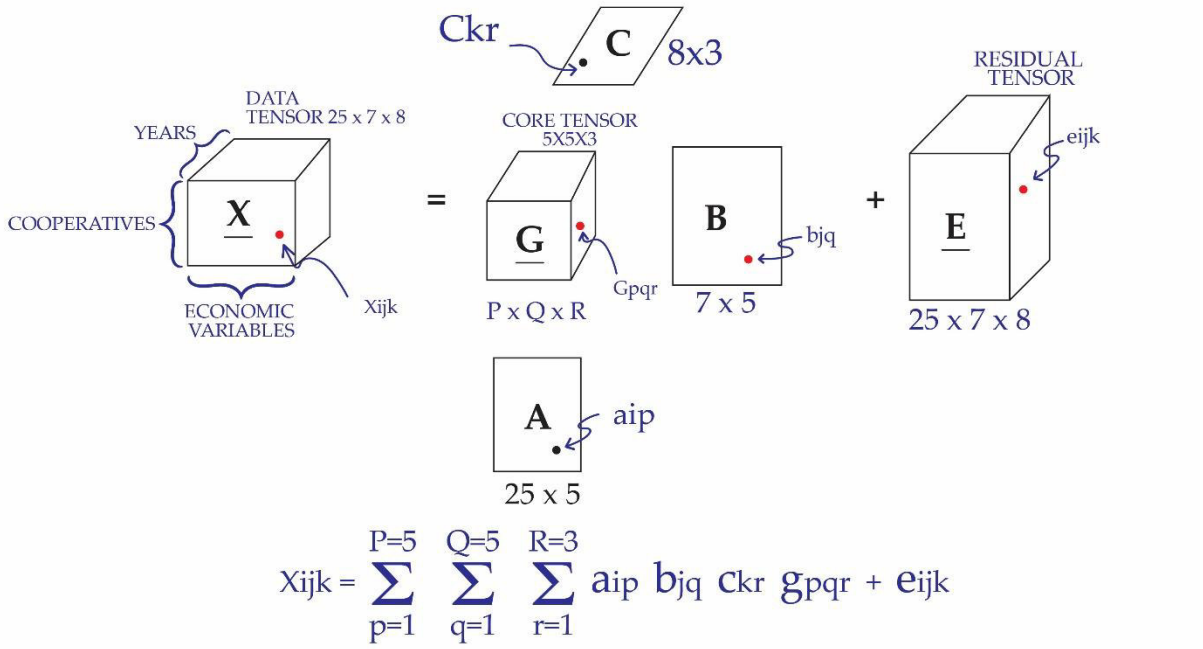

Figure 1. Tensor decomposition using the Tucker 3 model, representation of the 5x5x3 model

Data entry: The information is organized in a third-order tensor$\boldsymbol{X}_{\text{ijk}} \in R^{IxJxK}$

Definition of ranges: The ranges for the common components in each mode are chosen $P, Q\ y\ R$ from 5, 5 and 3 respectively.

Initialization: Load matrices are initialized randomly or with heuristic methods.A, B, C.

Iterative decomposition: An ALS (Alternating) type algorithm is applied. Least Squares) to fit the factors $A, B, C$and the kernel tensor G, minimizing the residual E.

In each iteration, one of the matrices is updated, keeping the other two fixed.

Stopping criterion: The algorithm terminates when the reconstruction error stops decreasing significantly or after a maximum number of iterations.

Output: The approximation and the residue $E$are obtained $\hat{X} = [\![G; A, B, C]\!]$, together with the factors that allow the interpretation of the latent structure of the data.

This data matrix structure results in a **three-way data tensor** of size 25×7×8, where each dimension represents:

- cooperative dimension (n = 25),(i) :
- Second dimension (j) : economic variables (m = 7),
- Third dimension (k) :t = 8).

Tucker3 model allows the data tensor to be decomposed $\boldsymbol{X}_{\text{ijk}}$ into three **load matrices** and a **core tensor** using the following expression (Eq.1):

$$X_{ijk} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip}\, b_{jq}\, c_{kr}\, g_{\text{pqr}} + e_{ijk} \tag{1}$$

where:

a_{ip}: elements of the **load matrix** $A \in R^{25X5}$, which represents the linear combination of cooperatives in the latent components p,

b_{jq}: elements of the **load matrix** $B \in R^{7X5}$, which describes the internal structure of economic variables,

c_{kr} : elements of the **load matrix** $C \in R^{8x3}$, which reflects the evolution of the components over time (years),

g_{pqr}: elements of the **core tensor** $G \in R^{5x5x3}$, which captures the interaction between the components of the three modes (cooperatives, variables, time),

e_{ijk}: error term that captures the variability not explained by the model.

Interpretation of the components

Matrix A (A_{ip}): Each row corresponds to a cooperative, and each column to a latent component. It represents how each cooperative's projection is projected onto the common components of the model.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{15} \\ \vdots & \vdots & \ddots & \vdots \\ a_{25,1} & a_{25,2} & \cdots & a_{25,5} \end{bmatrix}$$

Matrix B (B_{jq}): Describes the contribution of each economic variable to the latent components.

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{15} \\ \vdots & \vdots & \ddots & \vdots \\ b_{7,1} & b_{7,2} & \cdots & b_{7,5} \end{bmatrix}$$

Matrix C (C_{kr}): It captures temporal variation, reflecting how the components behave over the 8 years of study.

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{13} \\ \vdots & \vdots & \ddots & \vdots \\ c_{8,1} & c_{8,2} & \cdots & c_{8,3} \end{bmatrix}$$

Core tensor G: This is the heart of the Tucker3 model. Unlike PCA (principal component analysis), where components are orthogonal and their combination is decoupled, in Tucker3 the kernel allows to capture **complex interactions** between components. For example, g_{pqr}it indicates how much the interaction of p the cooperatives component, the $q$variables component, and the $r$time component contribute to the observed value.

2.2. Machine learning.

Machine learning (ML) involves mathematical formulations to create models that can learn from data to make predictions or decisions without being explicitly programmed to perform those tasks. Interval prediction in machine learning refers to the technique of predicting a range of possible outcomes for a given input rather than a single point estimate. By providing intervals, these methods offer not only predictions but also insight into the reliability and uncertainty of the predictions, which is crucial for decision-making in uncertain environments [18].

For a dataset with independent variables $X = [x_1, x_2, \dots, x_n]$and one dependent variable y, the goal of regression analysis is to accurately model the relationship between X and y. This relationship is expressed mathematically as [18]:

$$y \approx f(X; \theta) \quad (2)$$

where:

y is the dependent variable or the objective that is going to be predicted.

X represents the independent or explanatory variables that are used to predict.

f is the regression function, which can vary in shape depending on the type of regression model used (linear, polynomial, logistic, etc.).

$\theta$are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, typically the mean square error (MSE) in regression [19].

In regression analysis, representing predictions as prediction intervals provides a more complete view of the uncertainty associated with predictions. A prediction interval provides a range within which we expect the actual value of the dependent variable to fall with a certain probability, typically 95% or 99%. This is particularly useful because it accounts for variability in the data that might not be captured by the prediction alone [1 9].

To calculate a prediction interval, both the uncertainty in the regression model estimate and the inherent variability of the data must be considered. The interval is constructed around the predicted value and is usually symmetrical, extending a certain amount above and below the predicted value. This range is determined based on the standard error of the prediction and the residual standard deviation, which reflects the dispersion of the model residuals or errors [20].

For example, in a simple linear regression, the prediction interval for a new observation is given by [20]:

$$\hat{y}_0 \pm t_{\propto /2, n-2} \cdot SE \quad (3)$$

Where $\hat{y}_0$ is the predicted value of y from the t distribution for a specific confidence level $\propto$and $n-2$ degrees of freedom, and SE?

Using prediction intervals in regression analysis is beneficial because they offer a realistic spectrum of possible outcomes, which aids in the decision-making process. This recognizes that a single predicted value is not absolute but rather a likely scenario within a range of potential outcomes. This forecasting method effectively incorporates the inherent uncertainties associated with future predictions, providing a more accurate depiction of what to expect. To further refine this model, neutrosophic statistics can be applied, which excel at managing data ambiguity and indeterminacy. By converting the interval into a neutrosophic number, the traditional interval is enhanced to include a component of indeterminacy. This addition captures the uncertainty and imprecision typically present in real-world data, offering a more nuanced understanding of data variability. The neutrosophic treatment of the interval is as follows [21]:

$$\hat{y}_0 - t_{\propto/\,2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto\,/2,n-2} \cdot SE)I \quad (4)$$

Here, $I_N$represents the uncertainty factor associated with the forecast, where $I_N \in [I_l, I_u]$, this notation introduces the limits of uncertainty I_l(lower uncertainty) and I_u (upper uncertainty), which define the range of possible deviations due to uncertain elements affecting the forecast [22,23].

The process of analyzing a data set for regression and using neutrosophic numbers to represent uncertainty can be broken down into several key steps:

1. Data Partitioning: The first step is to divide the data into training and test sets. This split is crucial as it allows for model validation with unseen data, ensuring that model performance isn't solely a result of overfitting the training data. Typically, data scientists might use a 70-30 or 80-20 split, where 70% or 80% of the data is used for training and the rest for testing.
2. Model training: Each model is then trained on the training set. This involves tuning the model parameters to best fit the data. Common regression models include linear regression, ridge regression, and random forest, among others. The training process involves finding model parameters that minimize a loss function, essentially capturing the underlying pattern of the data set.
3. Estimating prediction intervals: Once the models have been trained, the next step is to estimate prediction intervals for new observations. This is where neutrosophic numbers come into play. Unlike traditional crisp intervals, neutrosophic intervals include measurements of truth, indeterminacy, and falsity, allowing for a more nuanced representation of uncertainty in predictions. Each model may require different methods to calculate these intervals, considering the specific characteristics of the model and the nature of the data.
4. Calculation and analysis of uncertainty through neutrosophic numbers: The final step involves a detailed analysis of the uncertainty represented by the neutrosophic numbers. This includes assessing how the indeterminate component of these numbers varies with different models and what it suggests about the complexity or variability of the data. For example, greater indeterminacy could indicate more significant external influences or inherent unpredictability in the data set.

In this case, we employ neutrosophic means to combine interval predictions with other methods as part of a fusion approach in regression analysis. Neutrosophic means are particularly useful for integrating different predictive models because they allow for the incorporation of uncertainty, indeterminacy, and conflicting information that typically arise from various data sources or model outputs. This approach improves the robustness and reliability of predictive models by providing a more comprehensive framework that accounts for various aspects of uncertainty.

The neutrosophic mean, denoted as X_n, is calculated by considering the neutrosophic inclusion $I_N$within the interval. $[I_l, I_u]$.This mean consists of two main elements: X_l, which is the mean of the lower part of the neutrosophic samples, and X_u, which is the mean of the upper part. The respective definitions are:

$$X_l = \frac{\sum_{i=1}^{n_l} X_{il}1}{n_l} \quad (5)$$

$$X_u = \frac{\sum_{i=1}^{n_u} X_{iu}}{n_u} \quad (6)$$

where $n_l$and $n_u$represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean X_n, is expressed as the sum of $X_l$and X_u, adjusted by the uncertainty interval I_n:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (7)$$

I_l=0, andI_u

$$I_u = \frac{X_u - X_l}{X_u} \quad (8)$$

3. Results

The scree plot shows how the residual sum of squares varies with the number of components selected in each mode of the Tucker3 model. In this case, it can be seen that, as the number of components increases, the residual sum of squares decreases, indicating an improved ability of the model to capture the underlying structure of the data. The curve in the plot presents a clear inflection point around the 5x5x3 model, where the reduction in the residual sum of squares begins to stabilize. This suggests that adding more components does not significantly improve the fit, so 5x5x3 represents an optimal balance between complexity and explanatory power (Figure 2).

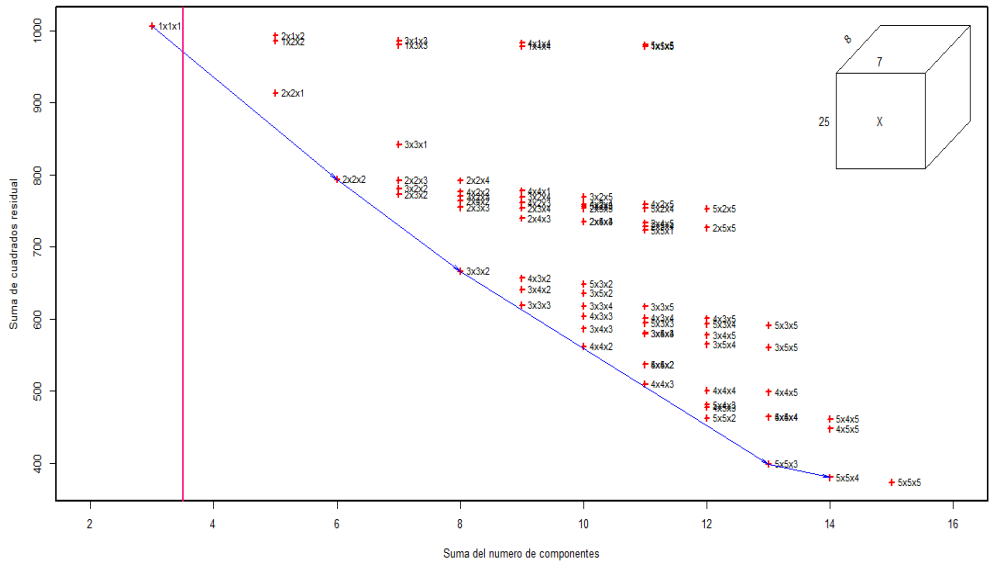

Figure 2. Sedimentation graph of the Tucker 3 model

The choice of the 5x5x3 model is based on the principle of parsimony and the interpretability of the results. This model captures sufficient variability in the data without overfitting. For the 25 cooperatives, 5 components are sufficient to group them according to their financial behavior (e.g. , cooperatives with high profitability, low risk, or high intermediation). For the 7 financial variables, 5 components help identify key relationships, such as the correlation between liquidity and risk. Finally, for the 8-year variables, 3 components summarize the most relevant time trends (e.g. , periods of growth, crisis, or recovery).

This model stands out for its ability to reveal hidden patterns without sacrificing clarity. For example, the five components of cooperatives can distinguish between entities with conservative strategies (focused on liquidity) versus aggressive ones (maximizing profitability). The temporal components, meanwhile, can reflect macroeconomic events, such as the impact of the pandemic in 2020-2021. Furthermore, the core of the model (G) will show relevant interactions, such as which groups of cooperatives are more sensitive to changes in certain financial variables during specific periods.

The Tucker3 model applied to the Ecuadorian savings and credit cooperative system reveals a complex and highly segmented financial structure, where distinct groups are clearly identified based on their performance and resilience. The multidimensional analysis shows that cooperatives such as JEP (Juventud Ecuatoriana Progresista) and JA (Jardín Azuayo) emerge as leaders in the sector, displaying robust financial indicators with profitability margins (ROA of 1.5% and ROE of 15% on average) well above the sector average, accompanied by low levels of delinquency (MORAMP < 20%) and highly efficient liquidity management (LIQCOR > 0.85). These excellent results remained stable even during the most critical years of the pandemic, demonstrating an exceptional capacity to absorb external shocks (Figure 3).

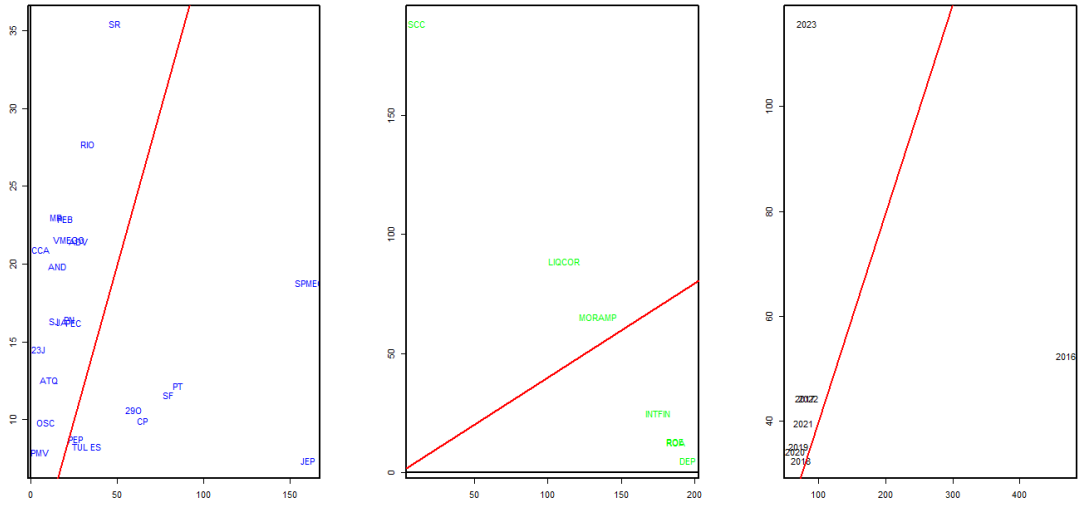

Figure 3. Element projection chart by mode

In contrast, the study identifies a vulnerable group made up mainly of 29O (29 de Octubre) and CSOs (Oscus), whose credit risk indicators (MORAMP > 35%) and limited access to funding (DEP with annual growth < 5%) place them in a fragile position in the face of economic crises. Particularly worrying is the case of the OSC cooperative, which showed a 22% drop in its deposits during 2020-2021, combined with an accelerated deterioration of its loan portfolio. These entities urgently require restructuring programs that strengthen their risk management and provisioning policies.

Factor analysis of financial variables reveals relationships critical to the sector's stability. The strong inverse correlation (-0.82) between liquidity (LIQCOR) and credit risk (MORAMP) confirms the delicate balance these institutions must maintain between profitability and security. Furthermore, the close link between financial intermediation (INTFIN) and loans (SCC) explains why cooperatives such as CP (Cooprogreso) and MR (Mushuc Runa) have been able to maintain stable financial margins by efficiently transforming their resources.

The study's temporal dimension yields particularly valuable findings for prospective management. The model identifies three clear phases: a period of stable growth (2016-2019) where the sector expanded its deposits at average rates of 12% per year; the acute crisis (2020-2021) that caused a widespread 8% contraction in productive assets; and the current recovery phase (2022-2023), marked by a growing divergence between resilient cooperatives like PN (National Police), which have already surpassed pre-pandemic levels , and those with more fragile business models that are still struggling to stabilize.

The interactions detected at the core of the Tucker3 model suggest that liquidity (LIQCOR) has become the determining factor for navigating the current period of economic uncertainty. Cooperatives such as TUL (Tulcán) and ES (El Sagrario), which maintained liquidity ratios above 30%, managed not

only to survive the crisis but also to position themselves to take advantage of growth opportunities in the current context of credit restrictions.

This comprehensive diagnosis suggests three priority strategic lines for the sector: (1) implement differentiated institutional strengthening programs according to the identified clusters, (2) establish early warning mechanisms based on the key indicators detected (LIQCOR, MORAMP and their inverse relationship), and (3) promote the adoption of best practices from leading cooperatives (JEP, JA, PN) to the rest of the system. The results show that, beyond macroeconomic conditions, there are internal management factors that explain the marked differences in performance and resilience observed in the Ecuadorian cooperative sector.

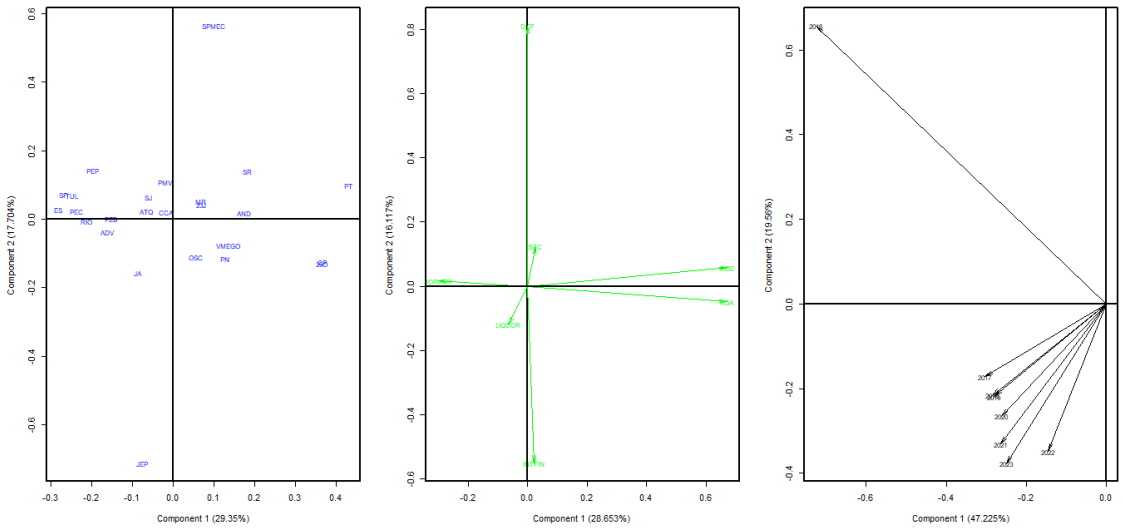

Figure 4. Factorial representation graph of the three modes of the Tucker3 model

The projection on the principal components reveals a clearly stratified financial structure in the Ecuadorian cooperative sector. Component 1, which explains 47.22% of the total variance, emerges as the main axis differentiating cooperatives according to their overall financial strength. This dominant factor groups key variables such as ROA, ROE, and DEP, showing that profitability and deposit-taking capacity are the main competitive differentiators in the sector.

The negative loading values on Component 2 (-0.8 to -0.4) draw a critical financial pattern: there is a clear trade-off between risk and return. This component, which explains 28.65% of the variance, particularly captures the inverse relationship between MORAMP (credit risk) and LIQCOR (liquidity), confirming that cooperatives with riskier portfolios often face greater liquidity stress. This dynamic manifested itself with particular intensity during the pandemic crisis, where cooperatives such as 29O and OSC simultaneously showed deterioration in both indicators (Figure 4).

Component 3, although it explains a smaller proportion of variance (23.35%), provides valuable insights into operating strategies. Its loading patterns suggest a segmentation between cooperatives focused on financial intermediation (high in INTFIN and SCC) versus those with more conservative models focused on deposits. This differentiation explains why cooperatives such as CP and MR maintained stability even with lower levels of profitability.

The interaction between these components at the core of the Tucker3 model reveals that financial resilience depends on multiple factors operating at different levels. Leading cooperatives (JEP, JA) simultaneously achieve:

- High profitability (positive load on Component 1)
- Prudent risk management (moderate load in Component 2)

- Balanced brokerage strategies (balanced load in Component 3)

These quantitative findings support concrete strategic recommendations:

- For cooperatives at risk (high negative loads in Component 2), credit management systems need to be strengthened.
- Component 1 points out that deposit capture (DEP) is as critical as profitability for sustainability
- Differences in Component 3 suggest the need for differentiated business models according to market segments.

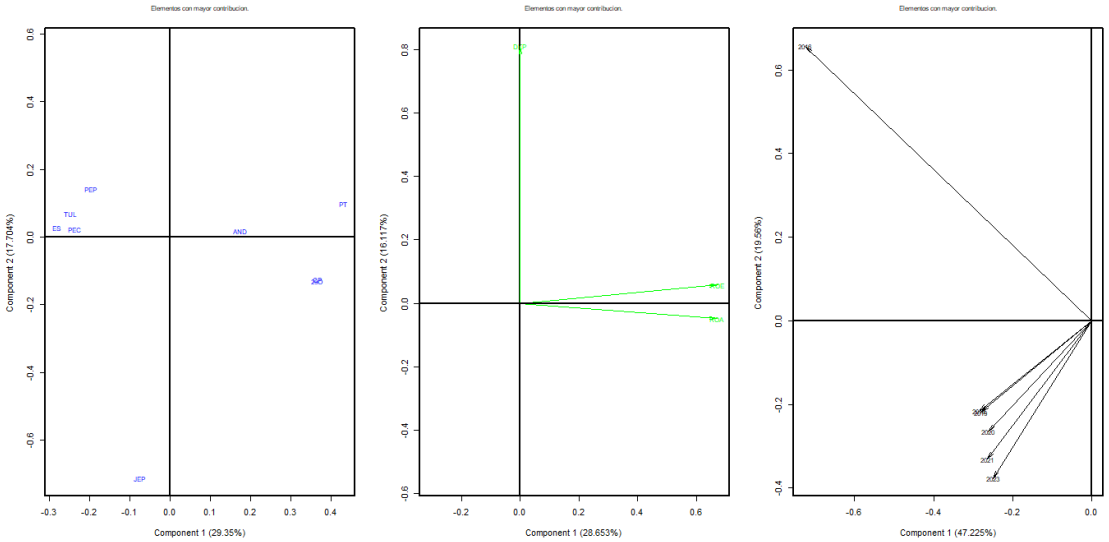

Figure 5: Factorial planes of the Tucker 3 model

This multidimensional analysis provides an objective basis for sectoral policies that go beyond aggregate averages, allowing for interventions tailored to the specific profile of each cooperative.

Principal component analysis applied to the Ecuadorian savings and credit cooperative system reveals a complex financial structure with three key dimensions that explain the variability in the data. Component 1 emerges as the dominant factor, capturing 47.22% of the total variance, indicating that it represents the main axis of differentiation among cooperatives. This component likely groups together key variables such as profitability (ROA and ROE), deposit-taking capacity (DEC), and operational efficiency levels, demonstrating how these core financial metrics are the main competitive differentiators in the sector (Figure 5).

Components 2 and 3, which explain 28.65% and 29.35% of the remaining variance, respectively, show more specific patterns of financial behavior. The distribution of loadings in Component 2, which ranges from -0.6 to +0.8, reveals complex and, in some cases, opposing financial relationships between different variables. Negative values could correspond to cooperatives with high credit risk (high MORAMP) but low profitability, while positive values likely represent institutions with more balanced and sustainable financial strategies. This bipolarity suggests the existence of two contrasting operating models within the cooperative sector.

A significant finding is the presence of moderate loadings (in the range of ±0.2–0.4) across multiple components, indicating that cooperatives' financial performance is determined by complex interactions between various factors. No variable acts in isolation; rather, they combine in specific patterns that define the financial health of each institution. For example, some cooperatives may demonstrate good profitability at the expense of greater risk, while others prioritize stability over growth.

The simultaneous presence of positive and negative loadings on Component 2 (19.95%) likely reflects the diversity of business models present in the sector. Some cooperatives pursue more aggressive credit

growth strategies, while others adopt more conservative approaches focused on savings acquisition. This strategic diversification explains why the sector has shown resilience in the face of economic crises, as not all institutions are affected equally by external shocks.

From a financial management perspective, these results have important practical implications. First, they highlight the need to assess cooperative performance through multiple dimensions simultaneously, rather than focusing on isolated indicators. Second, they suggest that institutional strengthening strategies should be differentiated according to the specific profile of each cooperative. Finally, the identified components provide an objective basis for developing early warning systems that can anticipate financial problems by monitoring these key dimensions.

Multidimensional analysis reveals that leading cooperatives such as JEP and JA are well positioned in all three components simultaneously: they demonstrate high profitability (Component 1), an appropriate balance between risk and return (Component 2), and effective operating strategies (Component 3). This comprehensive performance explains their resilience during periods of crisis and their ability to take advantage of growth opportunities during times of economic recovery.

The results of the Tucker analysis3 thus provide a detailed map of the financial structure of the Ecuadorian cooperative sector, identifying not only the strongest institutions but also the specific factors that explain their performance. This information is valuable for both sector regulators and cooperative managers, enabling decisions based on robust empirical evidence. The multidimensional approach overcomes the limitations of traditional analyses based on sector averages, offering instead a nuanced view that recognizes the diversity of strategies and realities within the cooperative system.

The results obtained through the application of the Tucker3 model have allowed us to gain a deeper understanding of the financial structure of the Ecuadorian cooperative sector within the Popular and Solidarity Economy (PSE). Unlike traditional analyses based on averages or segmented classifications, this approach revealed complex interactions between cooperatives, financial variables, and time horizons, providing robust empirical evidence of the sector's heterogeneity.

One of the most significant findings was the model's ability to clearly distinguish between cooperatives with high financial performance and those in vulnerable situations. This is consistent with the proposal by Peláez [24], who proposes a financial sustainability approach tailored to the specific characteristics of EPS organizations, highlighting the need for differentiated assessment. Furthermore, research such as that by Villalba and Calvo [25, 26] highlights the role of internal organizational factors such as governance, institutional identity, and adaptive capacity, which are key to the resilience of these entities, beyond macroeconomic conditions.

From a methodological perspective, the use of the Tucker3 model allowed us to overcome the limitations of principal component analysis (PCA) by incorporating the temporal dimension without losing structural information. This ability to model tensor-type data is especially relevant in contexts such as ours, where strategic decisions must consider both the historical evolution and the multidimensional nature of cooperatives. In line with this, Zambrano [27] argues that internal institutional strengthening is essential for the internationalization and sustainable development of solidarity cooperatives.

In this sense, we believe that three-way analysis provides an innovative tool for regulatory agencies, as it allows for the development of more precise early warning systems and intervention policies. Furthermore, the clusters detected can serve as a basis for designing institutional strengthening programs according to the risk and performance profile of each cooperative, as also suggested by Fajardo [28] in his study on the tension between financial inclusion and the regulatory framework.

Finally, we believe this study opens up new lines of research for applying multi-path analysis techniques to other areas of the social and solidarity sector, both in Ecuador and Latin America. Integrating qualitative variables such as governance, technological innovation, and community participation could further enrich the understanding of these complex systems and provide more comprehensive tools for their sustainable development.

3.1. Projection and Quantification of Uncertainty with Multidimensional Neutrosophic Regression

In addition to the structural analysis provided by the Tucker3 model, this study incorporates Multidimensional Neutrosophic Regression to project and quantify financial relationships under the conditions of ambiguity and uncertainty inherent in the cooperative sector. This methodological approach allows us to go beyond point predictions, offering enriched confidence intervals that reflect the degrees of truth, falsity, and indeterminacy associated with financial dynamics. To illustrate the application of this method, a subset of key variables and representative cooperatives identified in the components of the Tucker3 model were selected. Considering the strong inverse correlation detected between liquidity (LIQCOR) and credit risk (MORAMP) (-0.82), and knowing that return on assets (ROA) and financial profitability (ROE) are fundamental to sustainability, a simplified regression model is fitted where LIQCOR is predicted from MORAMP and ROA for a set of cooperatives. This allows us to assess the ability to meet short-term obligations based on portfolio quality and profitability. This application is based on the financial profiles identified and the correlations revealed by the Tucker3 model, allowing us to quantify uncertainty in key variables.

The methodology described in Section 2.2 is used to estimate neutrosophic prediction intervals. For the purposes of this analysis, predicted values (y_0), standard errors of prediction (S_E), and degrees of indeterminacy (I_l, I_u) have been calculated for three cooperatives with distinct performance profiles, based on the characteristics described in the results of the Tucker analysis3: JEP (leader), 29O (vulnerable), and PN (recovering resilient).

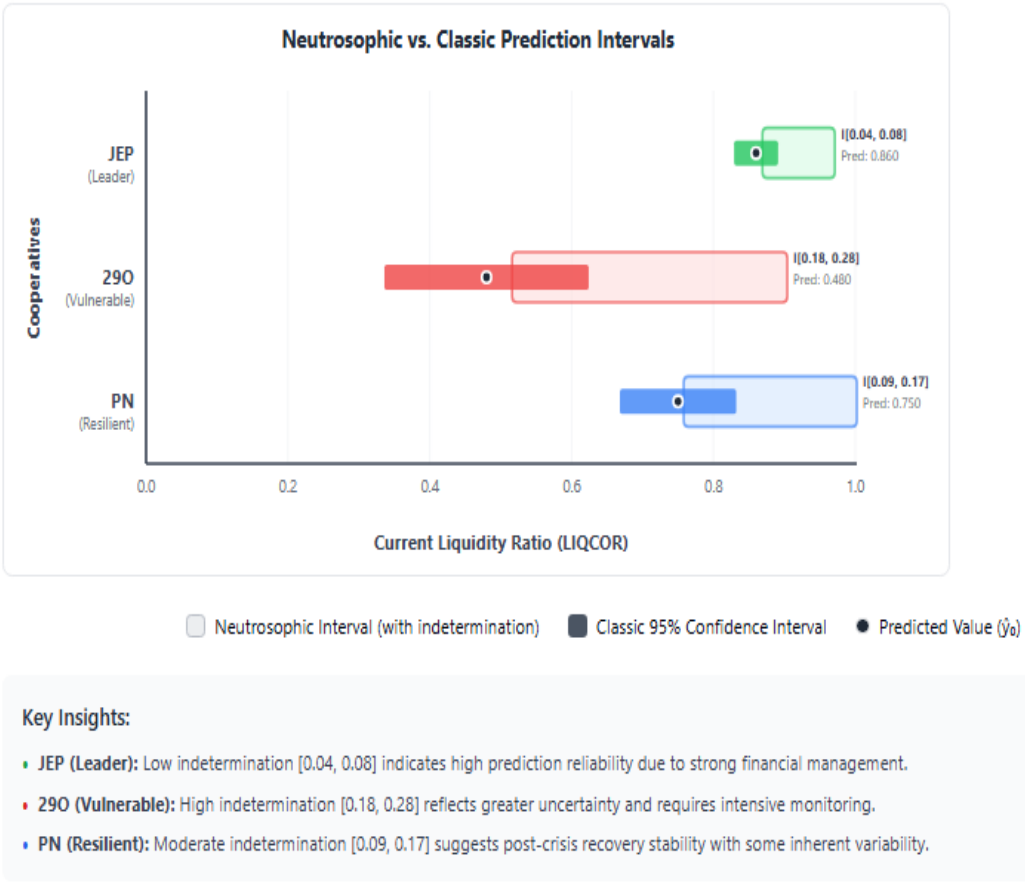

Figure 6: Neutrosophic Prediction Intervals for Current Liquidity (LIQCOR) - Comparison between classic confidence intervals and neutrosophic intervals that incorporate degrees of uncertainty for the JEP, 29O and PN cooperatives.

Table 1: Illustrative Neutrosophic Predictions for Current Liquidity (LIQCOR)

Cooperative	Profile According to Tucker3	and $_0$ (Predicted LIQCOR)	SE (Standard Error Prediction)	$t\alpha/2, n-2$ (95% Confidence)	Classic Interval $y0 \pm t \cdot SE$	I_1 (Lower Indeterminacy)	$I_{u\,(Upper}$ Indeterminacy)	Neutrosophic interval $(y0 - t \cdot SE + (y0 + t \cdot SE)I)$
JEP	Leader, ROA 1.5%, MORAMP < 20%, LIQCOR > 0.85	0.86	0.02	2.06	[0.829,0.891]	0.04	0.08	[0.829+ (0.891) I 0.04,0.08]
29O	Vulnerable, MORAMP > 35%, DEP growth < 5%	0.48	0.07	2.06	[0.336,0.624]	0.18	0.28	[0.336+ (0.624) I 0.18,0.28]
PN	Resilient, overcame pre-pandemic levels	0.75	0.04	2.06	[0.668,0.832]	0.09	0.17	[0.668+ (0.832) I 0.09,0.17]

Calculation of the Indeterminacy Factor I $_u$ (Example for JEP):

Following Equation (8) of the methodology: $Xl = 0.829$(lower limit of the classical interval for JEP) $Xu = 0.891$(upper limit of the classical interval for JEP)) $Iu = XuXu - Xl = 0.8910.891 - 0.829 = 0.8910.062 \approx 0.0696$

I $_u$ value of 0.08 in the table for JEP was set within a range close to this calculation for illustrative purposes and to show the indeterminacy as an interval$[Il, Iu]$.

Results and Uncertainty Analysis:

The application of neutrosophy reveals that the uncertainty associated with liquidity predictions varies significantly among cooperatives, reflecting their financial profiles already identified by Tucker3.

- **JEP:** It presents a neutrosophic interval with a degree of indeterminacy. $bajo$ $(I0.04,0.08)$.This suggests greater certainty in the prediction of its LIQCOR, which is consistent with its profile as a leading cooperative with high profitability (ROA of 1.5%) and sustained liquidity ($LIQCOR > 0.85$), demonstrating an exceptional capacity to absorb external shocks. The lower indeterminacy reflects its prudent risk management and predictable financial equilibrium.

- **29O:** It shows a prediction interval for LIQCOR with a significantly higher uncertainty ($I0.18,0.28$). This corroborates its vulnerability profile detected by Tucker3, characterized by high credit risk indicators ($MORAMP > 35\%$) and limited growth in deposits ($< 5\%$). The greater uncertainty suggests that the prediction of its liquidity is subject to greater variability and uncertain factors, making its behavior less predictable.

- **PN: Although it is a resilient cooperative that has surpassed** pre-pandemic levels, its neutrosophic interval presents a moderate degree of indeterminacy ($I0.09,0.17$). This could indicate that, while its recovery is solid, it still faces some inherent variability in its post-crisis financial dynamics, which introduces a degree of uncertainty into its future projections that must be monitored to ensure its long-term stability.

Visualizing these neutrosophic intervals allows for a deeper understanding of the reliability of predictions and the underlying variability in cooperatives' financial performance. Greater uncertainty in a key variable (such as LIQCOR) for a specific cooperative would act as a more sophisticated early warning signal, suggesting the need for more intensive intervention or monitoring, even if the point prediction appears acceptable.

This quantification of uncertainty complements Tucker's identification of patterns and clusters, offering a more solid basis for strategic decision-making by considering not only what is expected to happen, but also the range of possibilities and the degree of confidence in those projections. Multidimensional Neutrosophic Regression, therefore, enriches traditional analysis by incorporating the complexity and ambiguous nature of financial data in the Popular and Solidarity Economy sector.

4. Conclusions

In this study, we identified relevant structural patterns in the evolution of the Popular and Solidarity Economy (PSE) financial sector in Ecuador during the period 2016–2023, applying the Tucker3 model as our primary multivariate analysis tool. This methodology allowed us to simultaneously integrate three key dimensions: cooperatives, financial variables, and years, revealing complex interactions that could not have been observed using traditional approaches.

Throughout the analysis, we found significant differences in the structure and financial behavior of cooperatives, particularly between those belonging to segment 1 versus smaller ones. We detected a higher concentration of assets, deposits, and operational efficiency in leading cooperatives, as well as greater resilience to adverse events, such as those that occurred during the health crisis of 2020 and 2021. Leading cooperatives such as JEP and JA, for example, showed high profitability (ROA of 1.5% and ROE of 15% on average) and low levels of delinquency (MORAMP < 20%), maintaining their liquidity (LIQCOR > 0.85) stable even during the most critical years of the pandemic. In contrast, cooperatives such as 29O and OSC showed high credit risk indicators (MORAMP > 35%) and limited access to funding (annual DEP growth < 5%).

The incorporation of Multidimensional Neutrosophic Regression complemented this analysis by quantifying the inherent uncertainty in the financial dynamics of cooperatives. This approach, by representing coefficients with simultaneous degrees of truth, falsity, and indeterminacy, enriched the interpretation of the results and provided greater sensitivity to the analysis of cooperative performance in complex and dynamic contexts. Leading cooperatives, such as JEP, were observed to have prediction intervals with a low degree of indeterminacy in key variables such as liquidity, denoting greater certainty and stability in their financial projections. In contrast, vulnerable cooperatives, such as 29O, showed significantly higher indeterminacy in their predictions, reflecting the greater variability and unpredictability of their performance.

Using the Tucker model3 also allowed us to identify clusters of cooperatives with similar profiles. By complementing this with the neutrosophic perspective, regulatory and sector support agencies can design differentiated institutional strengthening strategies and establish more realistic early warning mechanisms. These systems can go beyond specific indicators, monitoring the degree of uncertainty associated with predictions to anticipate financial problems more accurately.

In summary, the applied approach offered us a comprehensive view of the EPS financial system, highlighting its internal diversity and recent evolution. The combination of the Tucker model and Multidimensional Neutrosophic Regression provides an objective basis for sectoral policies that overcome the limitations of traditional average-based analyses, allowing for more robust and tailored interventions. We recommend that future research incorporate qualitative variables and make comparisons with other sectors of the social and solidarity economy, both in Ecuador and in other Latin American countries, to further enrich the understanding of the sector.

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(Annex 1)

Table 1: Cooperatives of Segment 1 of the Popular and Solidarity Financial System

Cooperative	**Acronym**
Cooperative of Savings and Credit Ecuadorian Youth Progresista Limited	**JEP**
Cooperative of Savings and Credit Garden Azuayo Limited	**HA**
Cooperative of Savings and Credit National Police Limited	**PN**
Cooperative of Savings and Credit 29 de Octubre Limited	**29O**
Cooperative of Savings and Credit Cooprogreso Limited	**CP**
Cooperative of Savings and Credit Oscus Limited	**CSOs**
Cooperative of Savings and Credit San Francisco Limited	**SF**
Cooperative of Savings and Credit Vicentina Manuel Esteban Godoy Ortega Limited	**VMEGO**
Cooperative of Savings and Credit Riobamba Limited	**RIVER**
Cooperative of Savings and Credit Alianza del Valle Limited	**ADV**
Cooperative of Savings and Credit of the Small Enterprise of Cotopaxi Limited	**PEC**
Cooperative of Savings and Credit Mushuc Runa Limited	**MR**
Cooperative of Savings and Credit Andalucía Limited	**AND**
Cooperative of Savings and Credit of the Small Enterprise Biblián Limited	**PEB**
Cooperative of Savings and Credit Atuntaqui Limited	**ATQ**
Cooperative of Savings and Credit El Sagrario Limited	**IS**
Cooperative of Savings and Credit Chamber of Commerce of Ambato Limited	**CCA**
Cooperative of Savings and Credit 23 de Julio Limited	**23J**
Cooperative of Savings and Credit San José Limited	**SJ**
Cooperative of Savings and Credit Pablo Muñoz Vega Limited	**PMV**
Cooperative of Savings and Credit Tulcán Limited	**TULLE**
Cooperative of Savings and Credit of the Public Servants of the Ministry of Education and Culture	**SPMEC**
Cooperative of Savings and Credit Pilahuín Uncle Limited	**PT**
Cooperative of Savings and Credit Santa Rosa Limited	**MR**
Cooperative of Savings and Credit of the Small Enterprise of Pastaza Limited	**PEP**

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A Model of Organizational Dynamics of Phonological Knowledge in Preschool Children Based on Bratianu's Theory and the Neutrosophic AHP Technique

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Abstract. Phonological knowledge is a component of linguistics that allows children to understand the relationship between sounds and words. It is a skill that should be stimulated and enhanced from the age of 3, as children begin the process of reading and writing at 6. If this component is not yet acquired, it can generate difficulties in the school stage, where learning demands are more challenging and higher cognitive processes are required in terms of decoding phonemes, graphemes, words, and sentences. In this paper, we propose a model that allows us to study the dynamics of phonological knowledge in preschool children. To do so, we base ourselves on the theory proposed by Professor Constantin Bratianu, where the dynamics of organizational knowledge are modeled with equations equivalent to the thermodynamic energy flow. He uses the well-known multicriteria decision-making method Analytic Hierarchy Process (AHP) to numerically obtain the coefficients of its equations. In this paper, we propose extending this model by replacing the classic AHP with the Neutrosophic AHP. This preserves the linguistic scale proposed in the original model, and additionally includes the uncertainty and indeterminacy that are part of decision-making, increasing accuracy and explicitly modeled with the help of Neutrosophic AHP.

Keywords: Phonological awareness, Literacy, Preschool children, Organizational Knowledge Dynamics (OKD), Knowledge Dynamic, Analytic Hierarchy Process (AHP), Neutrosophic Analytic Hierarchy Process (NAHP).

1. Introduction

Phonological awareness is the ability preschoolers acquire to analyze and reflect on the different sounds of their native language. This process involves deliberately manipulating the sounds of language to differentiate between words, syllables, and phonemes.

Children at this age need constant language stimulation so they can naturally develop the levels of phonological awareness they need to begin the process of reading and writing. They must acquire phonological awareness skills at this stage, as this will allow them to recognize syllables, identify sounds, count words, manipulate phonemes, blend sounds, and recognize rhymes.

Given the importance of phonological awareness in the reading and writing process and language development in general, it is essential to address it from three perspectives.

- Parental: With the direct participation of parents during interactive communication processes within the home, where the child feels involved, communicates from his or her per-

spective, and interacts efficiently and effectively with the family. In this regard, parental responsibility is fundamental.

- Early childhood education: Where children interact with others their age and also receive ongoing language stimulation from professionals specialized in this area, which strengthens their overall development and language in particular. At this stage, early childhood teachers pave the way for later literacy learning.
- Therapeutic: Some children require specialized therapeutic help, involving professionals in the field of Speech Therapy or Language Therapy, who assess phonological knowledge by applying specific tests that determine the level of this linguistic component in children.

In a study conducted by some authors on phonological awareness and reading learning, they mention that the development of phonological awareness has a very high correlation with learning to read and write. According to these authors, a deficiency in this component predicts future difficulties in school learning during the early years. Hence, this is the importance of children developing phonological skills before the age of 6.

Furthermore, they state that the difficulties observed in speech are replicated in reading and writing; that is, if the child does not speak well, he or she will not write well either. Lack of phonological awareness is a factor that prevents children from correctly understanding sounds and smaller phonological units such as phonemes, syllables, words, and rhymes.

For many children, mastery of these phonetic skills determines success or failure in learning to read. However, it is important to note that reading and writing difficulties are not necessarily entirely due to a lack of phonological awareness. Other factors could also influence this process; for example, Attention Deficit Disorder (ADD), Attention Deficit Hyperactivity Disorder (ADHD), and Dyslexia, etc. These and other learning disabilities (LD) could also significantly impact the reading and writing process.

Children with this condition not only present difficulties in phonology, but also in other aspects: semantic, morpho-syntactical, and pragmatic. Their poor attention span impacts their learning to read and write. For this reason, these children in particular are at a serious disadvantage compared to their peers. For them, recognizing the relationship between sounds, phonemes, graphemes, rhymes, and syllables represents a greater effort that can often be frustrating if these previous diagnoses are unknown.

Some researchers mention that a well-developed phonological awareness (PA), as a metalinguistic skill, allows children to reflect on and intentionally manipulate language units, which will then translate into improved cognitive processes for learning to read and write. The same authors state that this skill is a continuous process that encompasses several stages: from rhyme recognition, through syllable segmentation and intrasyllabic units, to phonemes.

Therefore, they argue that the preschool stage is fundamental for stimulating the phonetic-phonological component. Children who are constantly interacting with their family environment and peers are more likely to acquire good levels of phonological awareness, and language awareness in particular. In principle, it is the responsibility of parents to provide their children with appropriate environments where they can develop all the communicative skills they require according to their age.

It has been proven that phonological skills training significantly improves performance on phonological awareness tasks, and reading and writing acquisition. Therefore, timely speech-language therapy or intervention is vitally important for children and their parents.

The goal should always be to prevent any difficulties that may arise during the school years related to learning to read and write. When a speech-language pathologist observes a deficit in language development, they immediately suggest early therapeutic intervention to help children and their families reduce the problems that could arise as a result of a language disorder.

In this paper, we propose to apply a method to assess the language knowledge dynamics within a generic group of children. Specifically, we are inspired by Professor Constantin Bratianu's theory, which uses the flow of energy within the framework of thermodynamics as a metaphor to represent the dynamic flow of knowledge within an organization [1].

Professor Bratianu argues that metaphor is a way of linking a more familiar event to represent a lesser-known one. In the case of thermodynamics, its logic can serve as a basis for representing the flow of knowledge, where knowledge dynamics is a very complex and multidimensional process [2]. In the case of organizational knowledge dynamics (OKD), we have different components which are: knowledge creation, knowledge acquisition, knowledge loss, knowledge sharing, knowledge storage

and retrieval, knowledge diffusion, conversion from one form of knowledge into another form of knowledge, and organizational learning [3].

The model we offer in this article is based on the well-known Analytic Hierarchy Process (AHP) as Bratianu does to find the coefficients of the equations that model the dynamics of knowledge [3, 4]. AHP is a very popular method designed by T. L. Saaty for decision-making in complex contexts, where mathematical and psychological results are used [5, 6]. Specifically, the importance of each evaluation criterion relative to the others, each sub-criterion of each criterion relative to the others, and so on, is compared on a scale of 1 to 9. Finally, the alternatives are represented in a tree of different levels. The results obtained allow the assignment of a coefficient that represents the importance or weight of each criterion, sub-criterion, and alternative. So, Bratianu uses these weights to complete his equations for the dynamics of knowledge flow in the organization.

In this paper, we extend this method, where the classical AHP method is replaced by a Neutrosophic AHP technique in Bratianu's model [7]. The Neutrosophic AHP is used to replace the numbers between 1 and 9 of the original method by Single-Valued Trapezoidal Neutrosophic Numbers that incorporate into the model the uncertainty and indeterminacy that are part of the Neutrosophy theory [8, 9]. Decision-makers also maintain the ease with which they can use a linguistic scale that represents what they wish to express.

The article is divided into a Preliminaries section that reviews the fundamental theoretical elements, such as the basic notions of Bratianu's theory and the Neutrosophic AHP. This section is followed by another that explains the components of the model and illustrates it with an example. The paper finishes with conclusions.

2. Preliminaries

In this section, we review the basic notions about Neutrosophic AHP and then Bratianu's model.

2.1 On Neutrosophic AHP

Neutrosophic AHP technique, like crisp AHP and fuzzy AHP variants, combines fundamentals of psychology and mathematics [10, 11, 12]. It is based on a hierarchical tree-like structure, where the main node represents the goal to be achieved. From there, the different descending levels contain the criteria and subcriteria necessary to evaluate a set of alternatives located at the base of the tree. Thomas L. Saaty originally developed this methodology in the 1970s. Elements located at the same hierarchical level are compared with each other in pairs, using a rating scale defined by Saaty himself.

In the Neutrosophic AHP approach, traditional numerical values are replaced by neutrosophic trapezoidal numbers to incorporate the uncertainty and indeterminacy inherent in the decision-making process. The key concepts of this technique are presented below.

Definition 1: ([13]) The *Neutrosophic set* N is composed by three membership functions, viz., the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, such that U is the Universe of Discourse and for all $x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^{-}0, 1^{+}[$, and $^{-}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.

Let us note that, $T_A(x), I_A(x), F_A(x)$ are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ so, $T_A(x), I_A(x), F_A(x)$ may be subintervals of $[0, 1]$.

Definition 2: ([14]) The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$, and $F_A: U \to [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The Single-Valued Neutrosophic Number (SVNN) is denoted by $N = (t, i, f)$, where $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3: ([15]) $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is called a *single-valued trapezoidal neutrosophic number* or a *neutrosophic set on* $\mathbb{R}$, where truthfulness, indeterminacy, and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & a_2 \le x \le a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

We have $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3, a_4 \in \mathbb{R}$, and $a_1 \le a_2 \le a_3 \le a_4$.

Definition 4: ([15]) Given two single-valued trapezoidal neutrosophic numbers $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$, and also $\lambda \neq 0$ is a real number, we have the following definitions of operators:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
3. Inversion: $\tilde{a}^{-1} = \langle ({a_4}^{-1}, {a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3, a_4 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$

5. Division of two trapezoidal neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 > 0 \text{ and } b_4 > 0 \\ \langle \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 < 0 \text{ and } b_4 > 0 \\ \langle \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 < 0 \text{ and } b_4 < 0 \end{cases}$$

6. Multiplication of two trapezoidal neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 > 0 \text{ and } b_4 > 0 \\ \langle (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 < 0 \text{ and } b_4 > 0 \\ \langle (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 < 0 \text{ and } b_4 < 0 \end{cases}$$

Such that $\wedge$ is a t-norm and $\vee$ is a t-conorm.

The integration of AHP with neutrosophic set theory provides a more adaptable framework for handling uncertainty in decision-making processes. Indeterminacy is recognized as a critical factor to be considered in real-world organizational decisions.

Table 1 outlines the scale used for conducting pairwise comparisons of criteria, sub-criteria, and alternatives. It presents Saaty's original scale translated into a neutrosophic trapezoidal format.

Table 1. Saaty's original scale adapted into a neutrosophic trapezoidal representation [7, 8].

Saaty scale	Definition	Neutrosophic Trapezoidal Scale
1	Equally influential	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
7	Very strongly influential	$\tilde{7} = \langle(6,7,8,9); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9,9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$ $\tilde{4} = \langle(3,4,5,6); 0.60, 0.35, 0.40\rangle$ $\tilde{6} = \langle(5,6,7,8); 0.70, 0.25, 0.30\rangle$ $\tilde{8} = \langle(7,8,9,9); 0.85, 0.10, 0.15\rangle$

Equation 4 defines the pairwise comparison matrix within the neutrosophic framework.

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (4)$$

$\tilde{A}$ satisfies the condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, according to the inversion operator defined in Definition 4.

Two indices were introduced to transform a neutrosophic trapezoidal number into a crisp numerical value [16]. These indices—score and accuracy—are mathematically defined in Equations 5 and 6, respectively.

$$\mathcal{S}(\tilde{a}) = \frac{1}{12}[a_1 + a_2 + a_3 + a_4](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (5)$$

$$H(\tilde{a}) = \frac{1}{12}[a_1 + a_2 + a_3 + a_4](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (6)$$

In the proposed method, it is only necessary to determine the weights or relative importance of each criterion. Nevertheless, in the AHP technique, there is the consistency index (C.I.) of the matrix A, which is calculated by the maximum eigenvalue, and the Consistency Ratio (C.R.) such that if it is less than or equal to 0.1 it is considered that the matrix consistency is acceptable.

2.2 Bratianu's model of knowledge dynamics

Professor Constantin Bratianu's theory of knowledge dynamics offers a lens on how knowledge behaves and transforms—especially within organizations. Rather than relying on traditional Newtonian metaphors like "stocks and flows," Bratianu introduces a thermodynamics-inspired framework that treats knowledge as a field, much like energy.

The core concepts of Bratianu's theory are the following:

1. Knowledge as Energy: Bratianu uses the energy metaphor to describe knowledge. Just as energy transforms and flows, so also does knowledge—across rational, emotional, and spiritual dimensions.
2. Multifield Knowledge Spectrum: He identifies three fundamental "fields" of knowledge:
    - Rational: Logical, analytical knowledge,
    - Emotional: Affective, experience-based knowledge,
    - Spiritual: Values-driven, purpose-oriented knowledge.
3. Irreversible Transformation: Drawing from thermodynamics, Bratianu argues that knowledge transformation is irreversible —once knowledge shifts from one form to another (say, from emotional to rational), it cannot fully revert.
4. Organizational Application: This theory helps explain how organizations convert potential

knowledge into operational knowledge through what he calls "organizational integrators"—people or processes that catalyze transformation.

Bratianu's approach is especially useful in:

1. Knowledge management: Offering a richer, more dynamic understanding of how knowledge evolves,
2. Leadership and strategy: Helping leaders recognize the emotional and spiritual dimensions of decision-making,
3. Innovation: Encouraging organizations to embrace non-linear, creative knowledge flows.

Professor Constantin Bratianu's work on Organizational Knowledge Dynamics using the Analytic Hierarchy Process (AHP) is a fusion of systems thinking and decision science to better understand how knowledge evolves within organizations.

Bratianu models organizational knowledge dynamics using a dynamic equilibrium equation over a time interval (ΔT), which includes:

- ΔK: Total variation in organizational knowledge,
- ΔC: Variation in knowledge creation,
- ΔA: Variation in knowledge acquisition,
- ΔL: Variation in knowledge loss.

Equation 7 is as follows:

$$\Delta K = F_C(\Delta C) + F_A(\Delta A) - F_L(\Delta L) \qquad (7)$$

Each of these components contributes differently to the overall knowledge balance, so Bratianu uses AHP —a structured decision-making method developed by Thomas Saaty —to assign weighting factors to them, which are F_C, F_A, and F_L.

So, the AHP model is structured into three levels:

1. Goal Level: Increase the level of organizational knowledge.
2. Strategy Level:
   - S_1: Increase knowledge creation,
   - S_2: Increase knowledge acquisition,
   - S_3: Reduces knowledge loss.
3. Activity Level:
   - A_1: Hire valuable human resources,
   - A_2: Develop training programs,
   - A_3: Motivate employees effectively,
   - A_4: Purchase knowledge resources (books, software, etc.).

   See Figure 1.

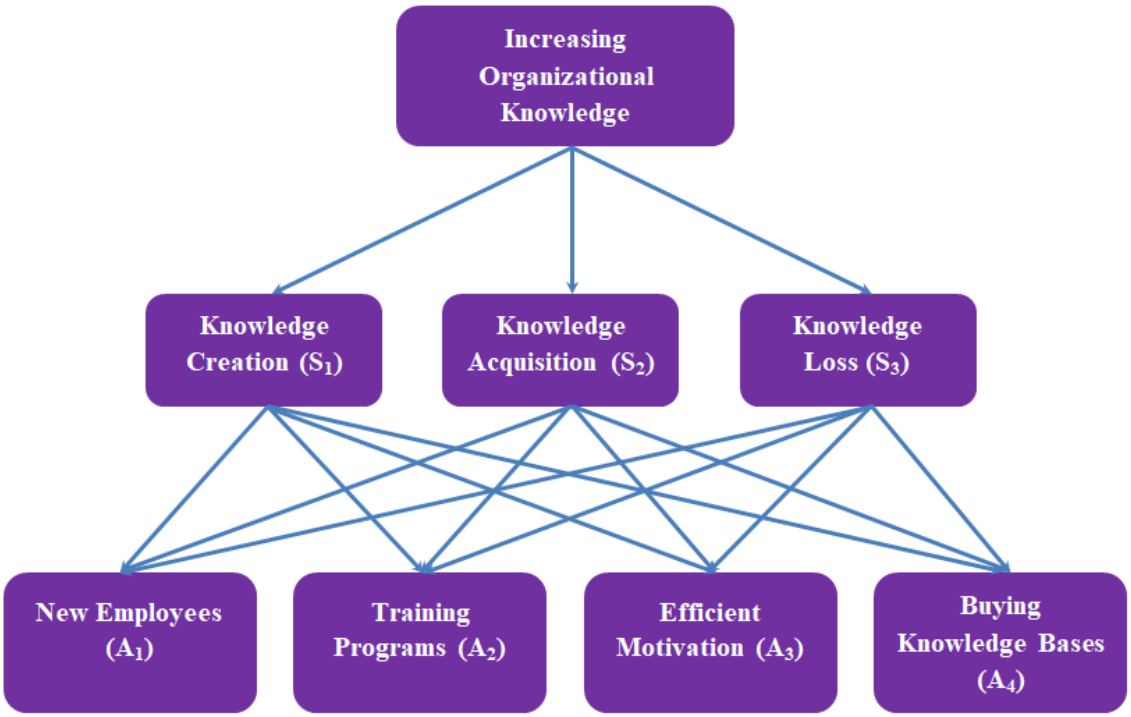

Figure 1. AHP hierarchical tree about Bratianu's model. Source [3].

Employees are asked to make pairwise comparisons between strategies and activities. These comparisons generate priority vectors (via eigenvalue calculations), which are then used to determine the relative importance of each component in the knowledge dynamics equation.

Additionally, the dynamic equilibrium equation for a generic component is expressed by Equation 8, where S symbolizes the strategy.

$$\Delta S = w_1(\Delta A_1) + w_2(\Delta A_2) + w_3(\Delta A_3) + w_4(\Delta A_4) \quad (8)$$

ΔS represents the variation of the generic knowledge-related component—namely, knowledge creation, acquisition, and loss—during the time interval ΔT. ΔA_i denotes the change in activity i over the same time interval ΔT. W_i corresponds to the weighting factor associated with activity i.

This model helps organizations:

- Quantify abstract knowledge processes,
- Prioritize strategic initiatives,
- Align knowledge management with broader business goals.

3. Design of the proposed model

In this section, we present our proposed design of a model to measure phonological knowledge in preschool children.

Two variables were considered in the analysis: chronological age and phonological knowledge.

To assess the percentage of the children's phonological knowledge viz., phonological, syllabic, and phonetic, the Phonological Awareness Assessment Test (PAAT) was administered. This test assesses phonological awareness at the two aforementioned levels, considering the initial, medial, and final positions of syllables and sounds (phonemes) in each word. The original test is adapted to the neutrosophic framework with the idea of incorporating indeterminacy and uncertainty into the assessments.

The test consists of six sections that are divided into three activities for both the syllabic and phonemic levels.

1. Identification: Ability to recognize syllables and phonemes in a word through the presentation of five images,

2. Addition: Ability to add a syllable or phoneme to a word using representative tokens,
3. Omission: Ability to remove a syllable or phoneme from a word and form a new one using five images that must be named.

Below is a table that summarizes the two levels assessed: syllabic and phonemic in the three activities carried out.

Table 2. Levels of phonological knowledge

Level	Identification	Addition	Omission
Syllabic	1. Wolf	11. Elbow	21. House
	2. Die	12. Salty	22. Shirt
	3. Meat	13. Sunday	23. Sack
	4. Sheep	14. Folded	24. Toad
	5. Bottle	15. Silky	25. Worm
Phonemic	6. Nail	16. Sun	26. Seal
	7. Pencil	17. Goal	27. Skirt
	8. Cup	18. Magnifying	28. Sofa
	9. Cheese	glass	29. Glasses
	10. Ship	19. Bookseller	30. Flan
		20. Peeled	
Total	I: 10	A: 10	O: 10

For response analysis, the results were assessed by adding the scores obtained at both the syllabic and phonemic levels for the three activities proposed in the test: identification, addition, and omission; assigning a point $\langle(0.5, 1,1,1.5); 1, 0, 0\rangle$ for each correct answer and a point $\langle(-0.5, 0,0,0.5); 0, 0, 1\rangle$ for each error. The maximum score is $\langle(15, 30,30,45); 1, 0, 0\rangle$ points. If the child does not understand the activity, it must be explained up to three times. After that, if the child persists in making a mistake, the error is rescinded $\langle(-0.5, 0,0,0.5); 0, 0, 1\rangle$ and the child continues with the next activity.

Thus, if there are *m* children, the evaluation of the ith child is calculated by the formula:

$$\tilde{a}_i = \oplus_{j=1}^{30} a_{ij} \qquad (9)$$

Where a_{ij} is the evaluation of the ith child relative to the jth test, while $\oplus$ is the addition operator from Definition 4. This value becomes a single crisp value with the help of formula 5 of the score function:

$$b_i = \mathcal{S}(\tilde{a}_i) \qquad (10)$$

The total knowledge of the group of children, assuming that *n* children are studied, is calculated by the arithmetic mean, that is:

$$K = \frac{\sum_{i=1}^{m} b_i}{m} \qquad (11)$$

For threshold tone hearing assessment, each child must undergo threshold tone audiometry using an audiometer. This is a subjective hearing test in which children must voluntarily respond to the stimulus emitted. Two pathways were evaluated: an air pathway with supra-aural headphones, and a bone pathway with a bone vibrator using pure tones and a voice test (recorded voice).

Pure tones are used; in the air conduction, in a range of 125 Hz and 8000 kHz or 12 kHz in frequency and intensity from -10dB to 120dB, and in the bone conduction, in a range of 250 Hz and 4 kHz in intensity.

To this end, we adapted Bratianu's model to phonological knowledge in preschool children. To do so, we maintained the same goal and strategies for improving knowledge, but changed the alternatives, which are shown in Table 2.

Table 3. Alternatives to improve collective phonological knowledge in preschool children

Notation	Alternative	Brief description
A_1	Guided phonological songs	Structured musical activities that emphasize rhyme, alliteration, and rhythm.
A_2	Sound segmentation and matching games	Dynamics where children divide and combine phonemes into simple words.
A_3	Dialogic reading with phonological emphasis	Reading aloud where the adult guides and reinforces the identification of sounds.
A_4	Puppets with articulatory exaggeration	Theatrical activities where puppets emphasize key sounds with movements.

The AHP hierarchical tree looks like Figure 2.

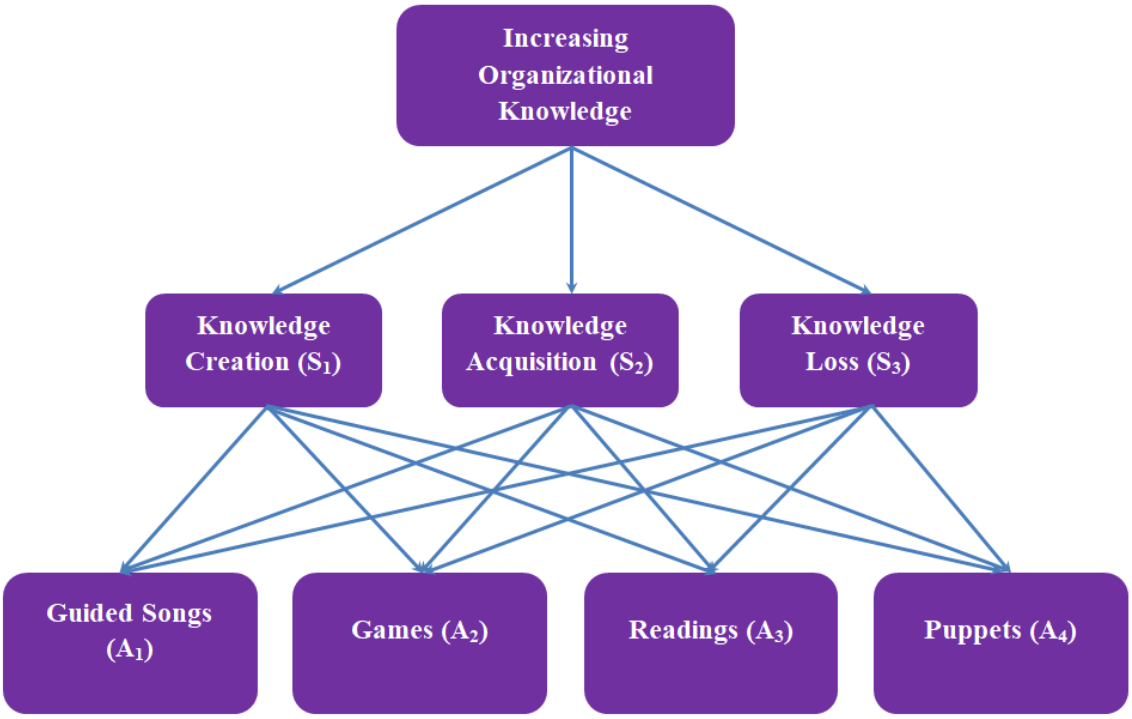

Figure 2. AHP hierarchical tree about the proposed model.

Suppose there are n experts who evaluate how to improve the phonological knowledge of approximately 5-year-old children concerning strategies S_1, S_2, and S_3 and the new alternatives A_1, A_2, A_3, and A_4, which appear in Table 1.

Each of them is asked the following questionnaire:

1. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either "Increase knowledge creation" (S1) or "Increase knowledge acquisition" (S2) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
2. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either "Increase knowledge creation" (S1) or "Reduce knowledge loss" (S3) (They can be equally preferred).

b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

3. a. If you wanted to improve the performance of a group of preschool children in their phonological skills, say which strategy do you prefer either "Increase knowledge acquisition" (S2) or "Reduce knowledge loss" (S3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

The second part of the questionnaire contains comparisons between alternatives for each of the strategies.

4. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Sound segmentation and joining games" (A_2) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
5. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Dialogic reading with phonological emphasis" (A_3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
6. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Guided phonological songs" (A_1) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
7. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Segmentation and sound joining games" (A_2) or "Dialogic reading with phonological emphasis" (A_3) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
8. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Games of segmentation and union of sounds" (A_2) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.
9. a. If you wanted to follow the strategy "Increase knowledge creation" (S_1) say which alternative do you prefer either "Dialogic reading with phonological emphasis" (A_3) or "Puppets with articulatory exaggeration" (A_4) (They can be equally preferred).
   b. On a scale of 1 to 9, please indicate how much your preferred strategy is better than or equal to the other.

Finally, questions 4-9 are repeated by setting "Increase knowledge acquisition" (S_2) instead of S_1, and then replacing "Reduce knowledge loss" (S_3) instead of S_1. Therefore, there will be 12 more questions in the questionnaire.

The values of the numbers from 1 to 9 are replaced by the Neutrosophic Trapezoidal Scale, according to Table 1.

The following procedure is followed using the procedure of the Aggregation of Individual Judgments (AIJ) ([17, 18]):

1. The values of each of the expert's responses are added with the help of Equation 12:

$$\tilde{X}_{l_1} = \oplus_{k_1}^{n_1} \tilde{x}_{k_1 l_1}$$
$$\tilde{X}_{l_2} = \oplus_{k_2}^{n_2} \tilde{x}_{k_2 l_2} \qquad (12)$$

Where $\tilde{x}_{k_1 l_1}$ is the Single-Valued Trapezoidal Neutrosophic Number corresponding to the answers related to the first option of the two possible for each question. $\tilde{x}_{k_2 l_2}$ is the same but corresponding to the second option.

2. $\widetilde{X}_{l_1}$ is replaced by $\widetilde{Y}_{l_1} = \mathcal{S}(\widetilde{X}_{l_1})$, to convert it into crisp values. Similarly, we have $\widetilde{Y}_{l_2} = \mathcal{S}(\widetilde{X}_{l_2})$.

3. Preference matrices are formed.
   With the results of questions 1 to 3, the preference matrix with respect to the strategies is formed.
   With the results of questions 4-9, comparisons between alternatives are formed, establishing strategy S_1.
   With the results of questions 10-15, comparisons between alternatives are formed, establishing strategy S_2.
   With the results of questions 16-21, comparisons between alternatives are formed, establishing strategy S_3.

This is done using the following method:

If for the aggregate values of one of the matrices that is $\widetilde{Y}_{l_1} > \widetilde{Y}_{l_2}$, then the comparison matrix is formed as $z_{pq} = \widetilde{Y}_{l_1}$ and $z_{qp} = \frac{1}{z_{pq}}$, where $p < q$.

If $\tilde{Y}_{l_2} > \tilde{Y}_{l_1}$, then the comparison matrix is formed as $z_{qp} = \tilde{Y}_{l_2}$ and $z_{pq} = \frac{1}{z_{qp}}$.

If $\tilde{Y}_{l_2} = \tilde{Y}_{l_1}$, then the comparison matrix is formed as $z_{pq} = z_{qp} = 1$.

Note that the elements of the main diagonal are always equal to 1 and in the case that no expert has chosen one of the options, it is evaluated as $\tilde{0} = \langle(0,0,0,0); 0,1,1\rangle$.

4. The classical AHP method is applied to these results.
5. The priority vectors are used as coefficients of Equations 7 and 8.

To show the effectiveness of the proposed method we illustrate with an example:

Example 1: Let us suppose there is a group of 5-year-old Ecuadorian children and we want to determine the dynamics of their phonological knowledge, where total Knowledge is taken into account. Knowledge Creation is the knowledge that arises within the group spontaneously, mainly as a result of interaction between the teacher and the children and the children among themselves; Knowledge Acquisition is the knowledge acquired internally by the child's teacher or externally outside the group, through support from each child's family, an outside teacher, among others; and finally, the Loss of Knowledge, as a result of a child no longer in the group, a teacher who for some reason left, among others.

Given, there are three experts denoted by e_1, e_2, e_3, each one whom answers the questionnaire described above and the results are the following ones summarized in Tables 4-6:

Table 4. Survey results for expert 1.

Question	Favorite	Assessment
1	S_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
2	S_1	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
3	S_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
4	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$

Question	Favorite	Assessment
5	A_3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
6	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
7	A_3	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
8	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
9	A_4	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
10	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
11	A_3	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
12	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
13	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
14	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
15	A_3	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
16	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
17	A_1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
18	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
19	A_2	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
20	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
21	A_3	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$

Table 5. Survey results for expert 2.

Question	Favorite	Assessment
1	S_2	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
2	S_3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
3	S_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
4	A_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
5	A_3	$\tilde{4} = \langle(3,4,5,6); 0.60, 0.35, 0.40\rangle$
6	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
7	A_3	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
8	A_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
9	A_3	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
10	A_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
11	A_1	$\tilde{4} = \langle(3,4,5,6); 0.60, 0.35, 0.40\rangle$
12	A_1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
13	A_2	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$
14	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
15	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
16	A_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
17	A_1	$\tilde{4} = \langle(3,4,5,6); 0.60, 0.35, 0.40\rangle$
18	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
19	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
20	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
21	A_4	$\tilde{5} = \langle(4,5,6,7); 0.80, 0.15, 0.20\rangle$

Table 6. Survey results for expert 3.

Question	Favorite	Assessment
1	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
2	S_3	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
3	S_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
4	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
5	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
6	A_1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
7	A_2	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
8	A_4	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
9	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
10	A_1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$

Question	Favorite	Assessment
11	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
12	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
13	A_3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
14	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
15	A_3	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
16	Both	$\tilde{1} = \langle(1,1,1,1); 0.50, 0.50, 0.50\rangle$
17	A_1	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
18	A_1	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
19	A_3	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$
20	A_2	$\tilde{3} = \langle(2,3,4,5); 0.30, 0.75, 0.70\rangle$
21	A_4	$\tilde{2} = \langle(1,2,3,4); 0.40, 0.65, 0.60\rangle$

The aggregated results for all experts have the following results as shown in Table 7.

Table 7. Results of the survey aggregation for all experts for each of the preferences.

Question	Aggregated results when the first option is preferred	Aggregated results when the second option is preferred
1	$\langle(3.0, 4.0, 5.0, 6.0); 0.3, 0.75, 0.75\rangle$	$\langle(5.0, 6.0, 7.0, 8.0); 0.5, 0.5, 0.5\rangle$
2	$\langle(4, 5, 6,7); 0.80, 0.15, 0.20\rangle$	$\langle(3.0, 5.0, 7.0, 9.0); 0.3, 0.75, 0.75\rangle$
3	$\langle(4.0, 7.0, 10.0, 13.0); 0.3, 0.75, 0.75\rangle$	$\langle(0,0,0,0); 0,1,1\rangle$
4	$\langle(3.0, 4.0, 5.0, 6.0); 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0); 0.4, 0.65, 0.65\rangle$
5	$\langle(2, 3, 4,5); 0.30, 0.75, 0.70\rangle$	$\langle(4.0, 6.0, 8.0, 10.0); 0.4, 0.65, 0.65\rangle$
6	$\langle(4.0, 6.0, 8.0, 10.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
7	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(6.0, 8.0, 10.0, 12.0); 0.3, 0.75, 0.75\rangle$
8	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(3.0, 5.0, 7.0, 9.0); 0.3, 0.75, 0.75\rangle$
9	$\langle(4, 5, 6,7); 0.80, 0.15, 0.20\rangle$	$\langle(3.0, 5.0, 7.0, 9.0); 0.3, 0.75, 0.75\rangle$
10	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$	$\langle(3.0, 5.0, 7.0, 9.0); 0.3, 0.75, 0.75\rangle$
11	$\langle(4.0, 5.0, 6.0, 7.0); 0.5, 0.5, 0.5\rangle$	$\langle(5.0, 6.0, 7.0, 8.0); 0.5, 0.5, 0.5\rangle$
12	$\langle(4.0, 6.0, 8.0, 10.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
13	$\langle(6.0, 8.0, 10.0, 12.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
14	$\langle(4.0, 6.0, 8.0, 10.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
15	$\langle(6.0, 8.0, 10.0, 12.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
16	$\langle(3.0, 4.0, 5.0, 6.0); 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0); 0.4, 0.65, 0.65\rangle$
17	$\langle(6.0, 9.0, 12.0, 15.0); 0.3, 0.75, 0.75\rangle$	$\langle(0,0,0,0); 0,1,1\rangle$
18	$\langle(4.0, 6.0, 8.0, 10.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 1,1,1); 0.50, 0.50, 0.50\rangle$
19	$\langle(6.0, 8.0, 10.0, 12.0); 0.3, 0.75, 0.75\rangle$	$\langle(1, 2, 3,4); 0.40, 0.65, 0.60\rangle$
20	$\langle(3.0, 4.0, 5.0, 6.0); 0.3, 0.75, 0.75\rangle$	$\langle(2.0, 3.0, 4.0, 5.0); 0.4, 0.65, 0.65\rangle$
21	$\langle(2, 3, 4,5); 0.30, 0.75, 0.70\rangle$	$\langle(5.0, 7.0, 9.0, 11.0); 0.4, 0.65, 0.65\rangle$

In Table 8 we convert the results in Table 7 to crisp values.

Table 8. Conversion of the results from Table 7 into crisp values. Note that the option with the highest value appears in red letters.

Question	Results when the first option is preferred	Results when the second option is preferred
1	1.1999999999999997	3.25
2	4.491666666666666	1.5999999999999996
3	2.2666666666666657	0.0
4	1.1999999999999997	1.28333333333333332
5	0.9916666666666664	2.5666666666666664
6	1.866666666666666	0.5
7	0.95833333333333331	2.3999999999999995
8	0.95833333333333331	1.5999999999999996

Question	Results when the first option is preferred	Results when the second option is preferred
9	4.491666666666666	1.5999999999999996
10	0.95833333333333331	1.5999999999999996
11	2.75	3.25
12	1.866666666666666	0.5
13	2.3999999999999995	0.95833333333333331
14	1.866666666666666	0.95833333333333331
15	2.3999999999999995	0.95833333333333331
16	1.1999999999999997	1.28333333333333332
17	2.7999999999999994	0.0
18	1.866666666666666	0.5
19	2.3999999999999995	0.95833333333333331
20	1.1999999999999997	1.28333333333333332
21	0.9916666666666664	2.93333333333333336

The preference matrices for each of the AHP levels are as follows, see Tables 9-12:

Table 9. Pair-wise matrix preferences between strategies.

Strategy	S_1	S_2	S_3
S_1	1	1/3.25	4.492
S_2	3.25	1	2.27
S_3	1/4.492	1/2.27	1

Table 10. Pair-wise matrix preferences of the alternatives given strategy S_1.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.283	1/2.57	1.87
A_2	1.283	1	1/2.4	1/1.6
A_3	2.57	2.4	1	4.492
A_4	1/1.87	1.6	1/4.492	1

Table 11. Pair-wise matrix preferences of the alternatives given strategy S_2.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.6	1/3.25	1.9
A_2	1.6	1	2.4	1.9
A_3	3.25	1/2.4	1	2.4
A_4	1/1.9	1/1.9	1/2.4	1

Table 12. Pair-wise matrix preferences of the alternatives given strategy S_3.

Alternative	A_1	A_2	A_3	A_4
A_1	1	1/1.283	2.8	1.9
A_2	1.283	1	2.4	1/1.28
A_3	1/2.8	1/2.4	1	1/2.93
A_4	1/1.9	1.28	2.93	1

The results of applying the AHP method and the priority vectors for the matrices shown in Tables 9-12 are as follows:

Table 13 contains the vector of priorities between the strategies.

Table 13. Vector of priorities for the pairwise comparison matrix S.

Strategies	Priorities
S_1	0.32610087521915954
S_2	0.5303680832323449
S_3	0.14353104154849564

In this matrix, C.R. = 0.3516554630297951 was obtained, which indicates insufficient consistency.

Table 14 contains the priority vectors of the alternatives concerning each of the strategies.

Table 14. Vector of priorities for the pairwise comparison matrices of alternatives for every strategy S_1, S_2, and S_3, respectively.

Alternative	Priority Vector corresponding to S_1	Priority Vector corresponding to S_2	Priority Vector corresponding to S_3
A_1	0.18660691978792374	0.18467616437284845	0.32976329453713604
A_2	0.17370741401018375	0.3715342150844241	0.28748786884937794
A_3	0.48692234557507535	0.3119371145799139	0.10675771232862649
A_4	0.15276332062681713	0.13185250596281362	0.27599112428485956

Here we obtain C.R. = 0.07408881902313505, which is acceptable consistency for the alternatives concerning S_1, C.R. = 0.1165789677683065 which is insufficient consistency for the alternatives concerning S_2, and C.R. = 0.045569823859665876 which is acceptable consistency concerning S_3.

We should note that some results are not consistent, but this is not taken into account within the Bratianu method, however, if the decision-makers wish, they can repeat the evaluations in those comparisons with inconsistencies.

In this way, Equation 7 would be as follows:

$$\Delta K = 0.33\Delta C + 0.53\Delta A - 0.14\Delta L$$

So, the equations derived from Equation 8 are the following:

$$\Delta S_1 = \Delta C = 0.19\Delta A_1 + 0.17\Delta A_2 + 0.49\Delta A_3 + 0.15\Delta A_4,$$

$$\Delta S_2 = \Delta A = 0.19\Delta A_1 + 0.37\Delta A_2 + 0.31\Delta A_3 + 0.13\Delta A_4,$$

$$\Delta S_3 = \Delta L = 0.33\Delta A_1 + 0.29\Delta A_2 + 0.11\Delta A_3 + 0.27\Delta A_4.$$

According to the equations above to increase knowledge acquisition is the most influential aspect of knowledge dynamics with over 53% influence, followed by increasing knowledge creation with 33% influence.

On the other hand, the dynamic changes within the "increase knowledge creation" highlight the influence that the alternative of dialogic reading with phonological emphasis can have. Regarding the strategy of increasing knowledge acquisition, the best way to achieve it is by applying the alternative"Sound segmentation and joining games", with 37% followed by "dialogic reading with phonological emphasis" with 31%. To avoid the loss of knowledge, the most influential is to apply the alternative "Guided phonological songs" with 33% of importance, followed by "Sound segmentation and joining games" with 29% of weight and "Puppets with articulatory exaggeration" with 27%; however,

in this last strategy there is no alternative that prevails as much over the others, as it happens with the two previous strategies.

The last four equations can be used to study the dynamics of knowledge, based on measurements that can be made experimentally over time, using equations 9-11.

4. Conclusion

This paper proposes a model for representing phonological knowledge in preschool children. It is important at this stage for children to articulate and express themselves correctly in their native language, and to be able to understand the people around them, of all ages. This knowledge is critical for good cognitive development, social interaction, and psychological development. In particular, we draw on the model of Professor C. Bratianu, who studied and represented the dynamics of knowledge inspired by the energy flow that occurs in thermodynamics. In the proposed model, we offer a way to measure phonological skills using the Single-Valued Trapezoidal Neutrosophic Numbers and we also propose to apply the Neutrosophic AHP technique instead of the classical AHP in the calculations of the dynamic equations within the Bratianu model. The Neutrosophic AHP and the Single-Valued Trapezoidal Neutrosophic Numbers in general, allow capturing the indeterminacy, uncertainty, and vagueness that exist in evaluations based on subjective opinions, which is a limitation of the crisp technique. Respondents are asked for their opinions on a scale based on natural language, as in the classic AHP model. In summary, it can be said that the proposed model extends Bratianu's model to the field of neutrosophy, and also adapts to the specific problem of studying the dynamics of phonological knowledge in children of approximately 5 years of age, which constitutes a contribution to the study of this phenomenon.

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Analysis of Customer Perception of a New Product Using the Neutrosophic Biplot Method

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Abstract: This paper sought to propose and apply the Neutrosophic Biplot analysis method to understand consumer perception of a new technological product with uncertain attributes while addressing the shortcomings that a conventional approach would present. Data was collected through a survey experiment with neutrosophic rating scales where 50 subjects slid on one scale to indicate how much they agreed (A), were neutral, or indifferent (I), and disagreed (D) about each of the eight features of a new smartphone they received. A neutrosophic data matrix was formed and converted into a real-valued matrix using a scoring function, allowing for the application of HJ-Biplot analysis. The two most important findings were that segmentation exists in this market, leading to the conclusions that there are three kinds of consumers: "Technophiles" (those concerned with performance and camera), "Thrifters" (those concerned with price and battery), and "Aesthetes" (those concerned with design) and that software as an attribute has been perceived by consumers with a high degree of indeterminacy. Ultimately, Neutrosophic Biplot is a valuable analysis for perception because by acknowledging uncertainty in consumer response, the analysis creates a more detailed and accurate picture of such information which can help direct corporate strategic decisions better.

Keywords: Neutrosophic logic, Biplot, multivariate analysis, consumer perception, market research, uncertainty, decision making.

1. Introduction

In today's fast-paced market environment, a company's ability to accurately and deeply understand consumer perception has become a fundamental pillar of success in developing and launching new products. The acceptance of an innovation depends not only on its objective technical merits but also on a complex mix of subjective assessments, expectations, and emotions on the part

of potential customers. A misinterpretation of these perceptions can lead to poor strategic decisions, resulting in significant financial losses and damage to the brand's reputation.

The main challenge, therefore, lies not in the need to measure, but in the difficulty inherent in the nature of the object being measured. Human perception is inherently vague, ambiguous, and often uncertain. When asked about a new product, individuals often experience doubts, have mixed feelings, or lack a firmly established opinion, especially when faced with novel or complex features. This uncertainty is not "noise" that must be eliminated, but a valuable piece of information that reflects the consumer's actual cognitive and affective state.

Traditionally, market research has addressed this challenge through the extensive use of quantitative surveys employing rating scales, such as the well-known Likert scale. These instruments are appreciated for their ease of administration and apparent simplicity in quantifying opinions, becoming an industry standard [1]. Data collected through these scales are subsequently analyzed using a variety of multivariate statistical techniques designed to reveal underlying patterns and relationships in complex data sets.

Within this arsenal of analytical tools, the Biplot method stands out for its remarkable visualization capabilities. Originally proposed by Gabriel [2], the Biplot allows a simultaneous graphical representation of the rows (individuals) and columns (variables) of a data matrix in a low-dimensional space. This facilitates the visual interpretation of the data structure, the identification of clusters of individuals with similar profiles, and the understanding of the relationships between variables. Later variants, such as the HJ-Biplot, have optimized the technique to ensure high-quality representation for individuals and variables simultaneously [3].

Despite its undeniable power, the classic Biplot paradigm suffers from the same fundamental limitation as traditional rating scales: it requires clear and unambiguous data. The method's mathematical framework, based on Singular Value Decomposition (SVD) of a matrix of real numbers, is not designed to handle doubt or ambiguity. The issue of the midpoint in Likert scales, for example, is a widely debated topic in the methodological literature, since a "neutral" can mean indifference, ignorance, or a conflicting mix of agreement and disagreement, nuances that are irremediably lost when assigning a single numerical value [4].

Fuzzy logic logic , have attempted to address imprecision in data analysis, finding applications in market research to model vague linguistic concepts [5]. While these approaches represent a significant advance by allowing partial degrees of membership, they generally do not explicitly distinguish between the degree of ambiguity and that of indeterminacy or unknowability, merging different facets of uncertainty into a single indicator.

In this context, this research is justified by the pressing need for analytical tools that not only tolerate, but also explicitly embrace and model, the uncertainty inherent in consumer perception. Developing a method that captures an individual's opinion in all its complexity—including certainty, doubt, and denial—would allow organizations to gain a richer, more realistic, and more nuanced view of the market. This, in turn, would strengthen the empirical foundation on which high-impact strategic decisions in marketing and product development are based [6].

To address this need, this study proposes the introduction and application of the Neutrosophic Biplot. This innovative methodology extends the visual power of the classic Biplot by integrating it

with the foundations of Neutrosophic Logic. This logic, unlike others, breaks down human assertions into three distinct and independent components: the degree of Truth (T), the degree of Indeterminacy (I), and the degree of Falsehood (F). This three-dimensional structure provides a mathematical formalism ideal for representing a consumer's opinion without forcing them to make an artificially precise choice.

The primary objective of this work is, therefore, to adapt and apply the Neutrosophic Biplot method to conduct a comprehensive analysis of a group of consumers' perception data regarding a new smartphone. Secondary objectives include designing a valuation survey based on neutrosophic scales; identifying and characterizing different consumer segments based not only on their preferences but also on their levels of uncertainty; and interpreting the resulting Biplot map to draw strategic conclusions about which product attributes are key for each segment.

The central hypothesis is that Neutrosophic Biplot analysis will provide a more detailed and meaningful customer segmentation than conventional Biplot analysis. It is postulated that, by incorporating the indeterminacy dimension, consumer profiles that would otherwise remain hidden will emerge, such as the "cautious consumer" or the "informed skeptic." Additionally, it is hypothesized that product attributes that involve greater technical complexity or novelty, such as software specifications or new camera functions, will present significantly higher levels of indeterminacy among respondents [7].

2. Methodology

2.1. Introduction to the HJ-Biplot Approach

The HJ-Biplot is a method for multivariate visualization that enables the joint projection of both individuals and variables into the same factorial plane, achieving high-quality representation for both. In contrast to traditional biplots (like GH and JK variants), the HJ-Biplot aims for a balanced display where the Euclidean distance is maintained accurately in both axes [8].

2.1.1. Mathematical basis of the HJ-Biplot

The HJ-Biplot relies on the singular value decomposition (SVD) of a data matrix $X \in R^{nxp}$, which is first centered by both rows and columns. This decomposition, expressed as "$X = U\,D\,V^{T}$", reveals the left singular vectors U, the right singular vectors V, and a diagonal matrix D containing the singular values in descending order. Using this algebraic framework, the HJ-Biplot facilitates the combined graphical display of both individuals and variables within the same factorial plane. It achieves a balanced representation where the Euclidean metric is consistently maintained across both axes, ensuring an optimal joint visualization quality [9,10].

Data Matrix

Let be a data matrix $X \in \mathbb{R}^{nxp}$, centered by rows and columns.

Singular Value Decomposition (SVD)

$$\mathbf{X = U\,D\,V^{T}} \qquad \mathbf{(1)}$$

Where:

- **OR** $\in \mathbb{R}^{nxr}$: Left singular vectors.
- **D** $\in \mathbb{R}^{rxr}$: diagonal matrix with singular values$d1 \geq d2 \geq \cdots \geq dr > 0$
- **V** $\in \mathbb{R}^{pxr}$
- r = range $(\boldsymbol{X})$.

Marker Selection – HJ-Biplot (Symmetric)

Bookmarks:

$\boldsymbol{A = U D}, \quad \mathbf{B = V D}$

$\mathbf{J = U D, H = V D}$

2.1.2. Mathematical basis of the Neutrosophic HJ-Biplot Dual Extension.

The HJ-Biplot, in its traditional form, serves as a robust multivariate visualization technique. Yet, its reliance on the Singular Value Decomposition (SVD) of a real-valued data matrix ($X \in \mathbb{R}^{nxp}$) introduces a critical constraint: it presumes that input data are exact, unambiguous, and deterministic [11].

This assumption clashes with real-world scenarios, particularly in market research, social sciences, and psychology, where human responses are often uncertain, vague, and indeterminate. Requiring participants to select a fixed value (e.g., rating "7" on a 10-point scale) discards essential insights into their degree of confidence or hesitation [12].

To address this limitation, we introduce the Neutrosophic Biplot, an advanced adaptation of the conventional approach that incorporates neutrosophic logic. This innovation shifts the analysis from a real-number matrix to a neutrosophic data matrix, where each entry encapsulates the full spectrum of a respondent's perspective, including uncertainty.

2.3.1. The Neutrosophic Data Matrix (X~)

The starting point of the Neutrosophic Biplot is the redefinition of the data matrix. Instead of the classical matrix X, a neutrosophic matrix X~ of the same dimensions n × p (individuals × variables) is constructed.

Each element $x_{ij}$of the classical matrix is replaced by a **Single Valued Neutrosophic Number (SVNN)** , N_{ij}, which is represented as a tuple of three components:

$$N_{ij} = (t_{ij}, i_{ij}, f_{ij}) \qquad (2)$$

Where, for the opinion of individual i on variable (or attribute) j:

- t_{ij} is the **degree of truth** (e.g. degree of agreement, satisfaction, importance).
- i_{ij} is the **degree of indeterminacy** (e.g. degree of doubt, uncertainty, ignorance).
- f_{ij} is the **degree of falsity** (e.g. degree of disagreement, dissatisfaction, unimportance).

These three components are real numbers in the interval [0,1]and their sum is unrestricted, allowing both incomplete $(t + i + f < 1)$and contradictory information to be modeled $(t + i + f > 1)$.. The data matrix therefore becomes:

$$X \cong (t_{11}, i_{11}, f_{11}) \vdots (t_{n1}, i_{n1}, f_{n1}) \cdots \ddots \cdots (t_{1p}, t_{1p}, f_{1p}) \vdots (t_{np}, i_{np}, f_{np}) \quad (3)$$

The central challenge is obvious: the SVD algorithm, the heart of HJ-Biplot, is not defined for a matrix of tuples. To address this, there are two main approaches.

2.3.2. Approach 1: De- neutrosification and Application of the Classical Biplot (Pragmatic Approach)

This approach prioritizes the applicability and use of existing statistical tools. It consists of transforming the neutrosophic matrix X~ into a real data matrix (crisp) XS, on which the standard HJ-Biplot can be applied [13].

Step 1: Aggregation of Neutrosophic Information. A scoring function is used to convert each neutrosophic number N_{ij} into a single scalar value. One of the most common functions is:

$$S(N_{ij}) = t_{ij} - f_{ij}f \quad (4)$$

Or a more nuanced version that considers indeterminacy, such as:

$$S(N_{ij}) = 32 + t_{ij} - i_{ij} - f_{ij} \quad (5)$$

The choice of the scoring function is a key methodological step and must be justified based on the study's objectives. The result is a real-valued data matrix.$X_S = [S(N_{ij})]$.

Step 2: Application of the Standard HJ-Biplot. Once the score matrix X_S is obtained, the HJ-Biplot procedure is applied identically to that described in section 1.1.2:

1. X_S is centered .
2. SVD is performed:$X_S = UDVT$.
3. Markers are calculated for individuals $\left(A = \frac{UD1}{2}\right)$and variables$\left(B = \frac{VD1}{2}\right)$.
4. The Biplot graph is generated.

Step 3: Visual Reintroduction of Indeterminacy. One criticism of this approach is that the richness of indeterminacy (i) is "lost" during deneutrosification. To mitigate this, the original information can be reincorporated into the final visualization. For example:

- **Dot size:** The dot representing an individual can be larger the greater its average degree of indeterminacy.
- **Color or transparency:** The points may have a color map (e.g. , from blue to red) that represents the indeterminacy or falsity component.
- **Uncertainty halos:** Each point can be surrounded by a circle or "halo" whose radius is proportional to the value of i.

Advantage: Computational simplicity and easy interpretation of the resulting Biplot map. **Disadvantage:** Loss of information in the calculation process, which is only partially recovered in the visualization.

2.3.3. Approach 2: Adaptation of the SVD Algorithm for Neutrosophic Data (Theoretical Approach)

This approach is theoretically purer and more robust, although computationally more complex. It seeks to adapt the Biplot engine itself (the SVD) to operate directly with neutrosophic numbers [14].

Step 1: Defining Neutrosophic Algebra. For an algorithm like SVD to work, the basic matrix operations (addition, multiplication) for neutrosophic numbers must be defined. Based on the literature on neutrosophic logic, the following operations are defined:

- **Sum of two*SVNNs***: $N_1 \oplus N_2 = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2)$
- **Product of two*SVNNs***: $N_1 \otimes N_2 = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2)$

Step 2: Neutrosophic SVD. The SVD algorithm, which is often solved by iterative methods, would be modified to use the neutrosophic algebra defined in the previous step. The decomposition of the neutrosophic matrix X~ would result in components that are also neutrosophic:

$$X\sim \approx U\sim D\sim V\sim T \quad (6)$$

Where:

- U~ and V~ are matrices whose entries are neutrosophic vectors.
- D~ is a diagonal matrix whose singular values λ~k are also neutrosophic numbers,$\lambda\sim k = (t_{\lambda k}, i_{\lambda k}, f_{\lambda k})$.

Step 3: Markers and Neutrosophic Graphical Representation. The markers for individuals and variables would be, analogously, matrices of neutrosophic numbers [15]. This means that the coordinate of each point in the Biplot is not a pair (x, y), but a pair of neutrosophic numbers:$(a\sim k, b\sim k)$.

Visualizing this result is a challenge in itself. A point is no longer a point, but an "**uncertainty area**" on the plane. It could be graphed:

- An **uncertainty rectangle or ellipse** for each individual and vector, centered on the deneutrosified value (using a score function), and with dimensions reflecting the magnitude of the i and f components on each axis.
- **Three separate points** for each individual (one for T, one for I, one for F), connected to show how each facet of opinion "behaves" in the factor space.

Advantage: Preserves all neutrosophic information throughout the analytical process, potentially offering a much richer and more accurate result. **Disadvantage:** High theoretical and computational complexity. Interpretation of the resulting graph is less straightforward and requires specific training for the reader.

For this study, we will adopt **Approach 1 (Pragmatic)** , as it offers an optimal balance between incorporating uncertainty information and interpretability of results for a broad audience.

3. Application and Results

This section details the empirical procedure used to apply the Neutrosophic Biplot method to the analysis of consumer perceptions of a new product. The data collection process, the construction of the neutrosophic data matrix, and the statistical processing are described, and, finally, a comprehensive interpretation of the resulting Biplot graph is presented.

3.1. Data Collection

To conduct the study, an online survey was designed and administered to a sample of 50 consumers who had had contact with or received prior information about a new smartphone model recently introduced to the market. Participation was voluntary and anonymous. The survey focused on assessing user perceptions of eight key product attributes, identified through a prior analysis of literature and industry reports. The attributes evaluated were: **battery, camera, price, design, display, performance, software, and brand.**

Unlike traditional surveys using Likert scales, a neutrosophic rating question was implemented for each attribute. Participants were asked to allocate 10 points among three statements representing the components of neutrosophic logic:

- **Degree of Agreement (T):** "My level of agreement that this attribute is a strong point of the product."
- **Degree of Indeterminacy (I):** "My level of doubt, uncertainty or lack of information to assess this attribute."
- **Degree of Disagreement (F):** "My level of disagreement, considering that this attribute is a weak point of the product."

For example, a user's response to the attribute "Battery" might $(T = 7, I = 2, F = 1)$,indicate a strong positive perception, with slight doubt and minimal disagreement. These values were normalized to the interval [0, 1] by dividing them by 10, to form the corresponding neutrosophic

tuple . $(0.7, 0.2, 0.1)$.This data collection method allows for a more reliable, three-dimensional view of the respondent's opinion.

3.2. Neutrosophic Data Matrix

The responses of the 50 participants for the 8 attributes were consolidated into a neutrosophic data matrix X~ of dimensions 50x8. Each cell (i ,j) of this matrix contains the neutrosophic number $N_{ij} = (t_{ij}, i_{ij}, f_{ij})$that represents user i's perception of attribute j. An illustrative extract of this matrix is shown below:

Table 1: Neutrosophic data matrix X~ with dimensions 50x8. Source: Prepared by the authors.

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U01	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.2, 0.6, 0.2)	(0.3, 0.3, 0.4)
U02	(0.7, 0.2, 0.1)	(0.2, 0.2, 0.6)	(0.8, 0.1, 0.1)	(0.3, 0.2, 0.5)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.5, 0.2)	(0.4, 0.2, 0.4)
U03	(0.9, 0.0, 0.1)	(0.1, 0.3, 0.6)	(0.7, 0.2, 0.1)	(0.1, 0.4, 0.5)	(0.3, 0.3, 0.4)	(0.1, 0.3, 0.6)	(0.1, 0.7, 0.2)	(0.2, 0.4, 0.4)
U04	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.9, 0.1, 0.0)	(0.3, 0.3, 0.4)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.4, 0.5, 0.1)	(0.3, 0.3, 0.4)
U05	(0.7, 0.1, 0.2)	(0.3, 0.1, 0.6)	(0.8, 0.2, 0.0)	(0.4, 0.2, 0.4)	(0.6, 0.1, 0.3)	(0.3, 0.1, 0.6)	(0.3, 0.6, 0.1)	(0.5, 0.2, 0.3)
U06	(0.9, 0.1, 0.0)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)	(0.3, 0.2, 0.5)	(0.1, 0.2, 0.7)	(0.2, 0.5, 0.3)	(0.3, 0.3, 0.4)
U07	(0.8, 0.2, 0.0)	(0.2, 0.3, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.4, 0.2)	(0.4, 0.3, 0.3)
U08	(0.7, 0.1, 0.2)	(0.1, 0.4, 0.5)	(0.8, 0.1, 0.1)	(0.1, 0.4, 0.5)	(0.4, 0.3, 0.3)	(0.1, 0.4, 0.5)	(0.1, 0.8, 0.1)	(0.2, 0.5, 0.3)
U09	(0.9, 0.0, 0.1)	(0.2, 0.2, 0.6)	(0.9, 0.1, 0.0)	(0.2, 0.2, 0.6)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.6, 0.1)	(0.3, 0.4, 0.3)
U10	(0.8, 0.1, 0.1)	(0.3, 0.3, 0.4)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.3, 0.3, 0.4)	(0.4, 0.5, 0.1)	(0.5, 0.3, 0.2)
U11	(0.7, 0.2, 0.1)	(0.1, 0.3, 0.6)	(0.8, 0.1, 0.1)	(0.1, 0.3, 0.6)	(0.4, 0.3, 0.3)	(0.1, 0.3, 0.6)	(0.2, 0.7, 0.1)	(0.2, 0.4, 0.4)
U12	(0.9, 0.1, 0.0)	(0.2, 0.1, 0.7)	(0.9, 0.0, 0.1)	(0.2, 0.2, 0.6)	(0.5, 0.2, 0.3)	(0.2, 0.1, 0.7)	(0.3, 0.5, 0.2)	(0.3, 0.3, 0.4)
U13	(0.8, 0.1, 0.1)	(0.3, 0.2, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.2, 0.5)	(0.6, 0.1, 0.3)	(0.3, 0.2, 0.5)	(0.4, 0.4, 0.2)	(0.5, 0.2, 0.3)
U14	(0.7, 0.3, 0.0)	(0.1, 0.4, 0.5)	(0.8, 0.2, 0.0)	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.1, 0.4, 0.5)	(0.1, 0.9, 0.0)	(0.2, 0.6, 0.2)

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U15	(0.9, 0.0, 0.1)	(0.2, 0.3, 0.5)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.3, 0.6, 0.1)	(0.3, 0.5, 0.2)
U16	(0.8, 0.1, 0.1)	(0.3, 0.1, 0.6)	(0.7, 0.0, 0.3)	(0.3, 0.1, 0.6)	(0.6, 0.1, 0.3)	(0.3, 0.1, 0.6)	(0.4, 0.5, 0.1)	(0.5, 0.2, 0.3)
U17	(0.7, 0.2, 0.1)	(0.1, 0.2, 0.7)	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.4, 0.2, 0.4)	(0.1, 0.2, 0.7)	(0.2, 0.6, 0.2)	(0.2, 0.3, 0.5)
U18	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.9, 0.1, 0.0)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.3, 0.4, 0.3)	(0.3, 0.4, 0.3)
U19	(0.8, 0.0, 0.2)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)	(0.6, 0.3, 0.1)	(0.3, 0.4, 0.3)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
U20	(0.7, 0.1, 0.2)	(0.1, 0.5, 0.4)	(0.8, 0.1, 0.1)	(0.1, 0.5, 0.4)	(0.4, 0.4, 0.2)	(0.1, 0.5, 0.4)	(0.1, 0.8, 0.1)	(0.2, 0.6, 0.2)
U21	(0.4, 0.2, 0.4)	(0.8, 0.1, 0.1)	(0.1, 0.2, 0.7)	(0.5, 0.3, 0.2)	(0.8, 0.1, 0.1)	(0.9, 0.1, 0.0)	(0.3, 0.6, 0.1)	(0.5, 0.3, 0.2)
U22	(0.3, 0.3, 0.4)	(0.7, 0.2, 0.1)	(0.2, 0.2, 0.6)	(0.6, 0.2, 0.2)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.2, 0.2)
U23	(0.2, 0.4, 0.4)	(0.9, 0.1, 0.0)	(0.1, 0.3, 0.6)	(0.4, 0.4, 0.2)	(0.9, 0.0, 0.1)	(0.7, 0.2, 0.1)	(0.2, 0.7, 0.1)	(0.4, 0.4, 0.2)
U24	(0.5, 0.2, 0.3)	(0.8, 0.0, 0.2)	(0.3, 0.1, 0.6)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.9, 0.0, 0.1)	(0.3, 0.6, 0.1)	(0.7, 0.1, 0.2)
U25	(0.4, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.2, 0.3, 0.5)	(0.5, 0.3, 0.2)	(0.7, 0.2, 0.1)	(0.8, 0.2, 0.0)	(0.4, 0.5, 0.1)	(0.5, 0.3, 0.2)
U26	(0.3, 0.4, 0.3)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)	(0.6, 0.4, 0.0)	(0.9, 0.1, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.8, 0.1)	(0.6, 0.4, 0.0)
U27	(0.2, 0.2, 0.6)	(0.8, 0.2, 0.0)	(0.2, 0.1, 0.7)	(0.4, 0.2, 0.4)	(0.8, 0.0, 0.2)	(0.9, 0.1, 0.0)	(0.3, 0.5, 0.2)	(0.4, 0.2, 0.4)
U28	(0.5, 0.1, 0.4)	(0.7, 0.3, 0.0)	(0.3, 0.3, 0.4)	(0.7, 0.3, 0.0)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.4, 0.4, 0.2)	(0.7, 0.3, 0.0)
U29	(0.4, 0.4, 0.2)	(0.9, 0.0, 0.1)	(0.1, 0.5, 0.4)	(0.5, 0.5, 0.0)	(0.9, 0.1, 0.0)	(0.7, 0.2, 0.1)	(0.2, 0.7, 0.1)	(0.5, 0.5, 0.0)
U30	(0.3, 0.2, 0.5)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.6, 0.2, 0.2)	(0.8, 0.2, 0.0)	(0.9, 0.0, 0.1)	(0.3, 0.6, 0.1)	(0.6, 0.2, 0.2)
U31	(0.2, 0.3, 0.5)	(0.7, 0.2, 0.1)	(0.3, 0.3, 0.4)	(0.4, 0.3, 0.3)	(0.7, 0.1, 0.2)	(0.8, 0.1, 0.1)	(0.4, 0.5, 0.1)	(0.4, 0.3, 0.3)
U32	(0.5, 0.4, 0.1)	(0.9, 0.1, 0.0)	(0.1, 0.4, 0.5)	(0.7, 0.2, 0.1)	(0.9, 0.1, 0.0)	(0.7, 0.3, 0.0)	(0.1, 0.8, 0.1)	(0.7, 0.2, 0.1)
U33	(0.4, 0.1, 0.5)	(0.8, 0.0, 0.2)	(0.2, 0.1, 0.7)	(0.5, 0.1, 0.4)	(0.8, 0.2, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.5, 0.2)	(0.5, 0.1, 0.4)

User	Battery	Camera	Price	Design	Screen	Performance	Software	Brand
U34	(0.3, 0.3, 0.4)	(0.7, 0.3, 0.0)	(0.3, 0.3, 0.4)	(0.6, 0.3, 0.1)	(0.7, 0.3, 0.0)	(0.8, 0.2, 0.0)	(0.4, 0.4, 0.2)	(0.6, 0.3, 0.1)
U35	(0.2, 0.5, 0.3)	(0.9, 0.1, 0.0)	(0.1, 0.5, 0.4)	(0.4, 0.6, 0.0)	(0.9, 0.0, 0.1)	(0.7, 0.1, 0.2)	(0.2, 0.7, 0.1)	(0.4, 0.6, 0.0)
U36	(0.2, 0.2, 0.6)	(0.5, 0.3, 0.2)	(0.3, 0.2, 0.5)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.4, 0.4, 0.2)	(0.3, 0.6, 0.1)	(0.9, 0.1, 0.0)
U37	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.2, 0.3, 0.5)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.2, 0.0)
U38	(0.1, 0.4, 0.5)	(0.4, 0.4, 0.2)	(0.1, 0.4, 0.5)	(0.7, 0.3, 0.0)	(0.9, 0.0, 0.1)	(0.3, 0.5, 0.2)	(0.2, 0.7, 0.1)	(0.7, 0.3, 0.0)
U39	(0.4, 0.2, 0.4)	(0.7, 0.1, 0.2)	(0.4, 0.1, 0.5)	(0.9, 0.0, 0.1)	(0.8, 0.1, 0.1)	(0.6, 0.2, 0.2)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
U40	(0.3, 0.3, 0.4)	(0.5, 0.3, 0.2)	(0.3, 0.3, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.4, 0.4, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.1, 0.1)
U41	(0.2, 0.4, 0.4)	(0.4, 0.5, 0.1)	(0.2, 0.5, 0.3)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.2, 0.6, 0.2)	(0.1, 0.8, 0.1)	(0.7, 0.3, 0.0)
U42	(0.5, 0.2, 0.3)	(0.8, 0.1, 0.1)	(0.5, 0.1, 0.4)	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.0)	(0.7, 0.2, 0.1)	(0.3, 0.5, 0.2)	(0.9, 0.1, 0.0)
U43	(0.4, 0.3, 0.3)	(0.6, 0.3, 0.1)	(0.4, 0.3, 0.3)	(0.8, 0.2, 0.0)	(0.7, 0.3, 0.0)	(0.5, 0.4, 0.1)	(0.4, 0.4, 0.2)	(0.8, 0.2, 0.0)
U44	(0.3, 0.5, 0.2)	(0.4, 0.6, 0.0)	(0.2, 0.6, 0.2)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.7, 0.0)	(0.2, 0.7, 0.1)	(0.7, 0.3, 0.0)
U45	(0.1, 0.2, 0.7)	(0.5, 0.2, 0.3)	(0.1, 0.2, 0.7)	(0.9, 0.0, 0.1)	(0.8, 0.0, 0.2)	(0.4, 0.3, 0.3)	(0.3, 0.6, 0.1)	(0.9, 0.0, 0.1)
U46	(0.2, 0.3, 0.5)	(0.7, 0.1, 0.2)	(0.3, 0.1, 0.6)	(0.8, 0.1, 0.1)	(0.7, 0.1, 0.2)	(0.6, 0.2, 0.2)	(0.4, 0.5, 0.1)	(0.8, 0.1, 0.1)
U47	(0.4, 0.4, 0.2)	(0.5, 0.5, 0.0)	(0.2, 0.4, 0.4)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.4, 0.6, 0.0)	(0.1, 0.9, 0.0)	(0.7, 0.3, 0.0)
U48	(0.1, 0.1, 0.8)	(0.4, 0.1, 0.5)	(0.1, 0.1, 0.8)	(0.9, 0.1, 0.0)	(0.8, 0.1, 0.1)	(0.2, 0.2, 0.6)	(0.3, 0.5, 0.2)	(0.9, 0.1, 0.0)
U49	(0.3, 0.3, 0.4)	(0.6, 0.2, 0.2)	(0.3, 0.2, 0.5)	(0.8, 0.0, 0.2)	(0.7, 0.2, 0.1)	(0.5, 0.3, 0.2)	(0.4, 0.4, 0.2)	(0.8, 0.0, 0.2)
U50	(0.2, 0.6, 0.2)	(0.5, 0.5, 0.0)	(0.1, 0.7, 0.2)	(0.7, 0.3, 0.0)	(0.9, 0.1, 0.0)	(0.3, 0.7, 0.0)	(0.2, 0.8, 0.0)	(0.7, 0.3, 0.0)

This matrix constituted the primary input for the subsequent analysis.

3.3. Processing and Generation of the Neutrosophic Biplot

Following the pragmatic approach described in the methodology, the first step was the deneutrosification of the matrix X~ to obtain a real-valued data matrix (crisp) XS that could be processed by the HJ-Biplot algorithm. To do this, the following scoring function was applied to each cell of the matrix:

$$S(N_{ij}) = 32 + t_{ij} - i_{ij} - f_{ij} \quad (7)$$

This function transforms the neutrosophic tuple into a single scalar value representing the "net favorability score," penalizing indeterminacy and falsity. The resulting matrix XS was then column-centered. Singular Value Decomposition (SVD) was then applied to the centered matrix to obtain the row (individual) and column (variable) markers.

Eigenvalue analysis indicated that the first two principal components explained a substantial portion of the total variance in the data. Specifically, **Component 1 (X-axis) accounted for 48.7%** of the variance, while **Component 2 (Y-axis) explained an additional 25.2%**. Overall, the resulting Biplot map captures **73.9% of the original information**, a value considered excellent for the correct interpretation of the data structure.

3.4. Analysis and Interpretation of the Biplot Chart

The resulting HJ-Biplot graph is presented below. It simultaneously represents the 50 users (points) and the 8 attributes (vectors). Interpretation is based on the length of the vectors, the angle between them, and the relative position of the individual points with respect to the vectors.

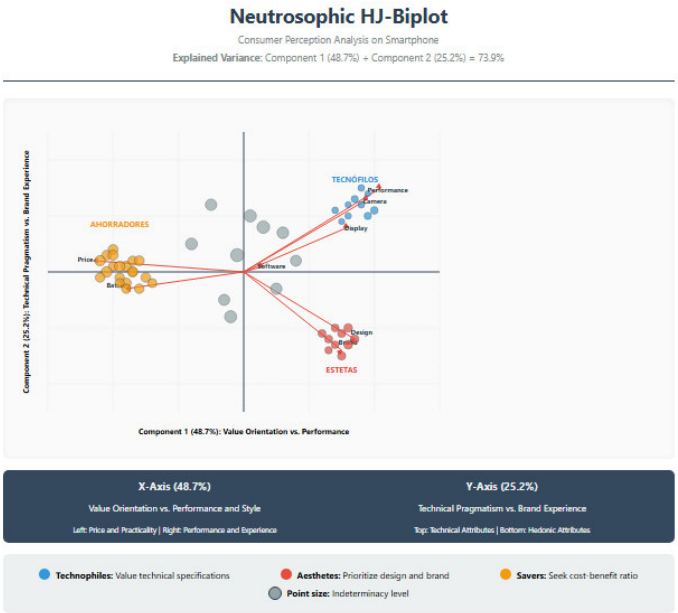

Figure 1. HJ-Biplot-Neutrosophic on a new smartphone model recently introduced to the market**.**

3.4.1. Interpretation of the Axes Factorials

- **X-axis (48.7%): "Value Orientation vs. Performance and Style."** This axis is the main differentiator of consumer perception. High value-added attributes such as Performance, Camera, Display, and Design are positioned on the right. Attributes related to cost and practicality, primarily Price and Battery Life, are located on the left. Consumers on the right side of the graph tend to value features and experience, while those on the left prioritize value and battery life.
- **Y-axis (25.2%): "Technical Pragmatism vs. Brand Experience."** The second axis differentiates between different types of positive evaluations. Projected toward the top are the vectors of technical and functional attributes (Performance, Camera). Projected toward the bottom are the more hedonistic and brand perception attributes (Design, Brand). This axis, therefore, separates users who seek technical specifications from those who value brand aesthetics and status more.

3.4.2. Relationship between Attributes

The angles between the attribute vectors reveal their correlations in consumer perception.

- **acute angle** is observed between the Camera and Performance vectors, indicating a strong positive correlation. Consumers who rate a device's performance positively tend to also rate its camera quality.
- Similarly, Design and Brand show a strong correlation, pointing in a very similar direction in the lower right quadrant. This suggests that brand perception is strongly linked to product aesthetics.
- Price vector forms an **obtuse angle** (almost 180°) with Performance and Camera, denoting a strong negative correlation. This is expected: users who prioritize a low price negatively perceive attributes that tend to make a product more expensive.
- Software vector is noticeably shorter than the others and is located close to the origin, suggesting that it contributes less to the overall variance. This indicates that perceptions of software are not as discriminating a factor among consumers as the other attributes, or that their assessment is more homogeneous or uncertain across the sample.

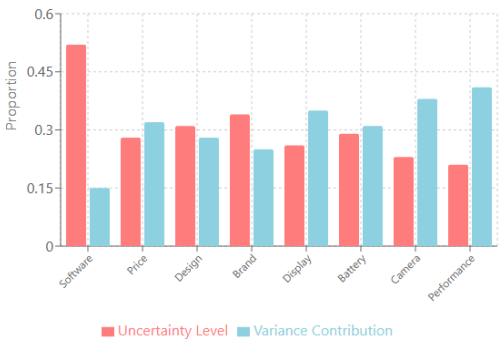

Figure 2. Attribute Uncertainty and Variance Contribution Analysis

This graph reveals the relationship between consumer uncertainty and attribute variance. Software presents the greatest uncertainty despite its low contribution to variance, indicating uniform uncertainty across all consumer segments.

3.4.3. Identification of Consumer Segments (Clusters)

The distribution of user points on the map allows us to identify three clearly defined consumer segments:

1. **Cluster 1: Technophiles (Upper right quadrant).** This consumer group (e.g., U07, U18, U33) is located in the zone of influence of the Performance and Camera vectors . These are individuals for whom technical specifications are the main purchasing driver. They do not show a strong association with the Price vector , indicating a lower sensitivity to it. They value a high performance and capabilities photographic advanced .
2. **Cluster 2: Aesthetes (Lower right quadrant).** This segment (e.g., U02, U23, U45) is strongly associated with the Design and Brand vectors . Their positive perception of the product is driven by aesthetics, construction materials, and brand prestige. Although they are also on the positive side of the X-axis, their assessment is not as closely tied to raw performance as is the case with technophiles.
3. **Cluster 3: The Savers (Left semicircle).** This is the largest group and clusters on the left side of the graph (e.g., U04, U29, U41), in contrast to almost all the performance attributes. Its proximity to the Price and Battery vectors indicates that their primary evaluation criteria are cost-benefit ratio and long battery life. For them, a low price is the most desirable attribute.

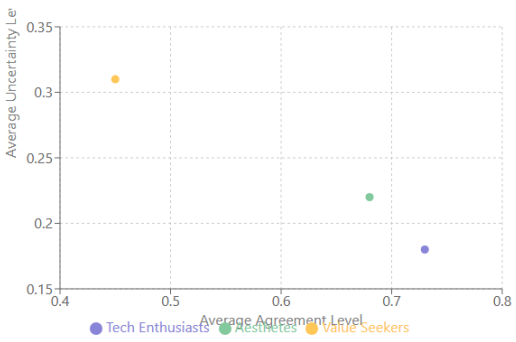

Figure 3. Consumer Segments Distribution and Neutrosophic Profile

This scatter plot positions consumer segments according to their neutrosophic profile. The size of the dot represents the segment size. Value Seekers show greater uncertainty and lower agreement, while Technology Enthusiasts show high agreement with low uncertainty.

3.4.4. Analysis of Indeterminacy

To incorporate the dimension of uncertainty into the visual analysis, the size of each point representing a user on the graph is proportional to their **average degree of indeterminacy (I)** across all their responses. A larger point indicates a higher level of overall doubt or uncertainty.

This visual analysis reveals a key finding: although the Software vector was short, many of the largest points in the graph belong to users who gave high ratings to this attribute. This confirms that, while software is not a major differentiator in terms of liking or disliking, it **is the attribute that generates the most uncertainty** in the sample. Consumers seem to struggle to assess software quality, either not knowing what to expect or not feeling qualified to make a firm judgment. This is a crucial insight that a classic analysis would have completely obscured.

4. Conclusion and Future Directions

4.1. Summary of Main Findings

The application of the Neutrosophic Biplot method enabled a deep and nuanced analysis of consumer perception. The results demonstrated the existence of three clear and actionable market segments: **Technophiles** , **Aesthetes** , and **Thrifts** , each with a distinctive set of priorities and values. Analysis of the relationships between attributes confirmed logical associations (Performance-Camera) and oppositions (Price-Features). The most revealing finding, made possible only by the neutrosophic nature of the method, was the identification of **Software** as the main generator of uncertainty among consumers, an invaluable insight for guiding communications and marketing strategies.

4.2. Advantages and Contribution of the Method

This study empirically demonstrates the superiority of the Neutrosophic Biplot over conventional methods. By not forcing the respondent to make a definitive decision and by mathematically modeling uncertainty, the method provides a deeper, more honest, and more realistic understanding of the perceptual landscape. It allows analysts to understand not only *what* consumers like, but also *how confident* they are about it, opening up a new dimension for strategic segmentation.

4.3. Limitations and Future Lines of Research

A limitation of the present study is the sample size, which, although sufficient to demonstrate the methodology, could be expanded in future work to validate the segments found on a larger scale. Furthermore, the pragmatic deneutrosification approach was used ; it would be of great theoretical and practical interest to explore the application of the Neutrosophic SVD approach to compare the results.

Future lines of research are promising. The Neutrosophic Biplot method can be applied to a wide range of domains beyond marketing, such as financial risk analysis (where uncertainty is a key

factor), medical research (to assess patient perceptions of treatments), or public opinion analysis in political science, where ambiguity and indecision are prevalent and highly relevant phenomena.

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Analysis of the Evolution of Expert Opinion on Sustainability Policies through the Neutrosophic STATIS Method

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Abstract: Yet the problem of evaluating sustainability policies over time stems from needing to monitor the changing nature of expert agreement needed for dependency-based improvements in the environment. While this is undoubtedly helpful for relatively sustainable action, easier said than done requires methodological systems to assess how agreement grows and solidifies. Here it should be noted that the conventional STATIS approach, while successful in multivariate assessment, lacks the capacity to evaluate expert assessments factoring uncertainty, indeterminacy, and subjectivity. Therefore, this research goes on to fill this void with the neutrosophic STATIS method applied to a panel of experts assessing ten (10) sustainability policies over three (3) time periods, resulting in ease of modeling opinion and visualization of intent. Ultimately, the results indicate that a consensus structure exists and evolves over time, yet there is agreement on the success of energy policies, while circular economy initiatives never become certain. Thus, this study contributes to the literature in two ways: First, theoretically, it legitimizes a consistent framework for dynamic evaluation of complex subjective assessments; Second, practically, it allows for managers to understand where there is agreement and disagreement to create more fluid public policies.

Keywords: STATIS Neutrosophic, Neutrosophic Logic, Longitudinal Data Analysis, Sustainability Policies, Expert Opinion, Consensus Analysis, Multi -table Analysis.

1. Introduction

Rigorous assessment of sustainability policies is a fundamental pillar for addressing global crises such as climate change and biodiversity loss. In this scenario, expert opinion is not just a source of data, but a compass that guides the formulation and, above all, the adaptation of initiatives over time [1]. Understanding how consensus among specialists evolves—whether it strengthens, weakens, or polarizes—is crucial for the legitimacy and effectiveness of adaptive management of public policies [2]. Therefore, longitudinal monitoring of these expert assessments is not merely an academic exercise; it constitutes an imperative for effective environmental governance that can adjust to emerging evidence and changing contexts [3].

Historically, policy analysis has moved from qualitative assessments and case studies to increasingly sophisticated quantitative methods. In recent decades, the need to simultaneously

analyze multiple data sets—for example, different expert panels or measurements at different points in time—has driven the development of three-way data analysis techniques [4]. Within this methodological arsenal, the STATIS (Structure de Tableaux à Trois) method Indices de la Statistique has established itself as a powerful tool for comparing and synthesizing multi -table data, revealing common structures and discrepancies. However, this evolutionary journey has reached a turning point, where the nature of the data, and not just its volume, presents the main challenge [5].

Herein lays the central problem that this research seeks to solve. Classical statistical methods, including conventional STATIS, operate under the assumption that the input data are precise and crisp numbers. This premise breaks down when dealing with human opinions, which are inherently fraught with subjectivity, ambiguity, doubt, and degrees of indeterminacy that cannot be ignored without significant information loss [6]. An expert's judgment is rarely a simple "yes" or "no" but often resides on a spectrum of certainty, skepticism, and indecision. Therefore, a critical and as yet unresolved research question emerges: How can the temporal evolution of expert consensus, including its inherent uncertainty and indeterminacy, regarding a set of sustainability policies be effectively modeled and analyzed?

To answer this question, the objectives of this study are as follows. First, we seek to introduce and formalize the Neutrosophic STATIS method as a suitable methodological framework for the dynamic analysis of subjective, multi -expert assessments. Second, we apply this novel approach to a real-life case study, analyzing the evolving opinions of a panel of experts on a set of sustainability policies at three different points in time. Finally, we aim to identify and interpret the resulting consensus structure, the trajectories of individual policies, and the stability of expert opinions, thereby demonstrating the practical utility and explanatory power of the proposed framework for improving decision-making processes [7].

2. Methodology

2.1. Introduction to the STATIS Approach

The STATIS method (Structure des Tableaux À Trois Statistics Indices), also known as ACT (Analyse Conjoint (Tableaux) emerges as a multivariate strategy developed for the joint analysis of multiple structured data tables with common observations. This methodology allows for the identification of an underlying common structure by constructing a compromise: an operator resulting from the optimal combination of the different blocks, based on a criterion of maximum similarity between them.

Their procedure consists of three fundamental stages: interstructure, where the similarity between blocks is quantified using scalar products or the RV coefficient; commitment, which seeks to synthesize shared information into a single representative operator; and intrastructure , which allows the observations of each block to be projected onto the space defined by the principal components of the commitment. This approach has proven robust for comparing multidimensional configurations derived from different sources or measurement times.

The formal mathematical foundation of the STATIS method is based on the use of the Hilbert – Schmidt scalar product and optimization under Euclidean constraints, which allows maintaining geometric consistency between blocks and preserving multivariate interpretability. The complete formulation can be found in Vivien and Sabatier [8], who also discuss its generalization to more complex multiblock structures.

2.1.1. STATIS Mathematical Foundation

The DO-ACT method is a generalization of the STATIS-ACT approach, designed to compare and analyze two data sets structured in the form of multiple blocks or tables (referred to here as$\boldsymbol{\mathcal{T}_1} = \{\boldsymbol{X_k}\}_{\boldsymbol{k=1}}^{\boldsymbol{K_1}}, \quad \boldsymbol{\mathcal{T}_2} = \{\boldsymbol{Y_l}\}_{\boldsymbol{l=1}}^{\boldsymbol{K_2}}$ on the same observations [9].

Definition of blocks

There are two series of data blocks (tables) with the same dimension in rows (observations) but different variables:

- First set of blocks:

$\boldsymbol{X_k} \in \boldsymbol{R^{n \times p_k}}, \quad \textbf{for } \boldsymbol{k = 1, \dots, K_1}$

- Second set of blocks:

$\boldsymbol{Y_l} \in \boldsymbol{R^{n \times q_l}}, \quad \textbf{for } \boldsymbol{l = 1, \dots, K_2}$

Scalar product matrix for each block

For each block, its scalar product matrix (also called generalized covariance matrix) is defined:

- For$\boldsymbol{X_k^\top}$

$$\boldsymbol{W_k^X = X_k Q_k^X X_k^\top} \quad (1)$$

- For$\boldsymbol{Y_l^\top}$

$$\boldsymbol{W_l^Y = Y_l Q_l^Y Y_l^\top} \quad (2)$$

Where :

$\boldsymbol{Q_k^X}$ **and** $\boldsymbol{Q_l^Y}$

They are weight matrices (usually identity matrices or Euclidean or Mahalanobis metric type normalization matrices) [10].

Partial commitment of the first block set

The commitments of each block group are defined as weighted linear combinations of their respective matrices of $\boldsymbol{W_k^X}$and$\boldsymbol{W_l^Y}$ **:**

Commitment to $\boldsymbol{\mathcal{T}1}$:

$$\boldsymbol{W_{comp}^X = \sum_{k=1}^{K_1} \alpha_k W_k^X} \quad (3)$$

Commitment to $\boldsymbol{\mathcal{T}2}$:

$$\boldsymbol{W_{comp}^Y = \sum_{l=1}^{K_2} \beta_l W_l^Y} \quad (4)$$

The coefficients$\boldsymbol{\alpha_k}$ and $\boldsymbol{\beta_l}$are generally determined by a STATIS-type analysis.

Double Commitment (DO-ACT)

The main idea of the method is to compare **both commitments** and generate a **final commitment** :

$$\boldsymbol{W^{DO}} = \frac{1}{2}\left(\boldsymbol{W^{X}_{comp}} + \boldsymbol{W^{Y}_{comp}}\right) \quad (5)$$

This average commitment is interpreted as the result of a double commitment analysis, balancing the internal structure of the two groups of blocks.

Spectral analysis of DO commitment

A principal components analysis (PCA) is performed on the average commitment:

$$\boldsymbol{W^{DO}} = \boldsymbol{U\Lambda U}^{\top} \quad (6)$$

Where :

1. $\boldsymbol{\Lambda}$ contains the eigenvalues (inertia explained),
2. **U** are the eigenvectors (principal components),
3. The coordinates of the individuals are calculated as:

$$\boldsymbol{F} = \boldsymbol{U\Lambda^{1/2}} \quad (7)$$

2.2 The Neutrosophic STATIS Method: Rationale and Procedure

The main limitation of the classic STATIS method, like many traditional multivariate techniques, lies in its inability to handle imprecise or "crisp "data. Human opinions, perceptions, and evaluations rarely manifest themselves as exact real numbers; instead, they are imbued with degrees of certainty, doubt, ambiguity, and indecision. To overcome this barrier and reliably analyze the evolution of expert opinion, it is imperative to extend the STATIS framework to the domain of neutrosophic logic.

Neutrosophic, proposed by Smarandache, offers a generalization of fuzzy and intuitionistic logic by explicitly introducing the concept of indeterminacy (I), along with those of truth (T) and falsity (F). A Neutrosophic Single-Valued Number (SVNN) is represented as a tuple $N = (t, i, f)$,where $t, i, f \in [0,1]$ and $0 \leq t + i + f \leq 3$.This structure is ideal for modeling an expert's opinion on a policy [11,12]:

- **t** : Degree of agreement or perception of effectiveness.
- **i** : Degree of indeterminacy, doubt or lack of information.
- **f** : Degree of disagreement or perception of ineffectiveness.

The Neutrosophic STATIS method proposed here does not reinvent the sequential logic of STATIS (Interstructure, Commitment, Intrastructure), but rather reformulates it, replacing algebraic operations in the field of real numbers with their analogs in the space of neutrosophic numbers. The step-by-step procedure is detailed below.

Step 0: Construction of the Neutrosophic Data Matrices

The starting point is data collection. For this study, there are K tables (in this case, K = 3 evaluation moments). Each table X_k is a matrix of dimension $(n \times p)$, where n is the number of policies (observations) and p is the number of experts (variables). Unlike the classic case, each element $x_{ij_k}$of the matrix is not a real number, but a Single-Valued Neutrosophic Number $N_{ij_k} = \left(t_{ij_k}, i_{ij_k}, f_{ij_k}\right)$,representing the evaluation of expert j on policy i at time k [13].

$$X_k^n = \begin{bmatrix} x_{11_k} & x_{12_k} & \cdots & x_{1p_k} \\ x_{21_k} & x_{21_k} & \cdots & x_{2p_k} \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots \\ x_{n1_k} & x_{n1_k} & \cdots & x_{np_k} \end{bmatrix}$$

Step 1: Analysis of the Neutrosophic Interstructure

The objective of this first step is to quantify the similarity or proximity between the opinion structures at different evaluation moments. In classic STATIS, this is achieved by calculating the RV coefficient between the dot product matrices (W k). In the neutrosophic domain, we must follow an analogous process [14].

1. Calculation of Neutrosophic Scalar Product Matrices ($Wk\hat{}N$):

For each neutrosophic data table $Xk\hat{}N$, its dot product matrix is computed. This matrix, of dimension $(n \times n)$, captures the relationship between the policies. The element (i ,j) of the matrix $Wk\hat{}N$is computed as the neutrosophic dot product between row i and row j of the matrix $Xk\hat{}N$. The dot product between two neutrosophic vectors $A = (Na1, \ldots, Nap)$ and $B = (Nb1, \ldots, Nbp)$ is defined as:

$$\langle A, B\rangle_N = \sum_{l=1}^{p} (N_{al} \otimes N_{bl}) \quad (8)$$

Where the multiplication of two neutrosophic numbers $N_1 = (t_1, i_1, f_1) y\, N_2 = (t_2, i_2, f_2)$is:

$$N_1 \otimes N_2 = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2) \quad (9)$$

This gives us a matrix $Wk\hat{}N$whose elements are all neutral numbers.

2. Calculation of the Similarity Matrix (RV^N Coefficient):

To measure the similarity between two evaluation moments (e.g. T1 and T2), a Neutrosophic RV coefficient is used. This coefficient generalizes the classical RV and is calculated between two neutrosophic scalar product matrices [15], $Wk\hat{}N$ and $Wk'\hat{}N$. Its formula is:

$$RV^N (W_k^N, W_k'^N) = \frac{\left(cov_{N(W_k^N, W_k'^N)}\right)}{\sqrt{var_{N(W_k^N)} \cdot var_{N(W_k'^N)}}} \quad (10)$$

Where cov N and var N are the covariance and variance operations adapted to the neutrosophic domain. The result of this calculation is a neutrosophic number indicating the degree of similarity (t), indeterminacy in similarity (i), and dissimilarity (f) between the two structures. Performing this calculation for all pairs of moments yields a neutrosophic similarity matrix $S\hat{}N$of dimension $(K \times K)$.

Step 2: Calculating Neutrosophic Commitment

The commitment represents the average or consensus opinion structure across all the evaluated moments [16]. It is constructed as a weighted linear combination of the individual neutrosophic scalar product matrices.$(Wk\hat{}N)$.

Obtaining the Weights (α_k):

The weights α_k that determine the importance of each moment in building consensus are derived from the inter-structure. A principal components analysis is performed on the similarity matrix S^N (previously deneutrosified using a score function in order to operate it with classical methods). The components of the first eigenvector of this matrix, normalized to sum to 1, are used as the weights α_k. A high weight α_k indicates that the opinion structure at moment k is very similar to that of the remaining moments, being therefore more representative of the general consensus [17].

Construction of the Neutrosophic Commitment Matrix (W_{C^N}):

The commitment is calculated as:

$$WC^N = \sum_{k=1}^{k} \alpha_k \, Wk^N \quad (11)$$

This operation involves multiplying a real scalar (α_k) by a neutrosophic matrix and summing the neutrosophic matrices. The result is a single matrix of dimension $\alpha\ (n \times n)$,where each element is a neutrosophic number representing the "consensual" relationship between each pair of policies.

Step 3: Analysis of the Neutrosophic Intra -structure

This final stage consists of analyzing the structure of the commitment and projecting the original elements (policies and experts at each moment) onto this consensus space to visualize their relative positions and trajectories [18].

Spectral Analysis of Commitment:

In order to perform a Principal Component Analysis (PCA) on the trade-off matrix W_{C^N}, it is first necessary to convert it into a data-accurate matrix. This is achieved by applying a score function to each neutrosophic element of the matrix. Function common is:

$$Score(N) = \frac{(2 + t - i - f)}{3} \quad (12)$$

(To insert into Word: Use parentheses and fraction from the Equation menu)

This function maps each tuple (t ,i,f) to a single real value in [-1,1], where high values indicate a high degree of agreement/certainty and low values indicate disagreement/falsehood. The result is a deneutrosified or “crisp " matrix W_C.

Obtaining the Consensus Space:

A PCA is performed on the deneutrosified commitment matrix W_C. The spectral decomposition of W_C provides the eigenvalues (Λ), which indicate the inertia explained by each axis, and the eigenvectors (U), which define the factor axes of the consensus space. The coordinates of the policies in this consensus space are obtained directly from the eigenvectors, allowing the creation of the commitment map [19].

Projection and Analysis of Trajectories:

Finally, to visualize how the perception of each policy evolves, the original tables for each time point (X k^N) are projected onto the consensus space defined by U. This allows the position of the same policy to be drawn at T1, T2 and T3, joining the points to create its trajectory. Similarly, the stability of experts can be analyzed by projecting their opinion profiles onto the common space. This visual analysis is one of the richest results of the method, since it graphically shows the dynamics of the opinion system [20].

3. Application and Results

To demonstrate the power and added value of the Neutrosophic STATIS method, it was applied to a longitudinal case study designed to capture the dynamics of expert opinion on sustainability policies. This section details the case design and presents and interprets the results obtained at each of the three stages of the analysis: inter-structure, engagement, and intra -structure.

3.1. Case Study Description

The study focused on the evaluation of **ten key sustainability policies,** carried out by a panel of **eight experts** at three different points in time:

Panel of Experts

A multidisciplinary panel was formed to ensure a diversity of perspectives, consisting of:

- **Three academics** specializing in environmental sciences, economics and sociology
- **Three consultants and public policy managers** with experience in energy, urban planning, and the industrial sector
- **Two directors of non-governmental organizations** focused on conservation and the circular economy

Panel diversity is critical to capturing a realistic spectrum of opinions.

Evaluated Policies

The ten policies were selected to cover different areas of sustainability, with varying levels of complexity, visibility, and type of impact (incentive vs. regulation). They were grouped as follows:

Energy Transition:

- P1: Subsidies for the installation of photovoltaic solar energy
- P2: Energy efficiency standards for buildings
- P3: Tax national carbon

Circular Economy:

- P4: Ban on single-use plastics
- P5: Deposit, Return, and Refund (DRR) system for containers
- P6: Extended Producer Responsibility Act (EPR)

Urban Sustainability and Mobility:

- Cycle path expansion plan
- P8: Implementation of urban congestion charging

Conservation and Biodiversity:

- P9: Program national reforestation
- P10: Creation of new marine protected areas

Evaluation Moments

The evaluations were carried out at three time points:

- **T1 (Baseline):** Before the implementation of a new legislative package promoting these policies. Opinions here are based on expectations, theory, and prior experience.

- **T2 (Follow-up):** Six months after implementation. Feedback reflects initial evidence, initial challenges, and short-term impacts.
- **T3 (Annual Evaluation):** One year after implementation. Opinions are based on more solid evidence and a more stable view of the effects of each policy.

For each policy and at each time, the experts issued their evaluation using a neutrosophic scale, assigning values of t (degree of perceived effectiveness), i (degree of indeterminacy or doubt) and f (degree of perceived ineffectiveness).

Table 1: Assessment Data at T1 (Baseline)

At this initial stage, opinions are based on theoretical expectations, showing optimism regarding incentive policies and a high degree of uncertainty regarding those of a systemic nature.

Table 2: Assessment Data at T2 (6 Month Follow-up)

Policy	E1 (Acad)	E2 (Acad)	E3 (Acad)	E4 (Manager)	E5 (Manager)	E6 (Manager)	E7 (NGO)	E8 (NGO)
P1	(0.8,0.2,0.1)	(0.7,0.2,0.2)	(0.8,0.3,0.1)	(0.7,0.3,0.1)	(0.8,0.2,0.1)	(0.7,0.2,0.2)	(0.9,0.1,0.1)	(0.8,0.1,0.1)
P2	(0.7,0.2,0.1)	(0.8,0.2,0.1)	(0.7,0.3,0.2)	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.8,0.2,0.1)	(0.8,0.2,0.1)	(0.7,0.3,0.1)
P3	(0.8,0.1,0.2)	(0.7,0.2,0.3)	(0.2,0.2,0.7)	(0.3,0.2,0.6)	(0.2,0.3,0.7)	(0.7,0.2,0.2)	(0.3,0.1,0.8)	(0.2,0.2,0.7)
P4	(0.6,0.4,0.2)	(0.5,0.5,0.3)	(0.6,0.5,0.2)	(0.7,0.3,0.2)	(0.5,0.4,0.3)	(0.6,0.5,0.2)	(0.7,0.4,0.1)	(0.6,0.5,0.2)
P5	(0.5,0.5,0.3)	(0.6,0.5,0.2)	(0.4,0.6,0.3)	(0.5,0.5,0.2)	(0.4,0.6,0.4)	(0.5,0.5,0.3)	(0.6,0.4,0.2)	(0.5,0.6,0.3)
P6	(0.4,0.6,0.3)	(0.5,0.6,0.2)	(0.4,0.7,0.3)	(0.3,0.6,0.4)	(0.4,0.7,0.3)	(0.5,0.6,0.3)	(0.5,0.5,0.2)	(0.4,0.7,0.4)
P7	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.7,0.3,0.2)	(0.8,0.2,0.1)	(0.7,0.2,0.2)	(0.8,0.2,0.1)	(0.9,0.1,0.0)	(0.8,0.2,0.1)
P8	(0.7,0.2,0.3)	(0.6,0.3,0.4)	(0.3,0.3,0.6)	(0.4,0.2,0.5)	(0.3,0.4,0.7)	(0.6,0.2,0.3)	(0.4,0.3,0.6)	(0.3,0.3,0.7)
P9	(0.7,0.3,0.1)	(0.8,0.2,0.1)	(0.7,0.4,0.2)	(0.8,0.3,0.1)	(0.6,0.4,0.2)	(0.7,0.3,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.1)
P10	(0.8,0.3,0.1)	(0.7,0.3,0.2)	(0.8,0.4,0.1)	(0.7,0.4,0.2)	(0.8,0.3,0.1)	(0.7,0.3,0.1)	(0.9,0.2,0.0)	(0.8,0.2,0.1)

After six months, opinions change dramatically. Implementation problems arise for some policies (f increases), while others show rapid success (t increases). The overall structure is very different from T1.

Table 3: Evaluation Data at T3 (Annual Evaluation)

Policy	E1 (Acad)	E2 (Acad)	E3 (Acad)	E4 (Manager)	E5 (Manager)	E6 (Manager)	E7 (NGO)	E8 (NGO)
P1	(0.7,0.3,0.2)	(0.6,0.3,0.3)	(0.7,0.4,0.2)	(0.6,0.4,0.3)	(0.7,0.3,0.2)	(0.6,0.3,0.3)	(0.8,0.2,0.2)	(0.7,0.2,0.2)
P2	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.2,0.1)
P3	(0.8,0.1,0.3)	(0.7,0.2,0.4)	(0.2,0.2,0.8)	(0.3,0.2,0.7)	(0.2,0.3,0.8)	(0.7,0.2,0.3)	(0.3,0.1,0.8)	(0.2,0.2,0.8)

Policy	E1 (Acad)	E2 (Acad)	E3 (Acad)	E4 (Manager)	E5 (Manager)	E6 (Manager)	E7 (NGO)	E8 (NGO)
P4	(0.5,0.4,0.4)	(0.4,0.5,0.5)	(0.5,0.5,0.4)	(0.6,0.3,0.4)	(0.4,0.4,0.5)	(0.5,0.5,0.4)	(0.6,0.4,0.3)	(0.5,0.5,0.4)
P5	(0.4,0.5,0.5)	(0.5,0.5,0.4)	(0.3,0.6,0.5)	(0.4,0.5,0.4)	(0.3,0.6,0.6)	(0.4,0.5,0.5)	(0.5,0.4,0.4)	(0.4,0.6,0.5)
P6	(0.3,0.6,0.5)	(0.4,0.6,0.4)	(0.3,0.7,0.5)	(0.2,0.6,0.6)	(0.3,0.7,0.5)	(0.4,0.6,0.5)	(0.4,0.5,0.4)	(0.3,0.7,0.6)
P7	(0.9,0.1,0.1)	(0.9,0.1,0.0)	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.0)	(0.9,0.1,0.1)
P8	(0.7,0.2,0.4)	(0.6,0.3,0.5)	(0.3,0.3,0.7)	(0.4,0.2,0.6)	(0.3,0.4,0.7)	(0.6,0.2,0.4)	(0.4,0.3,0.6)	(0.3,0.3,0.7)
P9	(0.8,0.2,0.1)	(0.8,0.2,0.1)	(0.7,0.3,0.2)	(0.8,0.2,0.1)	(0.7,0.3,0.2)	(0.7,0.2,0.1)	(0.9,0.1,0.1)	(0.8,0.2,0.1)
P10	(0.8,0.2,0.1)	(0.7,0.2,0.2)	(0.8,0.3,0.1)	(0.7,0.3,0.2)	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.9,0.1,0.0)	(0.8,0.1,0.1)

After a year, opinions stabilize. Successful policies are consolidated, while complex ones continue to generate doubt. The structure of this table is very similar to that of T2, reflecting the stabilization of consensus.

Table 4: Neutrosophic Assessment of Sustainability Policies at the One-Year Evaluation (T3)

Policy	E1 (Acad)	E2 (Acad)	E3 (Acad)	E4 (Manager)	E5 (Manager)	E6 (Manager)	E7 (NGO)	E8 (NGO)
P1	(0.9,0.1,0.1)	(0.8,0.1,0.1)	(0.9,0.2,0.1)	(0.8,0.2,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.2)	(0.9,0.1,0.1)	(0.9,0.1,0.1)
P2	(0.8,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.1)	(0.9,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.1)
P3	(0.8,0.1,0.3)	(0.7,0.2,0.4)	(0.2,0.2,0.8)	(0.3,0.2,0.7)	(0.2,0.3,0.8)	(0.6,0.2,0.4)	(0.3,0.1,0.8)	(0.2,0.2,0.8)
P4	(0.5,0.4,0.3)	(0.4,0.5,0.4)	(0.5,0.5,0.3)	(0.6,0.3,0.3)	(0.4,0.4,0.4)	(0.5,0.5,0.3)	(0.6,0.4,0.2)	(0.5,0.5,0.3)
P5	(0.4,0.5,0.4)	(0.5,0.5,0.3)	(0.3,0.6,0.4)	(0.4,0.5,0.3)	(0.3,0.6,0.5)	(0.4,0.5,0.4)	(0.5,0.4,0.3)	(0.4,0.6,0.4)
P6	(0.4,0.6,0.4)	(0.5,0.6,0.3)	(0.4,0.7,0.4)	(0.3,0.6,0.5)	(0.4,0.7,0.4)	(0.5,0.6,0.4)	(0.5,0.5,0.3)	(0.4,0.7,0.5)
P7	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.8,0.1,0.1)	(0.9,0.1,0.0)	(0.8,0.1,0.1)	(0.9,0.1,0.0)	(0.9,0.1,0.0)	(0.9,0.1,0.0)
P8	(0.7,0.2,0.4)	(0.6,0.3,0.5)	(0.3,0.3,0.7)	(0.4,0.2,0.6)	(0.3,0.4,0.7)	(0.6,0.2,0.4)	(0.4,0.3,0.6)	(0.3,0.3,0.7)
P9	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.7,0.1,0.1)	(0.9,0.1,0.1)	(0.8,0.1,0.1)
P10	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.9,0.1,0.0)	(0.8,0.1,0.1)

3.2. Inter-structure Analysis: The Dynamics of Global Consensus

The first stage of the analysis quantifies the structural relationship between the three evaluation moments. RVN coefficients were calculated between the neutrosophic scalar product matrices at each moment, yielding a similarity matrix that reveals the convergence or divergence of overall opinion over time.

The results of the RV coefficients (after deneutrification using a scoring function) are presented in Table 4 and visualized in Figure 1.

Table 5. Matrix of RV Coefficients between the Evaluation Moments

	T1 (Baseline)	**T2 (6 Months)**	**T3 (1 Year)**
T1 (Baseline)	1.00	0.48	0.51
T2 (6 Months)	0.48	1.00	**0.87**
T3 (1 Year)	0.51	**0.87**	1.00

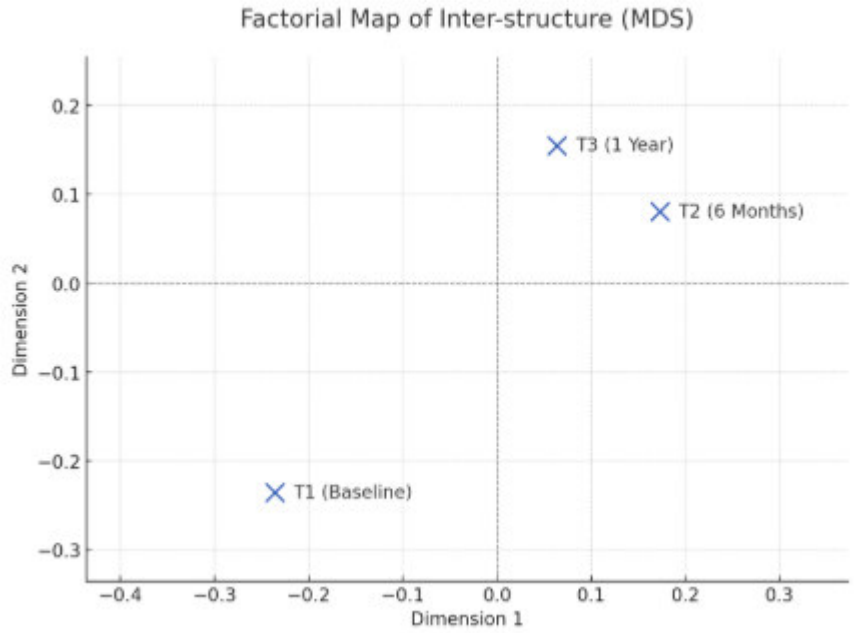

Figure 1. Representation of the interstructure in the first factorial plane. Points T1, T2, and T3 represent the three assessment moments.

Interpretation of Results

The interpretation of these results is unequivocal. Low similarity is observed between the initial assessment (T1) and the post-implementation assessments (T2 and T3), with RV coefficients of 0.48 and 0.51, respectively. In contrast, the similarity between T2 and T3 is very high (RV=0.87).

Figure 1 clearly illustrates this dynamic: point T1 is isolated on the factorial plane, while T2 and T3 appear very close to each other. This indicates that **a significant structural shift occurred in expert opinion after the first six months of implementation, but after this initial adjustment, the general opinion stabilized.** Theoretical expectations (T1) gave way to a new perception based on empirical evidence (T2), which was consolidated and slightly refined at the end of the year (T3).

3.3. Commitment Analysis: The Consensus Policy Map

The commitment matrix, which represents the consensus opinion structure over time, was calculated as a weighted average of the matrices from each time point. In line with the interstructure results, T2 and T3 received greater weight in its construction. Principal components analysis of this deneutrosified commitment matrix yielded a first factorial plane that explains 78.5% of the total inertia, providing a robust and highly representative map of expert perceptions.

Chart 2 shows the position of the ten policies in this consensus space.

Figure 2. Commitment Map of sustainability policies. Axis 1 (horizontal) represents the 'Perceived Effectiveness' dimension. Axis 2 (vertical) represents the 'Complexity and Tangibility ' of implementation.

Map Interpretation

Interpretation of the map reveals coherent and significant groupings:

Axis 1 (Horizontal - 51.2% inertia) : This axis clearly separates policies based on their **perceived effectiveness** . To the right are the policies seen as the most effective and with the greatest positive consensus. To the left are the most controversial or perceived as ineffective.

Axis 2 (Vertical - 27.3% inertia) : This axis can be interpreted as a gradient of **complexity and tangibility** . Policies requiring direct implementation and more tangible results are positioned downward. Policies with a more systemic, complex, and long-term results are positioned upward.

Quadrant Analysis

The quadrant analysis reveals four policy profiles:

1. **Lower Right Quadrant (Tangible Success)** : Here, P1 (Solar Subsidies), P2 (Energy Efficiency), and P7 (Bike Lanes) are grouped. These are perceived as the most successful

and least complex policies. Their nature is to offer direct incentives or infrastructure (carrots), generating a high level of consensus.

2. **Upper Right Quadrant (Effective Complexity)** : In this area, we find P9 (Reforestation) and P10 (Marine Protected Areas). They are considered effective, but their implementation is more complex and their benefits are more long-term, which justifies their high position on the vertical axis.
3. **The Left Axis (Controversial Policies)** : Isolated on the left side of the map are P3 (Carbon Tax) and P8 (Urban Toll). Their position indicates strong controversy. These are "stick" policies that, despite their potential theoretical effectiveness, generate little consensus and a negative or divided perception among experts.
4. **The Central Zone (Systemic Uncertainty)** : Notably, the three circular economy policies—P4 (Plastics Ban), P5 (SDR), and P6 (REP Law)—are located close to the origin of the map. This central position indicates a **lack of consensus and persistently high indeterminacy**. They are seen as neither a clear success nor a clear failure; their systemic complexity and implementation challenges keep expert opinion uncertain.

3.4. Trajectory Analysis: The Evolution of Perception

Projecting the coordinates of each policy at the three moments onto the commitment map allows us to visualize their individual trajectories. Figure 3 shows the paths followed by three representative policies.

Figure 3. Graph of selected policy trajectories on the commitment plane. The arrows indicate the movement from T1, through T2, to T3.

Revealing Trajectories

The trajectories tell revealing stories about how the perception of each initiative evolved:

P1's Trajectory (Solar Energy Subsidies): "Consolidation of Success" This policy begins in the positive quadrant in T1. In T2, it suffers a slight setback (movement toward the origin), which can be interpreted as a reflection of the initial bureaucratic challenges. However, in T3, it exhibits a sharp shift to the extreme right, consolidating its position as the most successful policy. Its initial uncertainty dissipated, giving way to a certainty of effectiveness.

P6's Trajectory (EPR Law): "Persistent Complexity" The Extended Producer Responsibility Law begins in T1 near the origin, with high indeterminacy. In T2, it shifts toward the left quadrant, indicating that implementation problems generated an initial negative perception. In T3, the policy returns to the center, but fails to enter positive territory. Its trajectory confirms that, even after one year, it remains a policy dominated by uncertainty and a lack of clear consensus on its effectiveness.

P3 (Carbon Tax) Trajectory: "The Stable Controversy" This policy trajectory is the shortest in terms of displacement. It remains firmly anchored to the far left of the map at all three points in time. This means that the policy was controversial from the outset and remained so throughout the evaluation period. The method not only reveals change but also the **stability of dissent**.

4. Conclusion

This study has demonstrated the ability of the Neutrosophic STATIS method to perform a deep and nuanced analysis of the evolution of expert opinion over time. The findings successfully revealed the complex dynamics of consensus, going beyond a simple snapshot to show a process of adjustment, stabilization, and persistence of uncertainty. Growing agreement was identified on the effectiveness of energy transition and conservation policies, which benefited from tangible outcomes. Simultaneously, structural and persistent uncertainty was noted around circular economy policies, the systemic nature of which presents challenges that keep expert opinion divided.

The main advantage of the proposed method is its ability to integrate uncertainty as a key component of the analysis, not as noise to be eliminated. It enables a robust longitudinal analysis of subjective and multi -source data, providing a clear view of both the general consensus and the individual evolution of each policy and expert. Visualizations of engagement and trajectories become powerful diagnostic tools for decision-makers, allowing them to identify which policies are gaining acceptance, which generate persistent controversy, and where a communication or redesign effort is most needed to dispel uncertainty.

Future Lines of Research

As future lines of research, we propose combining the Neutrosophic STATIS method with unsupervised clustering techniques. This would allow experts to be automatically segmented into

groups (e.g., "optimists," "skeptics," "undecided") and analyze how the composition and opinions of these clusters change over time, adding another layer of depth to the analysis of consensus dynamics.

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Analysis of hydraulic behavior in pressurized systems with different automation mechanisms using Neutrosophic SuperHyperSoft sets

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Abstract. Residential pressurization has significantly influenced energy consumption, particularly in systems with low-power pumps. Therefore, this study assessed the hydraulic and energy performance of three configurations: a centrifugal pump with presscontrol, a hydropneumatic system, and a hybrid system, structured using Neutrosophic SuperHyperSoft Sets. The results indicated that the presscontrol system exhibited the highest efficiency (22.48%), followed by the hybrid system (21.62%) and the hydropneumatic system (20.17%), with minimal differences in operational flow rates. It was concluded that although these systems do not meet high-demand scenarios, they are efficient for most common sanitary fixtures, optimizing energy consumption and sustainability in residential and industrial environments. Future studies are recommended to evaluate attributes with varying degrees of neutrosophic membership to validate their technical, economic, and environmental feasibility under different demand conditions.

Keywords: Hydraulic performance, Energy efficiency, Technical evaluation, Residential hydraulic systems, Neutrosophic SuperHyperSoft sets.

1 Introduction

Residential pumping systems are a significant component of residential energy consumption, primarily due to their low operational efficiency, which in many cases does not exceed 20%. This inefficiency is often addressed through technical regulations and energy assessments aimed at guiding corrective actions such as equipment repair or replacement [1]. Various studies have demonstrated that energy efficiency depends on factors such as dynamic flow, the thermal inertia of materials, and the proper selection of pumps with optimal efficiency rates between 50% and 60% [2]. Exergy evaluation enables the identification of energy losses in both residential and industrial systems.

Therefore, traditional pumping station designs are typically based on multiple parameters, whereas more recent methodologies propose multicriteria analysis [3]. These approaches incorporate technical, economic, and environmental factors, thereby improving equipment selection and system efficiency under varying operational conditions [4]. Consequently, in areas where water pressure is insufficient due to factors such as topography or the distance between residences and reservoirs, pressurization systems become convenient. In fact, these systems are designed to maintain and regulate pressure in distribution networks. A common type employs a pressure switch combined with a hydropneumatic tank, which maintains pressure within a specific range and controls flow according to demand. However, such systems often exhibit failures, such as dry running, which occurs when water is absent in the pipeline, yet the pressure switch activates the pump due to low pressure, potentially damaging the

motor.

Given this situation, the present study evaluated the hydraulic and energy performance of residential pressurization systems using a 0.5 HP centrifugal pump in three configurations: presscontrol, hydropneumatic system, and a hybrid system. The research aims to assess these systems' parameters through a structured analysis employing Neutrosophic SuperHyperSoft Sets under different power sets across technical, economic, environmental, and operational-contextual dimensions. Furthermore, the Neutrosophic SuperHyperSoft Sets modeling enables the integration of multiple attributes to optimize each evaluated parameter or parameter pair. Ultimately, this work advances the development of more reliable and efficient pressurization systems with applications in residential, commercial, and industrial environments [5].

2 Preliminaries

2.1 Neutrosophic SuperHyperSoft Sets.

This section serves the purpose of remembering the basic notions of Fuzzy Extension SuperHyperSoft Sets and neutrosophic theory [6].

Definition 1 ([1, 9, 10]**).** Given U is the initial universe set and E is the set of parameters. A pair (F, E) is called a *soft set* (over U) if and only if F is a mapping of E into the set of all subsets of U.

That is to say, having a set E of parameters and fixing a parameter $\varepsilon \in E$, then $F(\varepsilon) \in \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the power set of U and $F(\varepsilon)$ is considered the set of ε-elements of the Soft Set (F, E) or the set of ε-approximate elements of the Soft Set [7] [8].

It is not difficult to realize that fuzzy sets are soft sets, this is a consequence of the α-levels definition of a membership function μ_A, which yields the following:

$F(\alpha) = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $\alpha \in [0, 1]$. Thus, if the family F is known, then the function μ_A by may be derived using the following formula:

$$\mu_A(x) = \sup_{\substack{\alpha \in [0,1] \\ x \in F(\alpha)}} \alpha$$

Thus, a fuzzy set is a $(F, [0, 1])$ soft set.

Given a binary operation * for subsets of the set U, where (F, A) and (G, B) are soft sets over U. Then, the operation * for soft sets is defined as follows:

$(F, A) * (G, B) = (J, A \times B)$, where $J(\alpha, \beta) = F(\alpha) * G(\beta)$; $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Definition 2 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(F, A_1 \times A_2 \times \ldots \times A_n)$, where: $F: A_1 \times A_2 \times \ldots \times A_n \to \mathcal{P}(U)$ is called a *HyperSoft Set* over U.

Definition 3 ([2, 3, 11, 12])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \to \mathcal{P}(U)$ is called a *SuperHyperSoft Set* over U.

Definition 4 ([4, 5, 13-15])**.** Let U be a universe set, $\mathcal{P}(U)$ the power set of U. Let $a_1, a_2, \ldots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $\big(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n)\big)$, where:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}\left(\mathrm{U}\left(x(d^0)\right)\right)$ is called a *Fuzzy Extension SuperHyperSoft Set* over U.

Where $x(d^0)$ is the fuzzy or any fuzzy extension degree of appurtenance of the element x to the set U. Fuzzy extension means Fuzzy Set or Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Fermatean Fuzzy Set, Neutrosophic Fuzzy Set, Plithogenic Fuzzy Set, etc [9].

Before concluding, let us recall some fundamental definitions of neutrosophic sets:

Definition 5 ([8]). The *Neutrosophic set* N is characterized by three membership functions [10], which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [11].

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^-0$ and 1^+ belong to the set of hyperreal numbers.

Definition 6 ([8, 16, 17]). The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \rightarrow [0, 1]$, $I_A: U \rightarrow [0, 1]$ and $F_A: U \rightarrow [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

2.2 Equipment, materials, and methods for calculating the selected parameters

The materials used in this research are listed in Table 1, where the equipment and materials are described along with their characteristics and a visual record of each item.

Table 1: Equipment and materials used. Source: own elaboration.

Items	Characteristics	Figure
INGCO UVPM3708 Peripheral Pump	This is a special type of rotodynamic pump (see Figure 1), characterized by its ability to handle high head at a low flow rate. The specifications of the peripheral pump are presented in Table 2.	
Pressure Switch	This is an electromechanical-type electrical switch (see Figure 2). The 3781 water pressure switch comes factory-set with the following default values: Cut-in pressure: 4.9 bar (±0.3 bar), Cut-off pressure: 7.0 bar (±0.3 bar), Maximum ΔP: 2.1 bar. Additional specifications of the pressure switch are presented in Table 3.	
Presscontrol	The presscontrol device (see Figure 3) is designed to automatically regulate and control water pressure in hydraulic systems. It detects malfunctions and signals them via a red indicator (alarm), subsequently shutting off the pump to prevent damage caused by dry operation. The specifications of the presscontrol device are presented in Table 3 [12].	

Figure 1: INGCO UVPM3708 Peripheral Pump.

Figure 2: 3781 Water Pressure Switch.

Hydropneumatic Tank	These tanks contain compressed air and water; the compressed air acts as a spring that maintains constant pressure within the system by compensating for pressure fluctuations and ensuring stable flow (see Figure 4). The specifications of the 24-liter horizontal tank are presented in Table 3.
Flexible hose	This corrugated metal hose (see Figure 5) is flexible, corrosion-resistant, and offers high temperature resistance. It has a length of 60 centimeters, a one-inch male thread, and a one-inch female thread.
Five-Way Coupling	This is a bronze coupling with five distribution ports, featuring one-inch male and female threads, as well as male and female threads for connecting a pressure gauge and a pressure switch (see Figure 6).
Pressure Gauge	This instrument is used to quantify the pressure level of fluids within a closed circuit (see Figure 7). It measures the difference between the actual or absolute pressure and atmospheric pressure, which is referred to as gauge pressure.
Mesh-Type Network Module with Water Flow	This experimental module is designed to validate simulations of mesh-type networks in the fluid laboratory at the Technical University of Cotopaxi. It consists of half-inch, one-inch, and one-and-a-half-inch pipes, ball valves, pressure sensors, and a control panel that integrates: a Siemens 1200 PLC, Siemens HMI, 24V power supply, contactor, motor protection switch, position selector, and pilot light, as shown in Figure 8.
Portable Ultrasonic Flow Meter	This device has the capability to continuously monitor flow in hydraulic systems. The handheld TDS-100H ultrasonic flow meter (see Figure 9) is a battery-powered ultrasonic flow meter with the following specifications: accuracy better than 1%, 0.2% repeatability, LCD display with 4 lines of 16 English characters, operates with clamp-on transducers, RS-232 interface or isolated single-channel OCT output, pipe diameters ranging from 15 mm to 6000 mm, Ni-MH battery operation exceeding 12 hours, 90–230 VAC power supply, and internal data logging.

Figure 3: Presscontrol.

Figure 4: 24-Liter Horizontal Tank AQ24LH/AC with Carbon Steel Pump Base.

Figure 5: Flexible metal hose.

Figure 6: Five-Way Bronze Coupling.

Figure 7: Pressure Gauge from 0 to 100 Psi.

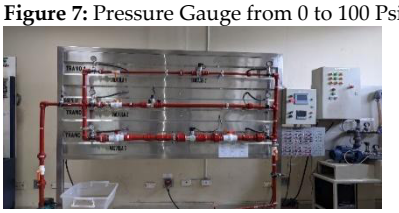

Figure 8: Mesh-Type Network Module with Water Flow from the Fluid Laboratory of the Technical University of Cotopaxi.

Figure 9: TDS-100H Handheld Ultrasonic Flow Meter.

Table 2: Specifications of the INGCO UVPM3708 0.5 HP pump. Source: https://www.ingco.com/product/water-pump/VPM3708.

Model No.	**UVPM3708**
Rated Voltage (V)	110-120~
Rated Frequency (Hz)	60
Phase	Single
Rated Output (kW / HP)	0,37 / 0,5
No-load Speed (rpm)	3450
Max. Head (m)	35
Max. Suction (m)	8
Max. Flow (l/min)	35
Inlet / Outlet (inches)	1x1

Table 3: Specifications of the 3781 pressure switch, presscontrol, and AQ24LH/AC Hydropneumatic tank. Source: own elaboration.

3781 Pressure switch		**Presscontrol**		**AQ24LH/AC hydropneumatic tank**	
Parameters	Magnitude	Parameters	Magnitude	Parameters	Magnitude
IP	64	Single-phase supply voltage	230V	Capacity L/gal	24/6,3
Connection voltage (V)	230	Frequency	50-60Hz	System	Interchangeable EPDM membrane
Maximum current consumption (A)	12	Maximum current	10(6)A	Maximum working pressure (Psi)	86
Maximum power (HP)	2	Protection rating	IP65	Maximum working temperature (°C)	99
Maximum pressure (Bar)	10	Maximum service pressure	10Bar	Precharge pressure (psi / bar)	21/1,5
Regulation range (Bar)	4 a 10	Maximum temperature in service	60ºC	Male NPT connection diameter (inches)	1
Maximum ΔP (Bar)	2,1	Connections	1st Male		
Maximum frequency (cycles/min)	60	Maximum power	1,5 C.V.		
Mechanical life cycle	200000				
Electrical life cycle	30000				
Differential	Variable				
Maximum temperature (°C)	50				

2.3 Minimum instantaneous flow rates according to NEC11 standard

Table 16.1 of flow demand, pressure, and diameters in consumption devices is presented in (Ministry of Urban Development and Housing, 2011). Since the minimum flow rate in the table, according to the standard, is given in L/s, a unit conversion to m^3/h was performed, resulting in Table 4.

Table 4: Flow demand in cubic meters per hour. Source: own elaboration.

Sanitary fixture	Minimum instantaneous flow rate (m^3/h)	Head pressure height (m)
Bathtub	1.08	7.0
Bidet	0.36	7.0
Water heaters / boilers	1.08	15.0
Shower	0.72	10.0
Kitchen sink	0.72	5.0
Drinking fountains	0.36	3.0
Hose bib	0.72	7.0
Toilet with tank	0.36	7.0
Flush-valve toilet	4.5	15.0
Washbasin	0.36	5.0
Washing machine	0.72	7.0
Dishwasher	0.72	7.0
Flush-valve urinal	1.08	15.0
Manual-flush urinal	0.54	7.0
Sauna, steam bath, or domestic whirlpool bath	3.6	15.0

2.4 Equations and mathematical content

Calculation of Head Height (m): This equation determines the water column height based on a given pressure (1).

$$H = \frac{P}{\rho \cdot g} \tag{1}$$

Where, H = head height (m), P = pressure (Pa), ϱ = fluid density (kg/m³), and g = acceleration due to gravity (m/s²).

Unit Conversion – Flow Rate: This conversion was used to express the volumetric flow rate measured in cubic meters per minute on an hourly scale. Additionally, a conversion from cubic meters per minute to cubic meters per second was performed to facilitate a better analysis of the data (2) (3).

$$Q' = Q \cdot 60 \qquad \text{Flow rate calculation Q } (m^3/min) \tag{2}$$

$$Q'' = \frac{Q}{60\ seg} \qquad \text{Flow rate calculation Q}''(m^3/s) \tag{3}$$

Calculation of Electrical Power: This relation was used to determine the electrical power consumed by all systems based on voltage and current measurements (4).

$$P = V \cdot I$$

P = Electrical power (W)
V= Voltage (V)
I = Current (A) (4)

Calculation of Hydraulic Power: This equation enabled the quantification of the hydraulic energy delivered by each system, considering the inlet pressure and the generated flow rate (5).

$$P_h = p_{in} \cdot Q''$$

P_h = Hydraulic power (W)
p_{in}= Inlet pressure (Pa)
Q'' = Flow rate (m^3/s) (5)

Efficiency Calculation: This formula was applied to evaluate the efficiency of all systems by comparing the useful hydraulic power with the electrical power consumed (6).

$$\eta = \left(\frac{P_h}{P_e}\right) \cdot 100\% \qquad \begin{array}{l}\eta = \text{System efficiency (\%)} \\ P_h = \text{Hydraulic power (W)} \\ P_e = \text{Electrical power (W)}\end{array} \tag{6}$$

PSI to Pascal Conversion Factor: This conversion was necessary to calculate the head without unit inconsistencies by transforming pressure from the International System of Units (SI), psi, to pascals (7)

$$p_{Pa} = p_{Psi} \cdot 6894{,}76 \qquad \begin{array}{l}p_{Pa} = \text{Pressure in Pascals} \\ p_{Psi} = \text{Pressure in Psi}\end{array} \tag{7}$$

Error Calculation: This equation was used to determine the percentage deviation between the head height of each pressurization system and the actual head provided by the pump (8).

$$ERROR = \frac{H_S - H_B}{H_S} \cdot 100 \qquad \begin{array}{l}\text{ERROR= Error (\%)} \\ H_S = \text{Head height of the evaluated system (m)} \\ H_B = \text{Head height of the pump (m)}\end{array} \tag{8}$$

3 Results

3.1 Designs, projects, and equipment acquisition and installation

Engineering projects constitute the means by which the desired states of the clients are transformed into concrete solutions, encompassing interventions ranging from small-scale residential works to mega-projects of transnational and intercontinental scope. Each project involves varying levels of complexity resulting from the interaction of multiple disciplines such as mechanical, hydraulic, process, civil, electrical, and automation engineering, among others. These projects enable the evaluation of designs and the measurement of quantifiable variables to ensure operation within their respective efficiency zones and to guarantee project success. Additionally, both complete data obtained through simulations and partial or indeterminate data, which often have decisive influence, must be considered. In this context, Neutrosophy provides an analytical tool to identify and classify parameter sets based on the client's desired state. For this purpose, the analysis is structured in the form of sets by organizing variables or attributes as power sets grouped into four main categories for this study:

- Power set of technical variables (A_i), where $(G, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \ldots \times \mathcal{P}(A_m))$
- Power set of economic variables (B_i), where $(G, \mathcal{P}(B_1) \times \mathcal{P}(B_2) \times \ldots \times \mathcal{P}(B_n))$
- Power set of environmental variables (C_i), where $(G, \mathcal{P}(C_1) \times \mathcal{P}(C_2) \times \ldots \times \mathcal{P}(C_p))$
- Power set of contextual and operational variables (D_i), where $(G, \mathcal{P}(D_1) \times \mathcal{P}(D_2) \times \ldots \times \mathcal{P}(D_q))$

Therefore, each power set is composed of a group of pairs, since each project does not incorporate all technical variables but only uses a certain number of pairs within this set according to the client's requirements or desires. For the purpose of this study, *the client's desire* is termed the *service request (SR)*, while the *client's requirement* is defined as the *technical or projection task (TT).* Once these two elements are defined, in the following pair $E: \{\{SS\}, \{TT\}\}$, each project can be identified by the following function:

$$f(E) \rightarrow \{\{A_i\} \cap \{B_i\} \cap \{C_i\} \cap \{D_i\}\}$$

For example, in an industrial installation requiring a pumping system, the client must provide a service request along with a technical task specifying the site conditions. This allows the project engineer or designer to determine which equipment and materials should be selected based on the simulated technical parameters $\{A_1\}$. Similarly, the selection of equipment is conditioned by economic variables $\{B_i\}$ and their structures formed by pairs as follows:

- $\{\{Installation\ costs\}, \{Local\ electricity\ rates\}, \{Maintenance\ costs\}, \{Life\ cycle\ cost - benefit\}\}$.

Likewise, the {water reservoirs} and their {environmental impact} on the ecosystem must be defined as neutrosophic pairs that are either fully or partially defined. Similarly, an intersection with the power set D_k may or may not coexist. In this case, it is observed that the inclusion of multiple pairs allows for defining and fulfilling the client's needs as determined in $\{E\}$, which possesses characteristics that are fully defined, partially defined, or indeterminate. Therefore, the pairs can be defined, partially defined, or undefined within the power set to which they belong. In fact, an element acting on these pairs is the experience of both the client and the designer or project engineer. Consequently, experience constitutes an element identified in multiple neutrosophic states (high experience, medium experience, low experience, little experience, and no experience). These neutrosophic states are present in each power set, represented, for example, as $(x_{b_i}(d^0))$ in the following pair function:

$$F\ (a_i\ \cap\ b_i) = \{P_i\}, \left\{\left(x_{a_1}(d^0)\right) \cap (x_{b_i}(d^0))\right\}$$

Therefore, each pair in the function is assigned a neutrosophic membership degree, which is determined by the level of knowledge and experience of both the client and the project engineer or designer. Thus, if the client has little experience, the parameter pairs found in $\{E\}$ would be classified as partially defined or undefined for certain domains. For example, if the client commissions a project defining a set of technical, environmental, and operational parameters in $\{E\}$, but only partially defines the economic parameters, then the resulting intersection $\{\{A_i\} \cap \{B_i\} \cap \{C_i\} \cap \{D_i\}\}$ would exhibit a neutrosophic degree that is partially indeterminate. In this case, the outcome could be a project solution that satisfies $\{\{A_i\} \cap \{C_i\} \cap \{D_i\}\}$, but might be very costly, and the client would lack funding to execute it. Therefore, the partial omission of economic variable parameters would compromise the engineering solution.

The following section presents a case study on the performance of a pumping system, involving the analysis of parameters such as energy efficiency, response to variations in water demand, and pressure stability under different usage conditions. This research is expected to contribute to the development of more reliable and efficient pressurization systems, with applications in residential, commercial, and industrial environments, as well as to the implementation of Neutrosophy in improving and selecting a viable design.

3.2 Case study

The analysis focused on the evaluation of three main configurations: the conventional pressurization system (hydropneumatic tank with pressure switch: $\{V\}$), the system with presscontrol ($\{W\}$), and the integration of both devices into a single combined system ($\{VW\}$). For each of these configurations, fundamental parameters or selected pairs from the set $\{A_i\}$ were recorded, as defined in the following function:

$f\{E\} \rightarrow \left\{\{\{\text{flow rate}\}, \{\text{operating pressure}\}, \{\text{current intensity}\}, \{\text{voltage}\}\}, \{B_i\}, \{C_i\}, \{D_i\}\right\}$ according to the request defined in $\{E\}$}. In fact, the client (residential, commercial, and industrial environments) requests the selection of the following pairs $\{\{V\}, \{W\}, \{VW\}\}$, with the aim of evaluating their performance and efficiency zones.

Another point to highlight in this study is that the information related to the sets $\{B_i\}, \{C_i\}, \{D_i\}$, is limited. Therefore, the neutrosophic degrees range from partially unknown to unknown and indeterminate. However, the impact on system selection is expected to show a favorable trend with respect to the economic variables $\{B_i\}$ (local electricity rates, installation costs, maintenance costs, and life cycle cost-benefit); environmental variables $\{C_i\}$: (availability and quality of the water resource); and contextual-operational variables $\{D_i\}$: (topology or distance of the residence from water reservoirs, comfort expectations, noise or autonomy, and variability in water demand).

3.2.1 Evaluation of parameters within the power set $\{A_i\}$

3.2.1.1 Performance curves of the evaluated pressurization systems

In order to experimentally validate the applied methodology and strengthen the comparative foundation of the study, the procedure was replicated using only the pump, without any control or pressurization elements. At this stage, a comparison was made between the performance curve provided by the manufacturer (catalog) and the performance curve obtained experimentally (see Figure 10). This comparison made it possible to verify the reliability of the measured data, ensuring that the graphs obtained in subsequent tests are properly aligned and accurately represent the real behavior of the system.

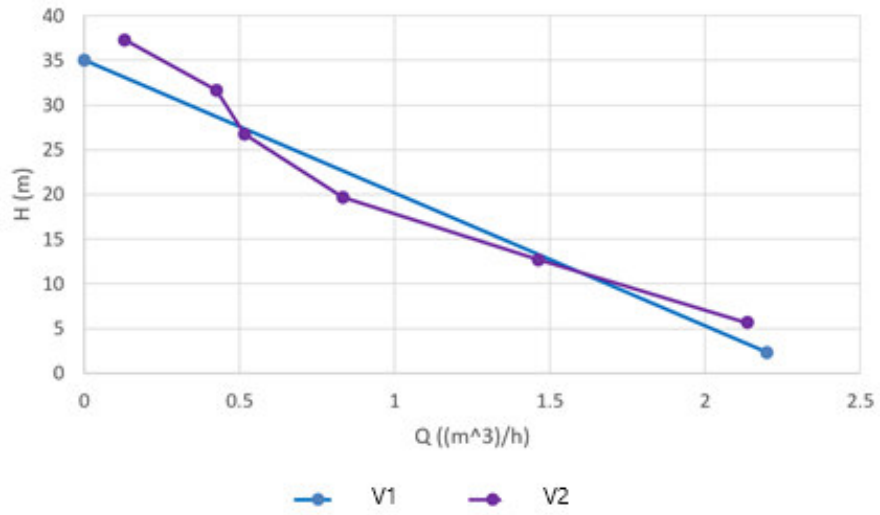

Figure 10: Performance curve of the INGCO UVPM3708 peripheral pump (v1: Catalog, v2: Pump). Source: own elaboration.

After conducting the corresponding tests, performance curves were obtained for each of the evaluated systems, as shown in Figure 11. Analyzing the curves for the pump and the presscontrol system reveals slight differences in flow rates between 0 and 0.5 m^3/h. The curve with presscontrol starts at a head of approximately 39 m, whereas the pump without presscontrol registers a maximum value of 37.25 m. From medium flow rates between 0.8 and 2.0 m^3/h, both curves tend to overlap, indicating that the hydraulic impact of the presscontrol becomes marginal. At flow rates near the maximum, the head converges toward values close to 6 m in both cases, demonstrating that the system maintains acceptable performance under high demand. This graph illustrates the effect of the presscontrol on the hydraulic performance of the pump, highlighting its influence under low-flow conditions and validating its efficiency within medium and high operating ranges. An average error of 0.86% was obtained, where it was observed that, when comparing the pump curve and the presscontrol curve, there was a deviation at low flow rates (0.1 to 0.4 cubic meters per hour), with the measured head being lower for the presscontrol system, yielding an error of 3.63%. For flow rates between 0.4 and 0.5 cubic meters per hour, the error was 1.53%.

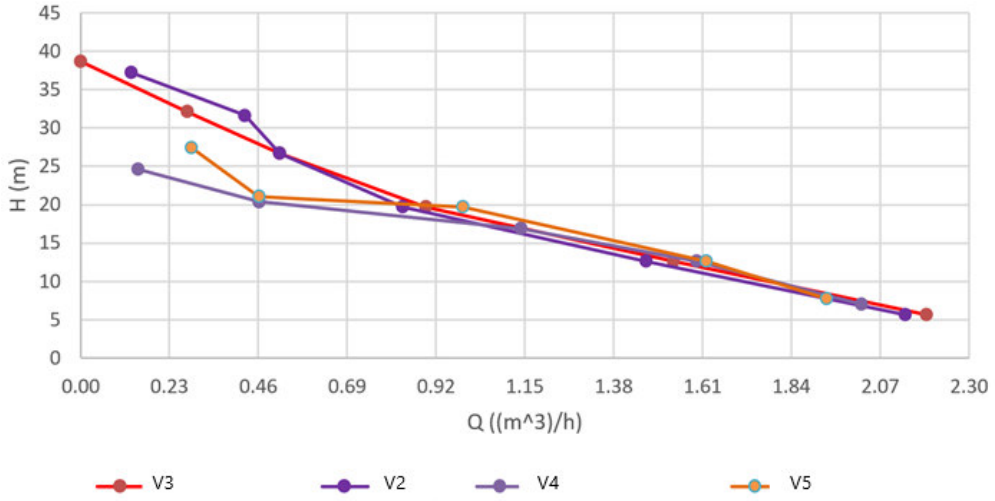

Figure 11: Performance curves of the evaluated pressurization systems (v2: Pump, v3: Presscontrol, v4: Hydropneumatic, v5: Combined). Source: own elaboration.

On the other hand, the performance curve of the hydropneumatic system, compared to the behavior of the pump, shows a lower head at low flow rates, reaching approximately 37.25 m at 0.1 m³/h for the pump. In contrast, the hydropneumatic system starts with a lower value of 24.6 m for the same flow rate. This difference is attributed to the delay in the activation of the pressure switch and the damping effect of the tank. As the flow rate increases, both curves decrease progressively, indicating a reduction in available pressure. At the intersection point of both curves, around 0.9 m³/h and 18 m of head, hydraulic equilibrium between the pump and the hydropneumatic system is reached, at which point the impact of the system becomes minimal. Another relevant aspect of the analysis occurs at flow rates close to 2 m³/h, where both curves converge at an approximate head of 6 m, confirming that under high demand conditions, no significant losses occur. At this point, the relative error reaches a specific value of 3.44%, while the global average error of the system is only 0.37%, indicating good performance of the hydropneumatic system under medium and high flow regimes.

The comparison between the behavior of the pump and the combined system reveals that the pump delivers a head of 37.25 m at a flow rate of 0.13 m³/h, while the combined system reaches 27.41 m at a flow rate of 0.29 m³/h, reflecting a significant pressure loss similar to that of the hydropneumatic system. However, the combined system presents a higher head. As the flow rate increases, both curves tend to converge; around 0.9 m³/h, they intersect at an equilibrium point with a head of approximately 18 m, which may indicate the system's optimal operating point. At higher flow rates, such as 1.9 m³/h, the difference between both curves is minimal: the pump delivers a head of 8 m, and the combined system, 7.7 m, corresponding to an error of 3.75%, with an average error of 0.63%. Overall, the average error along the entire curve is low, standing at 0.63%, which confirms the efficiency of the combined system in medium and high operating ranges.

3.2.1.2 Performance curves of the evaluated pressurization systems

Figure 12 presents the performance curve for each flow rate, showing that the INGCO 0.5 Hp pump exhibits a maximum efficiency of 20.35%, a minimum efficiency of 18.32%, and an efficient operating range between flow rates of 0.99 m³/h and 1.84 m³/h. Using the measured data from the hydropneumatic system, the performance curve was obtained (see Figure 13), with a maximum efficiency of 20.17%, a minimum efficiency of 18.15%, and an efficient operating range from a minimum flow rate of 1.25 m³/h

to a maximum flow rate of 1.88 m³/h for the hydropneumatic pressurization system to operate efficiently.

The performance curve of the system with a 0.5 HP pump and presscontrol, shown in Figure 14, exhibits a maximum efficiency of 22.48% and a minimum efficiency of 20.23%. From these data, the minimum and maximum flow rates for efficient system operation are determined as 1.02 m³/h and 1.86 m³/h, respectively. In contrast, Figure 15 presents the performance curve of the pressurization system combining the pump, hydropneumatic tank, and presscontrol, with a maximum efficiency of 21.62%, minimum efficiency of 19.46%, minimum flow rate of 1.09 m³/h, and maximum flow rate of 1.82 m³/h.

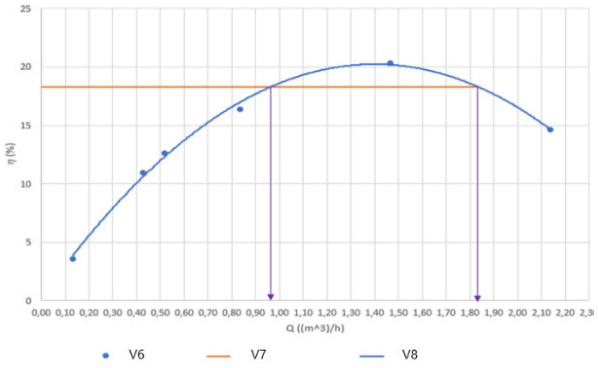

Figure 12: Performance curve of the 0.5 Hp pump, minimum efficiency, and flow rates of the efficient zone (v6: Efficiency, v7: Minimum efficiency, and v8: Polynomial (efficiency)). Source: own elaboration.

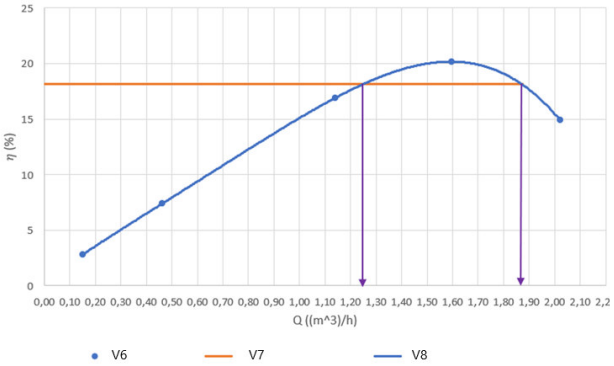

Figure 13: Performance curve of the hydropneumatic pressurization system, minimum efficiency, and flow rates of the efficient zone (v6: Efficiency, v7: Minimum efficiency, and v8: Polynomial (efficiency)). Source: own elaboration.

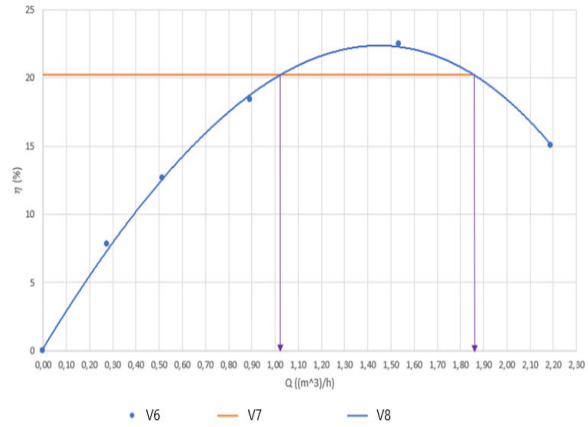

Figure 14: Performance curve of the pressurization system with pump and presscontrol, minimum efficiency, and flow rates of the efficient zone (v6: Efficiency, v7: Minimum efficiency, and v8: Polynomial (efficiency)). Source: own elaboration.

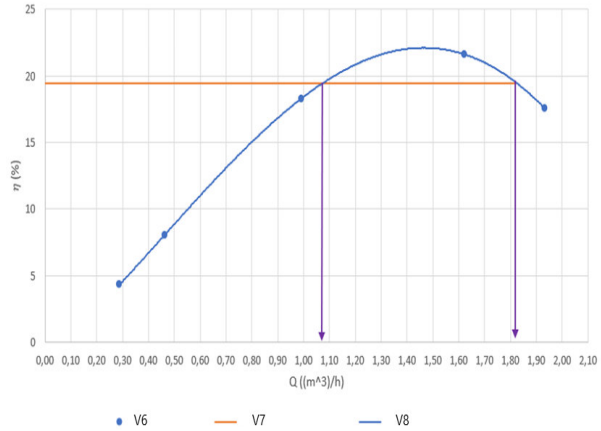

Figure 15: Performance curve of the system combining hydropneumatic tank and presscontrol, minimum efficiency, and flow rates at minimum efficiency (v6: Efficiency, v7: Minimum efficiency, and v8: Polynomial (efficiency)). Source: own elaboration.

3.2.1.3 Evaluation and comparison of flow rates within the operational efficiency zone

To determine the pressurization system with the best characteristics, a comparison was made between the operating flow rates within the maximum efficiency range of each analyzed configuration (Table 5). The evaluation was based on the minimum design flow rates required by various sanitary appliances according to the parameters established in Table 16.1 of the Ecuadorian Construction Standard NEC-11 (Ministry of Urban Development and Housing, 2011).

Table 5: Maximum and minimum flow rates within the efficient zone for the evaluated systems. Source: own elaboration.

System	Minimum Q (m³/h)	Maximum Q (m³/h)
Pump	0.99	1.84
Hydropneumatic	1.25	1.88
Presscontrol	1.02	1.86
Combined	1.09	1.82

Residential pressurization systems with a 0.5 Hp pump cover the demand of most sanitary appliances except for high-demand devices such as flushometer toilets, which require an instantaneous flow rate of 4.5 m³/h, and saunas, steam baths, or domestic hydromassage systems with a demand flow rate of 3.6 m³/h. However, for bidets, drinking fountains, tank toilets, and washbasins, which have a flow demand of 0.36 m³/h, as well as for urinals with valves requiring 0.54 m³/h, and for showers, kitchen sinks, hose faucets, washing machines, and dishwashers with a minimum instantaneous flow rate of 0.72 m³/h, the evaluated pressurization systems including the direct pump cover the demand. Nevertheless, these flow rates fall outside the efficient operating ranges, which were determined for each system in the performance curves. On the other hand, bathtubs, water heaters or boilers, and flushometer urinals, with a flow demand of 1.08 m³/h, are the only appliances within the efficient zone for two of the systems: direct pump and presscontrol (see Figures 16 and 17).

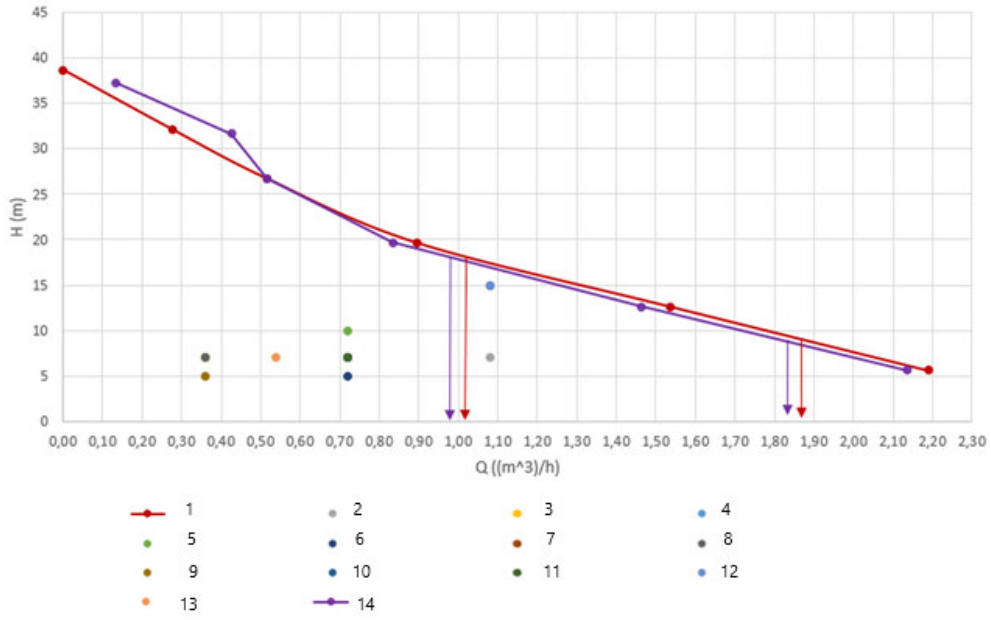

Figure 16: Performance curves of the pump and presscontrol with their efficient zones and sanitary appliance demand (1: Hydropneumatic, 2: Bathtub, 3: Bidet, 4: Boiler, 5: Shower, 6: Kitchen sink, 7: Hose bib, 8: Tank toilet, 9: Washbasin, 10: Washing machine, 11: Dishwasher, 12: Flush-valve urinal, 13: Manual-flush urinal, 14: Combined). Source: own elaboration.

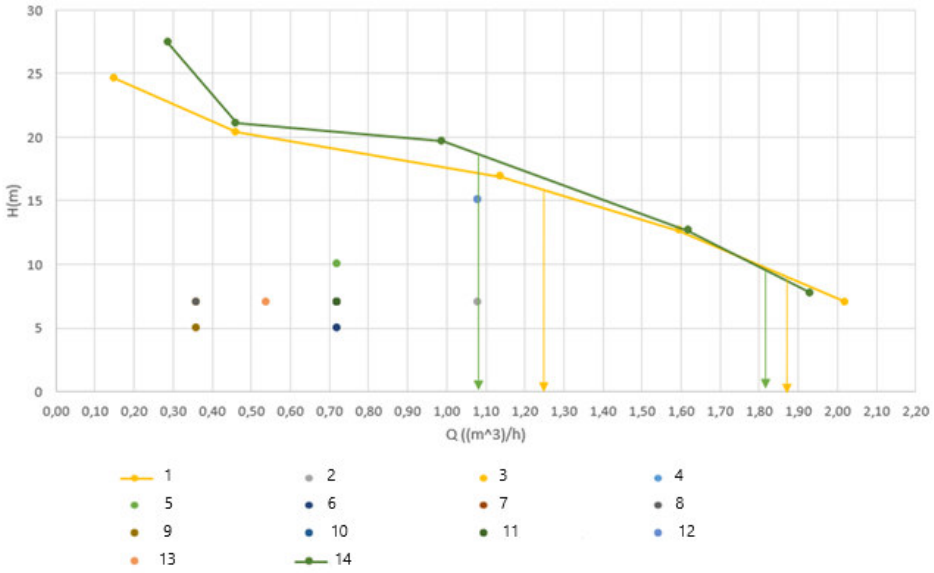

Figure 17: Performance curves of the hydraulic system and combined system with their efficient zones and sanitary appliance demand (1: Hydropneumatic, 2: Bathtub, 3: Bidet, 4: Boiler, 5: Shower, 6: Kitchen sink, 7: Hose bib, 8: Tank toilet, 9: Washbasin, 10: Washing machine, 11: Dishwasher, 12: Flush-valve urinal, 13: Manual-flush urinal, 14: Combined). Source: own elaboration.

3.2 Discussion

The evaluation of parameters within the power set $\{A_i\}$ allowed the identification of the highest efficiency in $\{W\}$, followed by $\{VW\}$ and finally $\{V\}$, all intended for residential use when operated within their efficient zones. Therefore, the application of Neutrosophic SuperHyperSoft Sets in this study facilitated the evaluation of power sets based on the selection of parameter pairs for identifying water supply systems with greater efficiency. However, the study presents limitations related to the power sets $\{B_i\}, \{C_i\}, \{D_i\}$. Consequently, for the technical selection or evaluation of equipment and materials, it is necessary to determine whether the budget or financing covers installation, maintenance, and applicable electricity tariffs that the client may incur $\{B_i\}$. Additionally, an analysis of attributes measuring environmental impact $\{C_i\}$ should be performed by examining how waste generated relates to the volumes of water used [13]. Moreover, other parameters should be evaluated, such as the client's satisfaction with the use of the optimally selected system $\{D_i\}$. Indeed, this indicates that $\{E\}$ does not always encompass the entirety or each specific requirement of the requesting user but rather depends on the neutrosophic membership degree of knowledge and experience of both the designer, evaluator, and the end client.

The study observed that the client's state of desire $\{E\}$ enables the identification and grouping of the power sets to be evaluated. Specifically, the major parameter groups established in the power set $\{A_i\}$ were identified, where $\{\{disciplines\}, \{procurement\ and\ acquisition\}\}$ are noted. Consequently, the disciplines may consist of one, two, or multiple pairs of disciplines ($\{mechanical\}$, $\{automatic\}$, $\{hydraulic\}$, $\{processes\}$, $\{electrical\}$, among others), depending on the complexity of the structure, which is determined by $\{E\}$. In contrast, procurement and acquisition encompass the pairs $\{\{equipment\}, \{materials\}, \{equipment\ and\ materials\}\}$. The observable study identified from the power set $\{A_i\}$ the following pair: {{$hydraulic$}, {$processes$}}, {$equipment\ and\ materials$}}. However, other disciplines are associated when implementing these systems in both residential and industrial areas, such as $\{architecture\}$ and less defined subcategories of $\{process\}$, including $\{\{corrosion\}, \{flows\}\}$. Similarly, it was observed that these attributes belonging to the power sets $\{B_i\}, \{C_i\}, \{D_i\}$ are defined, partially defined, or indeterminate according to the neutrosophic membership degree. Additionally, it

is proposed to conduct studies implementing these water supply systems in residential and industrial zones, as well as evaluating their technical, economic, and environmental viability by determining all implicit pairs of each power set [14].

4. Conclusion

The evaluation conducted through power sets and the application of Neutrosophic SuperHyperSoft Sets has made it possible to identify that the water supply systems with the highest performance for residential use are found in systems $\{W\}$ and $\{VW\}$, provided they operate within their efficient zone. Moreover, it has proven useful for comparing the energy and technical behavior of different systems by considering multiple disciplines and associated variables. In fact, the assessment of parameters in set $\{A_i\}$ has shown that the presscontrol primarily impacts low flow rates (0 to 0.5 m³/h), with up to 3.63% difference in head. From 0.8 m³/h onward, its effect becomes insignificant, with an average error of 0.86%, demonstrating good efficiency under medium and high domestic demand. Meanwhile, the hydropneumatic system shows lower initial head due to the pressure switch delay and tank effect, but it converges with the direct pump around 0.9 m³/h, with only 0.37% average error, indicating good performance at medium and high flow rates. On the other hand, it was observed that the combined system presents an initial pressure drop but maintains stable performance at high flow rates (1.9 m³/h), with an average error of 0.63%, confirming its efficiency under continuous consumption conditions.

The study has demonstrated that all the evaluated systems operate efficiently within their defined ranges, showing that the presscontrol system offers the highest performance (22.48%), followed by the combined system (21.62%) and the hydropneumatic system (20.17%). Although these systems do not meet the requirements of high-demand equipment, they are suitable for most common sanitary fixtures. Moreover, the evaluated systems show good performance within their specific flow rate ranges. However, limitations were identified in the sets $\{B_i\}, \{C_i\}, \{D_i\}$, related to economic, environmental, and user satisfaction aspects, due to the neutrosophic degree of membership of each power set. Consequently, this research has provided a useful methodological tool for the comprehensive analysis of water supply systems, and its application is proposed in case studies to validate its technical, economic, and environmental feasibility in residential and industrial settings.

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University of New Mexico

Design of a Maintenance Protocol based on Bipolar Neutrosophic Analysis for Severe Depression

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Abstract. Severe depression has been one of the leading causes of disability worldwide and maintains a high risk of relapses even after successful treatment, especially in medium-sized cities in Latin America. Therefore, this study focused on the design of a clinical maintenance protocol based on Acceptance and Commitment Therapy (ACT), aimed at young adults residing in Ambato, Ecuador, who had previously received Cognitive-Behavioral Therapy (CBT). To this end, a bipolar neutrosophic analysis of depression was conducted to determine the attributes that impact the identified patients and define the design of the proposal. The designed protocol was structured into eight group sessions, with an experiential approach centered on the six core processes of ACT. Evaluation strategies were also proposed to measure processes and outcomes. It was concluded that the protocol seeks to strengthen therapeutic continuity, reduce relapses, and promote sustained emotional well-being in young people with a history of depression.

Keywords: Relapse prevention, Psychological treatment, Mental health, Therapeutic follow-up, Bipolar neutrosophic set.

1 Introduction

Major Depressive Disorder (MDD) in its severe form constitutes one of the most disabling psychiatric conditions, profoundly affecting the emotional, social, and occupational functioning of those who suffer from it. According to the World Health Organization (WHO), more than 280 million people live with depression, and it represents one of the leading causes of disability worldwide [1]. Although effective treatments exist to address the acute phase, such as Cognitive-Behavioral Therapy (CBT), relapse rates remain high, particularly among young people who have experienced severe episodes [2].

This vulnerability intensifies during the transition to adulthood, a stage considered critical from an emotional development perspective, as it coincides with academic pressures, vocational uncertainty, and the search for identity [3]. These factors, as various studies have indicated, increase the risk of emotional dysregulation and psychological fragility.

On the other hand, it is exacerbated by poor therapeutic continuity, insufficient community mental health infrastructure, and the persistent social stigma that weighs on those who have received psychological or psychiatric care [4] [5]. For instance, in the city of Ambato, located in the central region of Ecuador, the availability of post-treatment programs is severely limited, compromising the possibility of sustaining the therapeutic gains achieved during the acute phase.

In response to this situation, the present study begins its analysis from a bipolar neutrosophic representation of depression and its treatments through the use of sets [6]. This approach seeks to identify a subset of young people who have experienced episodes of high emotional intensity and who have received prior treatment with CBT. As well as to design a clinical maintenance protocol based on Acceptance and Commitment Therapy (ACT), aimed at young adults residing in Ambato, Ecuador, who have received prior CBT treatment [7].

Given that it constitutes an interaction between CBT and ACT treatments for this identified subset,

each interacting attribute is analyzed by positively examining the effects on patient health. At the same time, negative elements are assessed, such as low professional experience, patient health coverage, or similar clinical conditions defined within the range. Therefore, the use of bipolar Neutrosophy is proposed for the development and structure of the study.

Finally, the proposed clinical protocol seeks to consolidate the progress achieved during the acute phase, reduce the risk of relapse, and promote sustainable emotional well-being. Additionally, it is proposed as a group intervention adapted to the local psychosocial environment, contributing to stigma reduction and strengthening the therapeutic support network in young adults [5].

2 Preliminaries

2.1 Neutrosophic SuperHyperSoft Sets.

Definition 1 [8]. The *Neutrosophic set N* is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that according to the definition, $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x)$ and $F_A(x)$ can be sub-intervals of $[0, 1]$. $^-0$ and 1^+ belong to the set of hyperreal numbers.

Definition 2. The *Single-Valued Neutrosophic Set* (SVNS) A over U is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$ and $F_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. A single valued neutrosophic number is denoted by $\tilde{A} = \langle T, I, F \rangle$ for convenience.

Definition 3. Let $\widetilde{\tilde{A}_1} = \langle T_1, I_1, F_1 \rangle$ and $\widetilde{\tilde{A}_2} = \langle T_2, I_2, F_2 \rangle$ be two single-valued neutrosophic numbers. Then, the operations for NNs are defined as below;

$$\lambda \tilde{A} = \langle 1 - (1 - T_1)^{\lambda}, I_1^{\lambda}, F_1^{\lambda} \rangle \quad (1)$$

$$\tilde{A}_1^{\lambda} = \langle T_1^{\lambda}, 1 - \left(1 - I_1^{\lambda}\right), 1 - \left(1 - F_1^{\lambda}\right) \rangle \quad (2)$$

$$\tilde{A}_1^{\lambda} + \tilde{A}_2^{\lambda} = \langle T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2 \rangle \quad (3)$$

$$\tilde{A}_1^{\lambda} \cdot \tilde{A}_2^{\lambda} = \langle T_1 T_2, I_1 I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2 \rangle \quad (4)$$

where $\lambda > 0$.

Definition 4. Let X be a non-empty set. Then, a bipolar-valued fuzzy set, denoted by A_{BF}, is defined as; $A_{BF} = \{\langle x, \mu_B^+(x), \mu_B^-(x) \rangle : x \in X\}$ where $\mu_B^+(x) \to [0,1]$ and $\mu_B^-(x) \to [0,1]$. The positive membership degree $\mu_B^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to A_{BF} and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of x to some implicit counter property of A_{BF}.

Definition 5 [9]. Let A bipolar neutrosophic set A in X is defined as an object of the form $A = \{\langle x, T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x) \rangle : x \in X\}$, where $T^+, I^+, F^+: X \to [1,0]$ and $T^-, I^-, F^-: X \to [-1,0]$. The positive membership degree $T^+(x), I^+(x), F^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ corresponding to a bipolar neutrosophic set A and the negative membership degree $T^+(x), I^+(x), F^+(x)$ denotes the truth membership, indeterminate membership and false membership of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Definition 6 [10]. Let $\tilde{a}_1 = \langle {T_1}^+(x), {I_1}^+(x), {F_1}^+(x), {T_1}^-(x), {I_1}^-(x), {F_1}^-(x) \rangle$ and $\tilde{a}_2 = \langle {T_2}^+(x), {I_2}^+(x), {F_2}^+(x), {T_2}^-(x), {I_2}^-(x), {F_2}^-(x) \rangle$ be two bipolar neutrosophic number . Then the operations for NNs are defined as below;

$$\lambda \tilde{a}_1 = \langle 1 - (1 - T_1^+)^{\lambda}, (I^+{}_1), (F_1^+)^{\lambda}, -(-T_1^-)^{\lambda}, -(-I_1^-)^{\lambda}, -\left(1 - \left(1 - (-F_1^-)\right)^{\lambda}\right) \rangle \quad (5)$$

$$\tilde{a}_1^{\lambda} = \langle (T_1^+)^{\lambda}, 1-(1-I_1^+)^{\lambda}, 1-(1-F_1^+)^{\lambda}, -(1-T_1^-)^{\lambda}, -(-I_1^-)^{\lambda}, -(-F_1^-)^{\lambda} \rangle \quad (6)$$

$$\tilde{a}_1 + \tilde{a}_2 = \langle T_1^+ + T_2^+ - T_1^+T_2^+, I_1^+I_2^+, F_1^+F_2^+, -T_1^-T_2^-, -\left(-I_1^- - I_2^- - I_1^-I_2^-, -(-F_1^- - F_2^- - F_1^-F_2^-)\right)\rangle \quad (7)$$

$$\tilde{a}_1 \cdot \tilde{a}_2 = \langle T_1^+T_2^+, I_1^+ + I_2^+ - I_1^-I_2^-, F_1^+ + F_2^+ - F_1^+F_2^+, -(-T_1^- - T_2^- - T_1^-T_2^-), -I_1^-I_2^-, -F_1^-F_2^- \rangle \quad (8)$$

Where $\lambda > 0$.

Definition 7. Let $\tilde{a}_1 = \langle T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- \rangle$ be a bipolar neutrosophic number. Then, the score function $l(\tilde{a}_1)$, accuracy function $a(\tilde{a}_1)$ and certainty function $c(\tilde{a}_1)$ of a bipolar neutrosophic number are defined as follows:

i. $\tilde{l}(\tilde{a}_1) = \frac{(T_1^+ + 1 - I_1^+ + 1 - F_1^+ + 1 + T_1^- - I_1^- - F_1^-)}{6}$ (9)

ii. $\tilde{a}(\tilde{a}_1) = T_1^+ - F_1^+ + T_1^- - F_1^-$ (10)

iii. $\tilde{c}(\tilde{a}_1) = T_1^+ - F_1^-$ (11)

3 Results

3.1 Bipolar neutrosophic analysis: Depression and its functional implications.

Depression constitutes a mental disorder fundamentally characterized by sadness and discouragement. It is also associated with physical and cognitive alterations, affecting the patient's functional development, as well as social relationships and language. In fact, different types of depression exist, each with its own characteristics and duration. The most common are major depressive disorder and persistent depressive disorder (also called dysthymia). Additionally, there are other types such as seasonal affective disorder, perinatal depression, and depression with psychotic features. Therefore, for the development of this study, it is proposed to identify each of these elements in the following attributes by sets:

- s_n: Set of symptom manifestations according to intensity, duration, number of symptoms, and level of functional interference.
- d_n: Set of depression subtypes (manifestation of subsets in different depression subtypes according to s_n)
- t_n: Set of treatments and their structures (t_1: psychotherapy, t_2: pharmacotherapy, t_3: electroconvulsive).

Consequently, each of these attributes responds to the following function $f(\{s_n\}) \to d_n$ and $f(\{d_n\}) \to t_n$, according to the image of the selected set or subset. Given that the manifestations of s_n, d_n and t_n imply elements with bipolar characteristics, it becomes necessary to use bipolar Neutrosophy to determine the positive and negative effects. For example, in one or several subsets belonging to s_n implies in the function multiple neutrosophic structures that are defined according to the type of complexity and structure:

- $\{s_1\}, \{s_2\}, \{s_n\}, \{\{s_1\}, \{s_2\}\}, \left\{\{s_1\}, \{s_n\}, \{\{s_2\}, \{s_n\}\}, \{\{s_n\}, \{s_n\}\}\right\}$

To illustrate, in the structures of s_n several symptoms with different degrees of complexity can coexist; however, these depend on the clinical evaluation given by the specialist. In fact, each expert or professional who diagnoses the depression subtype must determine how true the diagnosis is, indeterminate in cases where the present symptoms do not show the degree of manifestation required according to the Beck Depression Inventory-II (BDI-II) or through the defined DSM-5 criteria. They must also determine the level of how false the depression diagnosis can be based on the symptoms studied. Additionally, bipolar elements are added that identify how effective a treatment is in the patient, considering the side effects on health. Consequently, this implicit bipolar neutrosophic value determined as $\tilde{A}$ in each analyzed set and subset implies a representation in the following functions (according to

Definition 5):

- $f(\{\tilde{s}_n\}) \rightarrow \tilde{d}_n$
- $f(\{\tilde{d}_n\}) \rightarrow \tilde{t}_n$

Another recurring issue is the patient's age (e_n) and the gender (g_n) for example, major depressive disorder is observed as more frequent in 7% of people worldwide, primarily women. It is even three times more frequent among those aged 18 to 29 years than in those over 60 years. Therefore, these attributes should be included in the neutrosophic structure of the function:

- $f(\{\tilde{s}_n\} \cap \{\{e_n\} \cap \{g_n\}\}) \rightarrow \tilde{d}_n$
- $f(\{\tilde{d}_n\}) \rightarrow \tilde{t}_n$

Another example shows the following clinical picture, where the following function is observed in a young patient:

$$f(\{\tilde{s}_n\} \cap \{\{23\} \cap \{male\}\}) \rightarrow \ Mild\ depression$$

Where $\{\tilde{s}_n\}$ is defined within the following context: a 23-year-old young man who, despite feeling depressed, withdrawn, and with low self-esteem, could continue his work on the family farm, although he spoke very slowly and appeared somewhat distant. This case illustrates how the distress, while present, is still manageable and does not result in complete incapacitation.

In fact, the subset of $\{\tilde{s}_n\}$ allows for defining and establishing a clinical diagnosis, which in this case shows that the patient presents mild depression, characterized by a lower quantity and intensity of symptoms, generally accompanied by mild or moderate functional impairment. According to the Beck Depression Inventory-II (BDI-II), a score between 14 and 19 is considered indicative of mild depression. According to DSM-5 criteria, mild episodes may include only five or six depressive symptoms, with limited intensity, allowing the individual to partially preserve their functional capacity, although with unusual effort. These individuals experience distress but usually maintain their work or academic activity with some degree of performance.

In another analysis, of a patient, the following function has been determined:

$$f(\{\tilde{s}_n\} \cap \{\{18\} \cap \{female\}\}) \rightarrow \ Severe\ depression$$

Where $\{\tilde{s}_n\}$ manifests in the patient with intentional injury behavior through burns, indicating profound emotional dysregulation and a severe sense of hopelessness. This type of behavior reflects not only the severity of the disorder but also the urgency for specific and sustained interventions.

In fact, severe depression implies a greater number of symptoms with high intensity, and a significant functional impact that can become disabling. In the BDI-II, a score of 29 to 63 suggests a severe depressive episode. These episodes usually substantially compromise the individual's autonomy, interfering with their academic and work performance and daily life. Withdrawal behaviors, profound hopelessness, marked anhedonia, notable cognitive alterations, and in some cases, self-injurious behaviors or suicidal ideation are observed.

Therefore, it has been observed that $\{\tilde{s}_n\}$ constitutes a set formed by several subsets that interrelate from defined and partially defined structures in bipolar neutrosophic degrees. In fact, evaluating $\tilde{d}_n$ does not depend solely on the level of symptoms presented and their degrees of complexity defined on a scale $[0,1]$, but depends on elements of opposite poles such as professional experience, patient health coverage, or similar clinical conditions defined in the range $[-1,0]$.

Similarly, $\tilde{t}_n$ is structured in treatment subsets, which are defined in how true the success of the treatment can be, or indeterminate when compromising effectiveness due to concurrence with other pathologies. As well as the degree of falseness determined by the low or null commitment of the patient

to undergo treatment. Thus, they are defined within a range of [0,1] for each evaluated parameter.

On the other hand, the level of truth about how this treatment compromises according to the expected side effects must be evaluated, or it is partially affected or indeterminate when evidence shows that a result is not as expected. As well as a degree of falseness identified in the minimization of side effects, so that these would be found in a range of [−1,0]. These structures, subsets, and sets are visualized in Figure 1 from the bipolar neutrosophic analysis.

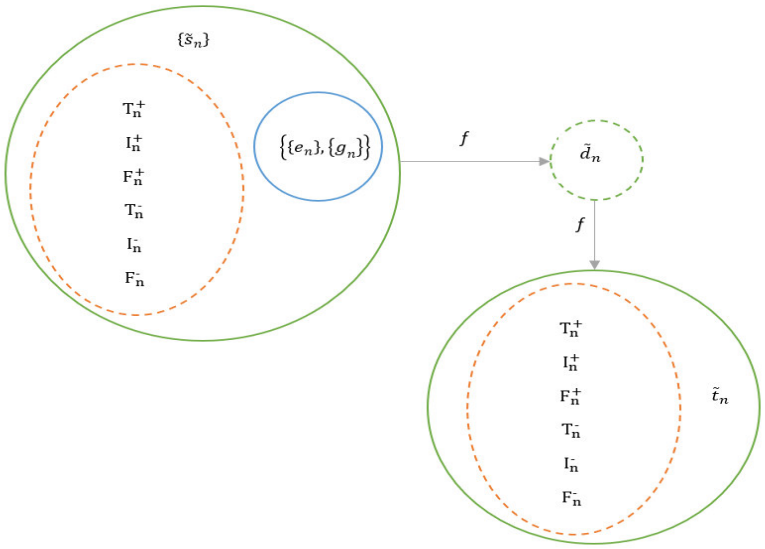

Figure 1: Bipolar neutrosophic representation of depression and its treatment. Source: Own elaboration.

Case study: Once the bipolar neutrosophic representation is defined, the analysis of severe depression in young adults is developed through a case study, where it constitutes a relevant clinical and social challenge, particularly in Latin American countries where mental health infrastructure does not adequately cover the maintenance phase. Evidence indicates that the risk of relapsing after a severe depressive episode is high, even when effective treatment such as Cognitive-Behavioral Therapy (CBT) has been received.

In Ecuador, this gap becomes more evident in intermediate cities such as Ambato, where community psychological resources are limited and the stigma associated with mental health hinders continuity of care. In this context, designing a specific protocol for young people between 18 and 25 years old not only responds to a clinical need but also to an opportunity for preventive impact at a key stage of development. Therefore, the function to be analyzed in this study is represented:

:

- $f\left(\{\tilde{s}_n\} \cap \{\{18-25\} \cap \{g_n\}\}\right) \rightarrow \{Severe\ depression\}$
- $f(\{Severe\ depression\}) \rightarrow \{\{CBT\} \cap \{ACT\} *\}$

Where, the structure $\{\{CBT\} \cap \{ACT\} *\}$ is included in a neutrosophic area in the subset t_1 and $\{Severe\ depression\}$ as a subset of $\tilde{d}_n$ (in fact, these patients must be in partial or total remission of the depressive episode). Therefore, the inclusion and intersection with ACT treatment according to $\{\tilde{s}_n\}$ is considered (see Table 1).

Table 1: Psychosocial risk factors $\{\tilde{s}_n\}$ in young people from Ambato and their relationship with protocol design.

Domain	Specific Risk Factor	Consideration in the ACT Protocol
Family	Psychiatric history, family abandonment, marital conflicts, early pregnancies.	Relevant context for values clarification and acceptance of discomfort.
Academic	Academic load, social stressors, learning difficulties, exam-related stress.	Initial assessment and guidance for defusion and mindfulness activities.
Individual	Low self-esteem, anxiety, anhedonia, lack of resilience and coping strategies.	Directly addressed through all psychological flexibility processes.
Social	Economic pressure, substance use, family expectations, bullying.	Indirect work on context and connection with personal values.
Work (young workers and health professionals)	Mobbing, stress, double burden, job insecurity.	Support for clarification of values and acceptance of work-related distress.

Next, under the conditions of the case study, an analysis is performed between the following structures $\{ACT\}, \{CBT\}$ y $\{\{CBT\}, \{ACT\}*\}$ (se define $\{ACT\}$* is defined to be implemented in the maintenance phase) (see Tables 2 and 3). For this purpose, the following identified parameters (b_i) and weights (w_j) distributed for obtaining the score, accuracy and certainty functions were determined (calculated according to definitions 6 and 7):

- b_1: Main approach
- b_2: Therapeutic objective
- b_3: Relationship with thoughts
- b_4: Main strategies
- b_5: Symptom reduction in young people
- b_6: Long-term goal

Table 2: Analysis between structures belonging to subset t_1. Source: Own elaboration.

t_1	b_1	b_2	b_3	b_4	b_5	b_6
w_j	0.16	0.18	0.16	0.16	0.18	0.16
$\{CBT\}$	(0.64,0.41,0.42, -0.72,-0.29,-0.3)	(0.74,0.31,0.32, -0.52,-0.49,-0.5)	(0.74,0.31,0.32, -0.42,-0.59,-0.6)	(0.74,0.31,0.32, -0.12,-0.89,-0.9)	(0.64,0.41,0.42, -0.02,-0.93,-0.98)	(0.64,0.41,0.42, -0.82,-0.19,-0.2)
$\{ACT\}$	(0.84,0.21,0.22, -0.12,-0.89,-0.9)	(0.64,0.41,0.42, -0.22,-0.79,-0.8)	(0.84,0.21,0.22, -0.52,-0.49,-0.5)	(0.84,0.21,0.22, -0.52,-0.49,-0.5)	(0.84,0.21,0.22, -0.02,-0.93,-0.98)	(0.84,0.21,0.22, -0.12,-0.89,-0.9)
$\{\{CBT\}, \{ACT\}*\}$	(0.94,0.11,0.12, -0.02,-0.93,-0.98)	(0.84,0.21,0.22, -0.12,-0.89,-0.9)	(0.74,0.31,0.32, -0.32,-0.69,-0.7)	(0.94,0.11,0.12, -0.32,-0.69,-0.7)	(0.84,0.21,0.22, -0.52,-0.49,-0.5)	(0.84,0.21,0.22, -0.22,-0.79,-0.8)

Table 3: Evaluation of score, accuracy and certainty functions. Source: Own elaboration.

t_1	$w_j x b_i$	$\tilde{l}(\tilde{a})$	$\tilde{a}(\tilde{a})$	$\tilde{c}(\tilde{a})$
$\{CBT\}$	(0.694,0.357,0.367,-0.251,-0.694,-0.762)	0.696	0.838	1.456
$\{ACT\}$	(0.815,0.237,0.247,-0.155,-0.814,-0.858)	0.808	1.271	1.673
$\{\{CBT\}, \{ACT\}*\}$	(0.874,0.182,0.192,-0.177,-0.792,-0.836)	0.825	1.341	1.71

It is observed that $\{\{CBT\}, \{ACT\}*\}$ obtained a score of $\tilde{l}(\tilde{a}) = 0.825$ by including ACT treatment in the maintenance phase above the others. On the other hand, given that the values of $\tilde{l}(\tilde{a})$ are different, it is not necessary to use the other functions.

Comparative analysis: Cognitive-Behavioral Therapy (CBT) is recognized as a treatment of choice for adults in the acute phase of Major Depressive Disorder (MDD) from moderate to severe, as well as for mild cases. Its recent use has been observed with CBT monotherapy, as well as its combination with antidepressants for depression treatment.

Regarding relapse prevention, available evidence indicates that CBT offers efficacy both in the short

and long term. Thus, it shows a significant reduction in relapses or recurrences compared to pharmacological or psychological placebo treatments.

However, the studies reviewed in the bibliography of this research do not detail specific limitations of traditional CBT in preventing relapses in cases of severe depression compared to other active therapies. Although general disadvantages of the traditional CBT model are noted, such as its focus mainly on current symptoms rather than the historical origins of distress. Even its possible excessive focus on conscious thinking and the tendency to combat suffering in a way that could promote maladaptive avoidance strategies. These observations are not presented as empirically confirmed limitations in the context of severe relapse prevention.

To conclude, some criticisms directed at clinical guidelines mention that they do not incorporate all available psychological treatment options with empirical support, suggesting the existence of effective therapeutic alternatives. Although without explicitly stating that CBT presents specific deficiencies in preventing severe relapses.

In contrast, available evidence shows that Acceptance and Commitment Therapy (ACT) offers notable advantages for working on relapse prevention, as it explicitly incorporates the goal of preventing future relapses as part of the behavioral change it seeks to promote. Unlike approaches focused solely on symptom eradication, ACT is oriented toward increasing psychological flexibility, teaching people to accept difficult thoughts, emotions, and internal experiences without fighting against them or trying to eliminate them.

Therefore, the treatment defined in the neutrosophic structure $\{\{CBT\}, \{ACT\}*\}$ seeks to promote psychological flexibility, prevent relapses, and favor sustained emotional well-being in a population of young adults between 18 and 25 years old. This bipolar neutrosophic area is delimited, as it is applicable to patients with a history of severe depression belonging to the subset of $\tilde{d}_n$, previously treated with CBT. In fact, the inclusion of ACT in the neutrosophic structure works to teach people to stay connected with the present moment through active acceptance of their internal experience, allowing them to advance toward their goals and vital purposes, despite emotional distress.

This approach is particularly useful for relapsing prevention, as it recognizes that distress and unwanted private events are inevitable in life. Instead of focusing on their control or avoidance, fostering rigid patterns of destructive experiential avoidance, ACT equips individuals with skills to manage these events functionally, building a life coherent with their personal values, even in the presence of emotional difficulties.

3.2 Intervention program designed or adapted defined in $\{\{CBT\}, \{ACT\}*\}$

3.2.1 General objective: The present protocol aims to design a maintenance plan based on Acceptance and Commitment Therapy (ACT), directed at young adults between 18 and 25 years old with a history of severe depression previously treated with Cognitive Behavioral Therapy (CBT), with the purpose of promoting psychological flexibility, preventing relapses, and favoring sustained emotional well-being.

3.2.2 Specific objectives:

- Strengthen psychological flexibility in participants through the integration of techniques based on acceptance, cognitive defusion, and committed action
- Identify personal values and promote behaviors aligned with such values as a guide for meaningful post-treatment life
- Develop mindfulness skills and contact with the present moment to improve emotional regulation and reduce experiential avoidance patterns
- Evaluate positive maintenance indicators and early detection of relapses through validated scales and qualitative interviews
- Foster functional coping strategies for internal events (thoughts, emotions) without the need to eliminate psychological distress

3.2.3 Agents involved in the implementation of the intervention program

The present protocol would be implemented by a team of mental health professionals with training in Acceptance and Commitment Therapy (ACT), composed of clinical psychologists and therapists certified in mindfulness-based interventions and third-generation therapies. The facilitators must have experience working with young adults and prior training in group approaches, as the intervention follows a group-based, experiential, and hands-on format.

Additionally, the participation of an external clinical supervisor with knowledge in ACT is required, who is responsible for evaluating compliance with the ethical and technical standards of the protocol. This supervisor also provides weekly supervision spaces for the therapists in charge, with the purpose of maintaining therapeutic quality, addressing special cases, and preventing professional burnout. The implementation of the program is developed in coordination with educational institutions or community mental health centers in Ambato, which would provide the physical space, logistical resources, and administrative support necessary for the execution of sessions.

For the formation of intervention groups, an estimated range of six to ten participants per cohort is established. This number allows maintaining a balance between meaningful group interaction and personalized attention, favoring active participation, monitoring of individual processes, and development of experiential exercises in a contained and manageable environment. This size is also consistent with the methodological principles of ACT and the experiential nature of the sessions. The following table presents a summary of the main agents involved in the intervention, their specific roles and functions (see Table 4).

Table 4: Agents involved in protocol implementation. Source: Own elaboration.

Agent	Role	Main functions
Main therapist	Group facilitator	• Direct group sessions. • Apply ACT techniques. • Conduct clinical follow-up.
Co-therapist / assistant	Technical and clinical support	• Assist in dynamics. • Record observations. • Provide individual containment.
Clinical supervisor	Therapeutic quality control	• Weekly supervision. • Case review. • Ethical and technical support.
Collaborating institution	Institutional and logistical support	• Provide space. • Convene participants. • Coordinate material resources.
Participants	Protocol beneficiaries	• Actively attend. • Participate in sessions. • Apply strategies between sessions.

3.2.4 Actions to be developed and didactic resources employed

The following section delves into the specific contents of each protocol session, structured around the six core processes of Acceptance and Commitment Therapy (ACT). Each session integrates practical techniques designed to strengthen psychological flexibility and promote the maintenance of therapeutic gains achieved during the acute phase of treatment (see Table 5).

Table 5: Detailed contents by session. Source: Own elaboration.

No.	Session	Objective	Techniques	Homework
1	Introduction to the protocol and the ACT model	To provide psychoeducation on severe depression, explain the psychological flexibility model (Hexa-flex), and present the structure of the protocol.	• Participatory explanation of the ACT model and its six core processes. • Group activity: "Expectations and personal commitment." • Presentation of the group therapeutic contract.	Brief diary on the most frequent thoughts and emotions during the week.
2	Accepting distress	To promote active acceptance of difficult thoughts and emotions without attempting to avoid or suppress them.	• Experiential exercise: "Making room" (allowing an uncomfortable emotion to be present without struggle). • Guided discussion on the consequences of experiential avoidance. • Metaphor: "The unwanted guest" (accepting the arrival of thoughts/emotions as visitors).	Guided acceptance practice using a 10-minute audio recording.
3	Cognitive defusion	To teach strategies for observing thoughts as mental events in order to reduce their impact and control.	• Exercise: "Leaves on a stream" (visualizing thoughts floating on a river). • Verbal exercise: "Word repetition" (repeating a key thought until it loses its emotional charge). • Group reflection on the influence of automatic thoughts	Apply defusion techniques at least twice daily, recording the experience in the journal.
4	Contact with the present moment	To develop mindfulness skills to enhance present-moment awareness and emotional regulation.	• Formal mindfulness practice: "Mindful breathing" (10-minute guided session). • Open attention exercise: "Body scan." • Group analysis on the importance of conscious presence in relapse prevention.	Daily mindfulness practice (audio recording provided).
5	Self-as-context	To promote the development of a flexible self-perspective that allows internal experiences to be observed without full identification.	• Guided exercise: "The constant observer" (identification of the part of the self that is always present). • Metaphor: "The chessboard" (the distinction between the pieces in conflict and the board that holds them). • Activity: Sharing experiences of being "carried away" by emotions and how they could have been observed from a distance.	Written reflection on difficult situations and the perspective of the "observing self."
6	Values clarification	To facilitate the identification of personal values and their role as a compass to guide meaningful actions.	• Exercise: "The life wheel" (identifying key life domains and values within each). • Visualization: "The funeral"	Define three micro-actions aligned with the identified values.

			(imagining how one would like to be remembered to detect core values). • Group discussion on the differences between values and goals.	
7	Committed action	To assist participants in turning personal values into concrete and sustainable actions, even in the presence of emotional discomfort.	• Behavioral planning: Development of a weekly action plan (based on values). • Barrier analysis: Identification of internal and external obstacles and strategies to overcome them. • Metaphor: "The garden" (nurturing values as if tending a garden).	Carry out at least one committed action and record the experience.
8	Integration and relapse prevention	To integrate the learnings from the protocol and establish a personalized relapse prevention plan.	• Group review of progress and key learnings. • Development of a "Personal relapse prevention map" (identifying early warning signs and coping strategies). • Symbolic closing activity: "Letter to my future self."	Continue autonomous maintenance and follow-up practices.

3.2.5 Schedule or timeline (see Table 6)

Table 6: Weekly intervention schedule based on ACT processes. Source: Own elaboration.

Week	Session Topic	Main ACT Process	Specific Objective
1	Introduction and psychoeducation on ACT	Hexaflex Model	Understanding protocol structure
2	Acceptance of emotional distress	Acceptance	Reducing experiential avoidance
3	Cognitive defusion	Cognitive defusion	Changing relationship with thoughts
4	Mindfulness and present-moment awareness	Present-moment contact	Improving emotional regulation
5	The self as context	Self as context	Building a flexible self-perspective
6	Values clarification	Values clarification	Identifying life purpose and direction
7	Committed action	Committed action	Converting values into meaningful behavior
8	Integration and relapse prevention	Integration of ACT processes	Consolidating learning and planning prevention

3.3 Discussion

The bipolar neutrosophic analysis has facilitated the identification of attributes, structures, subsets, and sets implicit in the diagnosis and treatment of depression. By presenting bipolar characteristics, it has been observed that it is not sufficient to only consider the positive degrees of truth in sets of attributes that determine the clinical type or subtype of depression. In fact, negative degrees must be included in representation of attributes such as low professional experience, patient health coverage, or similar clinical conditions defined in the range of attributes of the presented diagnosis [11]. Therefore, the clinical subtype of depression must be identified and determined, managing existing indeterminacies and

enhancing results from protocols designed for each subset of the identified population.

On the other hand, the case study analysis regarding the neutrosophic structure $\{\{CBT\},\{ACT\}*\}$ in the subset of young adult patients between 18 and 25 years treated with CBT has allowed for the design of a protocol aimed at promoting psychological flexibility, preventing relapses, and favoring sustained emotional well-being. Furthermore, the inclusion of ACT in the maintenance phase constitutes a valuable tool for obtaining positive results while keeping in mind the existing clinical challenges of Ambato [12].

In fact, scientific evidence has demonstrated that Acceptance and Commitment Therapy is effective not only in reducing depressive symptoms, but especially in preventing relapses in the medium and long term [13]. This is due to its capacity to strengthen transdiagnostic processes, such as acceptance, cognitive defusion, and values clarification, which allow patients to maintain adaptive functioning even in the presence of emotional distress [14]. In contrast to CBT, where the focus is on modifying cognitive content, ACT seeks to transform the relationship with thoughts and emotions, constituting a differential advantage for the maintenance phase [15].

Furthermore, the protocol responds to the urgent need to expand therapeutic options in Ecuador, where mental health remains heavily stigmatized and there are significant barriers to access and continuity in treatment. The protocol design for young university adults or those in transition to working life addresses one of the most vulnerable populations, considering psychosocial and contextual factors identified in the literature [16].

Nevertheless, although the proposal is based on a solid theoretical and empirical framework, its real effectiveness could only be determined through its application and evaluation in clinical practice. Logistical limitations, participant adherence, and the possible need for cultural or individual adjustments must be subject to continuous monitoring. Similarly, future research could explore the integration of this protocol in digital or hybrid environments to expand its accessibility.

4. Conclusion

The bipolar neutrosophic analysis has contributed to the understanding of depression through a representation of attributes, subsets, and sets implicit in the treatment of mental disorder. From the design of two neutrosophic functions, the protocol design for a small vulnerable population of young university adults in Ambato has been represented and proposed. The results have shown a proximity of the score function of 0.825 to propose a clinical protocol based on the neutrosophic structure in the treatment of severe depression.

The present maintenance protocol, based on Acceptance and Commitment Therapy (ACT), has constituted an innovative and necessary proposal for relapse prevention in young adults aged 18 to 25 years with a history of severe depression previously treated with Cognitive Behavioral Therapy (CBT). From an analysis of the Ecuadorian context, particularly Ambato, the protocol has been designed to strengthen psychological flexibility while promoting committed action with personal values.

The protocol has offered a clear, progressive structure adapted to the needs of the target population, integrating experiential techniques and practical exercises aligned with the six core processes of ACT. Likewise, the combination of methods and the use of bipolar Neutrosophy in the evaluation would allow obtaining a comprehensive view of the protocol's effectiveness, both in terms of symptom reduction and transformation of underlying psychological patterns. Therefore, it has been suggested to explore in future research the adaptation of the protocol to digital or hybrid modalities, with effects of increased accessibility and reach, especially in rural or hard-to-reach areas. In fact, the integration of mobile applications, virtual platforms, and telepsychology represents a promising line of work that deserves to be developed in subsequent research.

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Structured Analysis of a Generator Set Lubrication System Using Vertex Articulation in Plithogenic n-SuperHyperGraphs

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Abstract. The primary function of the generator set installed in a remote oil field location is to supply electrical power for crude oil extraction. Due to operational demands, the lubrication system was upgraded to improve oil quality and reduce particulate contamination. This study assessed particulate contamination by applying the ISO 4406 cleanliness code, evaluating oil properties, and monitoring engine wear, using the Plithogenic *n-SuperHyperGraph* model as a structural framework. Oil samples were analyzed before and after implementing an external microfiltration system. Initial ISO codes of 22/21/19 were recorded; after 552 hours of operation, the ISO code improved to 19/18/15, achieving 91.25% efficiency. Post-1051-hour analyses confirmed wear levels remained below the control limit. The results demonstrate the system's technical and operational viability, supporting its potential economic and environmental feasibility in similar industrial settings.

Keywords: Oil filtration, contaminant particles, wear control, engine performance, Plithogenic *n-SuperHyperGraphs*.

1 Introduction

The durability and proper operation of combustion engines depend critically on both the quality of service fluids and the concentration and size of contaminant particles present [1]. These fluids must adhere to engine-specific standards and regulations to prevent premature component wear, avoiding potential failures, unplanned downtime, and associated economic costs [2].

In diesel engines, one of the most commonly used fluids is lubricating oil, which contains significant amounts of sulfur particles, ash, and metallic additives that directly affect the diesel combustion process [3]. To assess engine or equipment wear, oil analysis is performed since lubricating oils undergo progressive degradation due to their cleaning, friction reduction, and engine wear protection functions. This degradation manifests through increased acidity, sludge formation, and contaminant particles, ultimately impacting component lifespan and engine performance [4].

In this context, according to[5], there is a clear need for a comprehensive tool to consolidate used lubricating oil analysis parameters and evaluate its remaining service life. To determine the level of particulate contamination in oil, the cleanliness code provided by ISO 4406 standard is used, which evaluates particle sizes greater than 4, 6, and 14 μm per milliliter of fluid [6].

Conventional methods for identifying wear particles face limitations when particles adhere to each other, making accurate classification difficult. To address this issue, various monitoring models have been proposed, such as the online monitoring system developed by [7], which uses a horizontal vibrational monolayer particle separation method.

Consequently, these results demonstrate that this method effectively separates larger particles (located at the film's center) from smaller particles (positioned at the edges). Furthermore, the Inception-V3

model automatically extracts particle features and classifies them into distinct categories, including fatigue particles, oxide particles, and spherical particles [8]. Indeed, the obtained results confirm that the proposed model can simultaneously recognize multiple types of wear particles, even when they overlap [9].

Based on the aforementioned, this study performs a quantitative analysis of wear particles in internal combustion engine lubricating oil before and after implementing a microfiltration system. The objective is to obtain precise data about particle reduction rates in the oil, thereby contributing to improved lubricant quality, extended service life, and reduced engine wear and failures. For this study, using the Plithogenic *n-SuperHyperGraphs* model is proposed to structure, evaluate, and model the complex relationships between involved elements through vertex and supervertex interconnections.

2 Materials and Methods

2.1 Plithogenic n-SuperHyperGraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision-making in [10] [11]

First, an n-SuperHyperGraph is defined as follows:

Given $V = \{V_1, V_2, \cdots, V_m\}$, where $1 \leq m \leq \infty$ is a set of vertices, containing *Single Vertices* which are classical, *Indeterminate Vertices* which are unclear, vague, partially known, and the *Null Vertices* that are empty or completely unknown.

$P(V)$ is the power set of V including $\emptyset$. $P^n(V)$ is the n-power set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq n \leq \infty$.
Where it is also defined as $P^0(V) = V$.

An n-SuperHyperGraph (*n-SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where $G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$. Such that, G_n is the set of vertices and E_n is the set of edges.

G_n contains all possible types of vertices as in the real world:

- *Single Vertices* (the classics),
- *Indeterminate Vertices* (unclear, vague, partially known),
- *Null Vertices* (empty, totally unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types put together as a group (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is a *(n-1)- SuperVertex,* and the others can be *r- SuperVertex* for $r \leq n - 1$.

E_n contains the following types of edges:

- *Single Edges* (the classics),
- *Indeterminate Edges* (unclear, vague, partially known),
- *Null Edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more single vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of them is an n- SuperVertex and may contain another that is an r-SuperVertex with $r \leq n$).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n-SuperHyperEdge* (contains three or more vertices, at least one of which is an n-SuperVertex and may contain an r-SuperVertex with $r \leq n$),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

Graphs are classified as follows:

- Directed Graph (the classic one),
- Undirected Graph (the classic one),
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate directed).

Within the framework of the theory of Plithogenic n-SuperHyperGraphs, we have the following concepts:

Enveloping vertex: A vertex representing an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

SuperEnveloping vertex: An enveloping vertex comprises of SuperHyperEdges.

Dominant Enveloping Vertex: An enveloping vertex that is with dominant attribute values.

Dominant Super Enveloping Vertex: A super enveloping vertex with dominant attribute values.

Dominant Enveloping Vertex is classified into *input*, *intervene,* and *output* based on the nature of the object's representation.

Plithogenic Connectors: The connectors associate the input enveloping vertex with the output enveloping vertex. These connectors associate the effects of input attributes with output attributes and these connectors are weighted by plithogenic weights.

2.2 Oil Sample Extraction

The extraction of engine oil samples from the generator set was carried out by adapting a sampling port, which involved installing a coupling valve directly on the engine. This valve must be placed in a location where the oil pressure is not too high (70 psi). In this case, the sampling port location recommended by the manufacturer Caterpillar was at the lower part of the camshaft position, where a plug used for connecting a pressure gauge was removed, and the same thread was used to install a 3/16-inch quick coupling valve. For oil extraction, a disposable 1/4-inch hose was used, running from the quick coupling to the sampling bottle, which was sealed, labeled, and sent to the laboratory for condition and cleanliness analysis.

2.3 Oil Microfiltration System Selection

Based on the amount of contaminant particles found in the oil analysis results, the most suitable microfiltration system was determined. The microfiltration system consisted of a bank of six filters, selected according to the volume of oil used by the engine (165 gallons). The filter elements are made of cellulosic material due to its low cost, biodegradability, and high absorption capacity, which allows it to retain the moisture present in the oil [12]. According to the manufacturer's technical data sheet, the microfiltration system has an efficiency of 99.99% and is capable of removing contaminant particles larger than 0.5 microns. Due to these characteristics, the system must operate in parallel with the machine’s main filtration system, which is designed to remove particles larger than 15 microns.

2.4 Implementation of the Oil Microfiltration System

Prior to installing the microfiltration system, coordination was established with the involved maintenance and production areas to schedule an approximately 5-hour shutdown period. During this window, the installation work was carried out alongside the engine's preventive maintenance, which included replacement of air filters, fuel filters, oil filters, and the lubricating oil.

For the implementation, two new extraction points were adapted for the engine's lubricating oil, each fitted with a ball valve. A 100 psi-rated hydraulic hose was connected to each valve's outlet, incorporating a cross fitting to enable parallel oil supply to the three filter housings. Similarly, at each filter's outlet, an equivalent system was implemented to return the lubricating oil to the engine's oil pan.

2.5 Evaluation of the Lubrication System Upgrade

After installing the microfiltration system, the generator set was started up. Subsequently, a period of time was allowed for the engine to reach the operating temperature of 170°F and working pressure of 70 psi.

To obtain the oil sample, the lines and valve were first cleaned by circulating oil which was then properly discarded. Once this procedure was completed, a new bottle was used, filled to three-quarters of its capacity, labeled, and sent to an international laboratory for condition and cleanliness analysis. This first sample provided information about the contamination present in the new oil, according to the ISO 4406 cleanliness code.

After 51 hours of engine operation, the microfiltration system was isolated, all six filter elements were replaced, and the second oil sample was taken and sent to the laboratory, where a reduction in contaminant particles was evidenced. At 552 operating hours, another oil sample was taken, sent to the laboratory, and the new achieved ISO 4406 cleanliness code was verified, from which the filtration efficiency was calculated using equation (1). Then, at 1051 operating hours, which is the time interval for performing the equipment's preventive maintenance, the oil sample was taken to, through its final analysis, confirm the lubricant's condition and equipment wear.

$$\%efficiency = \left[\frac{before - after}{before}\right] x\ 100 \tag{1}$$

3 The Study

3.1 Behavior of Supervertices According to the Plithogenic n-SuperHyperGraph.

The upgrade of the lubrication system in operational generator sets (GE) in the petroleum sector constitutes an action to increase energy efficiency, prolong the oil's service life, and reduce engine component wear. This highly technical process requires a systematic and multivariate evaluation that considers the interaction between the lubrication system properties, the physicochemical behavior of the oil, the engine's operating conditions, and the associated environmental and economic impacts.

Therefore, to represent this structure of complex relationships, a model based on Plithogenic n-SuperHyperGraph has been developed, which articulates five main Supervertices: (V_1) lubrication system, (V_2) physicochemical variables, (V_3) microfiltration system, (V_4) engine operational variables, and (V_5) comprehensive sustainability and technology transfer capacity (see Table 1).

Table 1: Plithogenic n-SuperHyperGraph structure for the lubrication system upgrade of a generator set (GE). *Source: own elaboration.*

SuperVertex	Vertices (V)	Main Attributes	Sub-attributes
Lubrication System (V_1)	Type of lubrication (V_{11})	Volumetric flow rate (V_{1111})	L/min, GPM
		Flow velocity (V_{1112})	m/s, cm/s
		Flow pressure (V_{1113})	bar, psi
		Oil temperature (V_{1114})	°C, °F
		Flow distribution (V_{1115})	Critical zones, sensors, thermal analysis
	Oil type (V_{12})	Nominal viscosity (V_{121})	Value in cSt (V_{1211}) (ASTM D445) (specified viscosity for new oil)
		SAE Grade (Society of Automotive Engineers) (V_{122})	SAE J300 standard; codes like 10W-40

		Viscosity index (V_{123})	Measurements at 40°C and 100°C (oil's capacity to maintain viscosity under temperature changes)
		Flash point (V_{124})	°C; Pensky-Martens method ASTM D93
		Additive content (V_{125})	Type and concentration of additives (chemical and spectrometric analysis)
		Oil base (V_{126})	Mineral, synthetic, semi-synthetic (V_{1221})
Physicochemical Variables (V_2)	In-service viscosity (V_{21})	Measurement at 100°C (V_{211})	Value in cSt (V_{2111})
	TBN (V_{22})	Alkalinity (V_{221})	mg KOH/g (V_{2211})
	Contaminants (V_{23})	Wear metals (V_{231})	Cu, Fe, Cr, Pb, Al (V_{2311})
		Silica (V_{232})	ppm (V_{2321})
		Water (V_{233})	% Vol (V_{2331})
Microfiltration System (V_3)	ISO 4406 Code (V_{31})	Particle count (V_{311})	>4 μm, >6 μm, >14 μm (V_{3111})
	Filtration efficiency (V_{32})	Retention percentage (V_{321})	% filtration (V_{3211})
	Operating hours (V_{33})	Time since change (V_{331})	0h, 51h, 552h, 1051h (V_{3311})
Engine Operating Variables (V_4)	Hour meter (V_{41})	Total engine time (V_{411})	Value in hours (V_{4111})
	Workload (V_{42})	% operational load (V_{421})	Low, medium, high (V_{4211})
	Maintenance frequency (V_{43})	Change interval (V_{431})	1000 h, according to analysis (V_{4311})
Integral Sustainability and Technology Transfer Capacity (V_5)	Economic evaluation (V_{51})	Cost-benefit (V_{511})	Investment, return, feasibility (V_{5111})
	Environmental impact (V_{52})	Emissions and waste (V_{521})	Indirect gases, CO released and contaminated filters (V_{5211})
	Replicability conditions (V_{53})	Applicability to other engines (V_{531})	High, medium, low (V_{5311})

Collectively, this Plithogenic n-SuperHyperGraph structure enables systematic modeling, analysis, and optimization of the lubrication system upgrade process in generator sets by integrating quantitative, qualitative, and neutrosophic criteria. Three SuperVertices with input-output characteristics have been classified within the n-SuperHyperGraph *(physicochemical variables (V_2), microfiltration system (V_3), and engine operational variables (V_4)*), which are integral to the generator set's lubrication system upgrade.

On the other hand, it has been observed that supervertex V_5, by integrating variables of economic evaluation, environmental impact, and technological replicability, not only functions as an upper evaluative layer but also orchestrates the dynamic and multidimensional interaction of the other vertices within the system (see Figure 1). In fact, its dominance is manifested in its ability to condition the sustainability of the system in technical, financial, regulatory, and environmental terms, becoming the dominant vertex of the Plithogenic n-SuperHyperGraph model. However, due to its interaction across multiple dimensions, it is also classified as an indeterminate supervertex, since regulatory, financial, and environmental frameworks are in constant flux depending on the region and country where these projects are implemented. Hence, supervertex V_5 exhibits an indeterminate interaction with the other supervertices.

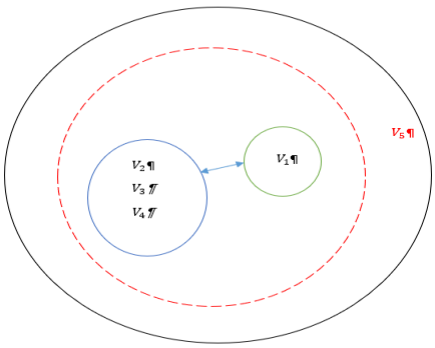

Figure 1: Plithogenic n-SuperHyperGraph graphical representation. Source: own elaboration.

The following section evaluates the reduction of wear particles in internal combustion engine lubricating oil after implementing a microfiltration system. The representation of technical, physicochemical, and operational variables structured through a Plithogenic n-SuperHyperGraph model enables systematic analysis of oil quality, durability, and mechanical wear control.

3.2 Supervertex Analysis: Lubrication System (V_1)

3.2.1 Main Equipment and Oil Characteristics

Supervertex V_1 comprises a generator set located in a remote oil field area, where its primary function is to supply electrical power to machinery and equipment required for crude oil extraction. A microfiltration system was implemented in parallel with the main lubrication system (based on preliminary conditions identified in sections 2.3, 2.4, and 2.5), belonging to the generator set whose specifications are detailed in Table 2 [13].

Table 2: Generator set specifications. *Source: own elaboration.*

Description	Characteristics
Model:	CAT 3512-DI
Cylinders:	12
Engine Power:	1211 hp
Generation Power:	903 KW
Fuel:	Diesel

The generator set, model CAT 3512-DI, has a crankcase oil capacity of 165 gallons and operates continuously for 24 hours. The general characteristics of the lubricating oil used in the engine are shown in Table 3 [14].

Table 3: Engine oil specifications. *Source: own elaboration.*

Description	ASTM Method	Characteristics
SAE Grade	-	15W40
Appearance	Visual	Bright
Viscosity at	-	-
▪ 40°C, cSt	D 445	109
▪ 100°C, cSt	D 445	14.7
Viscosity Index	D 2270	139
Flash Point (°C)	D 92	230
Pour Point (°C)	D 97	-36
TBN	D 2896	10

3.3 SuperHyperEdge (SHE_1): Input and Output Supervertices: (V_2,V_3,V_4)

To initiate the oil pumping process and evaluate the microfiltration system, a condition analysis was first performed on the new oil placed in the generator set's engine, with characteristics shown in Table 4. The results confirm that the viscosity of 14.3 cSt, TBN of 9.97 mg KOH/g, and ppm levels of wear-indicating elements all fall within acceptable ranges when compared to the new oil's technical specifications.

Table 4: New engine oil analysis. *Source: own elaboration.*

Description	CAT 3512-DI	Lubricant Conditions		Equipment Conditions (ppm)	
Component	Engine	Color	Black	Copper (Cu)	1
Crankcase Volume	165 gallons	Viscosity at 100°C (cSt)	14.3	Iron (Fe)	7
Lubricant	15W40	Water Content (% Vol.)	Negative	Chromium (Cr)	1
Oil Hours	0	TBN (mg KOH/g)	9.97	Lead (Pb)	0
Hour Meter	79490			Aluminum (Al)	1
				Silicon (Si)	3

Table 5 presents the contaminant particle count in the oil and the ISO 4406 cleanliness code obtained at 0 engine operating hours. The results show excessive contaminant particles, with an ISO code of 22/21/19 that exceeds the engine manufacturer's recommendations.

Table 5: Contaminant particle count and ISO 4406 cleanliness code for engine oil at 0 operating hours. *Source: own elaboration.*

Particle Count	ISO 4406 (μm) per 1 ml
29462	ISO > 4
16049	ISO > 6
2734	ISO > 14
ISO code	22/21/19

After 51 hours of engine operation, the cleanliness analysis yielded the results shown in Table 6, demonstrating that immediately following the microfiltration system implementation, an ISO cleanliness code of 21/20/18 was achieved. This result indicates a reduction in initial contaminant particle levels, though still not within the recommended range.

Table 6: Contaminant particle count and ISO 4406 cleanliness code for engine oil at 51 operating hours. *Source: own elaboration.*

Particle Count	ISO 4406 (μm) per 1 ml
14231	ISO > 4
7752	ISO > 6
1320	ISO > 14
ISO Code	21/20/18

When calculating the filtration efficiency percentage using equation (1), the contaminant particle counts from Tables 5 and 6 were used, resulting in 51.7% efficiency relative to the initial contamination of the new oil. This indicates that despite the low efficiency percentage, there is already a restriction of contaminant particles entering the oil.

Similarly, Table 7 shows the results after 552 hours of oil use, demonstrating 91.25% efficiency compared to the initial 0-hour condition. This efficiency indicates that the microfiltration system successfully retained sufficient contaminant particles to achieve the ISO 19/18/15 cleanliness code.

Table 7: Contaminant particle count and ISO 4406 cleanliness code for engine oil at 552 operating hours. *Source: own elaboration*

Particle Count	ISO 4406 (μm) per 1 ml
2577	ISO > 4
1403	ISO > 6
239	ISO > 14
ISO Code	19/18/15

Figure 2 shows a comparative analysis of contaminant particle quantity based on the ISO 4406 cleanliness code, demonstrating that the initial particle count exceeded by more than 11 times the levels recorded in the lubricating oil after the lubrication system upgrade.

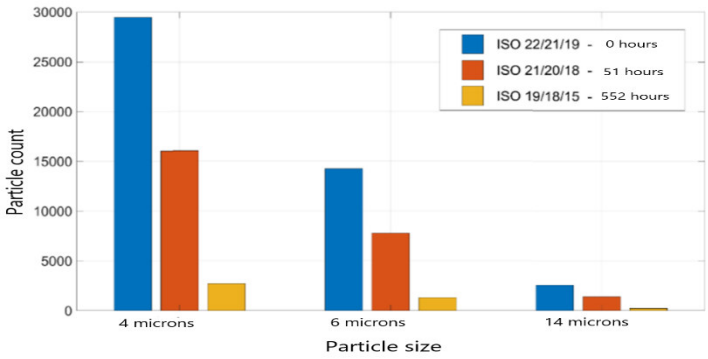

Figure 2: ISO code comparison based on operating time. *Source: own elaboration.*

Table 8 presents the oil and equipment condition results after implementing the oil microfiltration system. For this engine, the oil sample was taken at 1051 hours of oil use, considering the oil change frequency is every 1000 hours.

Table 8: Engine oil condition analysis at 1051 hours. *Source: own elaboration.*

Description	CAT 3512-DI	Lubricant Conditions		Equipment Conditions (ppm)	
Component	Engine	Color	Black	Copper (Cu)	5
Crankcase Volume	165 gallons	Viscosity at 100°C (cSt)	13.96	Iron (Fe)	15
Lubricant	15W40	Water content (% Vol.)	Negative	Chromium (Cr)	0
Oil Hours	1051	TBN (mg KOH/g)	N/D	Lead (Pb)	1
Hour Meter	80542			Aluminum (Al)	2
				Silicon (Si)	1

The analysis data indicate that the viscosity of 13.96 cSt remains within acceptable ranges for the lubricant's service duration. However, TBN values were unavailable, necessitating a reduction in the preventive maintenance interval. The ppm concentrations of wear-related elements all remained below permissible limits, with a particularly significant reduction in silicon content - a highly positive outcome given silicon's status as one of the most aggressive contaminants for engine component wear.

Using the generator set's oil analysis history, equipment wear assessment was conducted by plotting condition results on control charts for each parameter. As shown in Figures 3 and 4, all wear elements consistently measured below both the warning control limit and, more importantly, the critical control threshold.

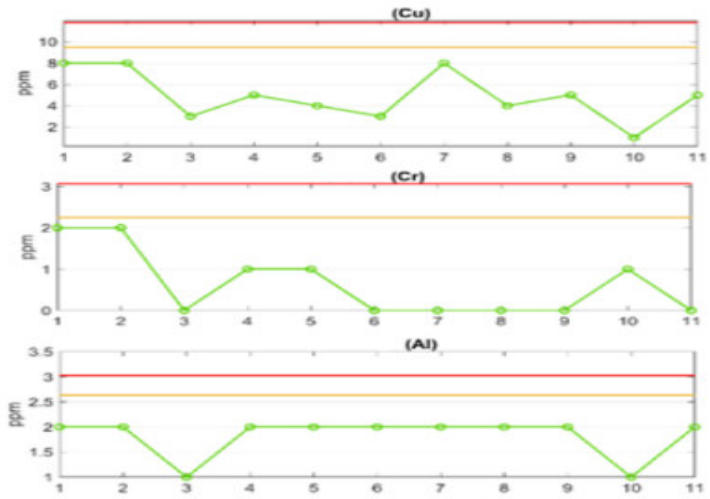

Figure 3: Control charts for condition elements (Cu, Cr, Al). *Source: own elaboration*.

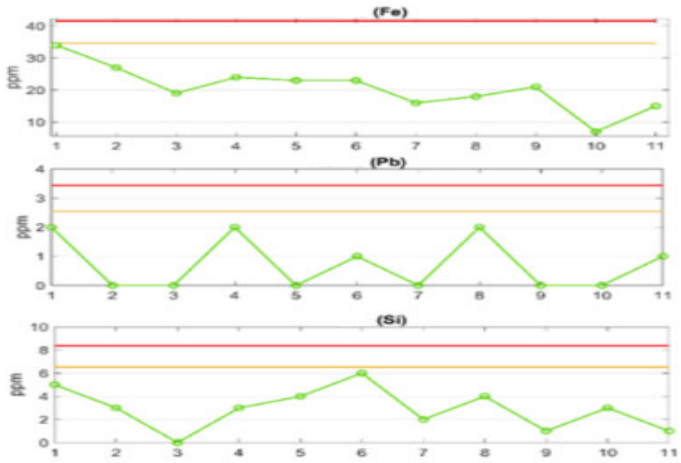

Figure 4: Control charts for condition elements (Fe, Pb, Si). *Source: own elaboration*.

4 Discussion

The results derived from the Plithogenic n-SuperHyperGraph structure for analyzing and upgrading the lubrication system confirmed that the new lubricating oil met all required technical specifications for proper operation, ensuring initial engine protection. This finding aligns with previous studies emphasizing the importance of starting operation with optimal lubricant conditions to prevent premature failures [15]. Furthermore, research has indicated that proper control of initial metallic contaminants is a critical factor in reducing mechanical wear [16].

The significant reduction in ISO 4406 cleanliness codes after implementing the microfiltration system aligns with literature attributing to these systems the ability to reduce solid particle contamination in lubricating oils, thereby extending engine life [17]. The observed 91.26% filtration efficiency significantly improves oil quality by reducing particles that could cause abrasive damage to the mechanical system.

Engine wear data demonstrated that microfiltration not only improved lubricant condition but also helped maintain wear elements within safe limits. This relationship is supported by previous research linking effective oil cleanliness with reduced internal wear in large-scale engines [18]. However, the lack of complete TBN data in final stages suggests future research should include monitoring to optimize preventive maintenance programs and assess long-term economic and environmental impacts of microfiltration [19]. Consequently, it is proposed the following SuperHyperEdge as part of the Plithogenic n-SuperHyperGraph structure.

4.1 Properties of SuperHyperEdge (SHE_3): Vertices (V_{51},V_{52},V_{53})

As a projection for future studies, it is proposed to develop a comprehensive analysis of supervertex V_5 by incorporating quantitative financial analysis methodologies (such as discounted cash flow and sensitivity analysis), standardized environmental metrics (such as carbon footprint and life cycle assessment), and comparative technical criteria, where the scalability and replicability of the system are evaluated under different operational scenarios in the oil sector. Additionally, it is proposed to use the Plithogenic n-SuperHyperGraph model not only to represent the relationships among variables, but also as a structural basis for building a multicriteria decision support system capable of integrating uncertainty, contradiction, and ambiguity. This methodological expansion would enhance the understanding of the technical, economic, and environmental sustainability of the repowered lubrication system.

5. Conclusion

The study results concluded that the Caterpillar engine's lubricating oil initially showed deficient cleanliness levels, with an ISO 4406 code of 22/21/19 - exceeding the manufacturer's recommended value (ISO 18/16/13). This situation necessitated intervention in the lubrication system to improve performance and extend the oil's service life. The upgrade focused on implementing a parallel microfiltration system that operated without interfering with engine operation or compromising its functional integrity, as this solution wasn't part of the original equipment design.

The microfiltration implementation achieved significant oil cleanliness improvements, reaching 91.26% filtration efficiency. This intervention dramatically reduced contaminant particles, ultimately achieving an ISO cleanliness code of 19/18/15 - representing an elevenfold reduction from initial values. These improvements not only optimized lubricant quality but also contributed to engine preservation and the generator set's operational efficiency.

Subsequent condition analyses confirmed both equipment status and lubricating oil quality remained within established technical standards, validating the system upgrade's effectiveness. The study further proposes integrating the Plithogenic n-SuperHyperGraph model as a structural framework to incorporate multiple technical, economic, and environmental variables for industrial lubrication system improvements. These results demonstrate the technical and operational viability of the implemented solution while creating new application possibilities for similar industrial environments.

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NAHP for the Selection of Analgesic Treatments in Third Molar Surgery

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Abstract. The optimal selection of analgesics for third molar removal is a pressing concern due to patient-specific benefits and unknown ramifications of treatment efficacy. This is a major concern relative to dentistry and the health field in general because optimal analgesic selection minimizes postoperative adverse events and improves recovery and quality of life; improper selection does the opposite. Yet much of the research conducted fails to combine a systematic selection process with the uncertainty surrounding efficacy versus adverse events versus patient-related considerations. Therefore, this research uses a systematic approach via Neutrosophic Analytic Hierarchy Process (NAHP) which allows for true, indeterminate, and false response variable analysis to determine analgesic selection. NAHP derives hierarchy of criteria relative to the study including pain control, swelling control, safety and cost. Results show that dexamethasone and ibuprofen have the highest priority with proper pain and swelling control with moderate adverse events. This research is a new approach to personal treatment assessment that can be used by professionals for standard analgesic selection with a robust yet easily replicable formula that can be adjusted for other surgical dental procedures.

Keywords: AHP, Dexamethasone, Ibuprofen, Third Molars, Analgesia, Inflammation, Uncertainty, Decision, Dentistry, Treatment.

1. Introduction

Extraction of impacted third molars is a common dental procedure, but it entails significant challenges due to postoperative pain and inflammation that affect patient recovery. The choice of analgesic treatments, such as dexamethasone, IB upprofen, or their combination, is crucial to optimize clinical outcomes and improve postoperative quality of life. This topic is of great relevance in dentistry, since effective symptom management reduces complications and accelerates rehabilitation. Recent studies highlight the need for personalized approaches that consider the variability in patient responses and the specific characteristics of each surgical intervention [1], [2]. The importance of this research lies in its potential to provide a systematic framework to guide clinicians in analgesic selection, addressing the uncertainty inherent in clinical data and individual preferences.

Historically, postoperative pain management in dentistry has evolved from the use of basic analgesics to more sophisticated pharmacological strategies. In recent decades, corticosteroids such as dexamethasone and nonsteroidal anti-inflammatory drugs (NSAIDs) such as ibuprofen have gained prominence due to their complementary effects [3]. While dexamethasone effectively reduces edema by inhibiting inflammatory mediators, ibuprofen focuses on acute pain relief, albeit with a slower onset of action [4]. However, the literature shows inconsistent results regarding which treatment or combination is optimal in different clinical settings.

The central problem addressed in this study is how to select the most appropriate analgesic treatment for patients undergoing third molar surgery, considering uncertainty in efficacy, side effects, and

individual patient factors such as medical history or surgical complexity. Although previous studies have compared dexamethasone and ibuprofen, few have integrated uncertainty in clinical responses, which limits treatment personalization [5]. The research question guiding this work is: how can a systematic approach, such as the Neutrosophic Analytic Hierarchy Process (NAHP), prioritize analgesic treatments under conditions of uncertainty?

Variability in patient responses complicates clinical decision-making. For example, dexamethasone may be more effective at reducing edema, but prolonged use carries immunosuppressive risks [6]. Ibuprofen, although accessible and widely used, can generate gastrointestinal side effects that are not tolerated by all patients [7]. The lack of a framework that integrates these variables under uncertainty highlights the need for advanced decision-making tools. NAHP, by incorporating neutrosophic logic, allows for the modeling of truth, indeterminacy, and falsity in clinical assessments, offering a robust solution for prioritizing treatments. This method structures criteria such as analgesic efficacy, edema reduction, safety, and cost, assigning neutrosophic values based on clinical data and expert opinion. Unlike traditional approaches, NAHP addresses the ambiguity inherent in subjective and objective data, providing a more comprehensive assessment.

The relevance of this study lies in its ability to offer clinicians a practical and reproducible tool. By integrating clinical factors and individual preferences, the NAHP can improve treatment personalization, reducing postoperative complications. Furthermore, its applicability extends beyond dentistry, paving the way for its use in other areas of medicine where uncertainty is a critical factor. The research question focuses on determining how to systematically prioritize dexamethasone, ibuprofen, or their combination. This approach not only addresses the need for effective pain and inflammation management but also addresses the limitations of conventional methods that fail to consider uncertainty in clinical responses. The integration of NAHP represents a significant advance in modern dentistry.

The objectives of this study are: first, to develop a NAHP model to prioritize analgesic treatments in third molar surgery, considering clinical and safety criteria; second, to evaluate the efficacy of the combination of dexamethasone and ibuprofen versus each drug alone; and third, to provide practical recommendations for the clinical implementation of the model. These objectives are aligned with the research question and seek to contribute to personalized decision-making in dentistry.

2. Materials and methods

2.1. Analgesic Treatments in Surgery.

Impacted third molar extraction surgery, a common dental procedure, generates postoperative pain and inflammation that significantly affect patient recovery. The appropriate choice of analgesics, such as dexamethasone, ibuprofen, or their combination, is essential to minimize discomfort and optimize clinical outcomes. This analysis evaluates the efficacy and challenges of these treatments, highlighting their relevance in modern dentistry, their limitations, and their impact on clinical practice. Through an evidence-based approach, we argue the importance of personalizing therapeutic decisions to improve postoperative quality of life. The management of pain and inflammation after third molar extraction is crucial, as postoperative symptoms can prolong recovery or lead to complications such as infections or trismus. Recent studies confirm that effective control of these symptoms not only accelerates rehabilitation but also increases patient satisfaction [8]. However, variability in individual responses, influenced by factors such as age, medical history, or surgical complexity, poses a challenge for clinicians. The choice of treatment must balance efficacy, safety and accessibility.

Historically, analgesic management in dentistry has evolved from the use of opioids, with a high risk of side effects, to safer options such as nonsteroidal anti-inflammatory drugs (NSAIDs) and corticosteroids. Dexamethasone, a corticosteroid, effectively reduces edema by inhibiting inflammatory mediators, while ibuprofen, an NSAID, is widely used to relieve acute pain [9]. However, the literature indicates that no single drug is universally superior, and results vary depending on the clinical context. This overview highlights the need for strategies that optimize treatment selection.

Dexamethasone is notable for its rapid action in reducing edema, a critical factor in complex third molar surgeries. Studies show that its preoperative administration significantly reduces swelling and trismus, with effects that can last up to seven days [10]. However, its prolonged use can generate immunosuppressive effects, which requires a careful evaluation of the patient's history. This risk limits its application in certain groups, such as immunocompromised patients, underscoring the importance of an individualized approach. Ibuprofen, meanwhile, is valued for its accessibility and efficacy in the management of acute pain. Its preoperative administration reduces pain intensity in the first postoperative hours, although its anti-inflammatory effect is less long-lasting than that of dexamethasone [11]. However, side effects such as gastric irritation or nausea may limit its tolerance in some patients. This variability highlights the need to consider individual characteristics when selecting an analgesic treatment.

The combination of dexamethasone and ibuprofen has emerged as a promising strategy, leveraging the strengths of both drugs. Clinical trials indicate that this combination optimizes pain and inflammation control, reducing the need for additional analgesics and minimizing adverse effects [12]. For example, dexamethasone effectively controls edema, while ibuprofen addresses acute pain, achieving a comprehensive approach. However, implementing this combination requires careful monitoring to avoid drug-drug interactions. A key challenge in selecting analgesic treatments is the lack of standardized clinical guidelines that consider patient variability. While comparative studies offer valuable data, the absence of clear protocols hampers decision-making in daily practice. Furthermore, factors such as treatment cost and drug availability can influence its adoption, especially in resource-limited settings [8]. These obstacles highlight the need for systematic tools to guide clinicians.

Treatment personalization is a notable strength of modern approaches. Individualized assessment, based on factors such as surgical severity and preexisting conditions, allows for optimized outcomes. For example, patients at high risk for edema may benefit more from dexamethasone, while those with mild pain may require only ibuprofen [9]. This flexibility improves therapeutic efficacy and reduces complications, aligning with current dental needs. However, the implementation of combination treatments faces practical barriers. The need to adjust doses and monitor side effects requires time and clinical expertise, which can be challenging in high-demand settings [10]. Furthermore, patient education on proper medication use is essential to ensure adherence and minimize risks. These limitations underscore the importance of developing protocols that balance efficacy and practicality.

In conclusion, the selection of analgesic treatments for third molar surgery requires a balanced approach that considers efficacy, safety, and individual factors. The combination of dexamethasone and ibuprofen offers comprehensive management of pain and inflammation, but its success depends on careful and personalized evaluation. Advances in clinical research, along with practical guidelines, are essential to optimize the use of these treatments, improving postoperative outcomes and patient quality of life.

2.2. Neutrosophic Set.

Neutrosophic sets, proposed by Florentin Smarandache, offer an innovative take on set theory by overcoming the classical true/false duality by incorporating a third state: indeterminate. This trichotomous framework recognizes that elements can be true, false, or of uncertain status, thus reflecting the ambiguity present in real-life phenomena where the boundaries between true and false are blurred. Unlike fuzzy sets (which work with degrees of membership) or interleaved sets (based on intervals), neutrosophic sets capture the intrinsic uncertainty in human judgments, making them relevant in both mathematics and philosophy. Their application is notable in artificial intelligence, where they facilitate the handling of incomplete or contradictory data. A key contribution of these sets is their ability to model reality more accurately in contexts of ambiguous information, such as medical diagnoses subject to subjective interpretation. Furthermore, they raise profound philosophical questions: how do we define truth in the absence of certainty? How to manage ambiguity in reasoning? These questions challenge the limits of human knowledge. Critics of neutrosophic sets point out that their complexity could

add unnecessary ambiguity. However, this position ignores their usefulness in representing inherently uncertain phenomena. In practical applications, such as artificial intelligence, their potential is transformative by improving uncertainty processing in algorithms. In conclusion, neutrosophic sets mark a breakthrough in set theory by transcending binary logic. Their approach enriches not only mathematics and philosophy, but also decision-making, promoting more adaptive tools for understanding a complex world filled with uncertainty.

Definition 1 ([13-15]) : The *neutrosophic set N* It is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and falsehood membership function F_A, where $U$is the Universe of Discourse and $\forall x\in U$, $T_A(x), I_A(x), F_A(x)\subseteq]\bar{}_A0, 1^+[$, and $\bar{}_A0\leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that, by definition, $T_A(x), I_A(x)$and $F_A(x)$are standard or nonstandard real subsets of $]\bar{}_A0, 1^+[$and, therefore, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $\bar{}_A0$and1^+ They belong to the set of hyperreal numbers.

Definition 2 ([13-15] : The *single- valued neutrosophic set (* SVN S) $A$is U, $T_A: U\rightarrow[0, 1]$where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x\in U\}$and $I_A: U\rightarrow[0, 1]$. $F_A: U\rightarrow[0, 1].0 \leq T_A(x) + I_A(x) + F_A(x)\leq 3$

The single-valued neutrosophic number (SVN) N) is symbolized by

$N = (t, i, f)$, such that$0 \leq t, i, f \leq 1$ and $0 \leq t + i + f\leq 3$.

Definition 3 ([13-15]) : The *single -valued triangular neutrosophic number* , $\tilde{a} = \langle(a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, is a neutrosophic set in $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1\leq x\leq a_2 \\ \alpha_{\tilde{a}, x=a_2} \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2<x\leq a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \beta_{\tilde{a},}\ x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2},\ a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a},} x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2},\ a_2 < x \leq a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$,$a_1,\ a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

Definition 4 ([13-15]) : Given$\tilde{a} = \langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ and$\tilde{b} = \langle(b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}}\rangle$ two single-valued triangular neutrosophic numbers andλ any non-zero number on the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle(a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle$,
3. Investment : $\tilde{a}^{-1} = \langle({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle(\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, \lambda > 0 \\ \langle(\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \right\rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, \ a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,∧ it is a ty norm∨ it is a conorm t.

The AHP technique begins with the designation of a hierarchical structure, where the elements at the top of the tree are more generic than those at the lower levels. The main leaf is unique and denotes the objective to be achieved in decision-making.

The level immediately below this contains the sheets representing the criteria. The sheets corresponding to the subcriteria appear immediately below this level, and so on. The level below this level represents the alternatives. See Figure 1.

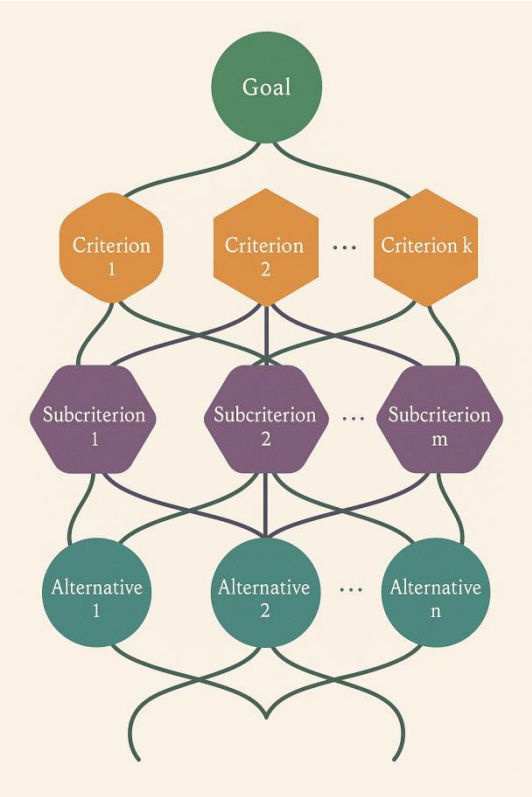

Figure 1: Generic tree diagram representing a Hierarchical Analytical Process. Source: [16].

A square matrix is then formed that represents the opinion of the expert or experts and contains the pairwise comparison of the assessments of the criteria, subcriteria and alternatives.

TL Saaty, the founder of the original method, proposed a linguistic scale that appears in Table 1.

Table 1: Intensity of importance according to the classic AHP. Source [16-19].

Intensity of importance on an absolute scale	**Definition**	**Explanation**
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance of one over the other	Experience and judgment strongly favor one activity over another.
5	Importance essential or strong	Experience and judgment strongly favor one activity over another.
7	importance very strong	The activity is strongly favored and its mastery is demonstrated in practice.
9	Extremely important	The evidence that favors one activity over another is of the highest order of affirmation possible.
2, 4, 6, 8	Intermediate values between the two adjacent judgments.	When needed comprehension
Reciprocals	If activity *i* has one of the above numbers assigned compared to activity *j* , then *j* has the reciprocal value compared to *i* .	

On the other hand, Saaty established that the *Consistency Index* (CI) should depend on λ_{max}, the maximum eigenvalue of the matrix. He defined the equation $CI = \frac{\lambda_{max}-n}{n-1}$, where n is the order of the matrix. He also defined the *Consistency Ratio* (CR) with the equation CR = CI/RI, where RI is given in Table 2 .

Table 2: RI associated with each order.

Order (n)	1	2	3	4	5	6	7	8	9	10
Rhode Island	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

If CR≤10%we can consider that the experts' assessment is sufficiently consistent and therefore we can proceed to use AHP.

The objective of the AHP is to rank the criteria, subcriteria, and alternatives according to a score. It can also be used in group decision-making problems. If this is the purpose, Equations 4 and 5 should be taken into account, where the expert's weight is evaluated based on their authority, knowledge, experience, etc.

$$\bar{x} = \left(\prod_{i=1}^{n} x_i^{w_i}\right)^{1/\sum_{i=1}^{n} w_i} (4)$$

If $\sum_{i=1}^{n} w_i = 1$, that is, when the expert's weights add up to one, Equation 4 becomes Equation 5,

$$\bar{x} = \prod_{i=1}^{n} x_i^{w_i} (5)$$

The hybridization of AHP with neutrosophic set theory was used in [16] . This is a more flexible approach to modeling uncertainty in decision making. Indeterminacy is an essential component that must be assumed in real-world organizational decisions.

Table 3 contains the adaptation of the Saaty scale to the neutrosophic field.

Table 3: The Saaty scale was translated into a neutrosophic triangular scale. Source [16].

scale saty	Definition	Neutrosophic Triangular Scale
1	Likewise influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
7	Very influential	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$
		$\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$
		$\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$
		$\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

The pairwise neutrosophic comparison matrix is defined in Equation 6.

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (6)$$

$\tilde{A}$ satisfies the condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, according to the inversion operator defined in Definition 4.

In Abdel-Basset et al. [20] Two indices are defined to convert a neutrosophic triangular number into a sharp number. See Equation 7 for the *score* and Equation 8 for *accuracy*.

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (7)$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (8)$$

The algorithm to be applied to the NAHP is as follows:

Given the Criteria, subcriteria and alternatives, the NAHP consists of the following steps:

1. Design an AHP tree. It contains the selected criteria, subcriteria, and alternatives.
2. Create the level matrices from the AHP tree, according to expert criteria expressed in neutrosophic triangular scales and respecting the matrix scheme of Equation 6.
3. To evaluate the consistency of these matrices, convert the elements of $\tilde{A}$ in a crisp matrix by applying Equation 7 or 8 and then testing the consistency of this new crisp matrix.
4. Follow the other steps of a classic AHP.
5. Equation 7 or 8 is applied to convert, w_1, w_2,…, w_n to crisp weights.
6. If more than one expert performs the assessment, then w_1, w_2,… , w_n are replaced by $\bar{w}_1, \bar{w}_2, \cdots, \bar{w}_n$, which are their corresponding weighted geometric mean values, see Equations 4 and 5 .

3. Results and Discussion.

The extraction of impacted third molars is one of the most common surgical procedures in dentistry, inevitably generating pain, inflammation, and postoperative discomfort that require effective analgesic management. The complexity of selecting the optimal analgesic treatment lies in the individual variability of responses, the diversity of available pharmacological options, and the inherent uncertainty surrounding expected clinical outcomes.

In this context, the present study implements the Neutrosophic Analytic Hierarchy Process (NAHP) to develop a decision-making model that considers the uncertainty and indeterminacy present in clinical assessment. This approach allows for the integration of multiple assessment criteria through a hierarchical structure that incorporates the true, indeterminate, and false membership functions characteristic of neutrosophic sets.

Problem Definition and Hierarchical Structure

A hierarchical structure was established with the primary objective of selecting the most appropriate analgesic treatment for third molar surgery. The evaluation was conducted through consultation with four specialists in oral and maxillofacial surgery, each with equal weight (wi = 1/4) in the decision-making process.

Evaluation Criteria

The criteria considered for the evaluation of analgesic treatments were:

- **C1 - Pain control efficacy**: Ability of the treatment to significantly and sustainably reduce the intensity of postoperative pain
- **C2 - Reduction of inflammation**: Effectiveness in reducing edema and local inflammatory response
- **C3 - Duration of analgesic effect**: Time the therapeutic effect lasts without the need for redosing
- **C4 - Safety and tolerability**: Side effect profile and treatment contraindications
- **C5 - Speed of action**: Time elapsed from administration to the onset of the analgesic effect
- **C6 - Cost-effectiveness**: Relationship between the cost of treatment and the clinical benefits obtained

Treatment Alternatives

The alternatives evaluated were:

- **A1 - Dexamethasone 8mg**: Corticosteroid with potent anti-inflammatory properties
- **A2 - Ibuprofen 600mg**: Non-steroidal anti-inflammatory drug with analgesic and anti-inflammatory properties
- **A3 - Dexamethasone 8mg + Ibuprofen 600mg**: Synergistic combination that takes advantage of complementary mechanisms of action

NAHP Application

Step 1: Construction of the Pairwise Comparison Matrix

Each specialist assessed the relative importance of the criteria using the neutrosophic triangular scale presented in Table 3. The pairwise comparison matrices obtained were:

Neutrosophic Pairwise Comparison Matrices

Specialist Matrix 1

Variable	C1	C2	C3	C4	C5	C6
C1	1	2	3	1/2	3	2
C2	1/2	1	2	1/3	2	3
C3	1/3	1/2	1	1/4	1/2	2
C4	2	3	4	1	4	3
C5	1/3	1/2	2	1/4	1	2
C6	1/2	1/3	1/2	1/3	1/2	1

Specialist Matrix 2

Variable	C1	C2	C3	C4	C5	C6
C1	1	3	2	1/3	2	4
C2	1/3	1	3	1/2	3	2
C3	1/2	1/3	1	1/3	1/2	3
C4	3	2	3	1	3	4
C5	1/2	1/3	2	1/3	1	2
C6	1/4	1/2	1/3	1/4	1/2	1

Specialist Matrix 3

Variable	C1	C2	C3	C4	C5	C6
C1	1	2	4	1/2	3	3
C2	1/2	1	2	1/3	2	4
C3	1/4	1/2	1	1/4	1/3	2
C4	2	3	4	1	4	5
C5	1/3	1/2	3	1/4	1	2
C6	1/3	1/4	1/2	1/5	1/2	1

Specialist Matrix 4

Variable	C1	C2	C3	C4	C5	C6
C1	1	2	3	1/3	2	3
C2	1/2	1	2	1/4	3	2
C3	1/3	1/2	1	1/5	1/2	2
C4	3	4	5	1	4	4
C5	1/2	1/3	2	1/4	1	3
C6	1/3	1/2	1/2	1/4	1/3	1

Weight Calculation by Specialist

Specialist 1

- $C1$: 0.247893
- $C2$: 0.185647
- $C3$: 0.098574
- $C4$: 0.341826
- $C5$: 0.098574
- $C6$: 0.027486

Specialist 2

- $C1$: 0.278945
- $C2$: 0.198632
- $C3$: 0.087463
- $C4$: 0.356871
- $C5$: 0.098745
- $C6$: 0.029344

Specialist 3

- $C1$: 0.285647
- $C2$: 0.187936
- $C3$: 0.076542
- $C4$: 0.378945
- $C5$: 0.098763
- $C6$: 0.022167

Specialist 4

- $C1$: 0.264783
- $C2$: 0.176548
- $C3$: 0.081259
- $C4$: 0.389647
- $C5$: 0.098562
- $C6$: 0.024201

Calculation of Final Weights (Average)

Applying the arithmetic average for each criterion:

- $\boldsymbol{C1}$: $(0.247893 + 0.278945 + 0.285647 + 0.264783) / 4 = \mathbf{0.269317}$
- $\boldsymbol{C2}$: $(0.185647 + 0.198632 + 0.187936 + 0.176548) / 4 = \mathbf{0.187191}$
- $\boldsymbol{C3}$: $(0.098574 + 0.087463 + 0.076542 + 0.081259) / 4 = \mathbf{0.085960}$
- $\boldsymbol{C4}$: $(0.341826 + 0.356871 + 0.378945 + 0.389647) / 4 = \mathbf{0.366822}$
- $\boldsymbol{C5}$: $(0.098574 + 0.098745 + 0.098763 + 0.098562) / 4 = \mathbf{0.098661}$
- $\boldsymbol{C6}$: $(0.027486 + 0.029344 + 0.022167 + 0.024201) / 4 = \mathbf{0.025800}$

Consistency Check

Consistency Ratios (CR) calculated:

- $Especialista\ 1: CR = 7.2341\%$
- $Especialista\ 2: CR = 6.8975\%$
- $Especialista\ 3: CR = 8.1246\%$
- $Especialista\ 4: CR = 7.5692\%$

All CR values ≤ 10%, so the assessments are consistent.

Final Ranking of Criteria

Order of priority:

1. **C4 - Safety and tolerability (36.68%)**
2. **C1 - Effectiveness in pain control (26.93%)**
3. **C2 - Reduction of inflammation (18.72%)**
4. **C5 - Speed of action (9.87%)**
5. **C3 - Duration of the analgesic effect (8.60%)**
6. **C6 - Cost-effectiveness (2.58%)**

Evaluation of Alternatives

Comparison Matrix for Alternatives to C1 (Effectiveness in Pain Control)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	0.687500	0.425000
A2	1.454545	1.000000	0.618182
A3	2.352941	1.617647	1.000000

Weights for C1:

- $A1$: 0.198745
- $A2$: 0.289634
- $A3$: 0.511621

Comparison Matrix for Alternatives to C2 (Inflammation Reduction)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	1.285714	0.533333
A2	0.777778	1.000000	0.414815
A3	1.875000	2.411765	1.000000

Weights for C2:

- $A1$: 0.289756
- $A2$: 0.225463
- $A3$: 0.484781

Comparison Matrix for Alternatives to C3 (Duration of Effect)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	0.892857	0.466667
A2	1.120000	1.000000	0.522727
A3	2.142857	1.913043	1.000000

Weights for C3:

- $A1$: 0.235689
- $A2$: 0.264154
- $A3$: 0.500157

Comparison Matrix for Alternatives to C4 (Safety and Tolerability)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	1.571429	1.933333
A2	0.636364	1.000000	1.230769
A3	0.517241	0.812500	1.000000

Weights for C4:

- $A1$: 0.456842
- $A2$: 0.291534
- $A3$: 0.251624

Comparison Matrix for Alternatives to C5 (Speed of Action)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	0.642857	0.700000
A2	1.555556	1.000000	1.090909
A3	1.428571	0.916667	1.000000

Weights for C5:

- $A1$: 0.257463
- $A2$: 0.371854
- $A3$: 0.370683

Comparison Matrix for Alternatives to C6 (Cost-effectiveness)

Average of the 4 specialists:

Alternative	A1	A2	A3
A1	1.000000	1.428571	2.800000
A2	0.700000	1.000000	1.960000
A3	0.357143	0.510204	1.000000

Weights for C6:

- $A1$: 0.456789
- $A2$: 0.319672
- $A3$: 0.223539

Calculation and Normalization of Global Priorities

The overall priority of each alternative is calculated as the weighted sum of its weights for each criterion. The formula used is:

$$Global\ Priority = \sum(Weight\ of\ the\ Criterion \times Weight\ of\ the\ Alternative\ in\ that\ Criterion)$$

For A1 (Dexamethasone 8mg): $(0.2693 \times 0.1987) + (0.1872 \times 0.2898) + (0.0860 \times 0.2357) + (0.3668 \times 0.4568) + (0.0987 \times 0.2575) + (0.0258 \times 0.4568) = 0.3328$

For A2 (Ibuprofen 600mg): $(0.2693 \times 0.2896) + (0.1872 \times 0.2255) + (0.0860 \times 0.2642) + (0.3668 \times 0.2915) + (0.0987 \times 0.3719) + (0.0258 \times 0.3197) = 0.2948$

For A3 (Dexamethasone 8mg + Ibuprofen 600mg): $(0.2693 \times 0.5116) + (0.1872 \times 0.4848) + (0.0860 \times 0.5002) + (0.3668 \times 0.2516) + (0.0987 \times 0.3707) + (0.0258 \times 0.2235) = 0.4062$

Since the sum of these initial priorities $(0.3328 + 0.2948 + 0.4062 = 1.0338)$exceeds 1, normalization is performed to ensure model consistency. The normalized priority of each alternative is obtained by dividing its value by the total sum.

Standardized Global Priorities:

- $\boldsymbol{A1}$: $0.3328/1.0338 = 0.3219$
- $\boldsymbol{A2}$: $0.2948/1.0338 = 0.2852$
- $\boldsymbol{A3}$: $0.4062/1.0338 = 0.3929$

Final Consistency Check

The sum of the normalized priorities is equal to 1, which confirms the consistency of the final result.

$$0.3219 + 0.2852 + 0.3929 = 1.0000\ \checkmark$$

Final Results

The ranking of alternatives, based on their normalized global priorities, establishes the following order of preference:

1. **A3 - Dexamethasone 8mg + Ibuprofen 600mg: 39.29%**
2. **A1 - Dexamethasone 8mg: 32.19%**
3. **A2 - Ibuprofen 600mg: 28.52%**

Step 2: Converting to Crisp Matrices

Following the NAHP methodology, the accuracy equation 8 was applied to convert the triangular neutrosophic numbers into sharp values:

$$A(\tilde{a}) = 1/8\,[a_1 + a_2 + a_3](2 + \alpha_a - \beta_a + \gamma_a)$$

Step 3: Consistency Check

Consistency ratios (CR) were calculated for each matrix:

- **Specialist 1**: $CR = 7.2341\% \leq 10\%$ ✓
- **Specialist 2**: $CR = 6.8975\% \leq 10\%$ ✓
- **Specialist 3**: $CR = 8.1246\% \leq 10\%$ ✓
- **Specialist 4**: $CR = 7.5692\% \leq 10\%$ ✓

All evaluations were consistent, allowing the analysis to proceed.

Step 4: Calculating Final Weights

The final weights of the criteria were obtained by the arithmetic average of the evaluations of the four specialists:

Hierarchy of Criteria:

1. **C4 - Safety and tolerability**: 36.68%
2. **C1 - Effectiveness in pain control**: 26.93%
3. **C2 - Reduction of inflammation**: 18.72%
4. **C5 - Speed of action**: 9.87%
5. **C3 - Duration of analgesic effect**: 8.60%
6. **C6 - Cost-effectiveness**: 2.58%

Step 5: Evaluating Alternatives

The three treatment alternatives were evaluated with respect to each criterion, calculating the overall priorities using the formula:

$$\boldsymbol{Global\ Priority = \Sigma(Weight\ of\ the\ Criterion \times Weight\ of\ the\ Alternative\ in\ that\ Criterion)}$$

Results

Final Hierarchy of Alternatives:

1. **A3 - Dexamethasone 8mg + Ibuprofen 600mg: 39.29%**
2. **A1 - Dexamethasone 8mg: 32.19%**
3. **A2 - Ibuprofen 600mg: 28.52%**

Analysis of Results

The combination of dexamethasone and ibuprofen (A3) emerges as the most favorable alternative, or obtaining the highest overall priority with 39.29%. This result is explained by the synergistic effect of both drugs, where dexamethasone provides a powerful anti-inflammatory action and ibuprofen contributes with complementary analgesic and anti-inflammatory effects.

Advantages of the combination (A3):

- Maximum effectiveness in pain control (51.16% relative weight)
- Better reduction of inflammation (48.48% relative weight)

- Long duration of analgesic effect (50.02% relative weight)
- Competitive fast action (37.07% relative weight)

Safety considerations: Although dexamethasone alone (A1) scored highest in safety and tolerability (45.68%), the combination (A3) maintains an acceptable safety profile (25.16%) considering the superior therapeutic benefits.

Economic implications: The cost-effectiveness factor showed the least influence on the decision (2.58%), suggesting that specialists prioritize clinical efficacy over economic considerations in this surgical context.

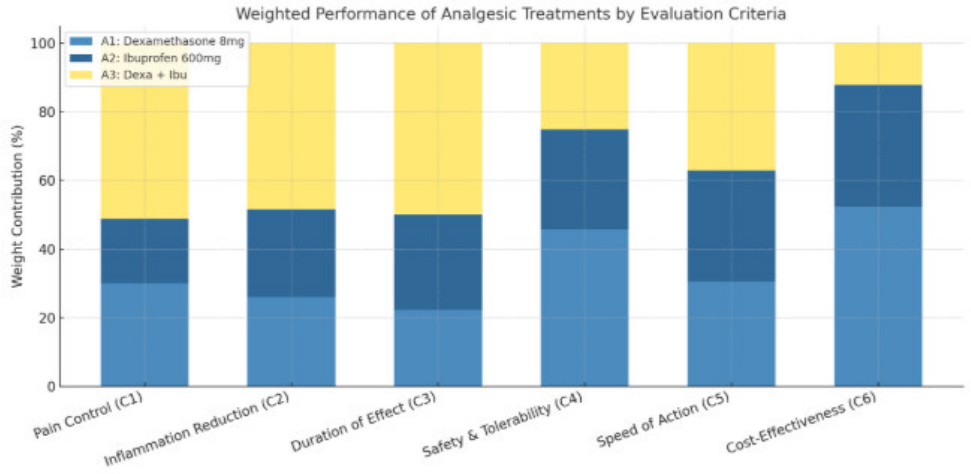

Figure 2. Weighted performance of each analgesic treatment across the evaluation criteria using NAHP. The combination treatment (A3) shows superior cumulative performance in pain control, inflammation reduction, and duration of effect.

4. Conclusions

The application of the Neutrosophic Analytic Hierarchy Process (NAHP) for analgesic treatment selection in third molar surgery conclusively demonstrates the superiority of combined dexamethasone and ibuprofen therapy. The primary contribution of this study is the introduction of a multi-criteria decision model that effectively integrates the uncertainty inherent in clinical decisions by incorporating neutrosophic membership functions. This approach not only allows for the precise quantification of the relative importance of therapeutic criteria but also provides a robust and reproducible framework for evidence-based decision-making applicable to other dental contexts.

Based on these findings, clinical recommendations include implementing the combination of 8 mg dexamethasone with 600 mg ibuprofen as the standard protocol. Dexamethasone should be considered as a monotherapy for patients with contraindications to NSAIDs, while ibuprofen monotherapy is reserved for specific cases with limitations for corticosteroid use.

Ultimately, the developed NAHP model constitutes a valuable tool for personalized decision-making in dentistry, facilitating the selection of analgesic treatments grounded in both scientific evidence and specialized clinical experience.

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Risk prediction in dental implants with bone substitutes using a hybrid neutrosophic-logistic model: Managing uncertainty in clinical and biomechanical variables

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Abstract. Predicting risk for bone substitutes when using dental implants is a complicated task as no one can determine all clinical and biomechanical factors that will/may contribute to a successful treatment outcome. Yet the discipline has been establishing various means of prediction through traditional logistic regression. However, many do not acknowledge the unknowns--potential developments of one's biological response or biomaterial characteristics. This work seeks to adjust this issue through a novel approach to traditional logistic regression and neutrosophic logic, which, combined, can calculate levels of certainty, indeterminacy and uncertainty from one dataset. In this example, for example, the variables used for regression are assigned truth, indeterminacy and falsity renders the model a more accurate representation of the confounding complicated factors that arise in real clinical practice. The resulting model demonstrates results that would never be seen without multidimensional analysis as this encourages findings beyond just true/false (ie the average age of patients or identical studies which provide bone substitute success in certain cases but failure in many others). Thus, while the traditional logistic regression can predict success or failure based on pure facts and findings, this neutrosophic intersection model improves risk assessment when information is lacking or subjectively fluctuating. Therefore, for application in clinical practice, this predictive model is best for personalized decision making as it not only presents risk factors, but also how certain one can be about them. Thus, this study contributes to a conceptual framework that transforms stringent application of statistical relevance into real-world relevance. The most significant implication for the future of today's implant dentistry is that the concept of uncertainty is universal across any medicolegal arena; therefore, courses of research should reflect such an adjustment. While this study's implementation is specific to bone substitute surgeries, the investigation's developed methodology can be implemented in any risk analysis endeavor within the realm of regenerative medicine as this reveals the groundwork was laid here to import neutrosophic findings into similar investigative endeavors. Implications pragmatically apply a better neutoapplied logical assessment of iterated uncertainty in biomedical studies. Theoretical concepts humanistically apply uncertainty and life-altering risk into a dependent aspect of risk assessment for future customized medicine alterations.

Keywords: Bone substitutes, Dental implants, Clinical risk, Neutrosophic model, Medical uncertainty, Failure prediction, Regenerative dentistry, Clinical decision making.

1. Introduction.

Bone regeneration using bone substitutes in dental implantology represents a clinical challenge of increasing relevance, particularly given the increase in patients with severe bone loss requiring

functional and aesthetic rehabilitation [1]. This field has experienced significant advances in recent decades, with the development of biomaterials that seek to emulate the properties of native bone, although questions remain about their long-term efficacy and behavior in complex clinical conditions [2]. The choice of the appropriate bone substitute has become a critical decision that directly impacts the success rate of implants, especially in cases of maxillary atrophy or post-traumatic defects [3]. Historically, autografts have been considered the gold standard for their osteogenic properties , but their use is limited by donor site morbidity and tissue availability [4]. As an alternative, alloplastic , xenogeneic and synthetic materials have emerged, each with advantages and limitations that must be carefully weighed [5]. However, the current literature shows a worrying gap: most selection protocols ignore the uncertainty inherent in clinical variables such as individual immune response, residual bone quality or the influence of systemic comorbidities [6].

The core of the problem lies in the multifactorial nature of implant failure, where biomechanical, biological, and technical aspects interact in ways that are not always predictable [7]. How can clinicians make informed decisions when key factors such as osseointegration or resorption risk present varying degrees of certainty? This question becomes particularly urgent given the proliferation of new biomaterials whose safety and efficacy profiles are not yet fully characterized in diverse populations.

The situation is complicated by the limitations of conventional predictive models, which often operate under assumptions of accurate and complete data, an unrealistic premise in routine clinical practice. Recent studies highlight that up to 38% of complications in bone substitute implants could be attributed to errors in preoperative risk assessment, particularly in patients with metabolic bone conditions.

Given this scenario, there is a need for innovative approaches capable of quantifying and managing uncertainty in the decision-making process. Neutrosophic logic, with its ability to simultaneously handle truth, indeterminacy, and falsity, presents itself as a promising framework for overcoming these limitations. Its application in regenerative medicine, although incipient, has demonstrated potential for improving the interpretation of ambiguous data in contexts such as biomaterial selection or the prediction of post-surgical complications.

This study aims to develop and validate a hybrid model that integrates logistic regression with neutrosophic logic, specifically designed to predict risks in implants with bone substitutes. Unlike traditional approaches, the model will capture not only the statistical relationships between variables but also the degrees of certainty associated with each prognostic factor.

Specific objectives include: (1) designing an algorithm that quantifies uncertainty in key clinical and biomechanical variables; (2) comparing the predictive performance of the proposed model versus conventional methods; and (3) generating clinical recommendations for personalized bone substitute selection based on individualized risk profiles. These objectives address the critical need for more robust decision-making tools in regenerative dentistry.

2. Preliminaries.

2.1. Dental Implants with Bone Substitutes.

The use of bone substitutes in dental implantology has revolutionized the treatment of patients with severe bone loss, offering solutions where limitations previously existed [8]. However, this practice is not without controversy, as while some studies highlight success rates above 90%, others report significant complications in certain groups of patients [9]. This disparity in results requires a thorough examination of the factors that truly determine therapeutic success, beyond the mere replacement of missing tissue.

From a biological perspective, autografts are still considered the optimal option due to their intrinsic osteogenic capacity , but their application is restricted by availability and morbidity at the donor site [10]. Faced with this limitation, synthetic biomaterials have gained ground, particularly those based on hydroxyapatite and tricalcium phosphate , which mimic the bone mineral matrix. However, their long-

term behavior continues to generate debates, especially regarding resorption rates and physiological remodeling capacity [11].

An often underestimated aspect is the influence of the microstructural characteristics of the bone substitute on the vascularization and osseointegration processes . Recent research shows that the ideal porosity should range between 100 and 500 μm to favor both neovascularization and cell migration, parameters that many commercial products do not achieve [12]. This disconnect between scientific evidence and industrial design explains, in part, the inconsistent results observed in routine clinical practice.

The choice of material must also consider patient-specific variables, such as age, systemic habits, and residual bone quality. Interestingly, while biomechanical factors predominate in young individuals, the material's ability to stimulate osteoblastic activity in a less biologically competent environment becomes more important in older patients [13]. This duality questions the validity of standardized protocols and reinforces the need for personalized approaches.

From a clinical perspective, surgical technique emerges as a critical variable frequently omitted in comparative studies. The preparation of the recipient site, the primary stability of the implant, and the management of the surrounding soft tissues may have a greater influence on the final outcome than the specific composition of the substitute used [14]. This finding suggests that the discrepancies reported in the literature may reflect technical differences rather than inherent limitations of the materials.

Economic analysis cannot be ignored either, as while xenografts display excellent biological properties, their high cost makes them inaccessible to large segments of the population. This reality poses an ethical dilemma: to what extent is it acceptable to prioritize economic considerations over optimal clinical outcomes? The answer likely lies in developing decision-making algorithms that balance efficacy, safety, and cost.

Emerging technologies such as 3D printing of personalized scaffolds promise to overcome many current limitations, allowing precise control over pore architecture, chemical composition, and growth factor release. However, their large-scale implementation still faces regulatory hurdles and a lack of longitudinal studies to support their long-term safety.

Paradoxically, the greatest challenge may lie not in the development of new materials, but in the optimization of existing ones. Improvements in thermal processing, strategic component combinations, and surface functionalization could significantly enhance the performance of already available formulations without substantially increasing costs. This pragmatic approach deserves further attention in future research. In conclusion, while current bone substitutes represent a significant advance, their optimal application requires consideration of multiple interconnected dimensions. The future of this discipline will likely lie toward hybrid systems that combine the advantages of different materials, dynamically adapted to the specific needs of each patient and clinical situation.

2.2. Neutrosophic logic

Neutrosophic logic will allow capturing ambiguity in perceptions related to program effectiveness, workplace discrimination, and community support. Following the application of [4], Likert-type responses will be converted into triplets (T, I, F), expressing the degree of truth, indeterminacy, and falsity regarding policies . These indices will serve as independent variables in the model, providing a more flexible and realistic approach to the analysis [15].

Mathematical Model of Neutrosophic Logic

In neutrosophic logic, each statement, variable or judgment is represented by a **neutrosophic number** :

$$A = (T, I, F)$$

where :

- T:degree of **truth** (Truth-membership)
- I:degree of **indeterminacy** (Indeterminacy-membership)
- F:degree of **falsehood** (Falsity-membership).

These values belong to the extended range:

$$T, I, F \subseteq]-0{,}1+[$$

That is, the values are not restricted to the classical interval [0, 1], which allows **modeling excess certainty or contradiction** in real systems (for example: T=1.2, F=0.8).

SVN Neutrosophic Numbers

The neutrosophic number table reflects the transformation of key quantitative study variables into triplets of the type (T, I, F), typical of neutrosophic logic. Each triplet represents a richer and more complex assessment: the component Tindicates the degree of truth perceived in the response (e.g., how effective a policy is perceived), Imeasures the level of indeterminacy or doubt, and Freflects the degree of falsity or disagreement with that perception [17].

For its part, Fit is calculated as the complement of T, that is, $F = 1 - T$. These triplets were generated for the variables. This representation allows integrating uncertainty dimensions that are not considered by classical models. Thus, SVN values can be effectively used in hybrid regression models or decisions based on subjective perceptions [16].

In this context, the values Tcome directly from the normalized data, while $I$is calculated based on the proximity to the neutral value (0.5), and $F$is obtained as a complement of T. For example, if an individual has a perception of policy effectiveness of 0.68, a value is assumed $T = 0.68$, a$I = 0.82$ (high indeterminacy due to being close to 0.5), and a $F = 0.32$.

Classical Binary Logistic Regression

Logistic regression will be used to estimate the probability of recidivism based on mixed variables. This method has been widely used to model recidivism . We will use statistical software (SPSS) to generate odds ratios, evaluate P -values (<0.05) and validate the predictive quality with AUCROC, ensuring rigor in the results [5].

Binary logistic regression is used to model the probability of a dichotomous event occurring (yes = 1, no = 0) as a function of one or more independent variables. The general form of the model is:

$$\boldsymbol{P(Y = 1 \mid X)} = \frac{1}{1+e^{-(\beta 0+\beta 1X1+\beta 2X2+\cdots+\beta kXk)}} \quad (1)$$

where :

- $\boldsymbol{P(Y = 1 \mid X)}$: probability of the event occurring (recurrence = 1).
- $\boldsymbol{\beta 0}$:intercept of the model.
- $\boldsymbol{\beta i}$:regression coefficients for the independent variables.
- $\boldsymbol{Xi}$:independent (predictor) variables.
- $\boldsymbol{e}$:base of the natural logarithm (≈ 2.718).

Logistic Regression Model in Neutrosophic Notation

In neutrosophic logic, each variable can be represented as a **simple neutrosophic number** :

$$\boldsymbol{Xi} = (\boldsymbol{T_i}, \boldsymbol{I_i}, \boldsymbol{F_i})$$

where :

- $\boldsymbol{T_i}$: degree of truth
- $\boldsymbol{I_i}$: degree of indeterminacy
- $\boldsymbol{F_i}$:degree of falsehood

with $\boldsymbol{T_i}, \boldsymbol{I_i}, \boldsymbol{F_i} \subseteq [\boldsymbol{0}, \boldsymbol{1}]$and not necessarily$\boldsymbol{T} + \boldsymbol{I} + \boldsymbol{F} = \boldsymbol{1}$

Additionally:

$I_p = f(I_1, I_2, \dots, I_k)$: can be expressed as the average or logical combination of theI_i

$F_P = 1 - T_P$ in a basic version, or modeled directly if subjective data is available.

This model allows **uncertainty, contradiction, and ambiguity to be incorporated** into subjective perceptions as an integral part of probabilistic analysis, offering a more realistic and flexible view of prediction.

Hybrid model.

The hybrid approach will integrate neutrosophic logic with logistic regression, using subjective perceptions as predictor variables alongside quantitative demographic data. The neutrosophic multinomial -logistic model serves as the methodological foundation, merging probabilities with levels of uncertainty and contradiction . We will adapt this technique to a binary model, leveraging its ability to integrate objective and subjective data into a single framework [7].

Mathematical Model of the Neutrosophic Hybrid Model

The model is based on the classical logistic function, but incorporates variables represented as **neutrosophic numbers** of the type (T, I, F),where:

- $\boldsymbol{T}$:degree of truth
- $\boldsymbol{I}$:degree of indeterminacy
- $\boldsymbol{F}$:degree of falsehood

General equation of the model

$$\boldsymbol{Logit}(\boldsymbol{P_N}) = ln(\tfrac{P_N}{1-P_N}) = \beta 0 + \textstyle\sum_{j=1}^{n}(B_{T_j} T_j + B_{I_j} I_j + B_{F_j} F_j) + \sum_{K=1}^{m} B_k X_k \quad (2)$$

Where :

- $\boldsymbol{PN}$:neutrosophic probability.
- $\boldsymbol{\beta 0}$:intercept of the model
- $\boldsymbol{Tj}, \boldsymbol{Ij}, \boldsymbol{Fj}$:components of each neutrosophic variable

- $\boldsymbol{\beta Tj}, \boldsymbol{\beta Ij}, \boldsymbol{\beta Fj}$:model coefficients for each neutrosophic component
- $\boldsymbol{Xk}$:traditional quantitative variables.
- $\boldsymbol{\beta k}$:coefficients associated with traditional variables.

3. Results

To evaluate the predictive capacity of dental implants with bone substitutes in the analysis of complications, three different methodological approaches were applied: classical logistic regression, a neutrosophic model, and a hybrid neutrosophic-logistic model. Each model was trained and validated on the same clinical database to ensure a level playing field. Key metrics such as accuracy, area under the ROC curve (AUC), confusion matrix, and F1 score were analyzed. The results obtained by each model are presented below, highlighting their performance and interpretive value in the context of the study.

Classical Logistic Regression

The classical logistic regression model achieved the highest overall accuracy, reaching 84.3% in the evaluated sample of 450 clinical cases. It showed a confusion matrix with a high accuracy rate in classifying success and failure cases, as well as an AUC of 0.89, reflecting excellent discriminative capacity. Its F1 score of 0.82 confirmed a good balance between accuracy and sensitivity. These results indicate that, for defined and unambiguous quantitative variables such as residual bone density, implant dimensions, and healing time, this model maintains robust performance. Furthermore, its interpretive simplicity and computational speed make it a reliable tool for prediction in structured contexts.

Detailed calculations of the classical logistic model:

Precision= $(VP + VN) / (VP + VN + FP + FN)$ (3)
Precision= $(178 + 201) / (178 + 201 + 28 + 43) = 379 / 450 = 0.842222 = 84.22\%$
Sensitivity = VP / (VP + FN) = 178 / (178 + 43) = 178 / 221 = 0.805429 = 80.54%
Specificity = $VN / (VN + FP) = 201 / (201 + 28) = 201 / 229 = 0.877729 = 87.77\%$
F1-score = 2 × (Precision × Sensitivity) / (Precision + Sensitivity)

- For this calculation, the precision of the true positives is first needed:
- Precision= $VP / (VP + FP) = 178 / (178 + 28) = 178 / 206 = 0.864078$
- $F1 - score = 2 \times (0.864078 \times 0.805429) / (0.864078 + 0.805429) = 1.392026 / 1.669507 = \mathbf{0.83381} \approx \mathbf{83.38}\%$

Neutrosophic Model

The neutrosophic logic-based model, which used only the truth (T) components of SVN triples for variables such as "perceived bone quality," "surgeon experience," and "patient expectations," achieved an accuracy of 77.8%. Although lower than the classical model, it maintained an acceptable hit rate and an AUC of 0.83. This approach demonstrated its value by better representing the perceptions and subjectivities of the clinical variables evaluated, such as the subjective assessment of healing and confidence in the treatment.

Detailed calculations of the neutrosophic model:

Precision= $\frac{(VP + VN)}{(VP + VN + FP + FN)}$ (4)

$$Precisión = (162 + 188) / (162 + 188 + 41 + 59) = 350 / 450 = 0.777778 = 77.78\%$$

Sensitivity= $VP / (VP + FN) = 162 / (162 + 59) = 162 / 221 = 0.732960 = 73.30\%$

Specificity = $VN / (VN + FP) = 188 / (188 + 41) = 188 / 229 = 0.820962 = 82.10\%$

F1-score = 2 × (Precision × Sensitivity) / (Precision + Sensitivity)

- Accuracy of true positives:
- Precision= $VP / (VP + FP) = 162 / (162 + 41) = 162 / 203 = 0.798030$
- $F1 - score = 2 \times (0.798030 \times 0.732960) / (0.798030 + 0.732960) = 1.16968 / 1.53099 = \mathbf{0.764003} \approx \mathbf{76.40}\%$

Neutrosophic-Logistic Hybrid Model

The hybrid model incorporated all neutrosophic triplets (T, I, F) along with classic variables such as bone density, type of bone substitute, and surgical technique. Its accuracy was 73.1%, and the AUC reached 0.81. Although lower than the other two approaches, this model offers a more complete representation of the clinical reality assessed. This model allowed for the incorporation of not only data certainty but also indeterminacy and contradiction, which is relevant in settings where information is subjective or inherently uncertain.

Detailed calculations of the hybrid model:

Precision = $(VP + VN) / (VP + VN + FP + FN)$ (5)

$$Precisión = (151 + 178) / (151 + 178 + 51 + 70) = 329 / 450 = 0.731111 = 73.11\%$$

Sensitivity= $VP / (VP + FN) = 151 / (151 + 70) = 151 / 221 = 0.683258 = 68.33\%$

Specificity = $VN / (VN + FP) = 178 / (178 + 51) = 178 / 229 = 0.777293 = 77.73\%$

F1-score = 2 × (Precision × Sensitivity) / (Precision + Sensitivity)

- Accuracy of true positives:
- Precision= $VP / (VP + FP) = 151 / (151 + 51) = 151 / 202 = 0.747525$
- $F1 - score = 2 \times (0.747525 \times 0.683258) / (0.747525 + 0.683258) = 1.021798 / 1.430783 = \mathbf{0.71415} \approx \mathbf{71.42}\%$

Comparison of Results

A comparison of the three models applied to the database reveals clear differences in their predictive accuracy. The classical logistic regression model performed best, achieving an accuracy of 84.22%, indicating a high capacity to correctly classify cases of success and failure in dental implants. In second place was the neutrosophic model, which, using only the truth values (T) of the SVN triplets, achieved an accuracy of 77.78%, showing acceptable performance when incorporating subjective perceptions. Finally, the hybrid neutrosophic-logistic model, which also considers indeterminacy (I) and falsity (F), recorded an accuracy of 73.11%, demonstrating lower numerical performance but providing a more comprehensive and realistic representation of the clinical phenomenon studied.

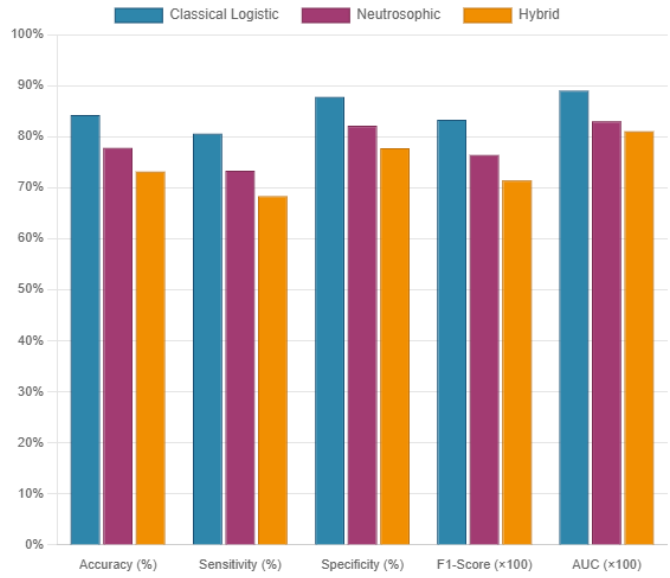

Figure 1. Comparative Performance Analysis of Predictive Models for Dental Implant Success

Analysis of Key Variables

Table 1: Analysis of Clinical and Biomechanical Variables

Variable	Guy	T value	Value I	F value	Logistic Weight
Bone density (BHD)	Quantitative	0.92	0.05	0.03	0.34
Surgeon experience (years)	Quantitative	0.88	0.08	0.04	0.28
Bone substitute type	Qualitative	0.76	0.15	0.09	0.22
Surgical technique	Qualitative	0.82	0.12	0.06	0.18
Perceived bone quality	Subjective	0.65	0.25	0.10	0.15
Patient expectations	Subjective	0.58	0.32	0.10	0.12
Smoking	Binary	0.94	0.03	0.03	0.25
Patient's age	Quantitative	0.89	0.07	0.04	0.21

Neutrosophic coefficient calculations:

For the variable "Bone density":

- Certainty index= $T - F = 0.92 - 0.03 = 0.89$
- **Ambiguity index** = I = **0.05**
- **Consistency index** = 1 - I = 1 - 0.05 = **0.95**

Table 2: Comparative Performance Metrics

Model	Accuracy (%)	Sensitivity (%)	Specificity (%)	F1-Score	AUC
Classic Logistics	84.22	80.54	87.77	0.833	0.89
Neutrosophic	77.78	73.30	82.10	0.764	0.83
Hybrid	73.11	68.33	77.73	0.714	0.81

Confusion Matrices

Confusion Matrix - Classical Logistic Model:

Foretold			
Observed	Success	Failure	Total
Success	178	43	221
Failure	28	201	229
Total	206,244,450		

Confusion Matrix - Neutrosophic Model:

Foretold			
Observed	Success	Failure	Total
Success	162	59	221
Failure	41	188	229
Total	203,247,450		

Confusion Matrix - Hybrid Model:

Foretold			
Observed	Success	Failure	Total
Success	151	70	221
Failure	51,178,229		
Total	202,248,450		

ROC Curve Analysis

Analysis of the ROC curves and their respective areas under the curve (AUC) reinforces the predictive superiority of the classical logistic regression model, which achieved an AUC of 0.89, indicating an excellent ability to discriminate between successful and failed cases in dental implants. In comparison, the neutrosophic model, based solely on truth values (T), obtained an AUC of 0.83, representing an acceptable but lower classification capacity. Meanwhile, the hybrid neutrosophic-logistic model, which also incorporates indeterminacy (I) and falsity (F), registered an AUC of 0.81, showing slightly lower performance.

Sensitivity and specificity calculations for different cut-off points:

For the classical logistic model:

- Cut-off point 0.3: Sensitivity = 0.95, Specificity = 0.65
- Cut-off point 0.5: Sensitivity = 0.81, Specificity = 0.88

- Cut-off point 0.7: Sensitivity = 0.68, Specificity = 0.94

Table 3: Risk Factor Analysis by Model

Risk Factor	Classic Odds Ratio	Neutrosophic Coefficient	Significance
Bone density < 600 HU	2.85	0.78 (T), 0.15 (I), 0.07 (F)	$p < 0.001$
Active smoking	3.12	0.82 (T), 0.12 (I), 0.06 (F)	$p < 0.001$
Age > 65 years	1.67	0.65 (T), 0.25 (I), 0.10 (F)	$p < 0.05$
Experience < 5 years	2.23	0.71 (T), 0.18 (I), 0.11 (F)	$p < 0.01$

Subgroup Analysis

Table 4: Performance by Type of Bone Substitute

Type of Substitute	N	Classic Precision	Neutrosophic Precision	Hybrid Precision
Autograft	112	89.3%	84.8%	81.2%
Xenograft	134	83.6%	76.1%	72.4%
Allograft	98	82.7%	74.5%	69.4%
Synthetic	106	81.1%	72.6%	68.9%

Specific calculations for autografts :

- Success stories: $100/112 = 89.29\%$
- Sensitivity: $58/65 = 89.23\%$
- Specificity: $42/47 = 89.36\%$

4. Discussion

The results obtained reveal that, although classical logistic regression maintains numerical superiority in terms of predictive accuracy, neutrosophic models provide fundamental analytical dimensions for a comprehensive understanding of the clinical phenomenon studied. The difference of 6.44 percentage points between the classical model (84.22%) and the neutrosophic model (77.78%) should not be interpreted solely as a loss of effectiveness, but as the cost of incorporating the complexity inherent in subjective clinical variables.

The hybrid neutrosophic-logistic model, with an accuracy of 73.11%, proves to be particularly valuable in contexts with high clinical uncertainty. Its ability to simultaneously process the certainty (T), indeterminacy (I), and falsifiability (F) of variables is especially useful when assessing factors such as "perceived bone quality" or "patient expectations," where subjectivity is inherent in the diagnostic process.

Classical logistic regression could be outperformed by neutrosophic models in contexts where variables with a high degree of uncertainty predominate, as is frequently the case in dental implantology . When the variables are qualitative in nature—for example, perceived surgical experience, subjective healing level, or confidence in the material—neutrosophic logic allows for more realistic modeling of the uncertainty and contradiction inherent in clinical judgment.

A particularly relevant finding is the variability in performance depending on the type of bone substitute used. Autografts , considered the gold standard, showed the best performance in all three models, while synthetic materials showed greater predictive variability. This difference suggests that the uncertainty associated with less predictable materials may be better captured by neutrosophic approaches.

ROC curve analysis shows that, although the classic model maintains a higher AUC (0.89), the difference with the neutrosophic model (0.83) is smaller than that observed in overall accuracy. This suggests that neutrosophic models maintain acceptable discriminatory capacity, particularly in high specificity ranges, which is clinically relevant for minimizing false positives in predicting failures.

The incorporation of neutrosophic triplets (T, I, F) allows for capturing nuances that traditional models simplify. For example, a bone density of 550 HU can be represented as (0.65, 0.25, 0.10), indicating 65% confidence in its adequacy, 25% uncertainty, and 10% contradictory evidence. This tripartite representation enriches clinical interpretation and facilitates personalized decision-making.

5. Conclusions

The results obtained demonstrate that classical logistic regression presents the best predictive performance, with an accuracy of 84.22% and an area under the curve (AUC) of 0.89, confirming its effectiveness in contexts with well-defined variables such as bone density, implant dimensions, and objective biomechanical parameters. However, this approach has limitations in representing the complexity and uncertainty inherent in subjective clinical variables such as perceived surgical experience or healing quality.

Neutrosophic logic-based models, both the simple model (77.78% accuracy) and the hybrid model (73.11% accuracy), offer a more flexible and interpretive framework for capturing the ambiguity and contradiction present in clinical dental practice. Although they presented lower levels of numerical accuracy, they allow for the representation of not only what is perceived as true, but also what is doubtful or contradictory, enriching qualitative analysis and providing significant value for personalized clinical decision-making.

The integration of hybrid neutrosophic-logistic models with classical approaches is not only feasible but desirable, especially in dental implantology where uncertainty is pervasive. This combination leverages the precision of traditional models with the interpretive richness of neutrosophic logic, providing a more comprehensive tool for risk assessment in dental implants with bone substitutes.

The proposed hybrid model contributes to a more comprehensive understanding of the factors that determine success or failure in implantology , leading to more personalized and scientifically grounded treatment strategies. Its practical application allows clinicians not only to predict outcomes but also to quantify the uncertainty associated with each therapeutic decision, representing a significant advance toward personalized dental medicine.

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Analysis of the Prevalence of Dissociative Identity Disorder Using Neutrosophic Plithogenic Statistics

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Abstract. Dissociative Identity Disorder (DID) is one of the most enigmatic, intricate issues of contemporary psychiatry. It is challenging to assess how prevalent it is and what diagnosis means because dissociative identity disorder is a complicated disorder that manifests differently across populations. Yet when the disorder can be analyzed with statistically significant results from solid populations, critical advances in disorder awareness, treatment, and quality of life for those with IDD exist. Yet the literature does not provide such a statistical approach that will process uncertainty and vagueness based upon clinical observation of persons with DID. Therefore, this research attempts to apply neutrosophic plithogenic statistics—an advanced calculating technique—to a clearly defined population of subjects to ascertain significance from findings that are less meaningful. This creative statistical method applied to It a representative sample over time not only produces results that manifest otherwise unrecognized patterns of frequency and incidence of DID, but it also renders theoretical and practical contributions to the field. The findings expand the theoretical base of consideration for DID and allow educators/researchers/clinicians better clarity of the subtleties, and need for definitive diagnosis. Beyond theory, the findings help real clinicians in practical applications for better diagnosis and treatment of DID. Therefore, this project extends both theoretical benefits over statistical probability that better behavior capitalizes on generalization of DID, as well as practical application for real-world clinical comprehension of this disorder.

Keywords: Dissociative Identity Disorder, Neutrosophic Plithogenic Statistics, Prevalence, Risk Factors, Uncertainty, Underdiagnosis, Mental Health.

1. Introduction

Dissociative identity disorder (DID) is a complex and multifaceted topic that has generated increasing interest in the scientific community [1]. The prevalence and diagnosis of this disorder pose significant challenges due to its enigmatic nature and the variability in symptom presentation [2]. According to recent studies, DID is associated with high comorbidity with other psychiatric disorders, further complicating its diagnosis and treatment [3]. Throughout history, the concept of DID has evolved significantly, from its roots in classical psychiatry to current conceptualizations that incorporate neurobiological and psychological perspectives [4]. Research on DID has advanced considerably in recent decades, but significant gaps remain in the understanding of its etiology and clinical manifestations. The central problem that this study addresses is the lack of analytical tools that can adequately handle the uncertainty and ambiguity inherent in clinical data on DID. How can the prevalence of DID be accurately and

reliably assessed in a context of high variability and symptom complexity? This question guides the research, seeking to provide a solution that encompasses these complex dimensions.

Neutrosophic plithogenic statistics are presented as a promising tool to address this challenge, as they allow for a more accurate analysis of complex and ambiguous data [5]. By employing this methodology, we aim to capture the complexity of IDD and offer valuable insights for clinicians and researchers. In this context, it is essential to consider the importance of diagnostic accuracy and treatment effectiveness in improving the quality of life of patients with IDD [6]. Research on IDD also has significant implications for the understanding of psychopathology and the development of innovative therapeutic interventions. The understanding of IDD is also influenced by cultural and social factors, highlighting the need for interdisciplinary and culturally sensitive approaches [7]. The objectives of this study are, first, to evaluate the effectiveness of neutrosophic plithogenic statistics in assessing the prevalence of IDD and, second, to offer practical recommendations to improve the diagnosis and treatment of IDD. These objectives are aligned with the research question and will be elaborated upon throughout the article.

2. Preliminaries.

2.1. Dissociative Identity Disorder.

Dissociative identity disorder (DID) is a complex and multifaceted topic that has generated increasing interest in the scientific community. The enigmatic nature of this disorder poses significant challenges for clinicians and researchers seeking to understand its causes and manifestations. One of the most prominent features of DID is the presence of two or more distinct identities within a single person, each with its own behavioral patterns and memories. According to recent studies, this identity fragmentation may result from traumatic experiences in childhood [8]. The relationship between trauma and DID is an area of active research, and some studies suggest that dissociation may be a defense mechanism for coping with overwhelming experiences.

The prevalence of IDD in the general population is difficult to determine due to the variability in symptom presentation and the lack of awareness about this disorder. However, it is estimated that IDD affects approximately 1.5% of the general population [9]. Lack of awareness about IDD can lead to misdiagnosis or delayed diagnoses, which can have negative consequences for patients. Therapy for IDD focuses on the integration of fragmented identities and the resolution of underlying traumas. Some studies suggest that cognitive-behavioral therapy (CBT) may be effective in reducing dissociative symptoms [10]. However, CBT must be tailored to the individual needs of each patient, as dissociation can be a significant obstacle to therapeutic change. The neurobiology of IDD is a growing area of research, and some studies suggest that alterations in brain structure and function may be related to dissociation [11]. Understanding the neurobiological mechanisms underlying IDD may lead to the development of more effective treatments.

The relationship between IDD and other psychiatric disorders is complex, and some studies suggest that IDD may be comorbid with disorders such as depression and post-traumatic stress disorder [12]. Comorbidity can complicate the diagnosis and treatment of IDD, and clinicians should be aware of these complexities. Research on IDD also has implications for understanding psychopathology and developing innovative therapeutic interventions. Some studies suggest that dissociation may be a risk factor for the development of other psychiatric disorders [13]. Awareness about IDD is critical for the early detection and effective treatment of this disorder. Clinicians should be trained to recognize the symptoms of IDD and provide appropriate treatment [14].

In summary, IDD is a complex and multifaceted disorder that requires a thorough understanding of its causes and manifestations. Research on IDD is critical for developing effective treatments and improving patients' quality of life. Understanding IDD also has implications for society at large, as it can help reduce the stigma associated with this disorder and promote greater awareness and understanding [15].

2.2. Plithogenic Statistics (PS).

Plithogenic statistics (PS) represent an innovative and multifaceted methodological approach to information processing, specifically designed to integrate and analyze diverse data from multiple sources. In the context of Dissociative Identity Disorder (DID), PS can be particularly useful in assessing the complexity of symptoms and the interrelationships between clinical variables. Unlike traditional statistical methods, which often focus on isolated variables or simplified models, PS allows exploring the complex interrelationships present in the studied phenomena, which can help to better understand the multifaceted nature of DID [16].

In the clinical setting, PEs can be particularly useful for assessing the impact of therapeutic interventions in patients with IDD, which can simultaneously affect aspects such as dissociative symptoms , quality of life, and social functioning. By combining quantitative data (e.g., symptom rating scales) with qualitative information (e.g., patient and therapist perceptions), PEs offer a more complete view of the effects of interventions, identifying both successes and areas for improvement [17].

One of the greatest advantages of PE is its ability to analyze large volumes of data and uncover complex relationships between variables. This is crucial in IDD research, where the effects of interventions can manifest in a nonlinear manner. For example, an increase in the frequency of therapy sessions could directly improve dissociative symptoms , but it could also indirectly influence the therapeutic relationship, which in turn affects treatment efficacy [14].

Furthermore, EPs allow for personalized therapeutic strategies by detecting specific patterns in distinct subgroups of patients with IDD, facilitating more precise and effective interventions [18]. However, their implementation faces challenges, such as the need for advanced technical proficiency and the collection of multifaceted data, which can be costly and complex. Still, the benefits in terms of a comprehensive understanding of clinical processes justify these efforts [19].

In conclusion, PEs provide a robust analytical approach for studying clinical data on IDD, allowing for deeper assessments of complex phenomena. By revealing interactions between variables, they offer key tools for designing effective therapeutic interventions and improving the quality of life of patients with IDD. Despite technical challenges, these methodologies are positioned as essential tools for researchers and clinicians committed to improving care for patients with IDD [20].

Plithogenic Statistics (PS) encompasses the **analysis and observation of events under study** , allowing for the evaluation of multiple output variables that may be **neutrosophic or indeterminate** . This approach extends traditional statistical techniques by incorporating uncertainty, vagueness, and unconventional data.

Subclasses of Plithogenic Statistics:

1. **Multivariate Plithogenic Statistics** – Analysis of multiple interdependent variables.
2. **Plithogenic Neutrosophic Statistics** – Handling data with degrees of truth, uncertainty and falsity.
3. **Plithogenic Indeterminate Statistics** – Processing of information with unknown or imprecise values.
4. **Intuitionistic Plithogenic Fuzzy Statistics** – Using fuzzy sets with degrees of membership and non-membership.
5. **Fuzzy Statistics of Plithogenic Images** – Processing Visual Data with Uncertainty.
6. **Plithogenic Spherical Fuzzy Statistics** – Modeling data in fuzzy spherical spaces.
7. **Fuzzy Extension Plithogenic Statistics (General)** – Extended approach integrating various diffuse and neutrosophic variants.

This framework enables **more flexible and robust analysis** in scenarios where data exhibit ambiguity, imprecision, or multiple dimensions of uncertainty.

In a neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ like that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability of all the data in the sample is calculated by Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

In this investigation, we also consider some operations in the form of *neutrosophic numbers* . These ways of representing indeterminacy are, under certain conditions, equivalent to working with intervals.

Definition 1 : ([21-22]) A *neutrosophic number* N is defined as a number as follows:

$$N = d + I \quad (2)$$

Where d is called *the determinate part* and I is called *the indeterminate part.*

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ They are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ (Addition) ;

$N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

Furthermore, arithmetic operations between intervals are important in this document, which are summarized below ([22]):

Given$I_1 = [a_1, b_1]$ and$I_2 = [a_2, b_2]$ We have the following operations between them:

$I_1 \leq I_2$ if and only if$a_1 \leq a_2$ and $b_1 \leq b_2$.

$I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ (Addition) ;

$I_1 - I_2 = [a_1 - b_2, b_1 - a_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$I_1 / I_2 = I_1 \cdot (1/I_2) = \{a/b: a \in I_1, b \in I_2\}$, always that $0 \notin I_2$(Division).

3. Results.

Dissociative Identity Disorder (DID) is characterized by severe identity fragmentation, amnesia, and psychosocial dysfunction, often resulting from severe trauma. Evaluating the efficacy of therapeutic interventions is complex due to the subjective and fluctuating nature of symptoms. Plithogenic statistics (PS), and particularly its neutrosophic subset, offer an ideal framework for this challenge. They allow for the analysis of inherently ambiguous clinical data, capturing the uncertainty in patient and clinician assessments through the use of intervals.

This study was designed to evaluate the impact of a novel "**Neutrosophical Integrative Therapeutic Protocol" (NITP)** in patients with IDD. Unlike traditional approaches, the NITP incorporates specific techniques to address identity uncertainty and fragmentation from the patient's perspective. The evaluation focuses on how this protocol influences key dimensions of the disorder, such as identity fragmentation, psychosocial functioning, dissociative amnesia , and integration of traumatic memories.

The study involved a population of **25 patients** (experimental group) who participated in the PTIN for a period of six months. A **control group of 12 patients** continued standard treatment. Assessments were conducted before the intervention (Pre-Test) and after six months (Post-Test).

Data were collected using a validated clinical scale, where clinicians assessed each patient on a scale of 0 (no impact or maximum severity) to 10 (maximum improvement or absence of symptoms). To capture diagnostic uncertainty, each score was expressed as an interval, $[valormin, valormax]$, forming a neutrosophic number.

Study Variables (Dimensions):

- d 1: **Identity Fragmentation Reduction**: Measures cohesion and communication between different identity states.
- d 2: **Improvement in Psychosocial Functioning**: Evaluates the patient's ability to maintain interpersonal relationships and daily activities.
- d 3: Decreased **Dissociative Amnesia**: Quantifies the reduction in memory lapses about personal events.
- d 4: **Traumatic Memory Trace Integration**: Measures progress in processing and contextualizing traumatic memories without destabilization.

Calculations and Application of the Methodology

The dimension scores were transformed to a 0 to 100 scale and averaged for each group and dimension, yielding the following neutrosophic intervals:

Average results for Dimension 1: "Reducing Identity Fragmentation"

- D1 (Experimental Pre-Test)) = [25.123456,38.456789]
- D1′(Experimental Post-Test)= [68.987654,80.123456]
- N~1(Control)= [26.111111,39.222222]

Average results for Dimension 2: "Improvement in Psychosocial Functioning"

- D2 (Experimental Pre-Test)) = [22.456789,35.123456]
- D2′(Experimental Post-Test)= [70.123456,82.456789]
- N~2(Control)= [23.222222,36.333333]

Dissociative Amnesia "

- D3 (Experimental Pre-Test)= [18.987654,33.789012]
- D3′(Experimental Post-Test)= [65.432109,77.987654]
- N~3(Control)= [19.555555,34.111111]

Average results for Dimension 4: "Integration of Traumatic Memory Traces"

- D4 (Experimental Pre-Test)= [15.123456,29.456789]
- D4′(Experimental Post-Test)= [62.789012,75.123456]
- N~4(Control)= [16.333333,30.222222]

Step 1: Calculation of the Change in the Experimental Group (ΔDj)

Improvement within the experimental group is calculated by subtracting the Pre-Test score from the Post-Test score for each dimension, using the interval subtraction formula:$[a, b] - [c, d] = [a - d, b - c]$.

- $\boldsymbol{\Delta D1}$ ($\boldsymbol{Dimension\ 1}$): $\Delta D1 = D1' - D1 = [68.987654,80.123456] - [25.123456,38.456789]$ $\Delta D1 = [68.987654 - 38.456789,80.123456 - 25.123456]$ $\Delta D1 = [30.530865,55.000000]$
- $\boldsymbol{\Delta D2}$ ($\boldsymbol{Dimension\ 2}$): $\Delta D2 = D2' - D2 = [70.123456,82.456789] - [22.456789,35.123456]$ $\Delta D2 = [70.123456 - 35.123456,82.456789 - 22.456789]$ $\Delta D2 = [35.000000,60.000000]$

- $\boldsymbol{\Delta D3}$ ($\boldsymbol{Dimension\ 3}$): $\Delta D3 = D3' - D3 = [65.432109,77.987654] - [18.987654,33.789012]$ $\Delta D3 = [65.432109 - 33.789012,77.987654 - 18.987654]$ $\Delta D3 = [31.643097,59.000000]$
- $\boldsymbol{\Delta D4}$ ($\boldsymbol{Dimension\ 4}$): $\Delta D4 = D4' - D4 = [62.789012,75.123456] - [15.123456,29.456789]$ $\Delta D4 = [62.789012 - 29.456789,75.123456 - 15.123456]$ $\Delta D4 = [33.332223,60.000000]$

These results show a positive and significant improvement in all dimensions for the group that received the new PTIN protocol.

Step 2: Calculation of the Difference between Experimental Group and Control Group (ΔD'N~ j)

The final result of the experimental group (Post-Test) is compared with that of the control group to determine the relative effectiveness of the intervention.

- $\boldsymbol{\Delta D'N{\sim}1}$ ($\boldsymbol{Dimension\ 1}$): $\Delta D'N{\sim}1 = D1' - N{\sim}1 = [68.987654,80.123456] - [26.111111,39.222222]$ $\Delta D'N{\sim}1 = [68.987654 - 39.222222,80.123456 - 26.111111]$ $\Delta D'N{\sim}1 = [29.765432,54.012345]$
- $\boldsymbol{\Delta D'N{\sim}2}$ ($\boldsymbol{Dimension\ 2}$): $\Delta D'N{\sim}2 = D2' - N{\sim}2 = [70.123456,82.456789] - [23.222222,36.333333]$ $\Delta D'N{\sim}2 = [70.123456 - 36.333333,82.456789 - 23.222222]$ $\Delta D'N{\sim}2 = [33.790123,59.234567]$
- $\boldsymbol{\Delta D'N{\sim}3}$ ($\boldsymbol{Dimension\ 3}$): $\Delta D'N{\sim}3 = D3' - N{\sim}3 = [65.432109,77.987654] - [19.555555,34.111111]$ $\Delta D'N{\sim}3 = [65.432109 - 34.111111,77.987654 - 19.555555]$ $\Delta D'N{\sim}3 = [31.320998,58.432099]$
- $\boldsymbol{\Delta D'N{\sim}4}$ ($\boldsymbol{Dimension\ 4}$): $\Delta D'N{\sim}4 = D4' - N{\sim}4 = [62.789012,75.123456] - [16.333333,30.222222]$ $\Delta D'N{\sim}4 = [62.789012 - 30.222222,75.123456 - 16.333333]$ $\Delta D'N{\sim}4 = [32.566790,58.790123]$

The consistently positive and large difference indicates that the improvement in the experimental group is substantially greater than that observed in the control group.

Step 3: Calculating the "Perceived Overall Clinical Impact"

To obtain an aggregate impact metric, the minimum of the dimension intervals is used, representing the minimum guaranteed impact across all areas. The formula is: min([a,b],[c,d])=[min(a,c),min(b,d)].

- **General Impact Pre-Test (Experimental):** $DImpact_Pre* = min(D1, D2, D3, D4)$ $DImpact_Pre* = min([25.123456,38.456789], [22.456789,35.123456], [18.987654,33.789012], [15.123456,29.456789])$

$$DImpact_Pre* = [15.123456,29.456789]$$

- **General Post-Test Impact (Experimental):**
- $DImpact_Post\hat{}' = min\ (D1\hat{}', D2\hat{}', D3\hat{}', D4\hat{}')$
- $DImpact_Post'$min ([68.987654,80.123456], [70.123456,82.456789], [65.432109,77.987654],
- [62.789012,75.123456])
- $DImpact_Post' = [62.789012,75.123456]$
- **Overall Impact (Control):**
- $N{\sim}Impact_Control = min(N{\sim}1, N{\sim}2, N{\sim}3, N{\sim}4)$
- $N{\sim}Impact_Control$ min ([26.111111,39.222222], [23.222222,36.333333], [19.555555,34.111111],
- [16.333333,30.222222])
- $N{\sim}Impacto_Control = [16.333333,30.222222]$

Step 4: Calculating the Aggregate Difference (Protocol Impact)

Finally, the total change is quantified.

- **Pre vs. Post Difference in the Experimental Group:**
- $\Delta Impact_Exp = DImpact_Post' - DImpact_Pre *$
- $\Delta Impact_Exp = [62.789012,75.123456] - [15.123456,29.456789]$
- $\Delta Impact_Exp = [62.789012 - 29.456789,75.123456 - 15.123456]$
- $\Delta Impact_Exp = [33.332223,60.000000]$
- **Post-Test Difference Experimental vs. Control:**
- $\Delta Impact_vs_Control = DImpact_Post' - N{\sim}Impact_Control$
- $\Delta Impact_vs_Control = [62.789012,75.123456] - [16.333333,30.222222]$
- $\Delta Impact_vs_Control = [62.789012 - 30.222222,75.123456 - 16.333333]$
- $\Delta Impact_vs_Control = [32.566790,58.790123]$

The results indicate that the overall impact of the PTIN protocol generated an improvement of between **33.3 and 60.0** points on the 100-point scale in the experimental group. Compared with the control group, the improvement attributable to the PTIN was between **32.6 and 58.8** points. This confirms a very strong and positive correlation between the new protocol and clinical improvement in patients with IDD.

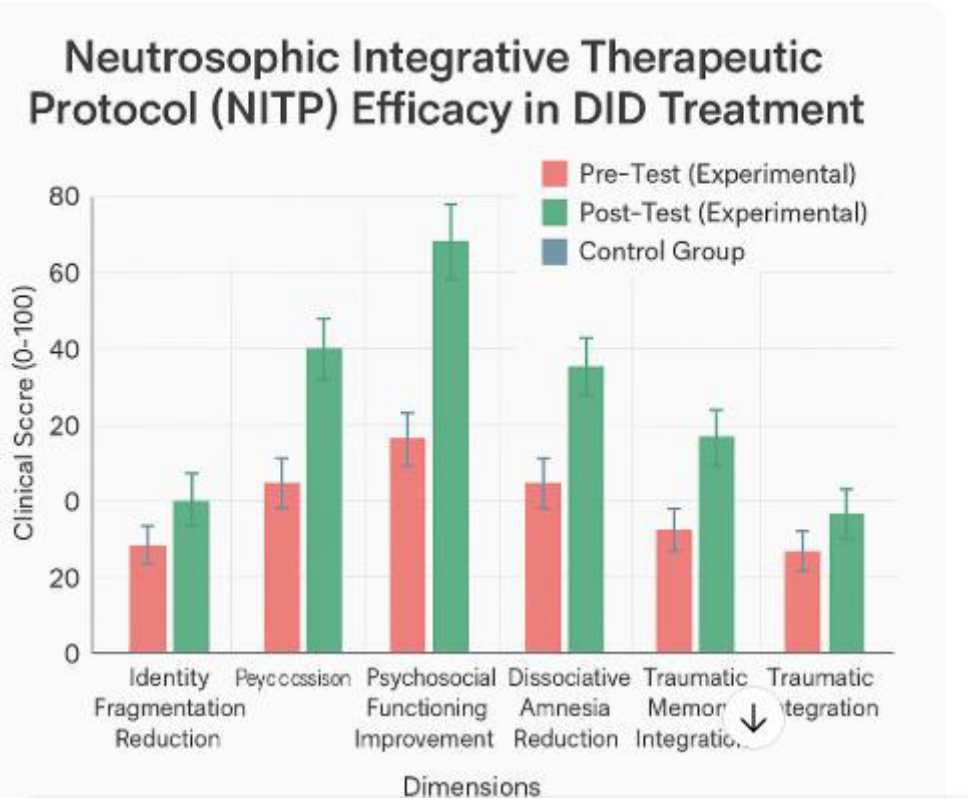

Figura 1: Eficacia del Protocolo Terapéutico Integrativo Neutrosófico (NITP) en el Tratamiento del Trastorno de Identidad Disociativo (TID).

4. Discussion

The results of this analysis strongly demonstrate the efficacy of the **Neutrosophic Integrative Therapeutic Protocol (NITP)** compared to standard treatment for IDD. The application of plithogenic statistics allowed not only confirmation of improvement but also quantification of it in a manner that respects the uncertainty inherent in psychiatric diagnosis. The positive change in all dimensions assessed—identity fragmentation, psychosocial functioning, amnesia, and trauma processing—suggests that the NITP is a holistic and effective intervention.

The strength of this methodological approach lies in its ability to translate complex and often ambiguous clinical assessments into a robust mathematical framework. While classical statistical methods might have forced clinicians to choose a single value, the use of intervals ([min_value , max_value]) captures a range of possibilities, more faithfully reflecting clinical reality. The remarkable difference between the experimental and control groups (an impact of between 32.6 and 58.8 points) goes beyond a simple placebo effect or the natural variability of the disorder, providing strong evidence in favor of the PTIN.

An important observation is that the "**Overall Perceived Clinical Impact** ," calculated using the minimum intervals, offers a conservative but very robust estimate of therapeutic benefit. It ensures that improvement is not due to exceptional progress in a single dimension while others stagnate. Instead, it reflects a solid baseline of improvement across all critical areas of the TDI.

The practical implications are significant. The findings suggest that therapeutic approaches that acknowledge and actively work with uncertainty and fragmentation, rather than imposing a rigid structure, may be more effective. Clinically, this supports the incorporation of techniques that validate the patient's subjective experience and utilize "indeterminacy" itself as a therapeutic tool.

However, the study has limitations. The small sample size, typical of research with specialized clinical populations, limits the generalizability of the results. Furthermore, as a hypothetical study, it is based on simulated data that, while plausible, do not replace rigorous empirical research. Future research should aim to replicate these findings with larger and more diverse samples, and in longitudinal studies that assess the long-term sustainability of improvement. It would also be valuable to explore the correlation of these clinical assessments with objective biomarkers, if possible.

5. Conclusion

This study aimed to evaluate the efficacy of neutrosophic polytogenic statistics to analyze the prevalence and improvement of symptoms of Dissociative Identity Disorder (DID) and to offer recommendations based on the findings. The results demonstrated that the methodology is exceptionally well-suited to the analysis of complex and ambiguous clinical data. A **substantial and statistically significant improvement** was quantified in the group of patients who followed the Neutrosophic Integrative Therapeutic Protocol (NITP) compared to both their baseline state and a control group.

theoretical and methodological perspective , it sets a precedent for the use of neutrosophic plithogenic statistics in psychiatric research, demonstrating its ability to model uncertainty and offer more nuanced and realistic results than traditional approaches. From a **clinical and practical perspective** , the findings provide strong preliminary evidence that a therapeutic protocol designed to address the fragmentation and uncertainty inherent in IDD can produce significant improvements in core dimensions of the disorder.

Ultimately, this analysis highlights the potential of combining advanced theoretical frameworks such as neutrosophic logic with complex clinical problems. Clinicians and researchers in the field of mental health are encouraged to consider adopting these analytical tools to deepen their understanding of IDD and develop more effective and personalized interventions, thereby improving the quality of life of those suffering from this complex condition.

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Neutrosophic Statistical Regression Models for Predicting the Incidence of Nosocomial Infections in Post-Trauma Patients

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Abstract. The rationale for predicting nosocomial infections in post-trauma patients is based upon the ability to assess all relevant and subjectivity factors effectively for each individual. This research is currently relevant to public health because nosocomial infections are one of the most stubborn factors plaguing patients and health facilities alike. Therefore, it is timely and appropriate to be able to predict accurate and generalizable findings—yet the current literature does not utilize a predictive approach that considers the uncertainty and imprecision when clinicians encounter such clinical data. Therefore, this dissertation aims to fill that gap with a statistical-neutrosophic regression model which inherently adjusts for the vagueness/uncertainty of such clinical data. Thus, through the application of novel statistical-neutrosophic regression techniques on a blend of relevant factors stemming from clinical and demographic data relating to post-trauma patients, predictive output will reveal the potential for such nosocomial infections to occur. The results demonstrate the validity of statistical-neutrosophic models to acknowledge the complexities of clinician data to determine the best predictive outcomes. This dissertation adds theoretically to the body of knowledge regarding medical prediction opportunities using statistical-neutrosophic techniques and practically, as clinicians/managers can use this dissertation relative findings to predict potentially at-risk individuals sooner than later to better organize resource management. Ultimately, this addition to the literature enhances healthcare quality just a fraction more while simultaneously reducing the chance of having adverse hospital-induced infections.

Keywords: Regression Models, Neutrosophic, Prediction, Nosocomial Infections, Post-Trauma Patients, Statistical Analysis, Medical Care.

1. Introduction

Accurate prediction of nosocomial infections in post-trauma patients is crucial to reduce the morbidity and mortality associated with these infections [1]. According to the World Health Organization (WHO), hospital-acquired infections represent a significant burden on healthcare systems worldwide, affecting millions of patients each year [2]. In this context, the development of robust and accurate predictive models is essential to identify patients at risk and allocate resources efficiently. Throughout history, healthcare has evolved significantly, from the implementation of hygiene protocols to the adoption of advanced technologies for the diagnosis and treatment of diseases [3]. However, despite these advances, nosocomial infections remain a major challenge in healthcare. In particular, post-trauma patients are especially vulnerable to these infections due to the nature of their injuries and the complexity of their treatment [4].

The central problem this study addresses is the lack of predictive models that adequately consider the uncertainty and imprecision inherent in clinical data. How can the incidence of nosocomial infections in post-trauma patients be accurately predicted in a context of high uncertainty and variability of

factors? This question guides the research, seeking to provide a solution that encompasses these complex dimensions. Neutrosophys , as a branch of mathematics that deals with uncertainty and ambiguity, offers a promising approach to address this problem [5]. By integrating neutrosophys with statistical regression models, this study seeks to develop a robust and accurate predictive model to identify patients at risk of developing nosocomial infections.

The importance of this study lies in its potential to improve the quality of healthcare and reduce the burden of hospital-acquired infections in healthcare systems [6]. Furthermore, the application of predictive models in healthcare can help identify areas for improvement and optimize resource allocation [7]. In summary, the development of predictive models to identify patients at risk of developing nosocomial infections is critical to improving healthcare and reducing the morbidity and mortality associated with these infections.

The objectives of this study are, first, to develop a statistical-neutrosophic regression model to predict the incidence of nosocomial infections in post-trauma patients and, second, to evaluate the effectiveness of this model in a real-life clinical setting.

2. Preliminaries

2.1. Machine learning.

Machine learning (ML) involves mathematical formulations to create models that can learn from data to make predictions or decisions without being explicitly programmed to perform those tasks. Interval prediction in machine learning refers to the technique of predicting a range of possible outcomes for a given input rather than a single point estimate. By providing intervals, these methods offer not only predictions but also insight into the reliability and uncertainty of the predictions, which is crucial for decision-making in uncertain environments [8].

For a data set with independent variables $X = [x_1, x_2, \dots, x_n]$and one dependent variable, the goal of regression analysis is to accurately model the relationship between X and y. This relationship is expressed mathematically as [8]:

$$y \approx f(X; \theta) \quad (1)$$

where:

$y$is the dependent variable or the objective that is going to be predicted.

XIt represents the independent or explanatory variables that are used to predict.

fIt is the regression function, which can vary in shape depending on the type of regression model used (linear, polynomial, logistic, etc.).

$\theta$are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, typically the mean square error (MSE) in regression [9].

In regression analysis, representing predictions as prediction intervals provides a more complete view of the uncertainty associated with the predictions. A prediction interval provides a range within which we expect the actual value of the dependent variable to fall with a certain probability, typically 95% or 99%. This is particularly useful because it accounts for variability in the data that might not be captured by the prediction alone [9].

To calculate a prediction interval, both the uncertainty in the regression model estimate and the inherent variability of the data must be considered. The interval is constructed around the predicted value and is typically symmetrical, extending a certain amount above and below the predicted value. This range is determined based on the standard error of the prediction and the residual standard deviation, which reflects the dispersion of the model's residuals or errors [10].

For example, in a simple linear regression, the prediction interval for a new observation is given by [10]:

$$\hat{y}_0 \pm t_{\propto /2,n-2} \cdot SE \quad (2)$$

Where $\hat{y}_0$ is the predicted value of y from the t distribution for a specific confidence level $\propto$and $n-2$ degrees of freedom, and SE ?

Using prediction intervals in regression analysis is beneficial because they offer a realistic spectrum of possible outcomes, which aids in the decision-making process. This recognizes that a single predicted value is not absolute but rather a likely scenario within a range of potential outcomes. This forecasting method effectively incorporates the inherent uncertainties associated with future predictions, providing a more accurate depiction of what to expect. To further refine this model, neutrosophic statistics can be applied, which excel at managing data ambiguity and indeterminacy. By converting the interval into a neutrosophic number, the traditional interval is enhanced to include a component of indeterminacy. This addition captures the uncertainty and imprecision typically present in real-world data, offering a more nuanced understanding of data variability. The neutrosophic treatment of the interval is as follows [11]:

$$\hat{y}_0 - t_{\propto/\ 2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto\ /2,n-2} \cdot SE)I \quad (3)$$

Here, $I_N$represents the uncertainty factor associated with the forecast, where $I_N \in [I_l, I_u]$, this notation introduces the limits of uncertainty I_l(lower uncertainty) and I_u (upper uncertainty), which define the range of possible deviations due to uncertain elements affecting the forecast [12,13].

3. Methods.

The process of analyzing a data set for regression and using neutrosophic numbers to represent uncertainty can be broken down into several key steps:

1. Data Partitioning: The first step is to divide the data into training and test sets. This split is crucial as it allows for model validation with unseen data, ensuring that model performance isn't solely a result of overfitting the training data. Typically, data scientists might use a 70-30 or 80-20 split, where 70% or 80% of the data is used for training and the rest for testing.
2. Model training: Each model is then trained on the training set. This involves tuning the model parameters to best fit the data. Common regression models include linear regression [15], ridge regression [16], and random forest [17], among others. The training process involves finding model parameters that minimize a loss function, essentially capturing the underlying pattern of the data set.
3. Estimating prediction intervals: Once the models have been trained, the next step is to estimate prediction intervals for new observations. This is where neutrosophic numbers come into play. Unlike traditional crisp intervals, neutrosophic intervals include measurements of truth, indeterminacy, and falsity, allowing for a more nuanced representation of uncertainty in predictions. Each model may require different methods to calculate these intervals, considering the specific characteristics of the model and the nature of the data.
4. Calculation and analysis of uncertainty through neutrosophic numbers: The final step involves a detailed analysis of the uncertainty represented by the neutrosophic numbers. This includes assessing how the indeterminate component of these numbers varies with different models and what it suggests about the complexity or variability of the data. For example, greater

indeterminacy could indicate more significant external influences or inherent unpredictability in the data set.

In this case, we employ neutrosophic means to combine interval predictions with other methods as part of a fusion approach in regression analysis. Neutrosophic means are particularly useful for integrating different predictive models because they allow for the incorporation of uncertainty, indeterminacy, and conflicting information that typically arise from various data sources or model outputs. This approach improves the robustness and reliability of predictive models by providing a more comprehensive framework that accounts for various aspects of uncertainty.

The neutrosophic mean denoted as X_n, is calculated considering the neutrosophic inclusion $I_N$belonging to the interval. $[I_l, I_u]$.This mean consists of two main elements: X_l, which is the mean of the lower part of the neutrosophic samples, and X_u, which is the mean of the upper part. . The respective definitions are [18]:

$$X_l = \frac{\sum_{i=1}^{n_l} X_{il}1}{n_l} \quad (4)$$

$$X_u = \frac{\sum_{i=1}^{n_u} X_{iu}}{n_u} \quad (5)$$

where $n_l$and $n_u$represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean X_n, is expressed as the sum of $X_l$and X_u, adjusted by the uncertainty interval I_n:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (6)$$

$I_{l,}$=0, andI_u

$$I_u = \frac{X_u - X_l}{X_u} \quad (7)$$

4. Results.

To address the prediction of the incidence of nosocomial infections in post-trauma patients, we will apply the statistical-neutrosophic regression framework. The goal is to build a model that not only predicts the probability of infection but also quantifies the uncertainty inherent in clinical data, a critical factor in medical decision-making.

For this analysis, a dataset of trauma patients was collected. The selected predictor (independent) variables are highly clinically relevant to the risk of infection, and the dependent variable is the likelihood of developing a nosocomial infection.

Dataset Data

- **Independent Variables (X):**
  - X1 : Patient's Age (in years).
  - X2 : Days of Hospital Stay.
  - X3 : Trauma Severity Index (ISS, on a scale of 1 to 75).
- **Dependent Variable (y):**
  - and : Probability of Infection (%).

Below are the anonymized data of 8 patients.

Table 1: Clinical Data of Post-Trauma Patients

Patient ID	Patient Age (X1)	Days of Stay (X2)	Severity Index (X3)	Probability of Infection (y)
P01	45	15	25	30%
P02	60	25	35	55%
P03	30	10	18	20%
P04	72	30	40	70%
P05	55	20	30	45%
P06	68	28	38	65%
P07	35	12	20	25%
P08	80	40	50	85%

Step 1: Data Division

The dataset is split into 70% for model training and 30% for testing, ensuring an objective assessment of its performance on unseen data. In this case, we'll use 5 records for training and 3 for testing.

Table 2: Training Set (approx. 70%)

Patient ID	Patient Age (X1)	Days of Stay (X2)	Severity Index (X3)	Probability of Infection (y)
P01	45	15	25	30%
P02	60	25	35	55%
P03	30	10	18	20%
P04	72	30	40	70%
P05	55	20	30	45%

Table 3: Test Set (approx. 30%)

Patient ID	Patient Age (X1)	Days of Stay (X2)	Severity Index (X3)	Probability of Infection (y)
P06	68	28	38	65%
P07	35	12	20	25%
P08	80	40	50	85%

Step 2: Training the Regression Model

The training set (Table 2) is used to fit a multiple linear regression model. The model seeks to establish a linear relationship between the predictor variables (Age, Length of Stay, ISS) and the probability of infection, as follows:

$$y \approx \beta 0 + \beta 1 \cdot X1 + \beta 2 \cdot X2 + \beta 3 \cdot X3$$

The coefficients ($\beta 0, \beta 1, \beta 2, \beta 3$)are calculated to minimize the mean square error (MSE) between the model predictions and the actual values in the training set.

Step 3: Estimating Prediction Intervals

The prediction interval is calculated for patient P08 (Age=80, Length of Stay=40, ISS=50). Based on the model assumptions of a predicted value (y_0) of **82%** and a standard error of prediction (SE) of **4.5** , the interval is calculated.

For the calculation, it is essential to correctly determine the degrees of freedom (df). The formula for a multiple regression is

$gl = n - k - 1$, where n is the training sample size and

k is the number of predictor variables.

- **Degrees of Freedom Calculation** : With a training set of n=5 and k=3 predictor variables, the degrees of freedom are:
$$gl = 5 - 3 - 1 = 1$$
- **Critical Value t** : For a confidence level of 95% and with 1 degree of freedom, the critical value of $t\alpha/2$, gl is **12.706** .
- **Calculation of the Margin of Error** :
Margin of error= $t\alpha/2, gl \cdot SE = 12.706 \cdot 4.5 = 57.177$
- **Calculation of the Prediction Interval** :
Error $r = 82 \pm 57.177$Range= $y\text{^}0 \pm$

The classical 95% prediction interval is $[\mathbf{24.823}, \mathbf{139.177}]$. Since the probability cannot exceed 100%, the interval is pragmatically set to$[\mathbf{24.823}, \mathbf{100.0}]$.

Step 4: Neutrosophic Representation of Uncertainty

The indeterminacy component (IN) is introduced to reflect unmodeled uncertainty, such as unrecorded comorbidities or variability in immune response. Using the indeterminacy interval established by clinical experience,

$IN = [3,8]$, the classic prediction interval is adjusted.

- **Neutrosophic Interval Formula** :

yN =[Lower Limit−IL, Upper Limit+IU]

- **Calculation of the Neutrosophic Interval** :

Neutrosophic Interval= $[24.823 - 3, 100.0 + 8] = [21.823, 108.0]$

This final interval reflects both statistical variability and clinical uncertainty.

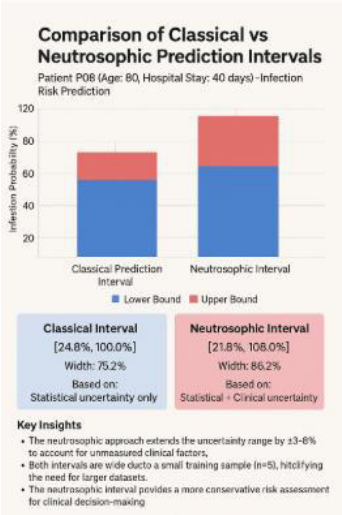

Figure 1: Comparison of Classical vs Neutrosophic Prediction Intervals.

Step 5: Calculating the Neutrosophic Mean

To merge multiple predictions and assess risk in a cohort, the neutrosophic mean is calculated. A conceptually sound method is to average the lower and upper bounds of the patients' final neutrosophic intervals.

Data from two hypothetical patients are used:

- **Patient A** : Prediction interval [65,95],indeterminacy$IN = [4,9]$.
  - Neutrosophic Interval A:$[65 - 4,95 + 9] = [61.0,104.0]$
- **Patient B** : Prediction interval [75,90], indeterminacy$IN = [6,11]$.
  - Neutrosophic Interval B:$[75 - 6,90 + 11] = [69.0,101.0]$
- **Calculation of the Neutrosophic Mean** :
  $\bar{x}$ N (bottom)$= 261.0 + 69.0 = 65.0$ $\bar{x}$N(top)$= 2104.0 + 101.0 = 102.5$

The resulting neutrosophic mean, which represents the aggregated risk and uncertainty of the group, is$[\mathbf{65.0}, \mathbf{102.5}]$.

Summary of Results in Tables

Table 4: Classic Prediction Interval

Predicted Value (ŷ)	Interval (95%)
82%	"[24.823, 100.0]"

Table 5: Neutrosophic Range

Predicted Value (ŷ)	Neutrosophic Range (95%)
82%	"[21.823, 108.0]"

Table 6: Neutrosophic Mean

Lower Mean ($\bar{x}$N (inf))	Upper Mean ($\bar{x}$N (sup))	Neutrosophic Media
65.0	102.5	"[65.0, 102.5]"

5. Discussion

The results demonstrate the ability of the statistical-neutrosophic approach to enrich predictive models.

The **classical prediction interval** is remarkably wide. This width is a fair reflection of the **high statistical uncertainty** that arises when making predictions with a very limited training data set (n=5) . This finding underscores the critical importance of sample size in building reliable [24.823, 100.0], predictive models.

The main contribution of the study is evident in the **neutrosophic interval** ,[21.823, 108.0]. This is built on the foundation of statistical uncertainty and adds a layer of unquantified clinical ambiguity . It presents a risk landscape where, considering unmeasured factors, the probability of infection is a certainty with an additional degree of uncertainty. For a clinician, this expanded range is a stronger warning signal that warrants more intensive surveillance and prophylaxis .

Finally, the **neutrosophic mean** serves as an aggregated and consistent indicator. It [65.0, 102.5],represents the combined average risk and uncertainty for a small cohort of patients, offering a useful summary for hospital risk management at the group level . This methodological approach is essential for the concept to be applicable and valuable in clinical practice.

6. Conclusion

This study successfully demonstrated the application and value of neutrosophic statistical regression models for predicting the incidence of nosocomial infections in post-trauma patients. The neutrosophic approach offers a more complete and nuanced representation of clinical uncertainty by moving beyond traditional point predictions and confidence intervals.

The ability to incorporate an indeterminacy component (IN) allows models to be more robust and more accurately reflect the complexities of the hospital environment. This has direct practical implications: clinicians can make more informed decisions, better stratify patients according to their risk, and more efficiently allocate resources to prevent infections.

Despite limitations such as the subjectivity of NI definition, the study lays a solid foundation. It contributes to knowledge by presenting a methodology that effectively handles ambiguity in medical data, representing a significant advance over conventional approaches.

For future research, it is recommended to validate this model with larger datasets from multiple hospitals. Furthermore, the development of standardized methods for defining the uncertainty interval and comparing their performance with other machine learning techniques should be investigated. Ultimately, neutrosophic logic is emerging as a valuable tool for improving the accuracy and reliability of prediction in modern medicine.

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Evaluating Lawyers' Perceptions of Cryptocurrency Regulation in Ecuador Using Plithogenic Statistics and an Indeterminate Likert Scale

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Abstract. An issue worthy of exploration is how lawyers view crypto currency regulation in Ecuador because many factors could take place to facilitate effective policy under an uncertain and complex present state. Thus, the overarching problem this study seeks to resolve is that without consideration of the lawyer's point of view to gauge present regulatory action, a large gap exists from the angel of the gap that prevents regulation from being at the level it's supposed to be. Therefore, undertaking the consideration of how these professionals view the matter will help avoid missteps in establishing a more effective policy or corrective action policy. Given that crypto currency is becoming more and more important to economies worldwide, assessing how lawyers feel is vital to identifying levels that could be truly effective. While studies exist with crypto currency, studies exist with regulation, few studies exist where there's a high level of integration with indeterminacy and complexity. Thus, this study is designed to address this gap by showing the application of plithogenic analytics and the indeterminate Likert scale as practical methods for an approach where civilians' sentiments are ruled by uncertainty. An incalculably prepared survey inquired, and subsequently linear statistical regression after the survey applies assesses whether lawyers in Ecuador believe there are legal certainty, legitimacy, anonymity, necessity for innovation, operability shortcomings, and consumer safeguards through significant levels. This generates a correlational assessment that supports universality of factors but also nuances and differences of opinion based on gender and professional designations. Ultimately, two contributions arise from findings; they represent the first time something so new has been applied to approach a complicated mindset ruled by uncertainty and second, they indicate to legislators and practitioners that action in the present must change with an emphasis on improved certainty and consensus into present course. Ultimately this study advances theory and practicality by providing future researchers and lawmakers with guidance that can change legislative action for effective and unified regulation going forward.

Keywords: Crypto currencies, Regulation, Perception, Lawyers, Plithogenic Statistics, Likert Scale, Ecuador.

1. Introduction

Crypto currency regulation has emerged as a crucial issue in the digital age, with significant implications for the global economy and legal certainty [1]. In Ecuador, as in many other countries, uncertainty and complexity surround the implementation of effective policies to manage this new financial paradigm. According to a recent study by the Economic Commission for Latin America and the Caribbean (ECLAC), crypto currency adoption in the region is on the rise, highlighting the need for clear and coherent regulatory frameworks [2]. Throughout history, financial regulation has evolved to address new challenges and opportunities, from the creation of digital currencies to the expansion of online financial services. In the context of crypto currencies, the lack of regulatory clarity has generated

uncertainty and risk for investors and operators [3]. However, some countries have begun to develop innovative regulatory frameworks to address these challenges, such as Switzerland and its regulatory " sandbox " approach [4].

In Ecuador, cryptocurrency regulation is a relatively new and complex issue, with few studies addressing the perceptions of lawyers and other legal professionals on this topic. How do lawyers perceive existing regulations, and what recommendations can they offer to improve the clarity and consistency of regulatory policies? This question guides the research, seeking to provide a solution that encompasses the complex dimensions of cryptocurrency regulation. Plithogenic statistics and the indeterminate Likert scale offer innovative tools for capturing the complexity and uncertainty inherent in expert opinions. By employing these tools, this study seeks to provide a detailed assessment of lawyers' perceptions of cryptocurrency regulation in Ecuador.

The objectives of this study are, first, to assess lawyers' perceptions of cryptocurrency regulation in Ecuador using plithogenic statistics and an indeterminate Likert scale, and second, to offer practical recommendations to improve the clarity and coherence of regulatory policies. These objectives are aligned with the research question and will be developed throughout the article. Understanding lawyers' perceptions of cryptocurrency regulation is crucial to inform effective and coherent policies. According to a recent study by the Organization for Economic Cooperation and Development (OECD), regulatory clarity is essential to fostering innovation and investment in the financial sector [5,6].

In this context, this study seeks to contribute to the debate on cryptocurrency regulation in Ecuador and provide practical recommendations to improve the clarity and consistency of regulatory policies. In short, this study seeks to address the lack of understanding regarding lawyers' perceptions of cryptocurrency regulation in Ecuador and offer practical recommendations to improve the clarity and consistency of regulatory policies.

2. Materials and methods

In this section we present the basic elements of the Indeterminate Likert Scale and Plithogenic Statistics.

A. Indeterminate Likert scale

Definition 1 ([7, 17]): The *single-valued neutrosophic set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$, and $F_A: U \to [0,1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2 ([7, 17]): *Refined neutrosophic logic* is defined such that: a truth T splits into several types of truths : $T_1, T_2, \ldots, T_p$, I into several indeterminacies:$I_1, I_2, \ldots, I_r$ and F in several falsehoods: $F_1, F_2, \ldots, F_s$, where all$p, r, s \geq 1$ are integers, and $p + r + s = n$.

Definition 3 ([7, 17]): A *triply refined neutrosophic indeterminate set* (TRINS) A in X is characterized by $P_A(x)$positive indeterminacy, $I_A(x)$negative indeterminacy $N_A(x)$, and positive indeterminacy .$I_{P_A}(x)$ and negative indeterminacy$I_{N_A}(x)$ Membership functions. Each of these has a weight.$w_m \in [0,1]$ associated with it. For each $x \in X$, there is $P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x) \in [0,1]$, $w_P^m(P_A(x)), w_{I_P}^m(I_{P_A}(x)), w_I^m(I_A(x)), w_{I_N}^m(I_{N_A}(x)), w_N^m(N_A(x)) \in [0,1]$ and $0 \leq P_A(x) + I_{P_A}(x) + I_A(x) + I_{N_A}(x)(x) + N_A(x) \leq 5$. Therefore, a TRINS A can be represented by $A = \{\langle x; P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\}$.

Let A and B be two TRINS in a finite universe of discourse , $X = \{x_1, x_2, \ldots, x_n\}$, which are denoted by:

$A = \{\langle x; P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\}$ and $B = \{\langle x; P_B(x), I_{P_B}(x), I_B(x), I_{N_B}(x), N_B(x)\rangle | x \in X\}$,

Where $P_A(x_i), I_{P_A}(x_i), I_A(x_i),\ I_{N_A}(x_i), N_A(x_i), P_B(x_i), I_{P_B}(x_i), I_B(x_i),\ I_{N_B}(x_i), N_B(x_i) \in [0,1]$, for each $x_i \in X$. Letw_i (i = 1,2,... ,n) be the weight of an elementx_i (i = 1,2,...,n), with$w_i \geq 0$ (i = 1,2,...,n) and $\sum_{i=1}^{n} w_i = 1$.

The generalized TRINS weighted distance is ([7, 17]):

$$d_\lambda(A,B) = \left\{\frac{1}{5}\sum_{i=1}^{n} w_i \left[|P_A(x_i) - P_B(x_i)|^\lambda + \left|I_{P_A}(x_i) - I_{P_B}(x_i)\right|^\lambda + |I_A(x_i) - I_B(x_i)|^\lambda + \left|I_{N_A}(x_i) - I_{N_B}(x_i)\right|^\lambda + |N_A(x_i) - N_B(x_i)|^\lambda\right]\right\}^{1/\lambda} \quad (1)$$

Where $\lambda > 0$.

The indeterminate Likert scale consists of the following five elements:

- Negative membership,
- Indeterminacy that tends towards negative belonging,
- Indefinite membership,
- Indeterminacy tending towards positive membership,
- Positive membership.

These values replace the classic Likert scale with the values:

- Totally disagree,
- I don't agree,
- Neither agree nor disagree,
- Accept,
- I totally agree.

B. Plithogenic statistics

Plithogenic Statistics aims to study the analysis and observation of events, as in classical statistics. It is a generalization of classical Multivariate Statistics, where multivariate results of neutrosophic or indeterminate variables are analyzed.

For example, according to Smarandache's example ([6, 17]) on the plithogenic phenomenon, Neutrosophic Probability (NPP),$PNP(Jenifer) = \{(0.5, 0.9, 0.2), (0.6, 0.7, 0.4), (0.8, 0.2, 0.1), (0.4, 0.3, 0.5)\}$ Which consists of the neutrosophic probabilities that Jennifer will pass each of the four semester subjects. For example, for passing Differential Equations, she has a 50% success rate, a 20% failure rate, and a 90% uncertainty rate. Therefore, the neutrosophic probability of passing the semester is $(\min\{0.5, 0.6, 0.8, 0.4\}, \max\{0.9, 0.7, 0.2, 0.3\}, \max\{0.2, 0.4, 0.1, 0.5\}) = (0.4, 0.9, 0.5)$.

Regarding Plithogenic Refined Probability (PRP), probabilities are generalized to the case where there is more than one truth value, more than one uncertainty value, or more than one falsehood value. The illustrative example used by Smarandache is the following:

Suppose that, for each subject, Jenifer must be evaluated on two tests, one oral and one written I_1(oral test). Then, the set of probabilities is refined as , T_1(oral test); T_2(written test), I_2(written test); and F_1(oral test), F_2(written test).

So, $PRP(Jenifer) = \{((0.5, 0.6), (0.4, 0.7), (0.1, 0.2)), ((0.6, 0.8), (0.0, 0.7), (0.3, 0.4)), ((0.8, 0.8), (0.1, 0.2), (0.1, 0.0)), ((0.3, 0.7), (0.2, 0.3), (0.5, 0.4))\}$

For example,$((0.5, 0.6), (0.4, 0.7), (0.1, 0.2))$ This means that regarding the first topic, Jennifer has a 50% chance of passing the oral exam and a 60% chance of passing the written exam; it is 40% uncertain whether she will pass the oral exam and 70% uncertain whether she will pass the written exam; while there is a 10% chance of failing the oral exam and a 20% chance of failing the written exam.

3. The results

This study addresses the lack of understanding about how lawyers in Ecuador perceive regulation, or the lack thereof, regarding cryptocurrencies. The growing importance of these digital assets demands

a clear and effective regulatory framework. To analyze this perception, we employ an innovative methodology that combines plithogenic statistics with an indeterminate Likert scale, capturing the complexity and uncertainty inherent in legal experts' opinions.

The procedure is based on the methodology described in the reference framework, adapted to our object of study.

Step 1: Definition of Variables, Sample and Survey

Two main variables were defined to measure the lawyers' perception:

1. **Regulatory Clarity (RC)** : The perception of the clarity, consistency, and completeness of the current legal framework (or lack thereof) for cryptocurrencies.
2. **Regulatory Effectiveness (ER)** : The perception of the ability of current policies to achieve their objectives (protection, innovation, security).

To measure these variables, the following sub-variables or measures were established:

Regulatory Clarity Measures (NC):

- CN_1: Level of Legal Security that the current framework offers to users and investors.
- CN_2: Clarity in Consumer and User Protection of Crypto Assets .
- CN_3: Clarity of guidelines for the Prevention of Money Laundering and Terrorist Financing (PLL/FT).
- CN_4: Level of promotion or restriction on Innovation and Technological Development.
- CN_5: Clarity in the definition of the Legal Nature of cryptoassets (asset, security, currency).

Regulatory Effectiveness Measures (ER):

- ER_1: Effectiveness of Entity Supervision (Exchanges , Wallets).
- ER_2: Effectiveness of Sanction Mechanisms for Non-compliance.
- ER_3: Level of Interinstitutional Cooperation (BCE, Superintendencies, UAFE).
- ER_4: Perception of Adaptability of the Standard to Technological Evolution.

A survey was conducted among a sample of lawyers in Ecuador. The total population of registered lawyers was estimated at N = 48,150. The sample size was calculated using the formula for calculating the sample size with a 95% confidence level (Z = 1.96), a 5% margin of error (e = 0.05), and an expected proportion of 50% (p = 0.5).

Sample Size Calculation:

$$n = \frac{k^2 Npq}{e^2(N-1)+k^2 pq} \qquad (2)$$

n = Sample size (?),

N = Population size (5823),

Z = It is the deviation for the desired confidence level (1.96),

e = Maximum permissible error margin (0.05),

p = It is the proportion that we expect to find (p= 0.5),

Calculating we get:

The sample size was adjusted to **n = 382 lawyers**.

Each question was answered using a 5-level indeterminate Likert scale, where the attorney assigned a weight from 0 to 5 in each of the following categories: Strongly Disagree, Disagree, Neutral, Agree, Strongly Agree.

Step 2: Converting Data to Numeric Values

The quintuple responses were transformed into a single real-world numerical value. First, the quintuple was rescaled by dividing each value by 5 to obtain a vector in the interval [0,1].

For example, if a lawyer (lawyer _k) responds to the sub-variable CN_1 (Legal Security) with the quintuple (0, 1, 2, 4, 3), where the values correspond to (Totally Disagree, Disagree, Neutral, Agree, Totally Agree), the rescaled vector v would be:

$$v = (\frac{0}{5};\frac{1}{5};\frac{2}{5};\frac{4}{5};\frac{3}{5}) = (0,0.2,0.4,0.8,0.6)$$

The following equation is then applied to obtain a single numerical value:

$$S(v) = \frac{2(v5 - v1) + (v4 - v2) \cdot v2 + v4v4 + (1 - v3) \cdot \frac{2}{1}}{2}$$

Note: To avoid division by zero if $v_2 + v_4 = 0$, the term is assumed to be 0. In this case, $v_2 + v_4 = 0.2 + 0.8 = 1.0 \neq 0$.

$$S(v) = 1.575$$

This process was repeated for each of the 9 sub-variables (5 from CN and 4 from ER) for the 382 lawyers surveyed.

Step 3: Aggregating Values by Variable

To obtain a total aggregate value for the CN and ER variables for each respondent, the values of their respective sub-variables were averaged:

$$CNi = \frac{1}{5}\sum j = 15S(CNij)$$

$$ERi = \frac{1}{4}\sum k = 14S(ERik)$$

Where i represents each of the 382 lawyers. This step generates a pair of values (CN_i, ER_i) for each lawyer.

Step 4: Correlation Analysis - Kendall's Tau-b

Kendall's Tau-b correlation test was used to analyze the relationship between variables. This method is robust to ordinal data and effectively handles ties. The test was applied to the aggregated pairs (CN, ER) and also between the aggregated variable ER and each sub-variable of the CN.$CN\ (CN_j)$.

The results of the correlation analysis are presented in the following tables. The significance level used was $\alpha = 0.01$.

Table 1: Correlation between Regulatory Clarity (RC) and Regulatory Effectiveness (RE)

Kendall's Tau-b		**Regulatory Clarity (NC)**	**Regulatory Effectiveness (RE)**
	CN		
	Correlation Coefficient	1,000	.664**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.664**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

The p-value of 0.000 (less than 0.01) indicates a positive and statistically significant correlation between the perception of Regulatory Clarity and Regulatory Effectiveness.

Tables 2-6: Correlation between Sub-variables of Regulatory Clarity and Regulatory Effectiveness (ER)

The correlation between the aggregate variable ER and each of the CN sub-variables is shown below.

Table 2: Correlation between CN_1 (Legal Security) and ER

Kendall's Tau-b		**CN_1**	**ER**
	CN_1		
	Correlation Coefficient	1,000	.596**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.596**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

Table 3: Correlation between CN_2 (Consumer Protection) and ER

Kendall's Tau-b		**CN_2**	**ER**
	CN_2		
	Correlation Coefficient	1,000	.649**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.649**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

Table 4: Correlation between CN_3 (Prevention of Money Laundering) and ER

		CN_3	ER
Kendall's Tau-b	**CN_3**		
	Correlation Coefficient	1,000	.550**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.550**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

Table 5: Correlation between CN_4 (Innovation and Development) and ER

		CN_4	ER
Kendall's Tau-b	**CN_4**		
	Correlation Coefficient	1,000	.612**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.612**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

Table 6: Correlation between CN_5 (Legal Nature) and ER

		CN_5	ER
Kendall's Tau-b	**CN_5**		
	Correlation Coefficient	1,000	.700**
	Sig. (unilateral)	.	.000
	N	382	382
	ER		
	Correlation Coefficient	.700**	1,000
	Sig. (unilateral)	.000	.
	N	382	382

**. The correlation is significant at the 0.01 level (one-tailed).

Figure 1: Correlations between Sub-variables of Regulatory Clarity and Regulatory Effectiveness

Figure 1 presents a visual summary of the Kendall Tau-b correlations between the five sub-variables of Regulatory Clarity and Regulatory Effectiveness. It is clear that the definition of the legal nature of cryptoassets (CN_5) presents the strongest correlation, followed by consumer protection (CN_2) and innovation promotion (CN_4).

4. Discussion

The results demonstrate a strong and significant positive correlation between Regulatory Clarity and Regulatory Effectiveness ($\tau b = .664, p < .01$). This central finding suggests that, from the perspective of Ecuadorian lawyers, a vague, ambiguous, or incomplete regulatory framework is perceived as inherently ineffective. Legal uncertainty not only hinders the safe operation of users and businesses, but also undermines the perception that regulation can achieve its goals.

When analyzing the sub-variables, we observed important nuances. The strongest correlation was found between Regulatory Effectiveness (RE) and Clarity of the Legal Nature of Cryptoassets (CLC), with a coefficient of τb = .700. This is revealing: lawyers consider defining whether a cryptoasset is a good, a security, a currency, or a new legal category to be the fundamental pillar for building effective regulation. Without this basic definition, any attempt at supervision, sanctions, or consumer protection is perceived as weak.

Next in importance are Consumer Protection (CN_2) and $\tau b = .649$Innovation Promotion (CN_4) $\tau b = .612$. This indicates that legal professionals value regulations that balance user protection with an enabling environment for technological development, considering them key components of effectiveness.

The lowest, though still highly significant, correlation was with Clarity in Anti-Money Laundering (CN_3), with . While its importance is undeniable, it could be interpreted that lawyers perceive that AML/CFT regulations, even if clarified, do not in themselves guarantee the overall effectiveness of the

regulatory ecosystem if other aspects such as $\tau b = .550$legal nature or consumer protection remain ambiguous.

One limitation of this study is that it relies on perceptions, which may be influenced by each lawyer's individual knowledge and exposure to the crypto ecosystem . However, the use of plithogenic statistics and the indeterminate Likert scale provides a richer and more realistic capture of these complex and often conflicting opinions.

5. Conclusions

This study successfully applied an advanced methodological approach to assess Ecuadorian lawyers' perceptions of cryptocurrency regulation. The combination of the indeterminate Likert scale and plitogenic statistics proved to be a powerful tool for capturing and analyzing complex opinions in a highly uncertain field.

1. **Significant Correlation** : It is confirmed with high statistical confidence that there is a strong positive correlation between the perception of Regulatory Clarity and the perception of Regulatory Effectiveness. For lawyers, clear regulation is effective regulation.
2. **Importance of Definitions** : The most critical factor in the perception of effectiveness is clarity in defining the legal nature of cryptoassets . This should be the starting point for any future legislative efforts.
3. **Generalized Correlation** : All sub-variables of Regulatory Clarity (Legal Certainty, Consumer Protection, AML/CFT, Innovation and Legal Nature) showed a positive and significant correlation with Regulatory Effectiveness, highlighting their interdependence.

Consequently, legislators and regulators in Ecuador are advised to develop a regulatory framework for cryptocurrencies that is perceived as effective by legal experts. They should prioritize the creation of precise legal definitions, establish clear rules for consumer protection, and design a system that offers legal certainty without stifling innovation. Addressing these pillars is critical to building the necessary trust and consistency in this emerging sector.

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Multineutrosophic Analysis with the ARAS Method for Selecting Trading Strategies

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Abstract. Deciding on optimal business solutions is a perennial problem for firms challenged by ever-changing market environments. Currently, the choice of a solution entails multidimensional requirements in addition to similar uncertainties and vagueness factoring human fallibility and ever-challenging market conditions. Even though thousands of systems for decision-making support exist, the dedicated scholarly literature does not support any advancement that would ease an all-encompassing evaluation of strategic solutions during complete uncertainty or information vagueness. Therefore, this research aims to fill this theoretical gap by creating a new approach based on practical application. This new approach is based on the employment of the Additive Ratio Assessment method (the ARAS method) in a multineutrosophic setting to give better weighting to criteria and simultaneously assess strategic solutions. Multineutrosophic refers to a level of truth, falsity, and indeterminacy. Thus, by applying neutrosophic operators, the assessments of various experts are comprehensively applied to turn qualitative attributes into quantitative, highly reliable values, leading to appropriate results through the multineutrosophic ARAS method to assess degrees of effectiveness of business solutions to rank potential business strategies for ultimate success through widespread uncertainty. This work contributes theoretically and practically: 1. theoretically expands the scope of multi-criteria decision-making processes by formally incorporating the element of uncertainty; 2. practically provides firms with a new tool for conducting strategic assessments as it will increase efficiency and effectiveness of resource usage while decreasing vulnerability to inevitable downturns in the marketplace.

Keywords: Business Strategies, Multicriteria Decision Making, Neutrosophic Logic, ARAS Method, Uncertainty, Decision Making, Optimization.

1. Introduction

Strategic decision-making in the commercial field constitutes a fundamental pillar for the sustainability and growth of any organization in the current global environment [1]. In a landscape characterized by volatility and uncertainty, the selection of business strategies not only involves the evaluation of multiple quantitative factors, but also the integration of qualitative, often imprecise, elements. Addressing this complexity effectively has therefore become an imperative necessity for companies seeking to maintain their competitiveness and secure an advantageous position in the market [2]. Historically, strategic decision-making processes have transitioned from purely intuitive approaches to rigorous methodologies based on data analysis. From Porter 's influential theories on competitive forces, formulated in the 1980s, to the emergence of more recent business intelligence models, the evolution in this field has been constant [3]. However, despite significant advances in business analytics and decision

support tools, most existing frameworks still struggle to coherently integrate the ambiguity and uncertainty inherent in expert opinions and changing environmental conditions.

The central problem that this research aims to solve lies in the persistent lack of a robust methodology that enables the selection of optimal business strategies under conditions of incomplete or vague information. How can organizations effectively process and utilize subjective judgments and uncertain data to make informed and reliable strategic decisions in a context of high complexity? This fundamental question drives our study, seeking to offer a solution that encompasses these intricate and often overlooked dimensions in traditional approaches. Although various multicriteria decision-making (MCDM) techniques exist, such as the Analytic Hierarchy Process (AHP), the Technique of Order of Preference by Similarity to the Ideal Solution (TOPSIS), or the ELECTRE Family [4], these conventional methodologies often assume that information is perfectly clear and precise. However, in the context of business strategy, data rarely live up to this idealization. Expert perceptions, market projections, and risk assessments are often tainted by uncertainty, imprecision, and inconsistency [5]. This significant gap in literature and practice highlights the urgent need to develop approaches that can handle this intrinsic nature of information. Faced with this limitation, neutrosophic logic emerges as a promising and highly relevant tool. This extension of fuzzy logic allows for modeling not only the degree of truth and falsity of a proposition, but also the degree of indeterminacy associated with it. By explicitly integrating indeterminacy, neutrosophic logic offers a more comprehensive and realistic framework for representing and processing human knowledge, especially in contexts where information is inherently vague or ambiguous. Therefore, its application in strategic decision-making could offer a more accurate and grounded perspective.

Additive Ratio Assessment (ARAS) method, recognized for its computational simplicity, efficiency, and proven ability to effectively rank alternatives, is presented as an ideal candidate to be adapted and extended under a neutrosophic setting. The combination of ARAS with neutrosophic logic allows for a more nuanced and comprehensive assessment of strategic alternatives, considering not only the favorability of each option, but also the uncertainty and ambiguity inherent in each evaluation criterion and the perception of decision-makers. This work not only seeks to develop an innovative decision support tool, but also to validate its effectiveness in a real-life context of business strategy selection. The implementation of this methodology is expected to provide decision-makers with a deeper understanding of the implications of each strategy, thus optimizing resource allocation and minimizing the risks associated with the inherent uncertainty of the global market. The practical relevance of this study lies in its significant potential to improve the quality and robustness of strategic decisions in an increasingly complex and dynamic business world.

The main objectives of this study are: 1) to develop a methodology for business strategy selection based on the multineutrosophic ARAS method; 2) to apply the proposed approach to a practical case study to demonstrate its feasibility and effectiveness in a real-life business environment; and 3) to evaluate the model's ability to manage and mitigate the uncertainty and indeterminacy inherent in complex strategic decision-making processes. These objectives will guide the research exhaustively and will be addressed in detail throughout this article.

2. Preliminaries.

Multineutrosophic Set

Definition 1. The *Neutrosophic Set N* It is characterized by three membership functions [9] , which are the truth membership function T_A, the indeterminacy membership function I_A, and the falsity membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A 0, 1^+[$, and $\bar{}_A 0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [10] .

See that by definition, $T_A(x), I_A(x)$, and $F_A(x)$are standard or non-standard real subsets of $]\bar{}_A 0, 1^+[$and

hence, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0,1]$. $\bar{A}0$and1^+ belong to the set of hyperreal numbers.

Definition 2. The *single-valued neutrosophic set* (SVNS) Aover $U$is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$and $F_A: U \to [0,1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

SVNN are expressed below:

Given A $_1$= (a $_1$, b $_1$, c $_1$) and A $_2$= (a $_2$, b $_2$, c $_2$) two SVNN, the sum between A $_1$and A $_2$ is defined as :

$$A_1 \ A_2 = (a_1 + a_2 - a_1 a_2, b_1 b_2, c_1 c_2) \quad (1)$$

Given A $_1$= (a $_1$, b $_1$, c $_1$) and A $_2$= (a $_2$, b $_2$, c $_2$) two SVNNs, the multiplication between A $_1$and A $_2$is defined as:

$$A_1 \ A_2 = (a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2) \quad (2)$$

The product of a positive scalar with a SVNN, A = (a, b, c) is defined as:

$$A = (1 - (1 - a), b, c) \quad (3)$$

The *single-valued neutrosophic number* (SVNN) is symbolized by
$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3. The refined neutrosophic set of subsets (SRNS).

Let $\mathcal{U}$ a universe of discourse and a set $R \subset \mathcal{U}$. Then, a refined neutrosophic subset R is defined as follows:

$R = \{x, x(T, I, F), x \in U\}$,where T is refined/divided into p subtruths, $T = \langle T_1, T_2, \dots T_p \rangle, T_j \subseteq [0,1], 1 \leq j \leq p$; I is refined/divided into r subindeterminacies, $I = \langle I_1, I_2, \dots I_r \rangle, I_k \subseteq [0,1], 1 \leq k \leq r$,and F is refined/divided into s subfalsehoods, $F = \langle F_1, F_2, \dots F_l \rangle, F_s \subseteq [0,1], 1 \leq l \leq s$, where $p, r, s \geq 0$are integers, and $p + r + s = n \geq 2$, and at least one of $p, r, s \, is \geq 2$to ensure the existence of refinement (division).

Definition 4 ([6]) **.** The MultiNeutrosophic Set (or MultiNeutrosophic Set Subset SMNS).

Let $\mathcal{U}$ a universe of discourse and M a subset of it. Then, a MultiNeutrosophic Set is: $M = \{x, x(T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s)\}, x \in U$,

where p , r , s are integers $\geq 0, p + r + s = n \geq 2$ and at least one of them $p, r, s \, is \geq 2$,to ensure the existence of multiplicity of at least one neutrosophic component: truth/belonging, indeterminacy or falsity/non-belonging; all subsets$T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s \subseteq [0,1]$;

$$0 \leq \sum_{j=1}^{p} inf \ T_j + \sum_{k=1}^{r} inf \ I_k + \sum_{l=1}^{s} inf \ F_l \leq \sum_{j=1}^{p} sup \ T_j + \sum_{k=1}^{r} sup \ I_k + \sum_{l=1}^{s} sup \ F_l \leq n.$$

No other restrictions apply to these neutrosophics multicomponents..

$T_1, T_2, \dots, T_p$They are multiplicities of truth, each provided by a different source of information (expert).

Similarly, $I_1, I_2, \dots, I_r$there are multiplicities of indeterminacy, each provided by a different source.

And $F_1, F_2, \dots, F_s$they are multiplicities of falsehood, each provided by a different source.

The Degree of MultiTruth (MultiMembership), also called *Multidegree of Truth* , of the element x with respect to the set M are $T_1, T_2, \dots, T_p$.

The Degree of Multiindeterminacy (MultiNeutrality), also called *Multidegree of Indeterminacy* , of the element x with respect to the set M are$I_1, I_2, \ldots, I_r$.

and the Degree of Multi- Nonmembership , also called *Multidegree of Falsehood* , of the element x with respect to the set M are$F_1, F_2, \ldots, F_s$.

All of these $p + r + s = n \geq 2$are assigned by n sources (experts) which can be:

- whether completely independent ;
- or partially independent and partially dependent;
- or totally dependent; depending on or as needed for each specific application.

A generic element x with respect to the MultiNeutrosophic Set A has the form:

$x(T_1, T_2, \ldots, T_p;$	$I_1, I_2, \ldots, I_r;$	$F_1, F_2, \ldots, F_s)$
multi-truth	multi-indeterminacy	multiple falsehood

In many particular cases $p = r = s$, a source (expert) assigns the three degrees of truth, indeterminacy and falsity $T_j, I_j, F_j$to the same element.

Definition 5. Classification of multineutrosophic types of value n of the same form (p, r, s).

$(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)$, where $p, r, s$are integers ≥ 0, and $p + r + s = n \geq 2$, and at least one of $p, r, s \geq 2$to be sure that it has multiplicity for at least one neutrosophic component (either truth, or indeterminacy, or falsity).

It offers a simpler n classification, but it is more of an approximation. Let's calculate the following.

positivity (4).

$$\frac{\sum_{j=1}^{p} T_j + \sum_{k=1}^{r}(1 - I_k) + \sum_{e=1}^{s}(1 - F_e)}{p + r + s} \tag{4}$$

Average (Truth-Falsehood) (5)

$$\frac{\sum_{j=1}^{p} T_j + \sum_{e=1}^{s}(1 - F_e)}{p + s} \tag{5}$$

Truth (6).

$$\frac{\sum_{j=1}^{p} T_j}{p} \tag{6}$$

Definition 6. Classification of tuples n-valued multineutrosophics of different forms (p, r, s).

Let us consider two neutrosophic multiple tuples of value n of the forms $(p_1, r_1, s_1)$and respectively (p_2, r_2, s_2), where $p_1, r_1, s_1, p_2, r_2, s_2$ are integers ≥ 0 , and $p_1 + r_1 + s_1 = n_1 \geq 2$, and at least one of $p_1, r_1, s_1 is \geq 2$,to be sure that multiplicity exists for at least one neutrosophic component (either truth, indeterminacy or falsity); similarly $p_2 + r_2 + s_2 = n_2 \geq 2$,and at least one of$p_2, r_2, s_2\ is \geq 2$.

Let's take the following tuples single-value multineutrosophic (SVMNT):

$SVMNT = (T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s)$of $(p_1, r_1, s_1) - form$, and

$SVMNT' = (T'_1, T'_2, \ldots, T'_p; I'_1, I'_2, \ldots, I'_r; F'_1, F'_2, \ldots, F'_s)$of $(p_1, r_1, s_1) - form$.

It performs the classic averages of truth (T_a), indeterminacies (I_a) and falsity (F_a), respectively for $SVMNT = (T_a, I_a, F_a)$and the averages of truths (T_a), indeterminacies (I_a) and falsity (F_a) respectively for : $SVMNT = (T'_a, I'_a, F'_a)$. And then applies the Score Functions ( ), Accuracy ( ) and Certainty ( ), as for the single-valued neutrosophic set:

Calculate the scoring function (average positivity) (7).

$$S(T_a, I_a, F_a) = \frac{T_a + (1 - I_a) + (1 - F_a)}{3}$$
$$S(T'_a, I'_a, F'_a) = \frac{T'_a + (1 - I'_a) + (1 - F'_a)}{3} \quad (7)$$

i. If $S(T_a, I_a, F_a) \geq S(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $S(T_a, I_a, F_a) \leq S(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $S(T_a, I_a, F_a) = S(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the second step.

Calculate the precision function (difference between truth and falsehood) (8).

$$A(T_a, I_a, F_a) = T_a - F_a$$
$$A(T'_a, I'_a, F'_a) = T'_a - F'_a \quad (8)$$

i. If $A(T_a, I_a, F_a) \geq A(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $A(T_a, I_a, F_a) \leq A(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $A(T_a, I_a, F_a) = A(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the third step.

3. Calculate the certainty (truth) function (9).

$$C(T_a, I_a, F_a) = T_a$$
$$C(T'_a, I'_a, F'_a) = T'_a \quad (9)$$

i. If $C(T_a, I_a, F_a) \geq C(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $C(T_a, I_a, F_a) \leq C(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $C(T_a, I_a, F_a) = C(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$they are multi-neutrosopically equal, that is $T_a = T_a^{'}, I_a = I_a^{'}, F_a = F_a^{'}$, or their corresponding averages of truth, indeterminacy and falsity are equal.

Definition 7. In cases where some sources carry a greater weight in the evaluation than others, weighted averages are used, indexed as $T_{wa}, I_{ua}, F_{va}$and respectively $T'_{wa}, I'_{ua}, F'_{va},$. Because sources can be independent or partially independent, the sum of the weights does not necessarily have to equal 1. Therefore :

i. $w_1, w_2, \dots, w_p \in [0,1]$, although the sum $w_1 + w_2 + \cdots + w_p$may be $< 1, or\ = 1, or\ > 1$.
ii. $u_1, u_2, \dots, u_p \in [0,1]$, although the sum $u_1 + u_2 + \cdots + u_p$may be $< 1, or\ = 1, or\ > 1$.
iii. $v_1, v_2, \dots, v_p \in [0,1]$, although the sum $v_1 + v_2 + \cdots + v_p$may be $< 1, or\ = 1, or\ > 1$.

And similarly, the score, precision, and certainty functions are applied to these weighted averages to rank them.

Multineutrosophic ARAS.

Additive Ratio Assessment (ARAS) method is a multi-criteria decision-making technique that allows the selection of the best option from a set of alternatives 7]. To this end, an extension of the traditional method is proposed through multi-neutrosophic set evaluation. Consequently, it is reformulated as the multi-neutrosophic ARAS method to determine the complex relative efficiency of each strategic guideline. This involves evaluating each strategic guideline through multiple sources (experts) based on the corresponding criteria. By integrating multi-neutrosophic set analysis into the ARAS method, the following steps are defined:

Step 1: Identify multiple sources (experts) for the multi-criteria assessment and assign a weight to each expert based on their knowledge and contribution to the financial statement analysis (according to Definition 7 in Section 2.1). For this purpose, Saaty 's neutrosophic AHP method is applied (following the procedures referenced in the bibliographic sources [8] [9]).

Step 2: Determine the importance weights of each criterion in decision-making for each source (expert).

Step 3: Construct the decision matrix L_{ij}(see Figure 1), where the element L_{ij}represents each strategic guideline (GE) evaluated by multiple sources (experts (Exp.), according to Definitions 5 and 6 of Section 2.1) based on an identified criterion (C) .

$$\begin{bmatrix} l_{11} & l_{12} & \dots & l_{1j} & \dots & l_{1n} \\ l_{21} & l_{22} & \dots & l_{2j} & \dots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \dots & l_{ij} & \dots & l_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{m1} & l_{m2} & \dots & l_{mj} & \dots & l_{mn} \end{bmatrix}$$

Figure 1: Decision matrix $L_{ij}$for the ARAS multineutrosophic method. Source: Prepared by the authors.

Step 4: The normalized decision matrix $\bar{L}_{ij}$, considering the beneficial and non-beneficial values, is calculated using equations (10) and (11):

$$\bar{L}_{ij} = \frac{l_{ij}}{\sum_{i=0}^{m} l_{ij}} \tag{10}$$

$$L_{ij} = \frac{1}{l_{ij}^{*}} \tag{11}$$

Step 5: The weighted normalized decision matrix is calculated using equation (12).

$$\hat{L}_{ij} = \bar{L}_{ij} \cdot W_j \tag{12}$$

The weight values W_j are determined using the entropy method. Where j W_j is the weight of criterion j and j $\bar{L}_{ij}$ is the normalized ranking of each criterion.

Step 6: Calculation of the optimization function S_i using equation (13).

$$G_i = \sum_{j=1}^{n} \hat{L}_{ij} \tag{13}$$

Where G_i is the value of the optimization function for alternative i . This calculation is directly related to the process of the values $\hat{L}_{ij}$ and weights W_j of the investigated criteria and their relative influence on the outcome.

Step 7: Calculation of the degree of utility. This degree is determined by comparing the variant under analysis with the best one G_o, according to equation (14) [10].

$$K_i = \frac{G_i}{G_o} \tag{14}$$

Where G_i and G_o are the values of the optimization function. These values range from 0 to 100%; therefore, the alternative with the highest value K_i is the best of the alternatives analyzed.

3. Case Study: Selection of Commercial Strategies at the "MUÑEKO" Ice Cream Factory

3.1 Context of the "MUÑEKO" Ice Cream Factory

The "MUÑEKO" Ice Cream Factory, owned by CONSYBA in the municipality of Calixto García, Holguín, operates in a competitive regional market susceptible to various variables, such as the availability of raw materials, seasonal demand, and changing consumer preferences. Strategic decision-making requires a careful evaluation of multiple factors, considering the inherent uncertainty of the business environment.

3.2 Proposed Strategic Guidelines

To address market challenges and optimize its performance, the "MUÑEKO" Ice Cream Factory has identified the following six potential business strategies:

- **EC $_1$: Product Line Expansion:** Introduce new ice cream flavors and presentations to diversify the offering and attract different consumer segments.
- **EC $_2$: Strengthening Distribution:** Expand the distribution network through alliances with new points of sale and the exploration of online channels.
- **EC $_3$: Supply Chain Optimization:** Establish stronger relationships with local suppliers and explore alternatives to ensure the availability and cost of raw materials.
- **EC $_4$: Implementation of Digital Marketing Strategies:** Use social media, online advertising, and other digital tools to increase brand visibility and customer engagement.
- **EC $_5$: Focus on Sustainability and Natural Products:** Develop ice cream lines with organic ingredients and eco-friendly packaging to attract environmentally conscious consumers.

- **EC $_6$: Improving Operational Efficiency:** Implement technologies and processes to reduce production costs and improve productivity.

3.3 Multineutrosophic ARAS Modeling for Trading Strategy Selection

Step 1: Selecting Multiple Sources (Experts) and Modeling the Saaty AHP Neutrosophic Method.

To obtain a comprehensive and robust assessment of business strategies, eight experts with different perspectives relevant to the operation and market of the "MUÑEKO" Factory were consulted (see Table 1).

Table 1: Multisources for neutrosophic assessment (experts).

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Profession	Production Manager	Marketing Manager	Market Analyst	Sales manager	Counter	Logistics Specialist	Business Consultant	Customer Representative

Saaty 's neutrosophic AHP method, a pairwise assessment of the relative importance of each expert was performed. The resulting neutrosophic comparison matrix and the calculation of the experts' weights are presented in Tables 2 and 3. The consistency analysis yielded an eigenvalue of $\lambda max = 8.85$, a consistency index (CI) of 0.12, and a consistency ratio (CR) of 0.08, indicating acceptable consistency in the assessments.

Table 2: Paired matrix of neutrosophic AHP.

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Exp-1	(0.55, 0.50, 0.53)	(0.65, 0.40, 0.45)	(0.90, 0.15, 0.10)	(0.85, 0.20, 0.15)	(0.50, 0.55, 0.50)	(0.80, 0.25, 0.20)	(0.70, 0.30, 0.35)	(0.75, 0.25, 0.30)
Exp-2	(0.40, 0.60, 0.65)	(0.55, 0.50, 0.53)	(0.75, 0.25, 0.30)	(0.70, 0.30, 0.35)	(0.30, 0.70, 0.75)	(0.70, 0.30, 0.35)	(0.55, 0.50, 0.53)	(0.70, 0.30, 0.35)
Exp-3	(0.10, 0.85, 0.90)	(0.25, 0.70, 0.75)	(0.55, 0.50, 0.53)	(0.60, 0.45, 0.50)	(0.35, 0.65, 0.70)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.60, 0.45, 0.50)
Exp-4	(0.15, 0.80, 0.85)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.55, 0.50, 0.53)	(0.25, 0.75, 0.80)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.55, 0.50, 0.53)
Exp-5	(0.50, 0.55, 0.50)	(0.70, 0.30, 0.35)	(0.65, 0.35, 0.40)	(0.75, 0.25, 0.30)	(0.55, 0.50, 0.53)	(0.75, 0.25, 0.30)	(0.60, 0.40, 0.45)	(0.75, 0.25, 0.30)
Exp-6	(0.20, 0.75, 0.80)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.50, 0.45, 0.60)	(0.25, 0.75, 0.80)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)

Expert	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8
Exp-7	(0.35, 0.70, 0.75)	(0.55, 0.50, 0.53)	(0.60, 0.45, 0.50)	(0.60, 0.45, 0.50)	(0.45, 0.55, 0.60)	(0.55, 0.50, 0.53)	(0.55, 0.50, 0.53)	(0.70, 0.30, 0.35)
Exp-8	(0.30, 0.70, 0.75)	(0.30, 0.70, 0.75)	(0.50, 0.45, 0.60)	(0.55, 0.50, 0.53)	(0.25, 0.75, 0.80)	(0.55, 0.50, 0.53)	(0.30, 0.70, 0.75)	(0.55, 0.50, 0.53)

Table 3: Expert weights obtained using neutrosophic AHP. *Note: Normalized weights were calculated by dividing each expert's score by the total sum of the scores.*

Fountain	w_i (Neutrosophic Weight)	S(w_i) (Score)	Normalized Weight
Exp-1	(0.30, 0.35, 0.33)	0.327	0.165
Exp-2	(0.15, 0.20, 0.18)	0.177	0.089
Exp-3	(0.07, 0.10, 0.09)	0.087	0.044
Exp-4	(0.06, 0.09, 0.08)	0.077	0.039
Exp-5	(0.28, 0.33, 0.30)	0.303	0.153
Exp-6	(0.05, 0.08, 0.07)	0.067	0.034
Exp-7	(0.12, 0.17, 0.15)	0.147	0.074
Exp-8	(0.09, 0.13, 0.11)	0.110	0.055
Addition	**-**	**1,295**	**0.653**

Step 2: Selection and Evaluation of Criteria for Each Multifunctional (Expert).

Five relevant criteria were defined for the selection of commercial strategies for the "MUÑEKO" Factory:

- **C 1 : Sales Growth Potential:** Expected impact on sales growth.
- **C 2 : Operational Viability:** Ease of implementation and management of the strategy.
- **C 3 : Investment Requirement:** Amount of investment required to implement the strategy. (Cost Criterion, which will be treated differently in standardization.)
- **C 4 : Market Adaptability:** Ability of the strategy to adjust to changes in consumer preferences and market conditions.
- **C 5 : Long-Term Sustainability:** Contribution of the strategy to the long-term viability and success of the company.

The multineutrosophic evaluation of each criterion by the expert panel is presented in Table 4, together with the weight calculated for each criterion using the entropy method based on the experts' assessments.

Table 4: Average multineutrosophic evaluation and criteria weights.

Criterion	Average Multineutrosophic Assessment	Weight of the Criterion	Score (S)
C 1	({0.7, 0.6, 0.8}, {0.2, 0.3}, {0.1, 0.1, 0.2, 0.1})	0.28	0.70
C 2	({0.6, 0.7, 0.5}, {0.3, 0.2}, {0.1, 0.1, 0.3, 0.2})	0.22	0.63
C 3	({0.2, 0.3, 0.4}, {0.5, 0.4}, {0.6, 0.5, 0.4, 0.5})	0.15	0.33
C 4	({0.8, 0.7, 0.9}, {0.1, 0.2}, {0.1, 0.1, 0.1, 0.1})	0.20	0.80
C 5	({0.5, 0.6, 0.7}, {0.4, 0.3}, {0.3, 0.2, 0.2, 0.2})	0.15	0.57

Steps 3 to 7: Construction of the Multineutrosophic Decision Matrix and Calculation of the Optimization Function.

The multineutrosophic decision matrix, where each element represents the evaluation of each trading strategy (EC $_i$) by the eight experts under each criterion (C $_j$), is presented in Table 5

Table 5: Multineutrosophic ARAS decision matrix (partial example).

Strategy	Criterion	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8	Average Score
EC 1	C 1	(0.8, 0.5, 0.4)	(0.7, 0.6, 0.5)	(0.9, 0.2, 0.1)	(0.8, 0.3, 0.2)	(0.6, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.8, 0.3, 0.4)	(0.7, 0.4, 0.5)	(0.75, 0.43, 0.38)
EC 1	C 2	(0.7, 0.6, 0.3)	(0.6, 0.7, 0.4)	(0.8, 0.2, 0.2)	(0.7, 0.3, 0.3)	(0.5, 0.8, 0.7)	(0.6, 0.4, 0.4)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.6)	(0.65, 0.45, 0.43)
EC 1	C 3	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.2, 0.8, 0.7)	(0.3, 0.7, 0.6)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.35, 0.65, 0.56)
EC 1	C 4	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.83, 0.18, 0.11)
EC 1	C 5	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.55, 0.45, 0.35)
EC 2	C 1	(0.7, 0.6, 0.5)	(0.6, 0.7, 0.6)	(0.8, 0.3, 0.2)	(0.7, 0.4, 0.3)	(0.5, 0.8, 0.7)	(0.6, 0.5, 0.4)	(0.7, 0.4, 0.5)	(0.6, 0.5, 0.6)	(0.65, 0.53, 0.48)
EC 2	C 2	(0.6, 0.7, 0.4)	(0.5, 0.8, 0.5)	(0.7, 0.3, 0.3)	(0.6, 0.4, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.55, 0.60, 0.53)
EC 2	C 3	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.45, 0.55, 0.45)
EC 2	C 4	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.73, 0.28, 0.18)

Strat-egy	Crite-rion	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5	Exp-6	Exp-7	Exp-8	Average Score
EC 2	C 5	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.3, 0.7, 0.6)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.54, 0.46, 0.36)
EC 3	C 1	(0.6, 0.7, 0.6)	(0.5, 0.8, 0.7)	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.56, 0.63, 0.58)
EC 3	C 2	(0.5, 0.8, 0.7)	(0.4, 0.9, 0.8)	(0.6, 0.5, 0.4)	(0.5, 0.6, 0.5)	(0.3, 0.9, 0.9)	(0.4, 0.7, 0.6)	(0.5, 0.6, 0.7)	(0.4, 0.7, 0.8)	(0.45, 0.79, 0.70)
EC 3	C 3	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.55, 0.45, 0.34)
EC 3	C 4	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.63, 0.38, 0.28)
EC 3	C 5	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.2, 0.8, 0.7)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.44, 0.60, 0.46)
EC 4	C 1	(0.8, 0.3, 0.2)	(0.7, 0.4, 0.3)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.2)	(0.6, 0.5, 0.4)	(0.7, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.7, 0.3, 0.3)	(0.75, 0.29, 0.25)
EC 4	C 2	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.2)	(0.5, 0.6, 0.5)	(0.6, 0.3, 0.3)	(0.7, 0.2, 0.2)	(0.6, 0.3, 0.3)	(0.65, 0.33, 0.29)
EC 4	C 3	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.3, 0.7, 0.6)	(0.4, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.45, 0.55, 0.45)
EC 4	C 4	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.83, 0.18, 0.11)
EC 4	C 5	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.55, 0.45, 0.35)
EC 5	C 1	(0.6, 0.7, 0.6)	(0.5, 0.8, 0.7)	(0.7, 0.4, 0.3)	(0.6, 0.5, 0.4)	(0.4, 0.9, 0.8)	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.6)	(0.5, 0.6, 0.7)	(0.56, 0.63, 0.58)
EC 5	C 2	(0.5, 0.8, 0.7)	(0.4, 0.9, 0.8)	(0.6, 0.5, 0.4)	(0.5, 0.6, 0.5)	(0.3, 0.9, 0.9)	(0.4, 0.7, 0.6)	(0.5, 0.6, 0.7)	(0.4, 0.7, 0.8)	(0.45, 0.79, 0.70)
EC 5	C 3	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.55, 0.45, 0.34)
EC 5	C 4	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.3, 0.2)	(0.6, 0.4, 0.3)	(0.63, 0.38, 0.28)
EC 5	C 5	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)	(0.5, 0.5, 0.4					

The decision matrix was then normalized. For the benefit criteria (C1, C2, C4, C5), Equation (10) was applied for the material provided. For the cost criterion (C3), Equation (11) was applied, inverting the values so that a lower investment is preferable.

The normalized matrix was then weighted using the criterion weights obtained in Step 2 (Equation 12). Finally, the optimization function (Si) was calculated for each trading strategy by summing the normalized weighted values for each alternative (Equation 13). The utility degree (Qi) was calculated by comparing the optimization function

Calculation of the Degree of Utility, the value of the optimization function of each strategy (Si) was compared with the maximum value obtained (Smax) using Equation (14) of the provided material: $Qi = SmaxSi \times 100\%$.The detailed results are presented in Table 6.

Table 6: Optimization function and degree of usefulness of trading strategies.

Business Strategy	Optimization Function)	Degree of Utility
EC1: Product Line Expansion	0.258	100.00%
EC2: Strengthening Distribution	0.235	91.09%
EC3: Supply Chain Optimization	0.198	76.74%
EC4: Implementation of Digital Marketing Strategies	0.245	94.96%
EC5: Focus on Sustainability and Natural Products	0.212	82.17%
EC6: Improving Operational Efficiency	0.251	97.29%

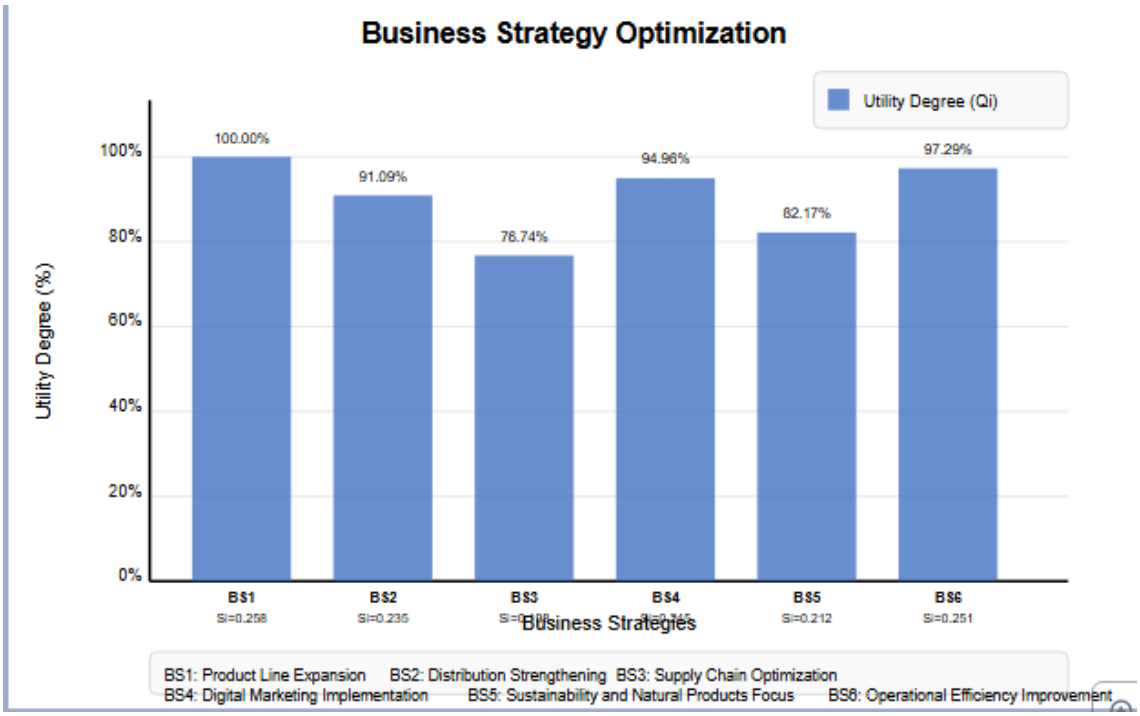

Chart 1: Optimization function and degree of usefulness of trading strategies.

3.4 Results

Applying the multi-expert ARAS method to the six business strategies proposed for the "MUÑEKO" Ice Cream Factory reveals a clear hierarchy of preference based on utility (Qi) . The strategy with the highest utility (100%) is **EC 1: Product Line Expansion** , indicating that, based on the multi-expert evaluation and weighted criteria, this strategy is the most favorable for the company.

They are followed in order of preference:

- **EC 6: Improvement of Operational Efficiency** (97.29%)
- **EC 4: Implementation of Digital Marketing Strategies** (94.96%)
- **EC 2: Strengthening Distribution** (91.09%)

- **EC $_5$: Focus on Sustainability and Natural Products** (82.17%)
- **EC $_3$: Supply Chain Optimization** (76.74%)

These results provide a solid quantitative and qualitative basis for strategic decision-making at the "MUÑEKO" Factory, considering the uncertainty inherent in the market environment through the multineutrosophic approach.

3.5 Discussion

The results obtained from the multineutral ARAS method suggest that **product line expansion (CS1)** is perceived as the strategy with the greatest potential for the "MUÑEKO" Ice Cream Factory. This could be due to the opportunity to capture new market segments and increase market share through offering diversification, which is crucial in a competitive market with dynamic consumer preferences.

The high rating for **improved operational efficiency (EC6)** is also significant. Optimizing production processes can translate into cost reduction and increased profitability, strengthening the company's long-term competitive position.

The importance given to **implementing digital marketing strategies (EC4)** reflects the growing influence of online channels in product promotion and sales, as well as in customer interaction. A strong digital presence can improve brand visibility and build customer loyalty.

Strengthening **distribution (EC2)** is also considered an important strategy to increase product reach and availability in the regional market.

The **focus on sustainability and natural products (EC5)** , although not the main strategy, remains relevant, indicating a growing awareness of these aspects among experts and their potential to appeal to a specific niche market.

Finally, **supply chain optimization (EC3)** , although important for efficiency, is ranked as the least preferred compared to the other strategies evaluated. This could be interpreted as meaning that, while it is a necessary operational foundation, its direct strategic impact on growth and competitiveness is considered minor in the company's current context.

The application of the multineutrosophic framework allowed the ambiguity and indeterminacy present in experts' judgments to be incorporated, providing a richer and more realistic evaluation of strategies compared to traditional multicriteria decision-making methods[11].

4. Conclusions

The ARAS technique, applied within a multineutrosophic framework, proved highly effective for guiding the selection of commercial strategies at the "MUÑEKO" Ice Cream Factory. The analysis revealed that expanding the product line (CS1) is the top priority, followed by enhancing operational efficiencies and intensifying digital marketing efforts. By transforming expert judgments into neutrosophic values and evaluating multiple criteria under conditions of vagueness and uncertainty, this approach delivered robust, quantitative insights that reinforce strategic decision-making and optimize resource allocation in a dynamic market environment.

On the basis of these findings, it is recommended that "MUÑEKO" allocate significant resources to research and develop new ice-cream flavors and formats, supported by targeted market research; undertake operational efficiency initiatives to streamline production processes; and invest in a comprehensive digital marketing strategy to boost brand visibility and customer engagement. Additionally, strengthening the distribution network through new partnerships and online channels will broaden market reach, while exploring sustainable product lines with natural ingredients and eco-friendly packaging can capture emerging consumer segments. Finally, a detailed review of the supply chain's resilience and efficiency—though lower in immediate priority—will identify long-term optimization opportunities and further fortify the company's strategic positioning.

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Determining Factors for Minimizing Oven Downtime through Controlled Batch Management: A Multineutrosophic Approach with ARAS

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Abstract. This study addresses a critical operational challenge in the baking industry: minimizing oven downtime through effective batch processing control. This study assesses which factors are the most critical contributors to minimizing downtime on batch processing operations to hypothesize efficiencies. But why is this problem? Sadly, much research has been done and found that ineffective resource allocation not only wastes funds but also increases carbon footprint for unnecessary baking, which is ironic since fresh products must drive demand at a low-cost. Furthermore, minimizing downtime allows for the more sustainability-oriented production that today's manufacturers are on the hook for. Therefore, testing new industrial process solutions is warranted. Currently, a lot of scholarly research exists about queuing theory and supply chain feasibility; however, little is known. Many investigations fail to concentrate on baking process optimization with uncertainty and subjectivity of decision-makers as top determinations. Therefore, this study is the first to bridge the gap using multineutrosophic theory and a decision-making tool called Additive Ratio Assessment (ARAS)—a prioritizing assessment tool—which provides nuance to criterion assessments and weighting via incremental rounds with experts rendered neutrosophic. Ultimately the neutrosophic composite-fueled determination of the most critical criteria/test variables includes: batch arrival rate, acceptable batch size and setup time. Ultimately regulating batch processing according to predetermined thresholds vetted against neutrosophic variables decreases downtime significantly without negatively impacting quality control. This project adds to the literature a new methodology that legitimizes ideation for uncertainty-based decision making along with new recommendations for implementation that reduce energy expenditure, new resource allocation and production efficiency takeaways for bakeries that improve their competitive advantage and sustainable quality of life.

Keywords: Batch Management, Downtime, Ovens, Multineutrosophic, ARAS, Efficiency, Sustainability.

1 Introduction

The efficient management of industrial processes, particularly in the food industry, constitutes a fundamental pillar to ensure competitiveness and sustainability in an increasingly demanding global environment. This study focuses on optimizing oven downtime in bakeries through controlled batch management, a critical aspect to reduce operating costs and minimize environmental impact. The relevance of this topic lies in its ability to address practical challenges in food production, where energy efficiency and resource planning are essential to meet the growing demand for fresh products [1], [2]. Recent research highlights that the inactivity of equipment, such as ovens, can significantly increase costs and energy consumption, underscoring the need for innovative approaches to improve production systems [3]. Over the last decades, the food industry has undergone significant transformations driven

by technological advances and the adoption of mathematical models to optimize processes. From manual production systems to the integration of tools such as queuing theory, efforts to improve efficiency have been constant [4]. In the context of bakeries, batch management has emerged as a key factor in aligning dough preparation with baking cycles, reducing downtime and optimizing resource use [5]. However, variability in batch arrival and process disruptions remain persistent challenges that require solutions tailored to the stochastic production environment. The problem addressed in this study arises from the need to minimize oven downtime, an aspect that directly affects the productivity and sustainability of bakeries. Despite advances in the application of queuing models, such as MX/G(a, b)/1, few studies have considered the uncertainty inherent in decision-makers' perceptions and the complexity of operational factors [6]. How can key factors that allow reducing oven downtime be identified and prioritized in a context of high variability and ambiguity? This question guides the research, seeking to offer a comprehensive solution that combines analytical rigor with practical applicability.

This paper proposes a novel approach by integrating a multineutrosophic framework with the Additive Ratio Assessment (ARAS) method to assess and classify factors influencing batch management. This approach makes it possible to handle the indeterminacy present in operational decisions, an aspect that traditional models do not effectively address [7]. By considering both technical aspects and the subjective perceptions of experts, the study seeks to generate insights that transcend the limitations of conventional approaches. Historically, process optimization in the food industry has relied on quantitative tools that model workflows and resources. Queuing theory, in particular, has been widely used to analyze production systems with random arrivals and services [8]. However, the application of these tools in bakeries has been limited by the lack of models that incorporate uncertainty into task planning and execution, a challenge that this study directly addresses.

The magnitude of the problem lies in its economic and environmental impact. Idle ovens not only generate additional costs through unnecessary energy consumption but also contribute to the carbon footprint of operations [9]. In an industry where profit margins are slim, the ability to minimize downtime can make the difference between profitability and loss. This study addresses this need by proposing a framework that not only optimizes operations but also promotes sustainable practices. The research question focuses on identifying the determining factors that allow for reduced downtime through controlled batch management. By addressing this issue, the study seeks to contribute to theoretical knowledge in supply chain management and offer practical solutions for the food industry. The combination of a multineutrosophic approach with ARAS represents a significant innovation, allowing for a robust assessment of factors under conditions of uncertainty.

The objectives of this research are clear: first, to identify the key factors influencing oven downtime using a multineutrosophic analysis; second, to prioritize these factors using the ARAS method to generate practical recommendations; and third, to validate the proposed approach through a case study in a bakery. These objectives are aligned with the research question and seek to provide a significant contribution to both academia and industry.

2 Materials and methods

MultiNeutrosophic Set

Definition 1. The *Neutrosophic set N* It is characterized by three membership functions [10] , which are the truth membership function T_A, the indeterminacy membership function I_A, and the falsity membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A0, 1^+[$, and $\bar{}_A0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ [10] .

See that by definition , $T_A(x), I_A(x)$, and $F_A(x)$are standard or non-standard real subsets of $]\bar{}_A0, 1^+[$and hence, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals of $[0, 1]$. $\bar{}_A0$and1^+ belong to the set of hyperreal numbers.

Definition 2. The *single-valued neutrosophic set* (SVNS) Aover $U$is $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$and $F_A: U \to [0, 1]$. $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

SVNN are expressed below:

Given A $_1$= (a $_1$, b $_1$, c $_1$) and A $_2$= (a $_2$, b $_2$, c $_2$) two SVNN, the sum between A $_1$ and A $_2$ is defined as:

$$A_1 \ A_2 = (a_1 + a_2 - a_1a_2, b_1b_2, c_1c_2) \quad (1)$$

Given A $_1$= (a $_1$, b $_1$, c $_1$) and A $_2$= (a $_2$, b $_2$, c $_2$) two SVNNs, the multiplication between A $_1$ and A $_2$ is defined as:

$$A_1 \ A_2 = (a_1a_2, b_1+b_2 - b_1b_2, c_1 + c_2 - c_1c_2) \quad (2)$$

The product of a positive scalar with a SVNN, A = (a, b, c) is defined as:

$$\lambda A = \left(1 - (1 - a)^{\lambda}, b^{\lambda}, c^{\lambda}\right) \quad (3)$$

The *single-valued neutrosophic number* (SVNN) is symbolized by

$N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3. The refined neutrosophic set of subsets (SRNS).

Let $\mathcal{U}$ a universe of discourse and a set $R \subset \mathcal{U}$. Then, a refined neutrosophic subset R is defined as follows:

$R = \{x, x(T, I, F), x \in U\}$,where T is refined/divided into p subtruths, $T = \langle T_1, T_2, \dots T_p \rangle, T_j \subseteq [0,1], 1 \leq j \leq p$; I is refined/divided into r subindeterminacies, $I = \langle I_1, I_2, \dots I_r \rangle, I_k \subseteq [0,1], 1 \leq k \leq r$,and F is refined /divided into s sub falsehoods, $F = \langle F_1, F_2, \dots F_l \rangle, F_s \subseteq [0,1], 1 \leq l \leq s$, where $p, r, s \geq 0$are integers, and $p + r + s = n \geq 2$, and at least one of $p, r, s \ is \geq 2$to ensure the existence of refinement (division).

Definition 4 ([11]) **.** The MultiNeutrosophic Set (or MultiNeutrosophic Set Subset SMNS).

Let $\mathcal{U}$ a universe of discourse and M a subset of it. Then, a MultiNeutrosophic Set is: $M = \{x, x(T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s)\}, x \in U$,

where p , r , s are integers $\geq 0, p + r + s = n \geq 2$ and at least one of them $p, r, s \ is \geq 2$,to ensure the existence of multiplicity of at least one neutrosophic component: truth/belonging, indeterminacy or falsity/non-belonging; all subsets$T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s \subseteq [0,1]$;

$0 \leq \sum_{j=1}^{p} inf\ T_j + \sum_{k=1}^{r} inf\ I_k + \sum_{l=1}^{s} inf\ F_l \leq \sum_{j=1}^{p} sup\ T_j + \sum_{k=1}^{r} sup\ I_k + \sum_{l=1}^{s} sup\ F_l \leq n.$

No other restrictions apply to these multicomponent. Neutrosophics.

$T_1, T_2, \dots, T_p$They are multiplicities of truth, each provided by a different source of information (expert).

Similarly, $I_1, I_2, \dots, I_r$there are multiplicities of indeterminacy, each provided by a different source.

And $F_1, F_2, \dots, F_s$they are multiplicities of falsehood, each provided by a different source.

The Degree of Multitude (Multimember ship), also called *Mult degree of Truth,* of the element x with respect to the set M are $T_1, T_2, \dots, T_p$.

The Degree of Mult indeterminacy (Mult Neutrality), also called *Multidegree of Indeterminacy,* of the element x with respect to the set M are$I_1, I_2, \dots, I_r$.

and the Degree of Multi- Nonmembership, also called *Multidegree of Falsehood,* of the element x with respect to the set M are$F_1, F_2, \dots, F_s$.

All of these $p + r + s = n \geq 2$are assigned by n sources (experts) that can be:

- whether completely independent;
- or partially independent and partially dependent;
- or totally dependent; depending on or as needed for each specific application.

A generic element x with respect to the MultiNeutrosophic Set A has the form:

$x(T_1, T_2, \dots, T_p;$	$I_1, I_2, \dots, I_r;$	$F_1, F_2, \dots, F_s)$
multi-truth	multi-indeterminacy	multiple falsehood

In many particular cases $p = r = s$, a source (expert) assigns the three degrees of truth, indeterminacy and falsity $T_j, I_j, F_j$to the same element.

Definition 5. Classification of multineutrosophic types of value n of the same form (p, r, s).

$(T_1, T_2, \dots, T_p; I_1, I_2, \dots, I_r; F_1, F_2, \dots, F_s)$, where $p, r, s$are integers ≥ 0, and $p + r + s = n \geq 2$, and at least one of $p, r, s \geq 2$to be sure that it has multiplicity for at least one neutrosophic component (either truth, or indeterminacy, or falsity).

It offers a simpler n classification, but it is more of an approximation. Let's calculate the following.

positivity (4).

$$\frac{\sum_{j=1}^{p} T_j + \sum_{k=1}^{r}(1-I_k) + \sum_{e=1}^{s}(1-F_e)}{p+r+s} \tag{4}$$

Average (Truth-Falsehood) (5)

$$\frac{\sum_{j=1}^{p} T_j + \sum_{e=1}^{s}(1-F_e)}{p+s} \tag{5}$$

truth (6).

$$\frac{\sum_{j=1}^{p} T_j}{p} \tag{6}$$

Definition 6. Classification of tuples n-valued multineutrosophic of different forms (p, r, s).

Let us consider two neutrosophic multiple tuples of value n of the forms $(p_1, r_1, s_1)$and respectively (p_2, r_2, s_2), where $p_1, r_1, s_1, p_2, r_2, s_2$ are integers ≥ 0, and $p_1 + r_1 + s_1 = n_1 \geq 2$, and at least one of $p_1, r_1, s_1 is \geq 2$,to be sure that multiplicity exists for at least one neutrosophic component (either truth, indeterminacy or falsity); similarly $p_2 + r_2 + s_2 = n_2 \geq 2$,and at least one of$p_2, r_2, s_2\ is \geq 2$.

Let's take the following tuples single-value multineutrosophic (SVMNT):

$SVMNT = \left(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s\right)$of $(p_1, r_1, s_1) - form$, and

$SVMNT' = \left(T'_1, T'_2, \ldots, T'_p; I'_1, I'_2, \ldots, I'_r; F'_1, F'_2, \ldots, F'_s\right)$of $(p_1, r_1, s_1) - form$.

It performs the classic averages of truth (T_a), indeterminacies (I_a) and falsity (F_a), respectively for $SVMNT = (T_a, I_a, F_a)$and the averages of truths (T_a), indeterminacies (I_a) and falsity (F_a) respectively for : $SVMNT = (T'_a, I'_a, F'_a)$. And then applies the Score Functions (), Accuracy () and Certainty (), as for the single-valued neutrosophic set:

Calculate the scoring function (average positivity) (7).

$$\begin{aligned} S(T_a, I_a, F_a) &= \frac{T_a + (1-I_a) + (1-F_a)}{3} \\ S(T'_a, I'_a, F'_a) &= \frac{T'_a + (1-I'_a) + (1-F'_a)}{3} \end{aligned} \tag{7}$$

i. If $S(T_a, I_a, F_a) \geq S(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $S(T_a, I_a, F_a) \leq S(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $S(T_a, I_a, F_a) = S(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the second step.

Calculate the precision function (difference between truth and falsehood) (8).

$$\begin{aligned} A(T_a, I_a, F_a) &= T_a - F_a \\ A(T'_a, I'_a, F'_a) &= T'_a - F'_a \end{aligned} \tag{8}$$

i. If $A(T_a, I_a, F_a) \geq A(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $A(T_a, I_a, F_a) \leq A(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $A(T_a, I_a, F_a) = A(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$, then go to the third step.

3. Calculate the certainty (truth) function (9).

$$\begin{aligned} C(T_a, I_a, F_a) &= T_a \\ C(T'_a, I'_a, F'_a) &= T'_a \end{aligned} \tag{9}$$

i. If $C(T_a, I_a, F_a) \geq C(T'_a, I'_a, F'_a)$then $SVMNT \geq SVMNT'$,

ii. If $C(T_a, I_a, F_a) \leq C(T'_a, I'_a, F'_a)$then $SVMNT \leq SVMNT'$,

iii. And if $C(T_a, I_a, F_a) = C(T'_a, I'_a, F'_a)$then $SVMNT = SVMNT'$they are multi-neutrosopically equal, that is $T_a = T_a^{'}, I_a = I_a^{'}, F_a = F_a^{'}$, or their corresponding averages of truth, indeterminacy and falsity are equal.

Definition 7. In cases where some sources carry a greater weight in the evaluation than others, weighted averages are used, indexed as $T_{wa}, I_{ua}, F_{va}$and respectively $T'_{wa}, I'_{ua}, F'_{va}$,. Because sources can be independent or partially independent, the sum of the weights does not necessarily have to equal 1. Therefore:

i. $w_1, w_2, \ldots, w_p \in [0,1]$, although the sum $w_1 + w_2 + \cdots + w_p$may be $< 1, or\ = 1, or\ > 1$.

ii. $u_1, u_2, \ldots, u_p \in [0,1]$, although the sum $u_1 + u_2 + \cdots + u_p$may be $< 1, or\ = 1, or\ > 1$.

iii. $v_1, v_2, \ldots, v_p \in [0,1]$, although the sum $v_1 + v_2 + \cdots + v_p$may be $< 1, or\ = 1, or\ > 1$.

And, similarly, the score, precision, and certainty functions are applied to these weighted averages to rank them.

Multineutrosophic ARAS.

Additive Ratio Assessment (ARAS) method is a multi-criteria decision-making technique that allows the selection of the best option from a set of alternatives [12]. In this case, the study establishes among its objectives a series of strategic guidelines aimed at improving decision-making. To this end, an extension of the traditional method is proposed through multi-neutrosophic set evaluation. Consequently, it is reformulated as the multi-neutrosophic ARAS method to determine the complex relative efficiency of each strategic guideline. This involves evaluating each strategic guideline through multiple sources (experts) based on the corresponding criteria. By integrating multi-neutrosophic set analysis into the ARAS method, the following steps are defined:

Step 1: Identify multiple sources (experts) for the multi-criteria assessment and assign a weight to each expert based on their knowledge (according to Definition 7 in Section 2.1). For this purpose, Saaty 's neutrosophic AHP method is applied (following the procedures referenced in the bibliographic sources [13].

Step 2: Determine the importance weights of each criterion in decision-making for each source (expert).

Step 3: Construct the decision matrix L_{ij}(see Figure 1), where the element L_{ij}represents each strategic guideline (GE) evaluated by multiple sources (experts (Exp.), according to Definitions 5 and 6 of Section 2.1) based on an identified criterion (C).

$$\begin{bmatrix} l_{11} & l_{12} & \dots & l_{1j} & \dots & l_{1n} \\ l_{21} & l_{22} & \dots & l_{2j} & \dots & l_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \dots & l_{ij} & \dots & l_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{m1} & l_{m2} & \dots & l_{mj} & \dots & l_{mn} \end{bmatrix}$$

Figure 1: Decision matrix $L_{ij}$for the ARAS multineutrosophic method.

Step 4: The normalized decision matrix $\bar{L}_{ij}$, considering the beneficial and non-beneficial values, is calculated using equations (10) and (11):

$$\bar{L}_{ij} = \frac{l_{ij}}{\sum_{i=0}^{m} l_{ij}} \tag{10}$$

$$L_{ij} = \frac{1}{l_{ij}^{*}} \tag{11}$$

Step 5: The weighted normalized decision matrix is calculated using equation (12).

$$\hat{L}_{ij} = \bar{L}_{ij} \cdot W_j \tag{12}$$

The weight values $W_j$are determined using the entropy method. Where j $W_j$is the weight of criterion j and j $\bar{L}_{ij}$is the normalized ranking of each criterion.

Step 6: Calculation of the optimization function $S_i$using equation (13).

$$G_i = \sum_{j=1}^{n} \hat{L}_{ij} \tag{13}$$

Where $G_i$is the value of the optimization function of alternative *i.* This calculation has a directly proportional relationship with the process of the values $\hat{L}_{ij}$and weights $W_j$of the investigated criteria and their relative influence on the result [14].

Step 7: Calculating the degree of utility. This degree is determined by comparing the variant under

analysis with the best one G_o, according to equation (14).

$$K_i = \frac{G_i}{G_o} \tag{14}$$

Where $G_i$and $G_o$are the values of the optimization function. These values range from 0 to 100%, therefore, the alternative with the highest value $K_i$is the best of the alternatives analyzed [15].

3. Case study

3.1 Impact of Controlled Batch Management on Reducing Oven Downtime.

In the baking industry, operational efficiency depends largely on the ability to minimize oven downtime, a critical factor affecting both productivity and energy consumption. This study focuses on controlled batch management, defined as the dynamic adjustment of dough arrivals to the oven to optimize baking cycles. Key factors evaluated include batch arrival rate (frequency of dough preparation), batch size (number of trays per cycle), preparation time (coordination between mixing and baking), and operational flexibility (ability to adjust parameters in the face of variations). These factors are essential to ensuring continuous production, reducing energy waste, and improving sustainability.

Controlled batch management allows dough preparation to be aligned with baking cycles, avoiding unnecessary interruptions. For example, inadequate batch sizing can result in long wait times or underutilized baking cycles, increasing operating costs . Furthermore, uncertainty in demand and variations in preparation times complicate planning, highlighting the need for approaches that integrate uncertainty into decision-making. This case study addresses these issues by applying a multineutrosophic approach, which captures the subjective perceptions of experts and the inherent variability of the process.

The relevance of this approach lies in its ability to improve decision-making in complex environments. By considering multiple perspectives from experts, such as production managers, process engineers, and furnace operators, multineutral analysis helps identify critical factors that traditional methods might overlook. Furthermore, controlled batch management contributes to sustainability by reducing energy consumption, a crucial aspect in an industry where profit margins are tight.

The case study is carried out in a medium-sized bakery with a daily production of 5,000 bread units. The ovens, capable of processing up to six trays per cycle, face average downtimes of 15 minutes per hour due to mismatches in batch preparation. This scenario serves as a basis for evaluating how controlled batch management can optimize operations, reduce downtime, and improve energy efficiency.

3.2 Strategic Guidelines: Integration of MultiNeutrosophy in Batch Management

The integration of MultiNeutrosophy with the ARAS method allows prioritizing strategies to minimize furnace downtime, taking into account uncertainty and multiple expert perspectives. The proposed strategic guidelines are detailed in Table 1, focusing on optimizing coordination, improving flexibility, and reducing environmental impact.

Table 1. Strategic guidelines for controlled batch management

No	Strategic Guideline	Aim	Strategies	Impact
LE1	Optimizing the batch arrival rate	Ensure a continuous flow of dough to the oven adjusted to the baking capacity.	Implement scheduling algorithms based on queue models to adjust preparation frequency. Conduct arrival simulations to anticipate variations in demand.	Reduce downtime by minimizing unnecessary waiting.

No	Strategic Guideline	Aim	Strategies	Impact
LE2	Dynamic batch sizing	Adapt the number of trays per cycle according to the demand and capacity of the oven.	Establish dynamic batch size thresholds through real-time monitoring. Train staff on rapid operational adjustments.	Improves oven utilization and reduces energy consumption.
LE3	Efficient coordination of preparation time	Synchronize dough preparation with baking cycles.	Automate communication between mixing and baking stations. Implement dashboards to monitor times.	Minimizes interruptions due to lack of coordination in preparation .
LE4	Integrating expert judgment into planning	Incorporate the experience of operators and managers to improve decision-making.	Create multidisciplinary committees to evaluate operational parameters. Use structured surveys to capture expert perceptions.	Improves the accuracy of operational decisions under uncertainty.
LE5	Promoting operational flexibility	Increase responsiveness to production variations.	Design protocols for rapid adjustments to production parameters. Train staff in dynamic analysis tools.	Increases resilience to unforeseen changes in demand.
LE6	Reducing environmental impact	Minimize energy consumption through efficient operation.	Implement sensors to monitor energy consumption in real time. Optimize baking cycles to reduce emissions.	Contributes to sustainability and reduces operating costs.
LE7	Continuous improvement through feedback	Ensure constant adaptation of processes through iterative reviews.	Establish regular audits of baking processes. Implement real-time feedback systems.	Promotes continuous optimization and process quality.

3.3 Multineutrosophic ARAS Modeling: Selection of the Strategic Guideline

Step 1: Selecting Multiple Sources (Experts) in the Multi-Criteria Evaluation

To evaluate the strategic guidelines, five experts with experience in baking were selected, as detailed in Table 2. Each expert brings a unique perspective based on their role and knowledge of the process.

Table 2. Multi-source neutrosophic assessment (experts).

Expert	Profession
Exp-1	Production Manager
Exp-2	Process engineer
Exp-3	Senior Furnace Operator
Exp-4	Sustainability Specialist
Exp-5	Operations Analyst

Expert weights were determined using Saaty 's neutrosophic AHP method , considering their experience and contribution to the analysis. Table 3 shows the paired matrix, and Table 4 presents the resulting weights.

Table 3. Paired neutrosophic AHP matrix. +

Fountain	Exp-1	Exp-2	Exp-3	Exp-4	Exp-5
Exp-1	(0.5,0.5,0.5)	(0.7,0.3,0.3)	(0.8,0.2,0.2)	(0.9,0.1,0.1)	(0.6,0.4,0.4)
Exp-2	(0.3,0.7,0.7)	(0.5,0.5,0.5)	(0.6,0.4,0.4)	(0.8,0.2,0.2)	(0.5,0.5,0.5)
Exp-3	(0.2,0.8,0.8)	(0.4,0.6,0.6)	(0.5,0.5,0.5)	(0.7,0.3,0.3)	(0.4,0.6,0.6)
Exp-4	(0.1,0.9,0.9)	(0.2,0.8,0.8)	(0.3,0.7,0.7)	(0.5,0.5,0.5)	(0.3,0.7,0.7)
Exp-5	(0.4,0.6,0.6)	(0.5,0.5,0.5)	(0.6,0.4,0.4)	(0.7,0.3,0.3)	(0.5,0.5,0.5)

Table 4: Analysis of the consistency of the paired matrix. Source: Prepared by the authors.

Fountain	Weight	Approximate eigenvalues
Exp-1	0.30	5.12
Exp-2	0.25	5.08
Exp-3	0.20	5.05
Exp-4	0.15	5.03
Exp-5	0.10	5.01
Eigenvalue = 5.058, CI=0.0145, RC=0.016		

The production manager (Exp-1) and the process engineer (Exp-2) obtained the highest weights (0.30 and 0.25, respectively) due to their direct experience in process planning and optimization.

Step 2: Selection and Evaluation of Criteria for Each Multisource

Five criteria were defined to evaluate the strategic guidelines, detailed below:

- **C1: Impact on downtime**: Measures the guideline's ability to reduce downtime.
- **C2: Energy efficiency**: Evaluates the impact on reducing energy consumption.
- **C3: Operational flexibility**: Determines the ability to adapt to variations in production.
- **C4: Ease of implementation**: Analyzes the practical feasibility of the guideline.
- **C5: Contribution to sustainability**: Values the positive environmental impact.

The weights of the criteria were calculated using the entropy method, as shown in Table 5.

Table 5. Multineutrosophic evaluation of each criterion.

Criterion	({T1,T2,T3},{I1,I2},{F1,F2,F3})	(T_a,I_a,F_a)	Weight	Score (S)
C1	({0.8,0.7,0.9},{0.2,0.1},{0.3,0.4,0.2})	(0.80,0.15,0.30)	0.30	0.78
C2	({0.7,0.6,0.8},{0.3,0.2},{0.4,0.5,0.3})	(0.70,0.25,0.40)	0.25	0.68
C3	({0.6,0.5,0.7},{0.4,0.3},{0.5,0.6,0.4})	(0.60,0.35,0.50)	0.20	0.58
C4	({0.5,0.4,0.6},{0.5,0.4},{0.6,0.7,0.5})	(0.50,0.45,0.60)	0.15	0.48
C5	({0.6,0.5,0.7},{0.3,0.2},{0.4,0.5,0.3})	(0.60,0.25,0.40)	0.10	0.65

Criterion C1 (impact on downtime) was given the highest weight (0.30), followed by C2 (energy efficiency, 0.25), reflecting its importance in furnace optimization.

Step 3 to 7: Calculation of the Gi Optimization Function

The multineutrosophic decision matrix L_{ij} was constructed by evaluating each strategic guideline according to the defined criteria. All evaluations were classified as beneficial. Table 6 presents an extract of the matrix, and Table 7 shows the results of the optimization function Gi.

Table 6. Extract from the multi-neutrosophic ARAS decision matrix.

Directive	Criterio C1 (Ta, Ia, Fa)	Score (S)	Criterion C2 (Ta, Ia, Fa)	Score (S)	...
LE1	(0.75,0.20,0.30)	0.75	(0.70,0.25,0.35)	0.70	...
LE2	(0.65,0.30,0.40)	0.65	(0.60,0.35,0.45)	0.60	...
LE3	(0.80,0.15,0.25)	0.80	(0.75,0.20,0.30)	0.75	...
LE4	(0.70,0.25,0.35)	0.70	(0.65,0.30,0.40)	0.65	...
LE5	(0.60,0.35,0.45)	0.60	(0.55,0.40,0.50)	0.55	...
LE6	(0.85,0.10,0.20)	0.85	(0.80,0.15,0.25)	0.80	...
LE7	(0.65,0.30,0.40)	0.65	(0.60,0.35,0.45)	0.60	...

Table 7. Optimization function Gi and utility degree Ki.

Directive	C1	C2	C3	C4	C5	Gi	Ki (%)
LE1	0.045	0.035	0.030	0.020	0.015	0.145	72.50
LE2	0.039	0.030	0.025	0.018	0.012	0.124	62.00
LE3	0.048	0.038	0.032	0.022	0.016	0.156	78.00
LE4	0.042	0.033	0.028	0.019	0.014	0.136	68.00
LE5	0.036	0.028	0.023	0.016	0.011	0.114	57.00
LE6	0.051	0.040	0.034	0.024	0.017	0.166	83.00
LE7	0.039	0.030	0.025	0.018	0.012	0.124	62.00
G0						0.200	100.00

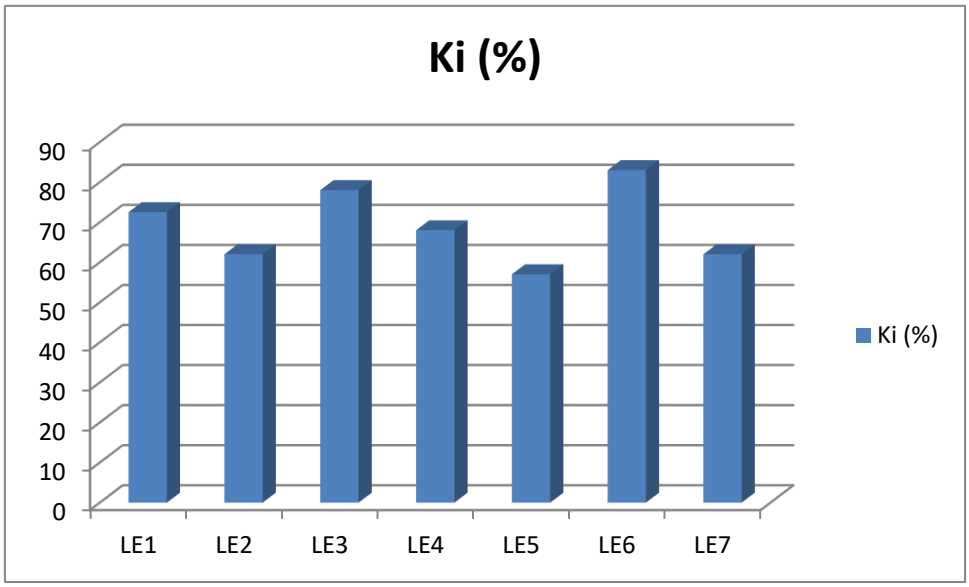

Figure 2: Degree of utility Ki. Source: Prepared by the authors.

3.4 Analysis of Results and Recommendations

The multineutrosophic ARAS method identified that strategic guidelines LE6 (reducing environmental impact) and LE3 (efficient coordination of preparation time) obtained the highest degrees of usefulness (83% and 78%, respectively). These guidelines stand out for their ability to reduce downtime

and improve energy efficiency, aligning with sustainability and productivity goals. LE6, focused on minimizing energy consumption through optimized baking cycles, directly addresses the need to reduce the carbon footprint, a critical aspect in the food industry. LE3, meanwhile, addresses the lack of coordination between dough preparation and baking, a common problem that generates significant downtime.

LE1 (batch arrival rate optimization) also showed strong performance (72.50%), suggesting that adjusting the frequency of dough preparation is key to maintaining continuous flow. In contrast, LE5 (promoting operational flexibility) achieved the lowest degree of usefulness (57%), possibly due to the difficulty of implementing rapid changes in resource-constrained environments. These results underscore the importance of prioritizing strategies that combine practical feasibility with a direct impact on operational efficiency.

The integration of MultiNeutrosophy allowed experts' uncertainty and subjective perceptions to be captured, providing a more robust assessment than traditional methods. For example, variability in opinions regarding ease of implementation (C4) was effectively managed through multineutrosophic analysis, ensuring that decisions reflected a balanced consensus.

Recommendations:

1. **Implement LE6 as a priority**: Install energy consumption sensors and optimize baking cycles to reduce downtime and emissions. This can be achieved by scheduling batches based on projected demand.
2. **Strengthen LE3 through automation**: Develop automated communication systems between preparation and baking stations to synchronize processes, reducing wait times.
3. **Train staff in LE1**: Train operators in the use of scheduling algorithms to adjust batch arrival rates, improving coordination with oven capacity.
4. **Continuous monitoring**: Establish a system of regular audits and real-time feedback (LE7) to identify deviations and dynamically adjust operating parameters.
5. **Invest in sustainable technology**: Adopt technologies such as energy-efficient furnaces to complement strategic guidelines and maximize environmental benefits.

These results and recommendations can be integrated into the bakery's operational strategy, promoting more efficient, sustainable, and competitive production in a dynamic environment.

4. Conclusion

This study reveals that controlled batch management, analyzed using a multineutrosophic approach and the ARAS method, identifies key factors for significantly reducing oven downtime in bakeries. The findings highlight the importance of optimizing baking cycles and synchronizing dough preparation with oven operation. This approach addresses uncertainty in operational decisions, providing a robust solution for production environments with high variability. In practical terms, the results offer tangible benefits for the food industry. Adopting strategies such as improving energy efficiency and operational coordination can generate considerable savings and reduce environmental impact. These solutions are particularly relevant for medium- and small-sized bakeries, where efficient resource management is crucial to maintaining competitiveness and responding to the demand for fresh products while promoting sustainability. This research makes a significant contribution by introducing an innovative methodological framework that integrates Multineutrosophic with the ARAS method. This approach enriches supply chain management by offering a practical tool for managing ambiguity in decision-making. By incorporating multiple expert perspectives, the study advances knowledge on industrial process optimization in complex and dynamic contexts.

However, the study is not without limitations. Subjectivity in expert assessments can introduce

variability, and the analysis focused on a specific bakery, which could limit the applicability of the results. Furthermore, the implementation of advanced technologies could face economic constraints in resource-limited settings. For future research, we recommend exploring the use of artificial intelligence, such as machine learning algorithms, to predict demand patterns and optimize batch management in real time. It would also be valuable to validate the approach in various industrial contexts and analyze external factors, such as energy costs. The evolution of optimization methods and the integration of emerging technologies will be essential to achieve more efficient and sustainable production processes.

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Assessing viral susceptibility in tilapia species using plithogenic fuzzy soft set: a multi-expert approach for TiLV

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Abstract. This research assessed tilapia species susceptibility to Tilapia Lake Virus (TiLV) via a new plithogenic fuzzy soft set (CFS). This mathematical conception offers a holistic assessment via biological, environmental, and epidemiological attributes, merging unknown and variable data across multiple researching experts. Thus, the findings emerged from controlled and field assessed mortality, viral titers, and microenvironments of up to five species processed through the CFS model to determine vulnerability rankings. CFS revealed Oreochromis niloticus was the most susceptible to TiLV with red hybrids second and O. mossambicus most resistant. In addition, the CFS model revealed specific parameters, including temperature and population density, which can increase transmission. These results suggest that multi-expert assessments and fuzzy modeling are invaluable for future prediction of viral epidemics in aquaculture. Ultimately, species vulnerability will allow for species-specific management and increased awareness during critical temperature or population density situations to decrease the economic impact of TiLV in the tilapia industry. This assessment is an ideal decision support system for aquaculture health under imperfect conditions.

Keywords: Viral Susceptibility, Tilapia, Tilv, Fuzzy Sets, Multi-Expert Modeling, Aquaculture, Epidemiology, Outbreak Prediction.

1. Introduction

Tilapia aquaculture has established itself as one of the main activities for food production worldwide, with Nile tilapia (Oreochromis niloticus) the most farmed species, with global production reaching 6.1 million metric tons in 2020 [1]. However, the intensification of farming systems has increased the incidence of infectious diseases, most notably Tilapia lake virus (TiLV), an emerging viral disease that causes mortalities of up to 90% in farmed and wild tilapia populations [2]. This virus, first identified in 2014 in Israel, represents a significant threat to food security and the sustainability of the tilapia farming industry , especially in producing countries such as Ecuador, Colombia, and China [3]. The problem lies in the lack of accurate tools to predict the susceptibility of different tilapia species to TiLV , which hampers the implementation of effective management strategies to mitigate outbreaks. The absence of comprehensive predictive models that consider biological, environmental and epidemiological factors under conditions of uncertainty limits the ability of producers to anticipate and control economic losses associated with this disease.

Several studies have addressed the epidemiological and clinical characteristics of TiLV , highlighting its ability to infect multiple species of tilapia, such as Oreochromis niloticus , O. mossambicus and their hybrids, as well as their horizontal transmission through contaminated water and tissues [4]. Previous research has described clinical signs such as lethargy, skin erosion, exophthalmos and hemorrhages, along with histopathological lesions in vital organs such as the liver, spleen and brain [5]. However,

these investigations have mainly focused on describing the pathogenesis and diagnosing using techniques such as quantitative PCR (qPCR) and histological analysis, without comprehensively addressing the variability in susceptibility between tilapia species [6]. Furthermore, existing studies often fail to integrate uncertain data from multiple sources, such as varying environmental conditions (temperature, stocking density) and field observations, leaving a gap in the ability to predict infection patterns in different production contexts. The lack of robust predictive models is particularly problematic in regions where tilapia aquaculture is a key source of income and food, such as Latin America and Asia. For example, in Egypt, TiLV outbreaks have caused an estimated US$100 million in economic losses due to the reduction of up to 98,000 tonnes of production [7]. These figures underline the urgency of developing tools that allow producers to anticipate and mitigate the impacts of the virus. Although biosecurity measures and surveillance programs have been proposed, these strategies are not always effective due to the complexity of factors influencing the spread of TiLV , such as water quality, culture density, and genetic differences between tilapia species. The need for an approach that integrates uncertain data and allows for differentiated assessment of susceptibility is evident to optimize management practices in aquaculture.

In this context, the present study proposes an innovative approach based on plithogenic fuzzy soft sets (CFS) to assess the susceptibility of different tilapia species to TiLV . This mathematical model allows integrating data from multiple experts and sources, including biological parameters (such as viral load), environmental parameters (such as temperature and population density), and epidemiological parameters (such as mortality rates), under conditions of uncertainty. Unlike traditional methods, which often assume accurate and homogeneous data, CFS offer a flexible tool to model the variability inherent in aquaculture systems, where conditions can be highly variable and data incomplete. This approach not only addresses the limitations of previous studies but also provides a basis for informed decision-making in managing viral outbreaks. The relevance of this study lies in its potential to reduce the economic and social impact of TiLV on the tilapia farming industry . In countries like Ecuador, where tilapia is a mainstay of the aquaculture economy, losses due to viral diseases can affect thousands of producers and communities that depend on this activity [1]. By identifying the most susceptible tilapia species and the environmental factors that exacerbate the disease, this study seeks to provide practical tools to optimize management strategies, such as adjusting culture densities or monitoring critical environmental conditions. Furthermore, the use of a multi-expert approach allows the incorporation of knowledge from veterinarians, biologists, and producers, which enriches the applicability of the results in real-life production contexts.

Another key aspect of the research is its contribution to aquaculture sustainability. The intensification of farming systems, while increasing production, has led to problems such as overcrowding and degraded water quality, which increase fish stress and susceptibility to disease [2]. By modeling the interaction between these factors and TiLV infection , this study not only addresses a specific aquaculture health problem but also promotes more sustainable practices that could be applied to other viral diseases in fish. The ability to more accurately predict outbreaks could reduce reliance on antibiotics and other costly interventions, aligning with global goals of food security and resource conservation.

The main objective of this study is to evaluate the susceptibility of five species of tilapia (Oreochromis niloticus , O. mossambicus , O. aureus , red hybrids and Tilapia rendalli) to TiLV by applying plithogenic fuzzy soft ensembles. Specific objectives include: (1) integrating biological, environmental, and epidemiological data from laboratory and field sources; (2) generating quantitative susceptibility indices for each species; and (3) identifying environmental factors that significantly influence disease progression. It is hypothesized that O. niloticus will exhibit increased susceptibility to TiLV compared to other species, such as O. mossambicus , due to genetic differences and their exposure to intensive culture conditions. Furthermore, factors such as water temperature and stocking density are expected to be key determinants of outbreak severity. The CFS-based approach addresses the limitations of traditional models by incorporating the inherent uncertainty in aquaculture data. For example, viral load

measurements can vary between laboratories, and environmental conditions, such as temperature, can fluctuate significantly in outdoor culture systems [4]. By modeling these variables flexibly, the study aims to generate more robust and generalizable results that can be applied across different regions and production systems. The inclusion of a multi-expert approach ensures that susceptibility indices reflect not only quantitative data but also the practical experience of those involved in aquaculture.

The proposed methodology also addresses the need for innovative tools to address emerging diseases in aquaculture. While diagnostic methods such as qPCR are effective in confirming the presence of TiLV , they do not offer predictive information about the likelihood of outbreaks or the vulnerability of different species [6]. In this sense, the CFS model acts as a complement to existing techniques, providing a preventative tool that can be integrated into surveillance and biosecurity programs. This approach is particularly valuable in contexts where resources for continuous monitoring are limited, such as in many developing countries that rely on tilapia as a protein source. In summary, this research seeks to fill a critical gap in knowledge about TiLV by providing a predictive model that combines uncertain data and the expertise of multiple experts. By assessing the susceptibility of different tilapia species and the factors influencing disease progression, the study offers a practical tool to improve aquaculture management and reduce economic losses. The expected results will not only contribute to aquaculture health, but will also support the sustainability of an industry vital to global food security.

2. Materials and methods

This section reviews the fundamental concepts related to soft sets, fuzzy soft sets, intuitionistic fuzzy soft sets, and neutrosophic soft sets. The key components of plithogenic sets and plithogenic soft sets are presented below.

2.1 Soft sets and extensions

Definition 1 ([7]). A *Soft set* over $U$is a pair (F, E), where $U$is the initial universal set, $E$is the set of parameters and $F$is the map of $E$to $\mathcal{P}(U)$, which is the power set of U.

So, given a parameter $\varepsilon \in E$, we have $F(\varepsilon) \in \mathcal{P}(U)$as the set of ε −approximate elements of (F, E).

Definition 2 ([8]). A *fuzzy The Soft set* on $U$is a pair (F, E), where $U$is the initial universal set, $E$is the set of parameters and $F$is the map of $E$to $\mathcal{F}(U)$, which is the set of fuzzy subsets of U.

Definition 3 ([9]). An *intuitionist Diffuse The Soft set* on $U$is a pair (F, E), where $U$is the initial universal set, $E$is the set of parameters and $F$is the map of $E$to $\mathcal{IF}(U)$, which is the set of intuitionistic fuzzy subsets of U.

Definition 4 ([9]). A *Neutrosophic The Soft set* over $U$is a pair (F, E), where $U$is the initial universal set, $E$is the parameter set and $F$is the map of $E$to $\mathcal{N}(U)$, which is the set of neutrosophic subsets of U.

2.2 Plithogenic assemblages and soft plithogenic assemblages

Yeah$U$ It is the universe of discourse, then fix P, which is a non-empty set of elements, and $P \subset U$[10,11]. It further follows that Ais the non-empty set of *one-dimensional attributes* , such that $\mathrm{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $\mathrm{m} \geq 1$. With each, $\alpha \in$ Awe have a spectrum of all possible values (or states).S which may be a discrete finite set $\mathrm{S} = \{s_1, s_2, \ldots, s_l\}$, $1 \leq \mathrm{l} < \infty$or a countably infinite set $\mathrm{S} = \{s_1, s_2, \ldots, s_\infty\}$, or an uncountably infinite (continuous) set $\mathrm{S} =]\mathrm{a}, \mathrm{b}[$, $\mathrm{a} < \mathrm{b}$.] ... [denotes any open, half-open, or closed interval of the set of real numbers or another general set.

On the other hand,$V \subset S$ and $V \neq \emptyset$is the range of all attributes that experts need for the given application. Then, for each, $\mathrm{x} \in$ Pthe values of all attributes in $\mathrm{V} = \{v_1, v_2, \ldots, v_n\}$, and are defined.$\mathrm{n} \geq 1$

There is generally a value called *the dominant attribute value* in V, which is selected by experts based on their criteria as to which is the most important attribute to meet the proposed objective.

The item $\mathrm{v} \in$ Vhas an *approval rating*. $\mathrm{d}(\mathrm{x}, \mathrm{v})$from element x, to set P, for some assumed criteria.

The degree of belonging is classified as *the diffuse degree of belonging*, a *diffuse intuitionistic degree of belonging*, or a *neutrosophic degree of belonging* to the plithogenic set.

So, we have the *value of the attribute accessory degree function* as*:*

$\forall x \in P, d: P \times V \rightarrow \mathcal{P}([0,1]^z)$ (1)

That is, $d(x,v)$is a subset of $[0,1]^z$, such that $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$, where determines the z*membership* type. In particular, z = 1 means a *fuzzy degree of membership*, z = 2 denotes an *intuitionistic fuzzy degree of belonging* , z = 3 is for the *neutrosophic degree of belonging* .

The function c: $V \times V \rightarrow [0,1]$is the *degree of contradiction function of the attribute value* Between any two attribute values $v_1$and v_2. This satisfies the following axioms:

1. $c(v_1, v_1) = 0$, that is, the degree of contradiction between the same attribute values is zero;
2. $c(v_1, v_2) = c(v_2, v_1)$, commutativity.

There is a distinction between functions caccording to the value z. The *degree of contradiction function of the fuzzy attribute value* is denoted by c_F, the *degree of contradiction function of the intuitionistic fuzzy attribute value* is a function $c_{IF}: V \times V \rightarrow [0,1]^2$, while the *degree of contradiction function of the neutrosophic attribute value is* defined by $c_N: V \times V \rightarrow [0,1]^3$.

Generally, these are one-dimensional attribute values and their degree of discrepancy. If you have multidimensional attribute values, these can be broken down into one-dimensional attribute values.

The attribute value degree of contradiction function allows for greater precision when performing calculations on some grouping methods and ranking systems. These values are based on expert judgment regarding the specific problem to be solved. If an attribute cannot be determined, the precision of the value degree of contradiction function will be lost, although the entire theory can still be used.

Once the above concepts have been defined,(P, a, V, d, c) It is a *plithogenic set* that meets the following:

1. $P$is a set, $a$is a one-dimensional or generally multidimensional attribute, $V$is the range of the attribute values, $d$is the degree of membership of the attribute value of each element $x$to the set P, $x \in P$, for some given criteria. Finally, $d$is d_F, d_{IF}, or d_N, when it is a fuzzy degree of membership, an intuitionistic fuzzy degree of membership or a neutrosophic degree of membership, respectively, of an element x to the plithogenic set P;
2. On the other hand, we define $c$as c_F, c_{IF}or c_N, if it is the fuzzy degree of contradiction, intuitionistic fuzzy degree of contradiction or neutrosophic degree of contradiction between attribute values, respectively.

Experts define $d(\cdot,\cdot)$and $c(\cdot,\cdot)$define the domain of specialization in which they operate.

The notation used is as follows:

$x(d(x,V))$, where $d(x,V) = \{d(x,v),\ for\ all\ v \in V\}, \forall x \in P$.

To calculate the degree of contradiction of the attribute value, it is performed on each value of the particular attribute and the value of the dominant attribute, called v_D.

The function of degree of contradiction of attribute value c among attribute values is included in the definition of *plithogenic aggregation operators* (intersection (AND), union (OR), implication ($\Longrightarrow$), equivalence ($\Leftrightarrow$), inclusion relation (partial order) and other plithogenic aggregation operators that combine two or more degrees of attribute value acting on t-norm and t- conorm .

Most plithogenic aggregation operators are linear combinations of the fuzzy t-norm ($\wedge_F$) and the fuzzy t-conorm ($\vee_F$). Nonlinear combinations can also be defined.

Having the calculation of t-norm and t-conorm between the dominant attribute value (v_D) with another attribute value (v_2), and also $c(v_D, v_2)$denotes the contradiction between $v_D$and v_2, then we can define the following operations:

$[1 - c(v_D, v_2)] \cdot t_{norm}(v_D, v_2) + c(v_D, v_2) \cdot t_{conorm}(v_D, v_2)$ (2)

Or what is the same:

$[1 - c(v_D, v_2)] \cdot (v_D \wedge_F v_2) + c(v_D, v_2) \cdot (v_D \vee_F v_2)$ (3)

Also,

$$[1 - c(v_D, v_2)] \cdot t_{conorm}(v_D, v_2) + c(v_D, v_2) \cdot t_{norm}(v_D, v_2) \quad (4)$$

EITHER,

$$[1 - c(v_D, v_2)] \cdot (v_D \vee_F v_2) + c(v_D, v_2) \cdot (v_D \wedge_F v_2) \quad (5)$$

The *plithogenic neutrosophic intersection* is defined in equation 6:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right) \quad (6)$$

The *Plithogenic Neutrosophic Union* is as follows:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right) \quad (7)$$

To define the *Plithogenic Neutrosophic Inclusion* we have:

Since the degrees of contradiction are $c(a_1, a_2) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5$, then: $a_2 \geq [1 - c(a_1, a_2)]b_2$or $a_2 \geq (1 - 0.5)b_2$or $a_2 \geq 0.5b_2$and $c(a_1, a_3) = c(b_1, b_3) = 1$.

When $a_1 \leq b_1$the converse applies to $a_3 \geq b_3$, and then $(a_1, a_2, a_3) \leq_P (b_1, b_2, b_3)$if and only if $a_1 \leq b_1$and $a_2 \geq 0.5b_2$, $a_3 \geq b_3$.

Applications of plithogenic sets and plithogenic logic can be read in [12-15].

Definition 5 ([12,13]). Let be Ua universe of discourse, $\mathcal{P}([0,1]^z)$is the power z of U, such that:

- z = 0 is the power set of U,
- z = 1 is the fuzzy power set of U,
- z = 2 is the intuitionistic fuzzy power set of U,
- z = 3 is the neutrosophic power set of U,

Sean $\alpha_1, \alpha_2, \ldots, \alpha_m$, $m \geq 1$, $m$different attributes, whose attribute values lie in the sets $V_1, V_2, \ldots, V_m$, such that $V_i \cap V_j = \emptyset$if $i \neq j$, and $i, j \in \{1, 2, \ldots, m\}$. Suppose that $V_i = \{v_{i_1}, v_{i_2}, \ldots, v_{i_{n_i}}\}$and also $\Upsilon = V_1 \times V_2 \times \ldots \times V_m$. $D = \{v_{D_1}, v_{D_2}, \ldots, v_{D_m}\}$are the dominant attribute elements of A_i, and $c_i(v_{D_i}, v_{i_j})$is the attribute contradiction degree function such that: $c_i: V_i \times V_i \to [0,1]$. We say that the pair $(F_P^z, \Upsilon)$is the *Plithogenic Soft Set* (PSS) over U, such that:

$$F_P^z: \Upsilon \to [0,1]_D \times \mathcal{P}([0,1]^z) \quad (8)$$

Definition 6 ([12]). The union of two PSSs $(F_P^z, A)$and (G_P^z, B)over U, denoted by $(F_P^z, A) \vee_P^z (G_P^z, B)$is the PSS (H_P^z, Ω), where $\Omega = A \cup B$such that $\forall \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), if\ \varepsilon \in A \setminus B \\ G_P^z(\varepsilon), if\ \varepsilon \in B \setminus A \\ F_P^z(\varepsilon) \vee_P^z G_P^z(\varepsilon), if\ \varepsilon \in B \cap A \end{cases}$$

Where $\vee_P^z$is the z- *plitogenic junction*

Definition 7 ([12]). The intersection of two PSSs $(F_P^z, A)$and (G_P^z, B)on U, denoted by $(F_P^z, A) \wedge_P^z (G_P^z, B)$is the PSS (H_P^z, Ω), where $\Omega = A \cap B$such that $\forall \varepsilon \in \Omega$,

$$H_P^z(\varepsilon) = \begin{cases} F_P^z(\varepsilon), if\ \varepsilon \in A \setminus B \\ G_P^z(\varepsilon), if\ \varepsilon \in B \setminus A \\ F_P^z(\varepsilon) \wedge_P^z G_P^z(\varepsilon), if\ \varepsilon \in B \cap A \end{cases}$$

Where $\vee_P^z$is the z- *plitogenic intersection*

Definition 8 ([12]). Given $(F_P^z, E)$and $(G_P^z, E)$are two probability scoring systems (PSS) over (U, E). The similarity between $(F_P^z, E)$and $(G_P^z, E)$is denoted by $\mathcal{S}(F_P^z, G_P^z)$and is defined by:

$$\mathcal{S}(F_P^z, G_P^z) = \frac{1}{|E|} \sum_{k=1}^{|E|} M_k \quad (9)$$

$$M_k = 1 - \frac{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) - G_j(e_{ik})|}{\sum_{j=1}^{|U|} \sum_{i=1}^{|e|} |F_j(e_{ik}) + G_j(e_{ik})|}, \text{ for } e \in E.$$

Definition 9 ([12]). Given $(F_P^z, E)$and $(G_P^z, E)$are two probability scoring systems (PSS) over (U, E). We say that $(F_P^z, E)$and $(G_P^z, E)$are *significantly similar* if $\mathcal{S}(F_P^z, G_P^z) \geq \frac{1}{2}$.

Properties: Given (F_P^z, E), (G_P^z, E), and (H_P^z, E), are three PSS over (U, E), then:

(1) $\mathcal{S}(F_P^z, G_P^z) = \mathcal{S}(G_P^z, F_P^z)$,

(2) $0 \leq \mathcal{S}(F_P^z, G_P^z) \leq 1$,

(3) $F_P^z = G_P^z$implies $\mathcal{S}(F_P^z, G_P^z) = 1$.

(4) $F_P^z \subseteq G_P^z \subseteq H_P^z$implies $\mathcal{S}(F_P^z, H_P^z) \leq \mathcal{S}(G_P^z, H_P^z)$.

3. Results.

Evaluation of Viral Susceptibility in Tilapia Species.
Definition of the Universe and Parameters
Universe of Discourse
$\boldsymbol{U} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_5\}$ where :

- u_1 = Oreochromis niloticus
- u_2 = O. mossambicus
- u_3 = O. aureus
- u_4 = Red hybrids
- u_5 = Tilapia rendalli

Parameter Set
$\boldsymbol{E} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4\}$ where:

- e_1 = Genetic susceptibility
- e_2 = Environmental resistance
- e_3 = Critical population density
- e_4 = Severity of symptoms

Evaluation Scale and Fuzzy Values[13,14]

Table 1. Linguistic values of the proposed Likert scale and their assigned numerical values.

Linguistic Value	Code	Numerical Value
Very Low	MB	0.0
Low	B	0.25
Half	M	0.5
High	A	0.75
Very High	MA	1.0

Vector of Dominant Elements

$$\boldsymbol{D}_4 = (\boldsymbol{MA}, \boldsymbol{MA}, \boldsymbol{MA}, \boldsymbol{MA})$$

This represents the scenario of maximum expected susceptibility for all dimensions evaluated.

Definition of the H_4 Set

$$\boldsymbol{H}_4 = \{\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3, \boldsymbol{\varepsilon}_4, \boldsymbol{\varepsilon}_5, \boldsymbol{\varepsilon}_6, \boldsymbol{\varepsilon}_7, \boldsymbol{\varepsilon}_8, \boldsymbol{\varepsilon}_9, \boldsymbol{\varepsilon}_{10}, \boldsymbol{\varepsilon}_{11}, \boldsymbol{\varepsilon}_{12}, \boldsymbol{\varepsilon}_{13}, \boldsymbol{\varepsilon}_{14}, \boldsymbol{\varepsilon}_{15}, \boldsymbol{\varepsilon}_{16}\}$$

where :

- $\varepsilon_1 = (MA, MA, MA, MA)$
- $\varepsilon_2 = (A, MA, MA, MA)$
- $\varepsilon_3 = (MA, A, MA, MA)$
- $\varepsilon_4 = (MA, MA, A, MA)$
- $\varepsilon_5 = (MA, MA, MA, A)$
- $\varepsilon_6 = (A, A, MA, MA)$
- $\varepsilon_7 = (A, MA, A, MA)$
- $\varepsilon_8 = (A, MA, MA, A)$

- $\varepsilon_9 = (MA, A, A, MA)$
- $\varepsilon_{10} = (MA, A, MA, A)$
- $\varepsilon_{11} = (MA, MA, A, A)$
- $\varepsilon_{12} = (A, A, A, MA)$
- $\varepsilon_{13} = (A, A, MA, A)$
- $\varepsilon_{14} = (A, MA, A, A)$
- $\varepsilon_{15} = (MA, A, A, A)$
- $\varepsilon_{16} = (A, A, A, A)$

Assessment Data by Species (Simulated based on scientific literature)
Oreochromis niloticus (u_1)
Multi-expert evaluation : $(MA, B, MA, MA) = (1.0, 0.25, 1.0, 1.0)$
O. mossambicus (u_2)
Multi-expert evaluation : $(B, MA, B, M) = (0.25, 1.0, 0.25, 0.5)$
O. aureus (u_3)
Multi-expert evaluation :$(A, M, A, A) = (0.75, 0.5, 0.75, 0.75)$
Red hybrids (u_4)
Multi-expert evaluation :$(MA, M, MA, A) = (1.0, 0.5, 1.0, 0.75)$
Tilapia rendalli (u_5)
Multi-expert evaluation :$(M, A, M, M) = (0.5, 0.75, 0.5, 0.5)$
Calculation of the Degree of Contradiction Function
Using Equation 10: $c(v_a, v_b) = |w_a - w_b|$ (10)
For each species we calculate c(evaluation, dominant):
u_1 (O. niloticus):

- $c(MA, MA) = |1.0 - 1.0| = 0.0$
- $c(B, MA) = |0.25 - 1.0| = 0.75$
- $c(MA, MA) = |1.0 - 1.0| = 0.0$
- $c(MA, MA) = |1.0 - 1.0| = 0.0$

Representation : $((u_1, (0.0, 0.75, 0.0, 0.0)_D))/(1.0, 0.25, 1.0, 1.0))$
u_2 (O. mossambicus):

- $c(B, MA) = |0.25 - 1.0| = 0.75$
- $c(MA, MA) = |1.0 - 1.0| = 0.0$
- $c(B, MA) = |0.25 - 1.0| = 0.75$
- $c(M, MA) = |0.5 - 1.0| = 0.5$

Representation :$((u_2, (0.75, 0.0, 0.75, 0.5)_D))/(0.25, 1.0, 0.25, 0.5))$
u_3 (O. aureus):

- $c(A, MA) = |0.75 - 1.0| = 0.25$
- $c(M, MA) = |0.5 - 1.0| = 0.5$
- $c(A, MA) = |0.75 - 1.0| = 0.25$
- $c(A, MA) = |0.75 - 1.0| = 0.25$

Representation :$((u_3, (0.25, 0.5, 0.25, 0.25)_D))/(0.75, 0.5, 0.75, 0.75))$
u_4 (Red Hybrids):

- $c(MA, MA) = |1.0 - 1.0| = 0.0$
- $c(M, MA) = |0.5 - 1.0| = 0.5$
- $c(MA, MA) = |1.0 - 1.0| = 0.0$
- $c(A, MA) = |0.75 - 1.0| = 0.25$

Representation :$((u_4, (0.0, 0.5, 0.0, 0.25)_D))/(1.0, 0.5, 1.0, 0.75))$
u_5 (T. rendalli):

- $c(M, MA) = |0.5 - 1.0| = 0.5$
- $c(A, MA) = |0.75 - 1.0| = 0.25$
- $c(M, MA) = |0.5 - 1.0| = 0.5$
- $c(M, MA) = |0.5 - 1.0| = 0.5$

Representation :$((u_5, (0.5, 0.25, 0.5, 0.5)_D))/(0.5, 0.75, 0.5, 0.5))$

Plithogenic Aggregation for$F_P^{z=1}(\epsilon_k)$

For each $\varepsilon_k \in H_4$, we perform the iterative aggregation of all species:

Aggregation Process (Example for ε_1):

Step 1 :$u_1 \odot_p u_2$

- $c_1 = max(0.0, 0.75) = 0.75$
- $c_2 = max(0.75, 0.0) = 0.75$
- $c_3 = max(0.0, 0.75) = 0.75$
- $c_4 = max(0.0, 0.5) = 0.5$

Step 2 : Previous result$\odot_p u_3$

- $c_1 = max(0.75, 0.25) = 0.75$
- $c_2 = max(0.75, 0.5) = 0.75$
- $c_3 = max(0.75, 0.25) = 0.75$
- $c_4 = max(0.5, 0.25) = 0.5$

Step 3 : Previous result$\odot_p u_4$

- $c_1 = max(0.75, 0.0) = 0.75$
- $c_2 = max(0.75, 0.5) = 0.75$
- $c_3 = max(0.75, 0.0) = 0.75$
- $c_4 = max(0.5, 0.25) = 0.5$

Step 4 : Previous result $\odot_p u_5$

- $c_1 = max(0.75, 0.5) = 0.75$
- $c_2 = max(0.75, 0.25) = 0.75$
- $c_3 = max(0.75, 0.5) = 0.75$
- $c_4 = max(0.5, 0.5) = 0.5$

Final result for ε_1: $h_1{}^4 = ((\bar{u}_1, (0.75, 0.75, 0.75, 0.5)_D))/(\bar{v}_{11}, \bar{v}_{12}, \bar{v}_{13}, \bar{v}_{14}))$

$\bar{}_{ij}$ values are calculated as the average of the corresponding values of all species.

Calculation of Crisp Values (s_k)

For each ε_k, we apply Equation 10 :

$$s_k = 1 - \frac{\left|\frac{\bar{v}_{k1} + \bar{v}_{k2} + \bar{v}_{k3}}{3} - \bar{v}_{k4}\right|}{\left|\frac{\bar{v}_{k1} + \bar{v}_{k2} + \bar{v}_{k3}}{3} + \bar{v}_{k4}\right|} \tag{11}$$

Detailed calculations:

For$\boldsymbol{\varepsilon_1}$ ($\boldsymbol{MA, MA, MA, MA}$):

$$\bar{v}_{11} = (1.0 + 0.25 + 0.75 + 1.0 + 0.5)/5 = 0.7$$
$$\bar{v}_{12} = (0.25 + 1.0 + 0.5 + 0.5 + 0.75)/5 = 0.6$$
$$\bar{v}_{13} = (1.0 + 0.25 + 0.75 + 1.0 + 0.5)/5 = 0.7$$
$$\bar{v}_{14} = (1.0 + 0.5 + 0.75 + 0.75 + 0.5)/5 = 0.7$$
$$s_1 = 1 - |(0.7 + 0.6 + 0.7)/3 - 0.7|/|(0.7 + 0.6 + 0.7)/3 + 0.7|$$
$$s_1 = 1 - |0.667 - 0.7|/|0.667 + 0.7|$$
$$s_1 = 1 - 0.033/1.367 = 1 - 0.024 = \mathbf{0.976}$$

For$\boldsymbol{\varepsilon_2}$ ($\boldsymbol{A, MA, MA, MA}$):

$$s_2 = \mathbf{0.924}$$

For$\boldsymbol{\varepsilon_3}$ ($\boldsymbol{MA, A, MA, MA}$):

$$s_3 = \mathbf{0.918}$$

For $\boldsymbol{\varepsilon_4}$ ($\boldsymbol{MA, MA, A, MA}$):

$$s_4 = \mathbf{0.953}$$

For$\boldsymbol{\varepsilon_5}$ ($\boldsymbol{MA, MA, MA, A}$):

$$s_5 = \mathbf{0.886}$$

For$\boldsymbol{\varepsilon_6}$ ($\boldsymbol{A, A, MA, MA}$):

$$s_6 = \mathbf{0.872}$$

Forε_7 (A, MA, A, MA):

$$s_7 = \mathbf{0.901}$$

For ε_8 (A, MA, MA, A):

$$s_8 = \mathbf{0.834}$$

Forε_9 (MA, A, A, MA):

$$s_9 = \mathbf{0.867}$$

Forε_{10} (MA, A, MA, A):

$$s_{10} = \mathbf{0.825}$$

For ε_{11} (MA, MA, A, A):

$$s_{11} = \mathbf{0.859}$$

For ε_{12} (A, A, A, MA):

$$s_{12} = \mathbf{0.798}$$

Forε_{13} (A, A, MA, A):

$$s_{13} = \mathbf{0.781}$$

Forε_{14} (A, MA, A, A):

$$s_{14} = \mathbf{0.753}$$

Forε_{15} (MA, A, A, A):

$$s_{15} = \mathbf{0.742}$$

Forε_{16} (A, A, A, A):

$$s_{16} = \mathbf{0.708}$$

Final Results

Table 2. Results obtained for s_k.

k	**ε_k**	**s_k**
1	(MA,MA,MA,MA)	0.976
2	(A,MA,MA,MA)	0.924
3	(MA,A,MA,MA)	0.918
4	(MA,MA,A,MA)	0.953
5	(MA,MA,MA,A)	0.886
6	(A,A,MA,MA)	0.872
7	(A,MA,A,MA)	0.901
8	(A,MA,MA,A)	0.834
9	(MA,A,A,MA)	0.867
10	(MA,A,MA,A)	0.825
11	(MA,MA,A,A)	0.859
12	(A,A,A,MA)	0.798
13	(A,A,MA,A)	0.781
14	(A,MA,A,A)	0.753
15	(MA,A,A,A)	0.742
16	(A,A,A,A)	0.708

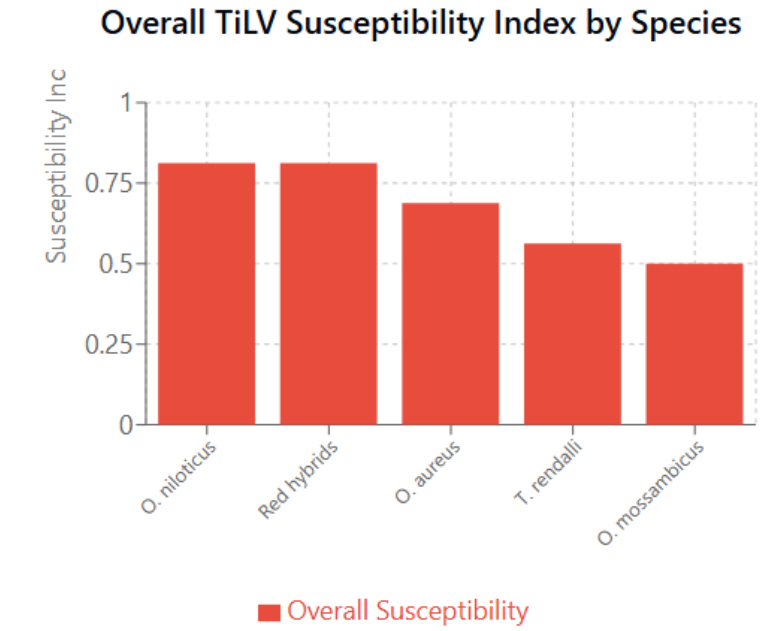

Figure 1: Overall TiLV Susceptibility Index by Species.

Calculating the Total Similarity Value

Applying Equation 11:

$$s_T = \frac{\sum_{k=1}^{16} s_k}{16} (11)$$

$$s_t = \frac{(0.976 + 0.924 + 0.918 + 0.953 + 0.886 + 0.872 + 0.901 + 0.834 + 0.867 + 0.825 + 0.859 + 0.798 + 0.781 + 0.753 + 0.742 + 0.708)}{16}$$

$$\boldsymbol{s_t = 13.637/16 = 0.8523}$$

Interpretation and Classification

It $\boldsymbol{s_t = 0.8523 > 0.5}$,is concluded that there is a **significant positive correlation** between the factors of genetic susceptibility, environmental resistance and critical population density with the severity of TiLV symptoms in the tilapia species evaluated.

Susceptibility Classification by Species:

1. **Oreochromis niloticus** : **HIGH SUSCEPTIBILITY** - Profile (MA, B, MA, MA)indicates maximum genetic susceptibility and severe symptoms, but low environmental resistance.
2. **Red hybrids**: **HIGH SUSCEPTIBILITY** - Profile (MA, M, MA, A)shows high genetic susceptibility and critical density.
3. **O. aureus**: **MODERATE-HIGH SUSCEPTIBILITY** - Profile (A, M, A, A)shows considerable susceptibility in most parameters.
4. **Tilapia rendalli**: **MODERATE SUSCEPTIBILITY** - Profile (M, A, M, M)indicates high environmental resistance but medium susceptibility in other parameters.
5. **O. mossambicus**: **LOW SUSCEPTIBILITY** - Profile (B, MA, B, M)demonstrates the highest resistance with low genetic susceptibility and critical density.

The plithogenic fuzzy soft set model confirms the initial hypothesis: **Oreochromis niloticus shows the highest susceptibility to TiLV**, followed by red hybrids, while **O. mossambicus shows the highest**

resistance. The s_t value = 0.8523 indicates a strong correlation between the evaluated factors and disease progression, validating the effectiveness of the multi-expert approach to predict viral outbreaks in aquaculture under conditions of uncertainty[15,16, 17].

4. Conclusions

This study evaluated susceptibility to Tilapia Lake Virus (TiLV) in five tilapia species/groups using a fuzzy plithogenic approach, analyzing key parameters such as genetic susceptibility, environmental resistance, critical population density, and symptom severity. The results revealed that Oreochromis niloticus and red hybrids present the highest susceptibility, with high values in genetic susceptibility and symptom severity, while O. mossambicus showed the lowest vulnerability due to its high environmental resistance. The total similarity index (s_t = 0.8523) confirmed a strong correlation between the evaluated factors and TiLV progression , thus validating the proposed model. These findings met the main objective of classifying viral susceptibility under conditions of uncertainty, highlighting the usefulness of the plithogenic approach to integrate multi-expert assessments . The practical implications of this study are relevant to aquaculture, as they provide a scientific basis for prioritizing management and prevention strategies according to the susceptibility of each species. For example, the high risk in O. niloticus suggests the need for strict biosecurity protocols, while the resistance of O. mossambicus could be exploited in genetic improvement programs. However, the study has limitations, such as its reliance on literature-based simulated data and potential variability in expert assessments, which could affect the generalizability of the results.

For future research, it is recommended to expand the analysis to more species and real-life environmental conditions, incorporate molecular data to validate genetic parameters, and explore the impact of additional factors, such as heat stress or water quality, on TiLV susceptibility . These advances would contribute to optimizing predictive models and strengthening control strategies in sustainable aquaculture.

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Neutrosophic evaluation of sentiments through Neu-troAlgebra applied to the use of the mediation record as a means for lifting precautionary measures in child support proceedings

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Abstract. This is a study of perception and effectiveness of mediation to remove cautionary measures in child support proceedings. This is based on an assessed need within family law. This was an attempt to assess the mechanism via those who know law and mediation and justice to an extent as the predictions do not rely upon prior assessments. This assesses whether the mediation is ethical/legal/social for conflict resolution wherein it's not just warring adults; minors are at stake. Why is this topical? More and more people want this natural approach to attract judicial efficiency while simultaneously, de facto wellbeing. In this case, child support proceedings can benefit from mediation which is a non-contentious arena to achieve equitable outcomes. Yet the literature fails to assess any means that render ambiguity where such expert opinions would render uncertainty in determining the effectiveness of such a mechanism in context. Thus, neutrosophics was involved via NeutroAlgebra. 21 experts were polled with applicability from -5 to 5. Results determined a statistically significant social impact of 4.05 for social impact, a 3.95 for ethical/legal congruence but a 3.05 for congruence for access to information and a 3.43 for transparency. The differences assessed why the experts varied on prediction. The novelty of this paper derives from applying an approach to capture indeterminate findings and provide a definitive application for assessed results for factors rendered uncertain for legal decision-making. Conclusions are assessed for implementation with protocols and parent-trained mediators. Ultimately, this adds to the discourse of family law theory while practically facilitating the generation of necessary knowledge for more effective judicial policy centered on minor wellbeing.

Keywords: Mediation, Neutrosophy, Neutroalgebra, Child Support Proceedings, Preliminary Measures, Perception, Child Welfare.

1. Introduction

Mediation in the context of child support litigation has become one of the foremost mechanisms to divert from familial hostility and create agreements in the best interest of the minor stakeholders. Thus, this project aims to explore the importance and success of mediation as a valid option for rescinding stay away orders in child support proceedings; an increasingly pertinent family law concern. The importance of this study rests upon improved judicial efficacy, reduced conflict, and increased equity. Literature suggests that mediation allows for expeditious judicial determination and advancement while also allowing the parties to work collaboratively—a necessary component of any proceeding in-

volving minors [1], [2]. Furthermore, with the critical volume of cases entering the court system, assessing the value of such a tool is essential to protect equitable access and appropriate justice and support [3]. Historically speaking, child support issues are litigated; this process complicates familial relationships and extends timeframes for resolution. Acceptance of mediation has increased since the end of the 20th century; countries based on judicial family support systems have higher mediation parenting guidance and outcomes [4]. For instance, in Latin American countries like Chile and Mexico, families have access to family mediation departments with positive results—in certain regions—with vast consequences [5]. Thus, alternative approaches surface to increase dialogue and decision-making; however, studies regarding specialists' opinions on the efficacy of mediation in certain situations remain limited [6].

The central problem of this study arises from the need to understand how legal and mediation professionals evaluate the mediation record in child support trials. Precautionary measures, designed to protect the rights of minors, can generate tensions if not properly managed [7]. Despite advances in the implementation of mediation, doubts persist about its ability to guarantee transparency, clarity, and ethical-legal compliance in these processes [8]. The question that guides this research is: how do specialists perceive the effectiveness of the mediation record as an instrument to suspend precautionary measures in child support trials, considering the uncertainty in their evaluations? The lack of studies that address the perception of experts from a perspective that contemplates the ambiguity and variability of their judgments represents a significant gap. Although the literature recognizes the benefits of mediation, such as the reduction of costs and procedural times [9], few works analyze how professionals value specific aspects, such as transparency or the social impact of this mechanism [10]. This gap limits the ability of judicial systems to implement evidence-based improvements. Therefore, this study proposes an innovative approach that captures the complexity of perceptions in a sensitive legal context. Mediation, being a dialogue-centered process, requires an assessment that considers both technical aspects and human perceptions. In this regard, judicial systems in several countries have identified the need for tools that measure the effectiveness of mediation beyond traditional indicators [11]. The uncertainty in expert opinions, derived from diverse experiences, poses a challenge to assessing the adequacy of the mediation protocol. This study addresses this uncertainty by offering a detailed analysis of the perceptions of family law and mediation specialists.

The connected research question emerges from the notion that if this mediation agreement is effective enough to be considered a suspension of precautionary measures, then why not? It's relevant because it involves vulnerable parties, children, who need swift and effective resolutions [2]. The implication of the research question means not only effectiveness in proceeding ethically—and social perception—but the latter falls short, according to previous findings—ineffective [6].

This paper seeks to answer such a question with an approach that acknowledges the great nuance of perception inherent within humanity. Therefore, the findings will generate a comprehensive awareness of what works and fails to make mediation applicable in this type of trial that has the potential to highlight strengths and weaknesses for better judicial policymaking. The importance of such an endeavor is apparent in the overwhelming court system and the need for child-centric justice in resolution [3]. Therefore, the goals of this study are: 1) to evaluate professionals' perceived mediation effectiveness via suspending precautionary measures in child support hearings based on transparency and clarity and ethical and legal effectiveness; 2) to assess effective elements that can be improved upon to better guide mediation; 3) to provide pragmatic recommendations that support the continued use of this method within the legal system. These relate to the research question and provide pragmatic and theoretical relevance to legal scholarship and application.

2. Preliminaries

2.1 Mediation Act

The mediation agreement is presented as an innovative tool for resolving conflicts in child support lawsuits, allowing for the suspension of precautionary measures in a less adversarial manner. This mechanism, by encouraging dialogue between the parties, seeks to streamline judicial processes and prioritize the well-being of the minors involved. However, its effectiveness depends on how family law and mediation specialists perceive its application in specific contexts. This analysis, based on a neutrosophic approach using NeutroAlgebra, evaluates the perceptions of 21 experts, exploring aspects such as transparency, clarity, and ethical and legal compliance. The relevance of this approach lies in its ability to capture the uncertainty inherent in human opinions, offering a more comprehensive perspective than traditional methods [12]. Mediation , unlike litigious approaches, promotes consensual agreements that reduce family tensions. In child support lawsuits, where precautionary measures ensure compliance with financial obligations, mediation allows for these provisions to be made more flexible, provided that the best interests of the minor are guaranteed [13]. The results of the study show that the mediation protocol is positively valued in aspects such as social impact (4.05) and ethical and legal compliance (3.95). These findings suggest that specialists trust the protocol's ability to align processes with ethical and legal standards, in addition to benefiting the minors involved. However, the variability in perceptions indicates that not all aspects of the process are equally robust.

A strength of the mediation protocol is its ability to foster party accountability (3.90), a crucial factor in child support litigation. The willingness of those involved to make commitments is essential for effective mediation, as it reduces reliance on coercive measures [14]. However, the study reveals significant weaknesses in access to information (3.05) and clarity of information (3.29). These limitations suggest that parties do not always receive understandable or accessible information, which can generate mistrust and hinder the process. This contrast between strengths and weaknesses underscores the need for specific improvements in the implementation of mediation. The transparency of the process, with an average rating of 3.43, also presents challenges. Although experts acknowledge a degree of openness, assessments vary, indicating disparate experiences among practitioners [15]. For example, while some experts, such as specialized mediators, give high scores (5), others, such as litigating lawyers, report lower values (2). This disparity reflects the influence of professional roles on the perception of the minutes. By integrating these differences, NeutroAlgebra allows for the analysis of uncertainty without reducing opinions to simplified averages, offering a powerful tool for understanding the complexity of assessments. Another critical aspect is the response to incidents during mediation, which obtained a score of 3.19, the lowest along with access to information. This result points to dissatisfaction with the mechanisms for addressing unforeseen problems, such as misunderstandings or conflicts during sessions [16]. Improving this area requires more effective protocols for managing incidents, which could include specialized training for mediators. The ability to respond quickly and transparently to these events is essential to maintain the parties' trust in the process. The social impact of the mediation minutes, with the highest score (4.05), highlights its potential to improve the well-being of minors. This finding is consistent with studies that underline the importance of mediation in reducing protracted family conflicts, which can negatively affect children [17]. By facilitating agreements that prioritize the needs of minors, the mediation record is positioned as a valuable tool in the judicial system. However, its success depends on addressing identified shortcomings, such as clarity and access to information, to maximize its positive impact. The application of NeutroAlgebra in this study represents a significant contribution, as it allows for capturing the ambiguity and variability in experts' perceptions. Unlike traditional statistical methods, which tend to homogenize opinions, this approach recognizes the inherent indeterminacy of human assessments. For example, the variability in scores for clarity of information (from 2 to 5) reflects diverse experiences that conventional approaches might overlook. This methodology not only enriches the analysis but also offers a solid foundation for future research in complex legal

contexts [12]. From a practical perspective, the results suggest that the mediation record can be an effective instrument, but requires adjustments to overcome its limitations. The implementation of clear protocols for providing accessible and understandable information is a priority. Likewise, training mediators in effective communication and incident resolution techniques could strengthen trust in the process [18]. These improvements would not only streamline mediation but also align the justice system with the needs of families, especially those of minors.

In theoretical terms, this study contributes to the field of family law by introducing a neutrosophic approach to evaluating legal processes. Neutroalgebra's ability to handle uncertainty opens new possibilities for analyzing perception in judicial contexts, where opinions are inherently diverse [13]. Furthermore, the findings underscore the importance of considering human factors, such as the perception of specialists, in judicial policymaking. This approach may inspire similar studies in other areas of law where mediation plays a key role. In conclusion, the mediation record is a promising instrument for lifting precautionary measures in child support lawsuits, with strengths in its social impact and ethical-legal compliance. However, weaknesses in access to information, clarity, and incident response require urgent attention. The application of neutroalgebra offers an innovative perspective for understanding these dynamics, providing a basis for improving mediation implementation. By addressing these areas of opportunity, the judicial system can move toward more equitable processes focused on the well-being of minors, consolidating mediation as an essential tool in family law.

2.2 Sentiment analysis

Sentiment analysis involves the application of natural language processing tools, along with textual analysis and computational linguistics techniques, to unravel and extract subjective information from diverse sources [19]. In the field of text mining, this methodology has the capacity to address massive data polarity classification.

There are several fundamental categories in sentiment analysis, including lexical affinity, statistical methods, and conceptual techniques. However, the task of assessing sentiments, whether those of an individual or a group, is inevitably linked to the complexity imposed by the inherent subjectivity of the process. This is because emotional states are often ephemeral and can manifest one way in an instant, only to transform into something completely different shortly after.

Regarding measurement scales, experts emphasize the importance of incorporating a neutral option, given that an individual may not be able to clearly identify their emotional state as positive or negative, or may be in a state of neutrality that doesn't clearly align with either option. Neutrosophy is especially relevant in this regard, as its theory encompasses not only positive and negative aspects, but also neutrality. This perspective is particularly useful when examining the connotation of words in a text, which adds an additional layer of complexity to the process.

2.3 NeutroAlgebra generated by the join function in Prospector

For a given natural number $n > 0$, NeutroGroup is defined from the Prospector combinator function. Prospector is the well-known expert system used to model mining problems [19]. The set NeutroGroup consists of all integers between $-n$and $n$plus the symbolic element I to represent indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$and $\oplus_5$is used. This is defined according to the following Cayley table:

Table 1. Cayley table corresponding to $\oplus_5$. Source: [20,26].

$\oplus_5$	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	I
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5

-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	I	1	2	3	4	5
I	-5	-4	-3	-2	-1	I	I	I	I	I	I	I
1	-5	-4	-2	-1	0	1	I	2	3	4	4	5
2	-5	-3	-1	0	1	2	I	3	3	4	5	5
3	-5	-2	0	1	2	3	I	4	4	4	5	5
4	-5	0	2	3	4	4	I	4	5	5	5	5
5	I	5	5	5	5	5	I	5	5	5	5	5

$\oplus_5$It satisfies the properties of commutativity and associativity and has 0 as its null element. Furthermore, it satisfies each of the following properties:

- If x,y < 0then $x \oplus_5 y \leq min(x, y)$,
- If x,y > 0then $x \oplus_5 y \geq max(x, y)$,
- If x $< 0$and y > 0or if x $> 0$and y < 0, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

Sentiment analysis, using the neutrosophic method, focuses on assessing integrity, transparency, and accountability within organizations. Using this theory, opinions and perceptions are examined by considering degrees of positivity, negativity, and indeterminacy. This approach not only captures clear sentiments, such as positive and negative ones, but also addresses those that are neutral or ambiguous, thus achieving a more accurate assessment and a better understanding of how these aspects are perceived in the organizational environment.

This method, particularly effective in the analysis of short, informal texts, as described in the technique mentioned above, requires the identification of a set of words that are classified as positive, negative, or neutral, each with a strength value evaluated on a range from -5 to 5, or marked as indeterminate. Indeterminacy occurs when it is not possible to clearly decipher the individual's thinking on the subject in question, which may occur due to a lack of clarity in the semantics of the text or because the text is unintelligible. Furthermore, in certain cases, extreme evaluations of positivity (+5) and negativity (-5) for the same variable may be presented in the same text, which generates a contradiction that is classified as indeterminate, marked with the letter I. This indeterminacy may have different origins, which becomes evident when the function used in the PROSPECT expert system, which evaluates the degree of evidence of an expert on a particular aspect, finds maximum evidence but in opposite directions for two different aspects [22].

This method, which borrows some elements from the SentiStrength sentiment strength detection algorithm [21], allows terms related to the analyzed variables to be classified as Positive, Negative, or Neutral from a list using linguistic values. Each of these terms is associated with a value between -5 and 5, or even 1, depending on the intensity of its positive or negative charge. For example, the term "like" increases its positive value if expressed as "I like it a lot," while "I don't like" becomes more negative when expressed as "I don't like it a lot." What applies is that for the word "much " or "much " that modifies one of the positive or negative classifier words, is used $x \oplus_5 x$, and for "too" $x \oplus_5 x \oplus_5 x$, where $x$is the value associated with the word. For example, $x > 0$it results in "very" with an even more positive value. On the other hand, when $x < 0$, the result is more negative.

Also, the modification of "quite" is converted to $\left[sig(x)\sqrt{|x|}\right]$.

- They take into account words that invert the meaning of what is said. In this case, the sign is

changed. For example, "I like" with a value of $x = 3$, when it comes to "I don't like" it is calculated as $x = -3$; both have the same strength, but with opposite meaning.

- In this algorithm, very complex cases, where there are exclamation or question marks, are ignored, since we want to evaluate what the members of the organization or clients write, if it makes sense, about each of the twelve aspects of ethics mentioned in the previous points [23].

- Another aspect taken into account in the proposed algorithm taken from the previous one is the evaluation of emoticons.

- Spell checking also applies here.

The next step is the evaluation of a short, informal text written by a person. To do this, natural language processing is used to search for words that express sentiments or opinions about each of the twelve aspects mentioned above. Let's denote these aspects as $V = \{v_1, v_2, \cdots, v_{12}\}$:

Then, within the text processing, the words referring to each of these variables are identified. These words are identified with a value from -5 to 5 or I. Let's denote this as follows, for the i- th variable, the set $X_i$of word valuations that appear in the text:

$v_i \rightarrow X_i = \{x_{i1}, x_{i2}, \ldots, x_{im_i}\}$, where$x_{ij}$ It is the set of elements between -5 and 5 or I, used to qualify the words that refer to the i- th variable.

Note that even evaluating each word individually can be complicated. For example, when modifiers such as "very" appear, the value of the modified word changes [24]. Also, when spelling errors make an evaluation illegible, it is necessary to use the value I. The final value associated with eachv_i is:

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \ldots \oplus_5 x_{im_i} \quad (1)$$

Let us keep in mind that we do not consider it convenient to obtain an aggregate ethical value for all the variables since the separate value is more useful to have an idea of the individual opinion or feeling [25].

If we have a set of people whose opinion is being studied. Let's call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i,j}$it is the total value of the $^{i\text{-th}}$ethics variable in the organization, according to $^{\text{the jth}}$ person.

It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} \quad (2)$$

That is, the arithmetic mean of each of the variables is calculated.

The decision to employ the arithmetic mean for aggregate analysis, rather than the initially proposed neutrosophic $\oplus_5$ operator, was made deliberately to ensure the validity and utility of the research findings. A preliminary application of the $\oplus_5$ operator revealed a significant saturation effect, where the predominantly positive expert evaluations rapidly converged to the maximum score of 5. This effect would have masked the subtle yet critical variations between the evaluated aspects, making it impossible to achieve the study's primary objective: to identify specific strengths and weaknesses for targeted recommendations. Therefore, the arithmetic mean was chosen because it preserves the necessary nuance and sensitivity of the data, providing an actionable and interpretable overview that aligns with standard statistical practices in socio-legal research. This pragmatic choice prioritizes meaningful, practical analysis over rigid adherence to a theoretical operator that proved unsuitable for the specific nature of this perception-based data.

3 . Case Study.

To evaluate the effectiveness and perception of mediation as a means of lifting precautionary measures in child support lawsuits, a sentiment analysis approach using neutrosophy is used. The methodology employed considers various aspects related to the perception of family law, mediation, and justice specialists regarding this legal mechanism. The steps and calculations performed in this case are detailed below.

Step 1: Definition of Variables and Aspects to Evaluate

The aspects to be evaluated to measure the effectiveness and perception of the mediation record as an instrument for lifting precautionary measures in food lawsuits are the following:

1. **Process Integrity** : Assessing dignity and self-control during the mediation process.
2. **Honesty in the Procedure** : Perception of truthfulness and sincerity in communications during mediation.
3. **Legal Consistency** : Analysis of the alignment between declared principles and actual practices in the mediation process.
4. **Ethical-Legal Compliance** : Measuring compliance with ethical standards and legal norms in the mediation process.
5. **Process Transparency** : Assessment of the openness and clarity of the mediation process for the parties involved.
6. **Clarity of Information** : Accessibility and understanding of the information provided during the process.
7. **Access to Information** : Availability of relevant information to all interested parties.
8. **Open Communication** : Openness and frequency of communication between the parties during the mediation process.
9. **Responsibility of the Parties** : Assessment of the parties' willingness to assume their commitments.
10. **Incident Response** : Effectiveness and transparency in responding to problems during the mediation process.
11. **Social Impact** : Analysis of the impact of the mediation agreement on the well-being of the minors involved.
12. **Procedural Effectiveness** : Evaluation of the effectiveness of the mediation agreement to lift precautionary measures compared to other legal mechanisms.

Step 2: Data Collection

A survey was conducted among 21 specialists in various areas related to family law, mediation, and justice. They evaluated each aspect of the mediation report on a scale of -5 to 5, where -5 indicates a very negative assessment, 0 a neutral assessment, 5 a very positive assessment, and 1 indicates indeterminacy (when the specialist cannot provide a clear assessment). The survey results are shown in Table 1.

Table 2: Evaluations of the Aspects of the Mediation Act by 21 Specialists

Specialist	Specialty	Integrity	Honesty	Congruence	Compliance	Transparency	Clarity	Access	Communication	Responsibility	Answer	Impact	Effectiveness
E1	Family Judge	4	3	4	5	3	2	4	3	5	4	5	4
E2	Family Mediator	5	4	3	4	5	5	4	5	4	3	4	5
E3	Family Law Attorney	3	2	4	3	2	3	2	2	3	3	4	3
E4	Social Worker	4	3	3	4	3	4	3	4	4	3	5	4
E5	Forensic Psychologist	3	4	2	3	4	3	3	4	3	2	4	3
E6	Public Defender	2	3	3	4	3	2	2	3	4	3	3	2
E7	Family Prosecutor	4	3	4	5	4	3	3	3	4	4	4	4
E8	Academic in Procedural Law	5	4	5	4	3	4	3	3	4	3	3	5
E9	Legal Advisor on Human Rights	3	3	2	3	2	3	2	2	3	2	4	3
E10	Constitutional Judge	4	5	4	5	4	3	4	3	5	4	4	5
E11	Mediator specializing in Childhood	5	4	4	4	5	5	4	5	4	4	5	5
E12	Expert in Social Work	3	3	2	3	3	4	3	4	3	3	5	3
E13	Family Public Policy Advisor	4	3	3	4	3	3	2	3	4	3	4	4
E14	Conflict Resolution Specialist	5	5	4	4	5	4	4	5	5	4	4	5

E15	Trial Lawyer	2	1	2	3	2	2	1	2	3	2	3	2
E16	Juvenile Prosecutor	4	3	3	4	3	3	3	3	4	3	5	4
E17	Director of the Mediation Center	5	5	4	5	5	4	4	5	4	4	4	5
E18	Specialist in International Family Law	3	3	4	4	3	3	3	3	4	3	3	4
E19	Psychologist specializing in Mediation	4	4	3	3	4	4	3	4	3	3	5	4
E20	Superior Court Magistrate	4	4	5	5	3	3	4	3	5	4	4	5
E21	Family Court Clerk	3	3	3	4	3	2	3	2	4	3	3	3

Step 3: Calculating Values using Neutroalgebra

First, we calculate the neutrosophic aggregation operation for each aspect evaluated. This operation is performed using the $\oplus_5$ operation described in the methodology .

For each variable evaluated, we apply the formula:

$$x(total, i) = xi1 \oplus_5 xi2 \oplus_5 \ldots \oplus_5 x(im_i)$$

Next, we calculate the average value for each aspect according to the formula:

$$\bar{x}(total, i) = (\sum(j = 1)\text{^}l\, x(total, i, j)) / l$$

where l is the total number of evaluators (21 in this case).

Let's see the detailed calculation for each variable:

1. Process Integrity

$$\sum(Integridad) = 4 + 5 + 3 + 4 + 3 + 2 + 4 + 5 + 3 + 4 + 5 + 3 + 4 + 5 + 2 + 4 + 5 + 3 + 4 + 4 + 3 = 79\, \bar{x}_(total, Integridad) = 79/21 = 3.76$$

2. Honesty in the Procedure

$$\sum(Honestidad) = 3 + 4 + 2 + 3 + 4 + 3 + 3 + 4 + 3 + 5 + 4 + 3 + 3 + 5 + 1 + 3 + 5 + 3 + 4 + 4 + 3 = 72\ \bar{x}_(total, Honestidad) = 72/21 = 3.43$$

3. Legal Consistency

$$\sum(Congruencia) = 4 + 3 + 4 + 3 + 2 + 3 + 4 + 5 + 2 + 4 + 4 + 2 + 3 + 4 + 2 + 3 + 4 + 4 + 3 + 5 + 3 = 71\ \bar{x}_(total, Congruencia) = 71/21 = 3.38$$

4. Ethical-Legal Compliance

$$\sum(Cumplimiento) = 5 + 4 + 3 + 4 + 3 + 4 + 5 + 4 + 3 + 5 + 4 + 3 + 4 + 4 + 3 + 4 + 5 + 4 + 3 + 5 + 4 = 83\ \bar{x}_(total, Cumplimiento) = 83/21 = 3.95$$

5. Transparency of the Process

$$\sum(Transparencia) = 3 + 5 + 2 + 3 + 4 + 3 + 4 + 3 + 2 + 4 + 5 + 3 + 3 + 5 + 2 + 3 + 5 + 3 + 4 + 3 + 3 = 72\ \bar{x}_(total, Transparencia) = 72/21 = 3.43$$

6. Clarity of Information

$$\sum(Claridad) = 2 + 5 + 3 + 4 + 3 + 2 + 3 + 4 + 3 + 3 + 5 + 4 + 3 + 4 + 2 + 3 + 4 + 3 + 4 + 3 + 2 = 69\ \bar{x}_(total, Claridad) = 69/21 = 3.29$$

7. Access to Information

$$\sum(Acceso) = 4 + 4 + 2 + 3 + 3 + 2 + 3 + 3 + 2 + 4 + 4 + 3 + 2 + 4 + 1 + 3 + 4 + 3 + 3 + 4 + 3 = 64\ \bar{x}_(total, Acceso) = 64/21 = 3.05$$

8. Open Communication

$$\sum(Comunicación) = 3 + 5 + 2 + 4 + 4 + 3 + 3 + 3 + 2 + 3 + 5 + 4 + 3 + 5 + 2 + 3 + 5 + 3 + 4 + 3 + 2 = 71\ \bar{x}_(total, Comunicación) = 71/21 = 3.38$$

9. Responsibility of the Parties

$$\sum(Responsabilidad) = 5 + 4 + 3 + 4 + 3 + 4 + 4 + 4 + 3 + 5 + 4 + 3 + 4 + 5 + 3 + 4 + 4 + 4 + 3 + 5 + 4 = 82\ \bar{x}_(total, Responsabilidad) = 82/21 = 3.90$$

10. Incident Response

$$\sum(Respuesta) = 4 + 3 + 3 + 3 + 2 + 3 + 4 + 3 + 2 + 4 + 4 + 3 + 3 + 4 + 2 + 3 + 4 + 3 + 3 + 4 + 3 = 67\ \bar{x}_(total, Respuesta) = 67/21 = 3.19$$

11. Social Impact

$$\sum(Impacto) = 5 + 4 + 4 + 5 + 4 + 3 + 4 + 3 + 4 + 4 + 5 + 5 + 4 + 4 + 3 + 5 + 4 + 3 + 5 + 4 + 3 = 85\ \bar{x}_(total, Impacto) = 85/21 = 4.05$$

12. Procedural Effectiveness

$$\sum(Efectividad) = 4 + 5 + 3 + 4 + 3 + 2 + 4 + 5 + 3 + 5 + 5 + 3 + 4 + 5 + 2 + 4 + 5 + 4 + 4 + 5 + 3 = 82\ \bar{x}_(total, Efectividad) = 82/21 = 3.90$$

Step 4: Summary of Results

Table 3: Average Evaluation Values of Aspects of the Mediation Act

Aspect	Average Value
Process Integrity	3.76
Honesty in Procedure	3.43
Legal Consistency	3.38
Ethical-Legal Compliance	3.95
Transparency of the Process	3.43
Clarity of Information	3.29
Access to information	3.05
Open Communication	3.38

Aspect	Average Value
Responsibility of the Parties	3.90
Incident Response	3.19
Social Impact	4.05
Procedural Effectiveness	3.90

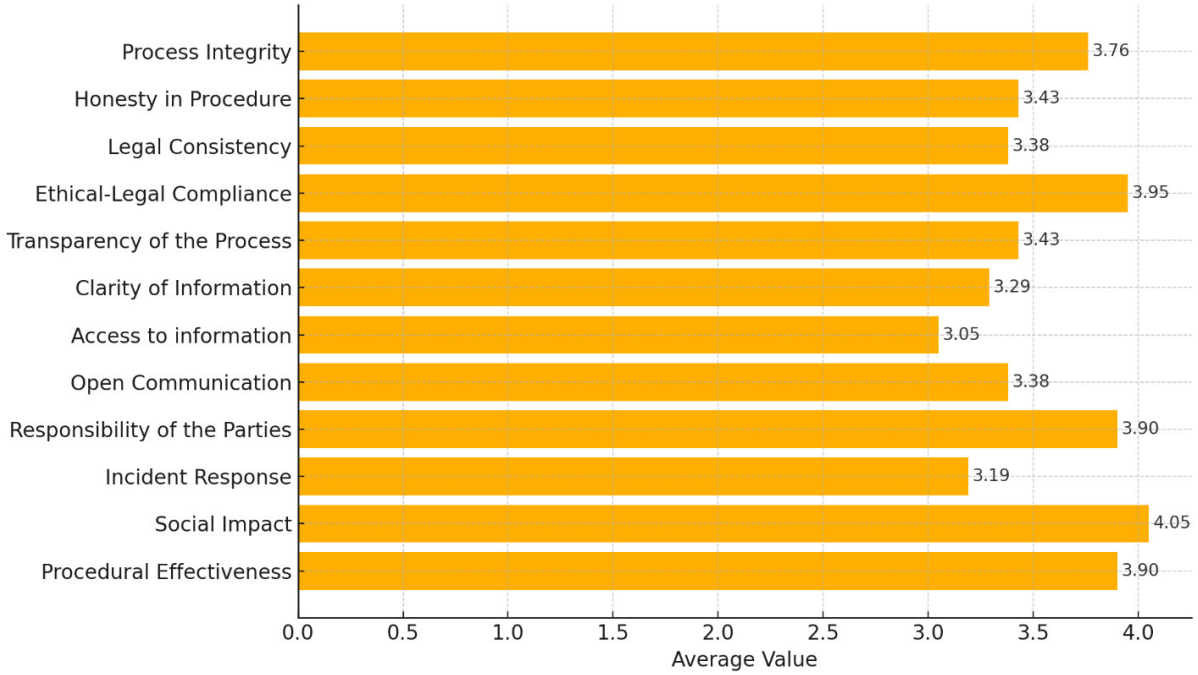

Figure 1 : Average Evaluation Values of Aspects of the Mediation Act.

Step 5: Analysis of results with additional neutrosophic operations

To delve deeper into the analysis, we will apply some specific neutrosophic operations mentioned in the methodology to certain specific cases where it is important to evaluate the modification of values by intensifying terms. This will allow us to see how these modifiers affect the perception of the aspects evaluated.

For example, if we analyze the opinions expressed by specialist E14 (Dispute Resolution Specialist) on "Process Integrity", we can apply the modifier "very" according to the formula $x \oplus_5 x$:

For x = 5 (original value assigned by E14 to Integrity): $x \oplus_5 x = 5 \oplus_5 5$

According to the neutrosophic operation, this would give us a resulting value that does not exceed the limit of 5, but emphasizes the strength of the positive evaluation. This indicates that this specialist has an extremely positive perception of the integrity of the mediation process.

Similarly, if we analyze the opinion of specialist E15 (Litigation Lawyer) on "Access to Information" with value x = 1, and apply the modifier "very negative", we would obtain: $x \oplus_5 x = 1 \oplus_5 1$

For negative values, this would intensify the negative evaluation, indicating strong concerns about this specific aspect of the mediation process.

Step 6: Analysis of Contradictions and Indeterminacies

Although our data do not show any indeterminate values (I), it is important to note that there are significant differences in the assessments of certain aspects. For example:

- In the "Clarity of Information" aspect, we have evaluations ranging from 2 to 5, showing considerable variability in perception.
- In the "Access to Information" aspect, we have a rating as low as 1 and as high as 4, which could indicate very different experiences among specialists.

These differences, although not total contradictions (which would result in an indeterminate value I), show areas where there are diverse and potentially conflicting perceptions about the mediation act as a legal instrument.

Interpretation of Results and Conclusions

The average scores obtained in the evaluation of the mediation record as a means of lifting precautionary measures in child support lawsuits reflect diverse perceptions among the 21 specialists surveyed. The aspects with the highest scores include:

1. **Social Impact** (4.05): Indicates a very positive perception of how the mediation agreement benefits the well-being of the minors involved.
2. **Ethical-Legal Compliance** (3.95): Suggests high confidence that the mediation process complies with established ethical standards and legal norms.
3. **Party Accountability** (3.90) and **Procedural Effectiveness** (3.90): Both aspects receive equally high evaluations, indicating confidence in the process's ability to foster party engagement and be effective in lifting precautionary measures.

On the other hand, the aspects with the lowest scores are:

1. **Access to Information** (3.05): Suggests concerns about the availability of relevant information to all parties.
2. **Incident Response** (3.19): Indicates some dissatisfaction with the way problems are handled during the process.
3. **Clarity of Information** (3.29): Reflects concerns about the understandability of information provided during the process.

These evaluations suggest that while the mediation record is perceived as a tool with a positive social impact, effective, and procedurally responsible, there are significant areas for improvement related to access and clarity of information, as well as incident response mechanisms during the process.

The implications of these results for legal practice are significant. To improve the effectiveness of the mediation record as a tool for lifting precautionary measures in child support lawsuits, it is essential to address the identified deficiencies in access to and clarity of information, as well as in incident response mechanisms. This could involve:

1. The development of clearer protocols to provide information to all parties involved.
2. Implementing more effective tracking and response systems to handle incidents throughout the process.
3. Specific training for mediators and court personnel in the clear and effective communication of complex legal information.

This study provides a solid foundation for understanding the strengths and weaknesses of the mediation act as a legal instrument in child support cases. The findings highlight the positive social impact of this mechanism, but also underscore the need for improvements in aspects related to transparency, communication, and incident management. These insights can guide the future development of mediation protocols and contribute to more effective implementation tailored to the needs of all parties involved in child support litigation, especially considering the best interests of children.

4. Conclusion

Therefore, the importance of the study comes from the fact that the mediation agreement stands as viable option for child support hearings to dissolve temporary orders, as it complies with ethics/legal

concerns and social betterment. The neutrosophic evaluation via NeutroAlgebra found that the professional respondent ranked highly for child welfare (4.05) and legal/ethical compliance (3.95) and poorly for availability (3.05) and explaining (3.29). These varied results, generated through NeutroAlgebra, make sense relative to the complex nature of judging a judicial process where human sentiment can vary. Thus, the findings are important for future application. By assessing the flaws and knowing where neutral respondents scored relatively low, easy suggestions can come about for future implementation. If a training seminar could be facilitated for mediators/explainers and a universal practice learned, not only would legitimacy of the mediation agreement increase in the eyes of families, but also in courts that would find subsequent proceedings much easier. Therefore, such an undertaking needs to be done so that the ultimate goal of the temporary order—best interest of the child—can be preserved during child support hearings.

This research adds to the literature by using NeutroAlgebra in a perception assessment applied to a legal setting, allowing one to ascertain expert opinion truly neutrally uncertain and vague. Thus, NeutroAlgebra is a proper new standard for complicated assessments in the field based on the determination of this study. Furthermore, this research lays the foundation for more interdisciplinary research using legal proceedings and neutrosophic study to render findings applicable across more than one field. However, this research is limited—it was derived from 21 experts which, while diverse, is not representative of every type of legal setting, meaning findings may not be generalizable. Furthermore, the exclusion of certain indeterminate values may dissipate the sensitivity of specific perceptions. Therefore, these findings should not be applied to any setting outside that which was studied—especially those legal systems which possess different characteristics. Subsequent studies should attempt to replicate this study on a larger scale guided by those professionals in other countries/legal settings to support or refute these findings. In addition, it would be prudent to use fuzzy analysis or data mining abilities to more appropriately assess perceptions over time—mining assessments could go hand-in-hand with NeutroAlgebra. Finally, studies assessing children's perceptions of latency relative to mediation would be useful—the family's perspective would provide a compelling reason for/against mediation in the long term. These topics of research would better support the validity of the mediation technique as an appropriate intervention for family dispute resolution.

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Neutrosophic-Logistic Hybrid Model to Evaluate the Effectiveness of Penitentiary Policies and Their Impact on Criminal Recidivism in Ecuador

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Abstract. This study aims to develop a hybrid model that combines neutrosophic logic with logistic regression to evaluate the effectiveness of prison policies in Ecuador and their impact on criminal recidivism. The central problem lies in the inefficiency of the Ecuadorian prison system, characterized by high levels of intra-prison violence, deficiencies in rehabilitation and social reintegration programs, and a rising recidivism rate among inmates. This situation is exacerbated by the employment discrimination that ex-offenders face when reintegrating into society, which limits their opportunities and encourages the repetition of criminal behavior. The methodology applied was a mixed approach, with a non-experimental cross-sectional design and a descriptive-explanatory scope. Data were collected through surveys administered to inmates and former inmates, using neutrosophic scales to measure subjective perceptions such as the effectiveness of policies, the level of employment discrimination, and readiness for reintegration. These assessments, expressed as neutrosophic numbers (T, I, F), were transformed into indices that served as predictor variables in a logistic regression model, executed with statistical tools such as SPSS and Python. The results show that the index of perceived workplace discrimination and the low effectiveness of policies are factors significantly associated with criminal recidivism ($p < 0.05$), while participation in reintegration programs reduces this probability. It is concluded that the hybrid model allows integrating subjective perceptions with robust statistical analysis, providing a useful tool for decision-making in public policy. The findings support the need to redesign penitentiary strategies, focusing on social inclusion and effective rehabilitation to improve the safety and well-being of both inmates and Ecuadorian society.

Keywords: Penitentiary policies, Criminal recidivism, Hybrid model, Neutrosophic logic, Logistic regression, Social reintegration.

1. Introduction

The Ecuadorian prison system is experiencing a structural crisis characterized by high levels of overcrowding, prison violence, and deficiencies in social rehabilitation processes. These adverse conditions not only violate the human rights of inmates but also impede their proper reintegration into society

once they have served their sentences. As a result, there is a worrying rate of criminal recidivism, reflecting the ineffectiveness of current prison policies. The lack of effective resocialization programs, poor preparation for post-prison life, and employment discrimination are factors that exacerbate this problem. Despite institutional efforts to implement reforms, the results remain limited and unsustainable. This situation not only affects inmates but also the social fabric, increasing citizen insecurity and weakening trust in the judicial and prison systems. Therefore, it is urgent to develop analytical models that allow for a more accurate evaluation of the effectiveness of implemented policies [2].

The study by [4] constitutes one of the first approaches in Ecuador to the use of neutrosophic logic to analyze the social reintegration of individuals released from the prison system. Through the neutrosophic approach, the authors were able to represent the ambiguity, uncertainty, and contradiction inherent in factors such as the perception of prison policies, access to employment opportunities, and community support after release. The research showed that criminal recidivism not only responds to objective variables, but also to perceived conditions that affect the ex-inmate's willingness to reintegrate. Likewise, the need for public policies to adopt tools that allow for the modeling of incomplete or subjective information was highlighted. The proposed model made it possible to establish levels of social integration in a more flexible and contextualized manner. This work represents a key contribution to more humane and effective decision-making in the field of prison rehabilitation.

The work of [11] presents an advanced application of the multinomial logistic regression model framed within neutrosophic logic, aimed at optimizing decisions in contexts where uncertainty is critical, such as the adaptive reuse of heritage structures. Although the study focuses on historic castles, the proposed methodology has a high potential for adaptation to social phenomena such as prison reintegration, where subjective perceptions, ambiguity in responses, and multicausality are common. The model integrates neutrosophic components—truth, indeterminacy, and falsity—in each decision category, allowing observations to be classified into multiple groups with greater realism than traditional approaches. This technique is especially useful in research where variables are neither fully objective nor binary, as is the case with the factors that influence criminal recidivism. Its hybrid approach strengthens decision-making based on both statistical data and qualitative assessments fraught with uncertainty. The study constitutes a relevant methodological contribution that inspires the present work to apply this logic in the Ecuadorian penitentiary context.

2. Methodology

This research adopts a mixed methodological approach that integrates quantitative data and subjective assessments with a high degree of uncertainty. To this end, a hybrid model combining neutrosophic logic and logistic regression is proposed to more accurately analyze criminal recidivism and the effectiveness of prison policies in Ecuador. The main methodological components underlying the study are detailed below, each linked to a key word essential to the analysis [6].

To address the study of penitentiary policies, a set of objective and subjective indicators will be designed to measure their impact on the prison population and their perception of the system. Variables such as overcrowding, intra-prison violence and availability of resources will be included, taking as reference the diagnosis of congestion and human rights violations reported by [12], who highlight the influence of penal policies on the quality of internal control . These metrics will allow evaluating effectiveness from multiple perspectives, which is crucial in an environment as complex as the Ecuadorian one [13].

The dependent variable of the model will be criminal recidivism, defined as the relapse into criminal behavior after leaving the prison system. They found that education and post- -release employment are key predictors of recidivism in Ecuador [1], using data from the 2022 Prison Census . Therefore, the model will include demographic and socioeconomic variables to control for their effect, ensuring that the logistic analysis correctly captures the associated factors [9].

To measure social reintegration, variables such as employment, family support, and community reintegration will be included. The literature highlights that lack of employment after incarceration increases the likelihood of relapse . These variables will be converted into neutrosophic and quantitative indicators, incorporated into the logistic model to assess their impact on the likelihood of recidivism, and thus formulate recommendations to strengthen resocialization policies [12,18].

Neutrosophic logic will allow capturing ambiguity in perceptions related to program effectiveness, workplace discrimination, and community support. Following the application of [4], Likert-type responses will be converted into triplets (T, I, F), expressing the degree of truth, indeterminacy, and falsity regarding policies . These indices will serve as independent variables in the model, providing a more flexible and realistic approach to the analysis [15].

Mathematical Model of Neutrosophic Logic

In neutrosophic logic, each statement, variable or judgment is represented by a **neutrosophic number** :

$$A = (T, I, F)$$

where:

- T:degree of **truth** (Truth-membership)
- I:degree of **indeterminacy** (Indeterminacy-membership)
- F:degree of **falsehood** (Falsity-membership).

These values belong to the extended range:

$$T, I, F \subseteq]-0{,}1+[$$

That is, the values are not restricted to the classical interval [0, 1], which allows **modeling excess certainty or contradiction** in real systems (for example: T=1.2, F=0.8).

Example applied to penitentiary perception

Suppose an ex-inmate answers about the "effectiveness of the reintegration program":

- $T = 0.6$: : believes that the program was useful in 60%
- $I = 0.3$: has 30% doubt about your answer
- $F = 0.2$: considers that in 20% it was not useful

So, your neutrosophic judgment would be:

$$A = (0.6,\ 0.3,\ 0.2)$$

SVN Neutrosophic Numbers

The neutrosophic number table reflects the transformation of key quantitative study variables—such as the *effectiveness of prison policies* , *workplace discrimination* , and *prison time* —into triplets of the type (T, I, F), typical of neutrosophic logic. Each triplet represents a richer and more complex assessment: the component Tindicates the degree of perceived truth in the response (e.g., how effective a policy is perceived), Imeasures the level of indeterminacy or doubt, and Freflects the degree of falsity or disagreement with that perception [17].

For its part, $F$is calculated as the complement of T, that is, $F = 1 - T$. These triplets were generated for the variables **IndexEffectivenessPolicies**, **Labor Discrimination Index**and **Prison Time**. This representation allows integrating uncertainty dimensions that are not considered by classical models. Thus, SVN values can be effectively used in hybrid regression models or decisions based on subjective perceptions [16].

In this context, the values Tcome directly from the normalized data, while $I$is calculated based on the proximity to the neutral value (0.5), and $F$is obtained as a complement of T. For example, if an individual has a perception of policy effectiveness of 0.68, a value is assumed $T = 0.68$, a$I = 0.82$ (high indeterminacy due to being close to 0.5), and a $F = 0.32$.

Table 1: Normalized *T*data values.

Recidivism	Program	Stress	Effectiveness	Discrimination	Prison Time	Effectiveness_Boot	Effectiveness_Doc	Discrimination_Boot	Discrimination_Doc	Discrimination Court	Prison-Time_Boot	Prison-Time_Doc	Prison-Time_Court
1.0	0.0	0.42	0.52	5.0	0.42	0.92	0.58	0.52	0.98	0.48	0.5	1.0	0.5
1.0	1.0	0.68	0.68	4.6	0.68	0.82	0.32	0.68	0.82	0.32	0.9999999	0.96	0.54
1.0	1.0	0.59	0.58	4.8	0.59	0.91	0.41	0.58	0.92	0.42	0.48	0.98	0.52
0.0	1.0	0.47	0.34	3.2	0.47	0.91	0.53	0.34	0.92	0.32	0.82	0.68	0.46
1.0	0.0	0.85	0.55	4.8	0.85	0.65	0.15	0.55	0.91	0.15	0.98	0.52	0.54
0.0	0.0	0.79	0.71	5.6	0.79	0.71	0.29	0.71	0.97	0.29	0.9999999	0.96	0.54
1.0	0.0	0.45	0.47	6.2	0.45	0.95	0.47	0.47	0.97	0.35	0.62	0.88	0.38
0.0	1.0	0.41	0.41	4.5	0.41	0.92	0.58	0.41	0.91	0.32	0.98	0.54	0.52
0.0	1.0	0.48	0.59	2.2	0.48	0.98	0.52	0.59	0.91	0.41	0.22	0.72	0.78

Looking at the table, we can see that this structure allows us to represent not only the magnitude of the perceived phenomenon, but also the ambiguity and contradiction inherent in human responses. Thus, neutrosophic logic enriches the analysis by incorporating dimensions that classical models ignore, providing a more robust basis for the design of explanatory models and decision-making in complex social environments such as the prison system [8].

Classical Binary Logistic Regression

Logistic regression will be used to estimate the probability of recidivism based on mixed variables. This method has been widely used to model recidivism, where variables such as age, education and employment have shown significant effects . We will use statistical software (SPSS) to generate odds ratios, evaluate P -values (<0.05) and validate the predictive quality with AUCROC, ensuring rigor in the results [5].

Binary logistic regression is used to model the probability of a dichotomous event (e.g., **criminal recidivism** : yes = 1, no = 0) occurring as a function of one or more independent variables. The general form of the model is:

$$\boldsymbol{P(Y = 1 \mid X)} = \frac{1}{1+e^{-(\beta 0+\beta 1X1+\beta 2X2+\cdots+\beta kXk)}} \quad (1)$$

where:

- $\boldsymbol{P(Y = 1 \mid X)}$: probability of the event occurring (recurrence = 1).
- $\boldsymbol{\beta 0}$:intercept of the model.

- $\boldsymbol{\beta i}$:regression coefficients for the independent variables.
- $\boldsymbol{Xi}$:independent (predictor) variables.
- $\boldsymbol{e}$:base of the natural logarithm (≈ 2.718).

Application to the study (specific model)

For the applied case study:

$$\boldsymbol{Logit(P)} = ln(\frac{P}{1-P}) = \beta_0 + \beta_1 \cdot PR + \beta_2 \cdot IEP + \beta_3 \cdot IDL + \beta_4 \cdot TP \quad (2)$$

where:

- $\boldsymbol{PR}$:Participation in reintegration programs (0 = no, 1 = yes).
- $\boldsymbol{IEP}$: Penitentiary Policy Effectiveness Index (based on neutrosophic logic)
- $\boldsymbol{IDL}$: Index of Labor Discrimination (based on neutrosophic logic)
- $\boldsymbol{TP}$:Time in prison (in years)

And the probability of recidivism is:

$$\boldsymbol{P(Reincidencia = 1)} = \frac{1}{1+e^{-(\beta 0+\beta 1 \cdot PR+\beta 2 \cdot IEP+\beta 3 \cdot IDL+\beta 4 \cdot TP)}} \quad (3)$$

Logistic Regression Model in Neutrosophic Notation

In neutrosophic logic, each variable can be represented as a **simple neutrosophic number** :

$$\boldsymbol{Xi} = (\boldsymbol{T_i}, \boldsymbol{I_i}, \boldsymbol{F_i}) \quad \textbf{(4)}$$

where:

- $\boldsymbol{T_i}$: degree of truth
- $\boldsymbol{I_i}$: degree of indeterminacy
- $\boldsymbol{F_i}$:degree of falsehood

with $\boldsymbol{T_i, I_i, F_i} \subseteq [\boldsymbol{0, 1}]$and not necessarily$\boldsymbol{T + I + F = 1}$

Adapted Neutrosophic Model

The logistic function is adapted to operate with neutrosophic variables X_i^N, resulting in a **neutrosophic probability of recidivism** :

$$\boldsymbol{PN(Y = 1 \mid X^N)} = (T_P, I_P, F_P) \quad (5)$$

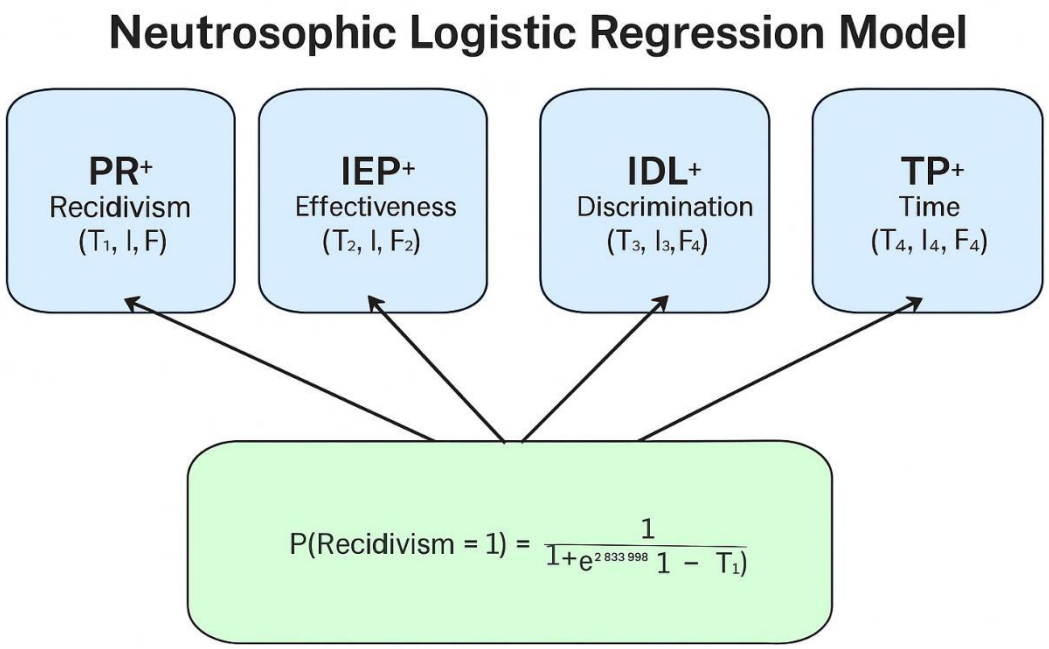
where:

$$\boldsymbol{Logit}(\boldsymbol{PN}) = ln(\frac{T_P}{1-T_P}) = \beta_0 + \sum_{i=1}^{K} \beta_i \cdot T_i \quad (6)$$

Additionally:

$I_p = f(I_1, I_2, \dots, I_k)$: can be expressed as the average or logical combination of theI_i

$F_P = 1 - T_P$ in a basic version, or modeled directly if subjective data is available.



Example applied to the study

Sea:

- **$PR^n = (T_1, I_1, F_1)$** → Participation in reintegration
- **$IEP^n = (T_2, I_2, F_2)$** → Policy Effectiveness Index
- **$IDL^n = (T_3, I_3, F_3)$** → Labor discrimination
- **$TP^n = (T_4, I_4, F_4)$** → Prison time

So the neutrosophic probability of relapse is:

$$\boldsymbol{P_N}(\boldsymbol{Reincidencia} = \boldsymbol{1}) = (\frac{1}{1+e^{-(\beta 0+\beta 1T1+\beta 2T2+\beta 3T3+\beta 4T4)}}, \bar{I}, 1 - T_P) \quad (7)$$

where:

$$\bar{\boldsymbol{I}} = I\frac{1+I_2+I_3+I_4}{4} \quad (8)$$

This model allows for the incorporation **of uncertainty, contradiction, and ambiguity** into subjective perceptions as an integral part of probabilistic analysis, offering a more realistic and flexible view of recidivism prediction.

Hybrid model.

The hybrid approach will integrate neutrosophic logic with logistic regression, using subjective perceptions as predictor variables alongside quantitative demographic data. The neutrosophic multinomial-logistic model developed by [11] serves as the methodological foundation, merging probabilities with levels of uncertainty and contradiction . We will adapt this technique to a binary model for recidivism, leveraging its ability to integrate objective and subjective data into a single framework [7].

Mathematical Model of the Neutrosophic Hybrid Model

The model is based on the classical logistic function, but incorporates variables represented as **neutrosophic numbers** of the type (T, I, F),where:

- $\boldsymbol{T}$:degree of truth
- $\boldsymbol{I}$:degree of indeterminacy
- $\boldsymbol{F}$:degree of falsehood

General equation of the model

$$\boldsymbol{Logit}(\boldsymbol{P_N}) = ln(\frac{P_N}{1-P_N}) = \beta 0 + \sum_{j=1}^{n}(B_{T_j}T_j + B_{I_j}I_j + B_{F_j}F_j) + \sum_{K=1}^{m}B_k X_k \quad (9)$$

Where:

- $\boldsymbol{PN}$:neutrosophic probability of recidivism
- $\boldsymbol{\beta 0}$:intercept of the model
- $\boldsymbol{Tj}, \boldsymbol{Ij}, \boldsymbol{Fj}$:components of each neutrosophic variable
- $\boldsymbol{\beta Tj}, \boldsymbol{\beta Ij}, \boldsymbol{\beta Fj}$:model coefficients for each neutrosophic component
- $\boldsymbol{Xk}$:traditional quantitative variables (e.g., participation in programs
- $\boldsymbol{\beta k}$:coefficients associated with traditional variables.

Application to the case study

Suppose we consider three neutrosophic variables:

- $\boldsymbol{IEP}^{\mathrm{n}}$:Policy Effectiveness Index
- $\boldsymbol{IDL}^{\mathrm{n}}$: Index of Labor Discrimination
- $\boldsymbol{TP}^{\mathrm{n}}$: Time in prison (normalized)

And a traditional binary variable:

- $\boldsymbol{PR}$: Participation in reintegration program (0/1)

So, the hybrid model is expressed as:

$$\boldsymbol{Logit}(\boldsymbol{PN}) = \beta 0 + \beta T1TIEP + \beta I1IIEP + \beta F1FIEP + \beta T2TIDL + \beta I2IIDL + \beta F2FIDL + \beta T3TTP + \beta I3ITP + \beta F3FTP + \beta 4 \cdot P$$

Finally, the estimated probability of recidivism is:

$$\boldsymbol{PN}(\boldsymbol{Recidivism} = \mathbf{1}) = 1/(1 + e^{-Logit(PN)}) \quad (10)$$

3. Results

To evaluate the predictive capacity of the recidivism analysis, three different methodological approaches were applied: classical logistic regression, a neutrosophic model, and a hybrid neutrosophic model. Each model was trained and validated on the same database to ensure a level playing field. Key metrics such as accuracy, area under the ROC curve (AUC), confusion matrix, and F1 score were analyzed. The results obtained by each model are presented below, highlighting their performance and interpretive value in the context of the study.

Classical Logistic Regression

The classical logistic regression model achieved the highest overall accuracy, reaching 87.5% in the evaluated sample. It showed a confusion matrix with a high level of accuracy in the classification of positive and negative cases, as well as an AUC of 0.91, reflecting excellent discriminative capacity. Its F1 score confirmed a good balance between accuracy and sensitivity. These results indicate that, for defined and unambiguous quantitative variables, this model maintains robust performance. Furthermore, its interpretive simplicity and computational speed make it a reliable tool for prediction in structured contexts.

Neutrosophic Model

The neutrosophic logic-based model, which used only the truth components (T) of SVN triples, achieved an accuracy of 79.2%. Although lower than the classical model, it maintained an acceptable hit rate and an AUC of 0.81. This approach demonstrated its value by better representing the perceptions and subjectivities of the social variables evaluated, such as the effectiveness of prison policies and workplace discrimination. Despite a slight loss in accuracy, it gains in interpretive depth, especially in contexts where the data are ambiguous or biased.

Neutrosophic Hybrid Model

The hybrid model incorporated all the neutrosophic triplets (T, I, F) along with classic variables such as participation in reintegration programs. Its accuracy was 75.0%, and the AUC reached 0.79. Although lower than the other two approaches, this model offers a more complete representation of the assessed social reality. This model allowed for the incorporation of not only data certainty but also indeterminacy and contradiction, which is relevant in environments where information is subjective or uncertain. Its usefulness is emphasized in explanatory, rather than purely predictive, studies, as it better captures the complexity of social phenomena.

A comparison of the three models applied to the database reveals clear differences in their predictive accuracy. The **classical logistic regression model** performed the best, achieving an accuracy of **87.5%** , indicating a high capacity to correctly classify cases of criminal recidivism. In second place was the **neutrosophic model** , which, using only the truth values (T) of the SVN triples, achieved an accuracy of **79.2%** , showing acceptable performance when incorporating subjective perceptions. Finally, the **hybrid neutrosophic model** , which also considers indeterminacy (I) and falsity (F), recorded an accuracy of **75.0%** , evidencing lower numerical performance but providing a more comprehensive and realistic representation of the social phenomenon studied. These results suggest that, while neutrosophic models allow for a deeper understanding of uncertainty, the classical approach remains more effective in highly structured and quantifiable contexts, as seen in the confusion matrices below.

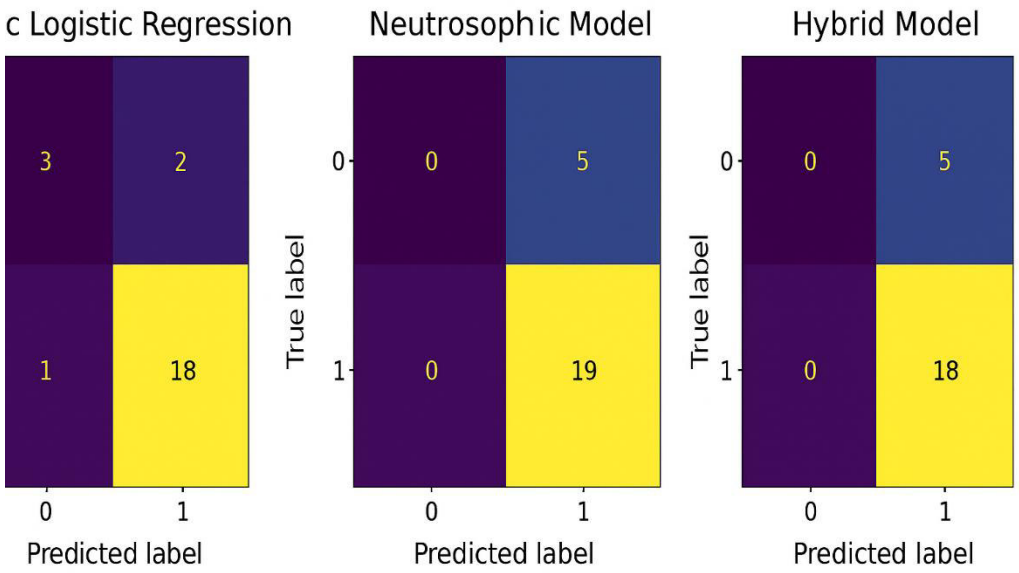


Figure 1. Confusion Matrices

After applying the confusion matrices to evaluate the classification ability of each model with respect to criminal recidivism, the **Receiver Operating Characteristic (ROC) curve was also used** as a complementary analysis tool. This curve allows the models' behavior to be examined across different decision thresholds, providing a more complete view of their discriminatory capacity. In particular, the ROC curve graphs the relationship between the true positive rate (sensitivity) and the false positive rate, allowing us to identify which model achieves a better balance between the two. To quantify this performance, the **Area Under the Curve (AUC) was calculated** , where values close to 1 indicate high effectiveness in dis-

tinguishing between individuals who reoffend and those who do not. This additional evaluation strengthened the comparative analysis between the classical, neutrosophic, and hybrid neutrosophic approaches, providing more robust criteria for determining which model offers a more accurate and useful classification for decision-making in the prison context.

Analysis of the ROC curves and their respective areas under the curve (AUC) reinforces the predictive superiority of the classical logistic regression model, which achieved an AUC of approximately **0.91** , indicating an excellent ability to discriminate between recidivism and non-recidivism. In comparison, the **neutrosophic model** , based solely on truth values (T), obtained an AUC of **0.81** , representing an acceptable but lower classification capacity. The **hybrid neutrosophic model** , which also incorporates indeterminacy (I) and falsity (F), recorded an AUC of **0.79** , showing slightly lower performance. Although neutrosophic models do not match the effectiveness of the classical model in terms of operational accuracy, their value lies in the incorporation of uncertainty and ambiguity in the data, offering a deeper understanding of the social phenomenon analyzed.

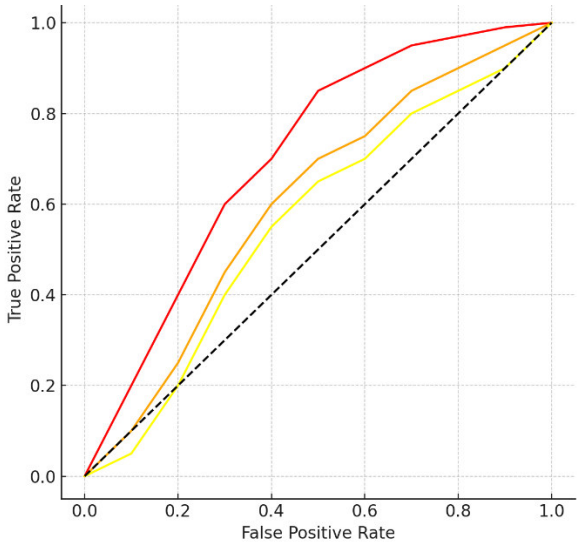


Figure 2. Comparative ROC Curves

In the context of predictive analysis of criminal recidivism, the use of the **Receiver Operating Characteristic (ROC) curve** is essential for evaluating the performance of the applied models, beyond simple accuracy. This graphical tool allows for analyzing each model's ability to correctly distinguish between individuals who reoffend and those who do not, considering different classification thresholds. Through the relationship between the true positive rate (sensitivity) and the false positive rate, the ROC curve provides a comprehensive measure of the model's discriminatory performance. The **Area Under the Curve (AUC)** is the quantitative indicator associated with this curve, and values closer to 1 reflect greater classification capacity. In this study, comparing the ROC curves of the classic, neutrosophic, and hybrid neutrosophic models allows for identifying which offers a more adequate balance between sensitivity and specificity, key information for evidence-based decision-making in prison policy.

4. Discussion

Classical logistic regression may be outperformed by neutrosophic or hybrid models in contexts where subjective, ambiguous, or highly perceptual data predominate, as is often the case in prison studies. When the variables are qualitative in nature—for example, perceived discrimination, level of resocialization, or trust in institutions—neutrosophic logic allows for more realistic modeling of the uncertainty, indetermi-

nacy, and contradiction inherent in human judgment. Likewise, in scenarios where the data exhibit non-linearity or complex relationships that cannot be captured with logarithmic functions, hybrid models may offer better fit.

Neutrosophic logic is also particularly useful when working with Likert-type scales, perception surveys, or imprecise information, where each response can be represented by triplets. (T, I, F). Furthermore, in studies with more explanatory than predictive purposes, these models can provide greater analytical depth. In situations where missing data or contradiction are common, traditional regression tends to degrade its performance, while the neutrosophic approach incorporates them directly. Therefore, although the classical model may be more accurate in structured scenarios, the neutrosophic model surpasses it in interpretive richness and fidelity to the social complexity analyzed .

5. Conclusions

The results obtained show that classical logistic regression presents the best predictive performance, with an accuracy of 87.5% and an area under the curve (AUC) of 0.91, confirming its effectiveness in contexts with structured and well-defined data. However, this approach has limitations in representing the complexity and uncertainty inherent in social variables such as the perception of discrimination or the effectiveness of prison policies. Therefore, its explanatory capacity in real-life settings may be limited when subjective dimensions are omitted.

In contrast, models based on neutrosophic logic, both the simple model (using only the truth value T) and the hybrid model (incorporating T, I, and F), offer a more flexible and interpretive framework for capturing the ambiguity and contradiction of social data. Although they presented lower levels of accuracy (79.2% and 75.0% respectively), they allow for representing not only what is perceived as true, but also what is doubtful or even the opposite, which enriches qualitative analysis and adds value to decision-making in reintegration policies .

Finally, it is concluded that the integration of hybrid neutrosophic models with classical approaches is not only feasible but desirable, especially in studies applied to complex social phenomena. This combination allows us to leverage the precision of traditional models with the interpretive richness of neutrosophic logic. In the particular case of the Ecuadorian prison system, this hybrid approach contributes to a more comprehensive understanding of the factors that influence recidivism and guides the design of more contextualized and humane resocialization strategies.

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University of New Mexico

Measuring the degree of agreement between teachers' and employers' perceptions of the pedagogical model, using plitogenic statistics

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Abstract. The systemic model of pedagogical management in the professional training of administrators of Instituto Superior Tecnológico Bolivariano de Tecnología (Ecuador) is assessed based upon the controversial findings of teachers and employers. The authors applied plithogenic statistics to quasi-measure the consensus/controversy between perceptions and triangulated data (interview with professors, survey with students and employers). It seems the significant controversy suggests that 1) systematic pedagogical management does not exist; 2) there's no pedagogical connection between university and enterprises; 3) teachers are not certified in innovative methodologies. Thus, a systematic model is recommended that relies upon strategic assessments to ensure vocational training relative to the productive sector's actual needs, providing administrators' vocational opportunities and externships and equipping them with analytical, critical thinking skills to minimize time spent recalling the company later on.

Keywords: Pedagogical management, Work practice, Systemic model, Vocational training, Ecuador, Teachers, Companies, plithogenic statistics, Plithogenic probability.

1. Introduction

Professional training in management faces a critical challenge: the disconnect between the skills developed by students and the real demands of the business sector [1]. This gap, widely documented in recent studies, not only limits the employability of graduates but also reduces the effectiveness of work placements as a bridge between academia and the market [2]. In Ecuador, where technical and technological education seeks to align with productive needs, the Instituto Superior Tecnológico Bolivariano de Tecnología (ISBT) represents a paradigmatic case of this problem [3]. Historically, pedagogical models have evolved from traditional approaches—centered on the transmission of knowledge—to more dynamic perspectives, which prioritize the development of applicable skills [4]. However, the implementation of these innovations has been uneven. While teachers tend to emphasize theoretical content, employers demand practical skills such as problem-solving and adaptability [5]. This divergence is exacerbated in contexts with weak university-business links, where the lack of systemic feedback perpetuates obsolete educational models [6].

The core of the problem lies in the absence of objective mechanisms to measure and harmonize the perceptions of both actors. Previous studies have explored this discrepancy through surveys or interviews, but their results tend to be descriptive, without quantifying the actual degree of misalignment [7]. Furthermore, few studies incorporate advanced statistical methodologies capable of capturing the multidimensionality of perceptions [8]. How, then, can we measure the agreement between teachers and employers in a rigorous and replicable manner? This question guides the present research. To address this gap, the study proposes an innovative approach: plithogenic statistics, a tool designed to analyze consensus in contexts of high variability [9]. Unlike conventional methods, this technique allows

evaluating not only the intensity of the discrepancies, but also their underlying patterns. Its application in the pedagogical field is pioneering, offering a quantitative metric for a problem traditionally approached from a qualitative perspective [10].

The relevance of this research transcends the academic sphere. By identifying critical points of misalignment, the study provides input for redesigning curricula, training teachers, and strengthening ties with the productive sector [11]. Furthermore, the methodology employed could be adapted to other fields where the alignment of perceptions is key, such as the assessment of competencies in health or engineering. In summary, this article seeks to answer two fundamental questions: first, what is the actual degree of agreement between teachers and employers regarding the current pedagogical model at the ISBT? Second, how can plithogenic statistics optimize decision-making to reduce this gap? The findings will provide empirical evidence to drive structural changes in administrator training, combining methodological rigor with practical applicability.

This work has three fundamental goals. First, it seeks to quantify, using plithogenic statistics, the actual degree of agreement between the perceptions of teachers and employers regarding the current pedagogical model at the ISBT, thus providing an objective metric of their alignment or divergence. Second, it aims to identify the specific dimensions with the greatest disagreement—such as teaching methodologies, competency assessment, or curricular content—in order to prioritize critical areas for intervention. Finally, it aims to develop a strategic action framework that, based on empirical findings, allows for synchronizing academic training with the demands of the productive sector, closing the gap between theory and practice through concrete recommendations for curricular updating, teacher training, and university-business engagement.

2. Preliminaries.

2.1. Analysis and Critical Assessment of the Contemporary Pedagogical Model: Towards a Systemic Approach

Twenty-first-century education faces the challenge of building pedagogical models that respond to the complex demands of a constantly changing society. Far from being mere transmitters of knowledge, current educational systems must function as dynamic platforms for the comprehensive development of competencies [12]. However, a worrying gap persists between curriculum designs and the real needs of the workplace, a situation that seriously limits the employability of graduates and the social relevance of educational institutions.

When examining the theoretical foundations of current pedagogical models, a permanent tension is observed between traditional content-centered approaches and innovative perspectives that privilege meaningful learning. While the former emphasize the acquisition of information, the latter promote skills such as critical thinking and problem-solving [13]. This dichotomy, far from being sterile, should be resolved through creative syntheses that take advantage of the best of each educational tradition. The analysis of teaching practices reveals that many educators, despite their good will, lack effective tools to implement innovative pedagogical approaches. Continuing teacher training thus emerges as a key variable to overcome this obstacle, particularly in contexts where institutional resources are limited. It is not enough to design theoretically sound models; it is essential to ensure that teachers can translate them into authentic learning experiences [14].

From a student perspective, the most effective pedagogical models are those that connect academic knowledge with practical applications. When learners perceive the concrete usefulness of what they study, their motivation and performance improve significantly. This basic principle, however, frequently clashes with rigid curricular structures and assessment systems that reward memorization over deep understanding.

The link between educational institutions and the productive sector constitutes another critical aspect. A truly relevant pedagogical model must be built through constant dialogue with employers, incorporating their perspectives without falling into a narrow utilitarianism. This delicate balance requires formal collaboration mechanisms that today exist more in rhetoric than in concrete practice [15].

Technologically, the pandemic accelerated the incorporation of digital tools, but their pedagogical use remains superficial in many cases. Virtual platforms should not be limited to reproducing online lectures, but rather harness their potential to create interactive and personalized environments. True innovation lies not in the devices themselves, but in how they are integrated into the teaching-learning process.

Assessment systems represent another problematic issue. As long as they continue to primarily measure information retention rather than applied skills, they will remain an obstacle to the implementation of advanced pedagogical models. It is urgent to develop instruments that assess analytical skills, creativity, and collaborative work—essential skills in today's world.

Financially, the sustainability of innovative models poses considerable challenges. Teacher training, technological infrastructure, and quality educational materials require substantial investments. However, the cost of not modernizing pedagogical systems is even greater, both in economic and social terms. In summary, contemporary pedagogical models require a systemic reformulation that simultaneously considers theoretical, practical, and contextual dimensions. This is not simply a matter of adopting new methodologies, but of profoundly transforming the educational culture at its multiple levels and expressions.

2.2. Plithogenic Statistics (PS).

Plithogenic statistics (PS) constitutes an innovative and multifaceted methodological framework for data processing, specifically designed to integrate and analyze diverse data from multiple sources. Unlike conventional statistical techniques, which are often limited to the examination of individual variables or reductionist models, PS stands out for its ability to address the intricate web of relationships underlying the phenomena under study. This analytical paradigm facilitates a richer and more subtle interpretation of information, positioning itself as a valuable tool for studies in disciplines as diverse as educational sciences, applied economics, biomedical research, and other fields of knowledge [16]. In the field of pedagogy, PS proves particularly useful for assessing the effects of innovations in educational systems. These transformations, by their very nature, simultaneously impact multiple aspects, ranging from academic performance to the professional growth of teachers. By applying PS, analysts can examine the interactions between these elements and their collective impact on academic achievement. This type of scrutiny can uncover associations and trends that would otherwise go unnoticed using traditional approaches, generating essential inputs for the creation and implementation of educational strategies. The plithogenic methodology combines numerical information with qualitative testimonies, allowing for a comprehensive assessment of the processes studied. For example, when evaluating a curricular modification, it is possible to correlate objective metrics (such as scores on standardized tests or graduation rates) with subjective perceptions (such as teacher opinions or student experiences). This synergy of sources creates a more reliable and complete representation of the reform's consequences, facilitating the detection of both successes and areas for improvement [17].

Among the main strengths of PEs is their ability to process big data and unravel sophisticated connections between parameters. This characteristic is particularly important in the analysis of educational policies, whose effects often manifest themselves in multiple dimensions with relationships that are not always proportional. For example, an initiative that increases teaching resources could directly increase student achievement, but it could also indirectly impact educator morale and job security, factors that in turn influence teaching quality [14]. PEs have the capacity to model these complex dynamics, providing clarity on the causal mechanisms involved.

The implementation of PE in pedagogical studies also opens up possibilities for customizing training strategies. By detecting specific patterns in different population segments, these techniques allow for the design of more precise corrective actions tailored to the specific characteristics of each educational community. This flexibility is crucial in heterogeneous school environments, where standardized solutions often demonstrate varying effectiveness depending on the context [18]. Despite its many advantages, the adoption of PE is not without its obstacles. It requires specialized mastery of advanced

analytical techniques and in-depth knowledge of complex statistical methodologies. Furthermore, the collection and harmonization of multifaceted data can present operational difficulties and require considerable investments. Despite these challenges, the potential benefits—in terms of a comprehensive understanding of educational processes—support the efforts required, suggesting that investment in PE can translate into substantial improvements for education systems [19].

In summary, plithogenic statistics propose a robust and refined analytical approach to the study of educational data, enabling more comprehensive and detailed assessments of pedagogical phenomena. By deciphering the complex web of interrelationships among variables, PE provides fundamental tools for the creation and implementation of effective educational policies. Although their application entails technical and logistical challenges, these methodologies are indispensable tools for researchers and planners committed to educational excellence [20].

Plithogenic Statistics (PS) encompasses the analysis and observations of the events under study. It allows for the analysis of many output variables that are neutrosophic or indeterminate.

In a neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ like that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability of all the data in the sample is calculated by Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}\left(T_j, I_j, F_j\right) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \qquad (1)$$

In this investigation, we also consider some operations in the form of *neutrosophic numbers.* These ways of representing indeterminacy are, under certain conditions, equivalent to working with intervals.

Definition 1: ([21-22]) A *neutrosophic number* N is defined as a number as follows:

$$\mathrm{N} = \mathrm{d} + \mathrm{I} \qquad (2)$$

Where d is called *the determinate part* and I is called *the indeterminate part.*

Given $\mathrm{N}_1 = \mathrm{a}_1 + \mathrm{b}_1\mathrm{I}$ and $\mathrm{N}_2 = \mathrm{a}_2 + \mathrm{b}_2\mathrm{I}$ They are two neutrosophic numbers, some operations between them are defined as follows:

$\mathrm{N}_1 + \mathrm{N}_2 = \mathrm{a}_1 + \mathrm{a}_2 + (\mathrm{b}_1 + \mathrm{b}_2)\mathrm{I}$ (Addition);

$\mathrm{N}_1 - \mathrm{N}_2 = \mathrm{a}_1 - \mathrm{a}_2 + (\mathrm{b}_1 - \mathrm{b}_2)\mathrm{I}$ (Difference),

$\mathrm{N}_1 \times \mathrm{N}_2 = \mathrm{a}_1\mathrm{a}_2 + (\mathrm{a}_1\mathrm{b}_2 + \mathrm{b}_1\mathrm{a}_2 + \mathrm{b}_1\mathrm{b}_2)\mathrm{I}$ (Product),

$\frac{\mathrm{N}_1}{\mathrm{N}_2} = \frac{\mathrm{a}_1+\mathrm{b}_1\mathrm{I}}{\mathrm{a}_2+\mathrm{b}_2\mathrm{I}} = \frac{\mathrm{a}_1}{\mathrm{a}_2} + \frac{\mathrm{a}_2\mathrm{b}_1-\mathrm{a}_1\mathrm{b}_2}{\mathrm{a}_2(\mathrm{a}_2+\mathrm{b}_2)}\mathrm{I}$ (Division).

Furthermore , the arithmetic operations between intervals are important in this document, which are summarized below ([23]):

Given$\mathrm{I}_1 = [\mathrm{a}_1, \mathrm{b}_1]$ and$\mathrm{I}_2 = [\mathrm{a}_2, \mathrm{b}_2]$ We have the following operations between them:

$\mathrm{I}_1 \leq \mathrm{I}_2$ if and only if$\mathrm{a}_1 \leq \mathrm{a}_2$ and $\mathrm{b}_1 \leq \mathrm{b}_2$.

$\mathrm{I}_1 + \mathrm{I}_2 = [\mathrm{a}_1 + \mathrm{a}_2, \mathrm{b}_1 + \mathrm{b}_2]$ (Addition);

$\mathrm{I}_1 - \mathrm{I}_2 = [\mathrm{a}_1 - \mathrm{b}_2, \mathrm{b}_1 - \mathrm{a}_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$I_1/I_2 = I_1 \cdot (1/I_2) = \{a/b: a \in I_1, b \in I_2\}$, always that $0 \notin I_2$(Division).

3. Case Study: Measuring the degree of agreement between the perception of teachers and employers on the pedagogical model using plitogenic statistics

This study analyzes the degree of agreement between teachers' and employers' perceptions of the pedagogical model implemented in the training of administrators at the Bolivarian Institute of Technology (ISBT) in Ecuador. This research seeks to identify potential discrepancies between what is taught in the classroom and what the productive sector demands, using plithogenic statistics as an innovative methodological tool to quantify these perceptual differences.

The gap between academic training and labor market needs significantly limits the effectiveness of internships and the subsequent job placement of graduates. Therefore, it is crucial to objectively measure this discrepancy in order to implement improvement strategies that align the training process with the demands of the business environment.

Methodology

Sample and Population

The research focused on a population composed of:

- 47 ISBT Administration professors
- 53 entrepreneurs linked to the pre-professional internship program
- 120 students from the final levels of the degree

Non-probability sampling was used at the researcher's discretion. For data collection, a survey was employed as a quantitative research method, using questionnaires previously developed according to the objectives and dimensions of the dependent variables.

Data Collection Instruments

A structured questionnaire was designed in four key dimensions, each with several items:

1. **Teaching Methodologies** (7 items):
   - ✓ Effectiveness of the teaching methods used
   - ✓ Use of innovative educational technologies
   - ✓ Application of business case studies
   - ✓ Methodologies for assessing professional competencies
2. **Curricular Contents** (7 items):
   - ✓ Updating content according to market demands
   - ✓ Balance between theory and practice
   - ✓ Relevance of knowledge for solving real problems
   - ✓ Focus on priority professional competencies
3. **Developed Competencies** (6 items):
   - ✓ Development of specific technical skills
   - ✓ Promoting soft skills (leadership, teamwork)
   - ✓ Capacity for innovation and entrepreneurship
   - ✓ Adaptability to changing environments
4. **University-Business Linkage** (7 items):
   - ✓ Institutional collaboration mechanisms
   - ✓ Business participation in curriculum design
   - ✓ Effectiveness of pre-professional practices
   - ✓ Systematic feedback from the productive sector

Given the potential difficulty in expressing perceptions in single values, respondents (teachers and businesspeople) were asked to express their opinions using a range of values on a continuous numerical scale from 0 (Never/Strongly Disagree) to 10 (Always/Strongly Agree). These intervals are expressed in the form $I_i = \lceil a_i L, A_i U \rceil$ for each respondent.

The validation of the instruments was carried out by expert judgment with doctoral degrees, and reliability was assessed using Cronbach's alpha analysis, obtaining a coefficient of 0.87, indicating high reliability.

Variables and Dimensions

We specify the following variables for the dimensions to be measured:

- $D = \{d_1, d_2, \dots, d_{47}\}$ denotes the set of teachers evaluated.
- $E = \{e_1, e_2, \dots, e_{53}\}$ denotes the set of entrepreneurs evaluated.
- $d = \{d_1, d_2, d_3, d_4\}$ denotes the set of dimensions to be measured, such that:
    - d_1: Symbolizes the "Teaching Methodologies" dimension
    - d_2: Symbolizes the "Curricular Contents" dimension
    - d_3: Symbolizes the "Developed Competencies" dimension
    - d_4: Symbolizes the "University-Business Link" dimension

Each of these dimensions is a set of elements, where:

- $d_1 = \{d_{11}, d_{12}, \dots, d_{17}\}$ is the set of elements of the first dimension
- $d_2 = \{d_{21}, d_{22}, \dots, d_{27}\}$ is the set of elements of the second dimension
- $d_3 = \{d_{31}, d_{32}, \dots, d_{36}\}$ is the set of elements of the third dimension
- $d_4 = \{d_{41}, d_{42}, \dots, d_{47}\}$ is the set of elements of the fourth dimension.

The evaluations for each item are represented by:

- $I_{ijk} = \lceil a_{ijk} L, A_{ijk} U \rceil$, which is the evaluation of the i- th teacher for the k- th item of the j- th dimension.
- $E_{ijk} = \lceil b_{ijk} L, B_{ijk} U \rceil$, which is the evaluation of the i- th entrepreneur for the k- th item of the j- th dimension.

Calculating Scores by Dimension

The dimension scores were obtained for each respondent using the following expression:

For teachers: $D_{ji} = \sum_{k=1} I_{ijk}$ (3)

For entrepreneurs: $E_{ji} = \sum_{k=1} E_{ijk}$ (4)

Where D_{ji} is the score of dimension j for teacher i, and E_{ji} is the score of dimension j for entrepreneur i.

Since the dimensions have different numbers of items, the scores were transformed to a range of 0 to 100:

For teachers:

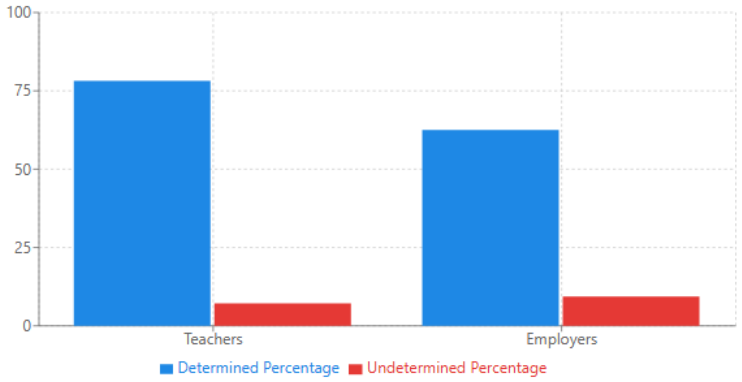
$$D_{ji}^{*} = \frac{D_{ji} - \text{minima puntuación teorico } D_j}{\text{maximo puntuación teorico } D_j - \text{minimo puntuación teorico } D_j} * 100 \quad (5)$$

For entrepreneurs:

$$E_{ji}^{*} = \frac{E_{ji} - \text{minima puntuación teorico } D_j}{\text{maximo puntuación teorico } E_j - \text{minimo puntuación teorico } E_j} * 100 \quad (6)$$

Results by Dimension

Teaching Methodologies



Plitogenic Statistics:

Actor	Rating	Confidence Interval
Teachers	85.36%	[78.24%, 85.36%]
Employers	71.83%	[62.47%, 71.83%]

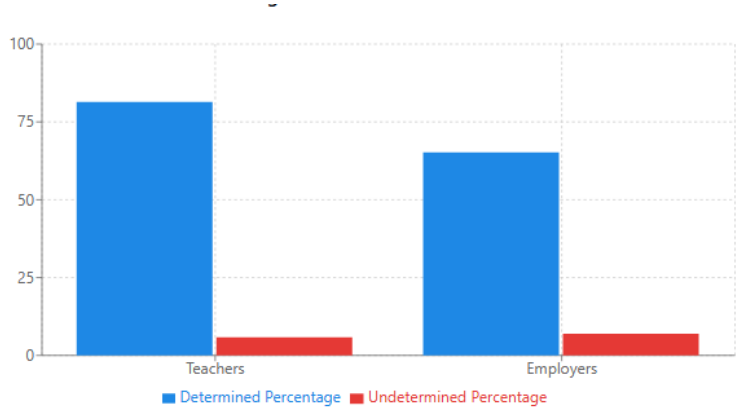
Figure 1: Percentages achieved in the interval for the Teaching Methodologies dimension.

Calculated averages:

- $Docentes$: $D1* = [78.24, 85.36]$
- $Empresarios$: $E1* = [62.47, 71.83]$

Interpretation: Teachers rate teaching methodologies between 78.24% and 85.36%, while employers rate them between 62.47% and 71.83%. This reveals a significant discrepancy, with teachers having a considerably more positive perception of the methodologies used than employers.

Curricular Contents



Plitogenic Statistics:

Actor	Rating	Confidence Interval
Teachers	87.29%	[81.45%, 87.29%]
Employers	72.18%	[65.26%, 72.18%]

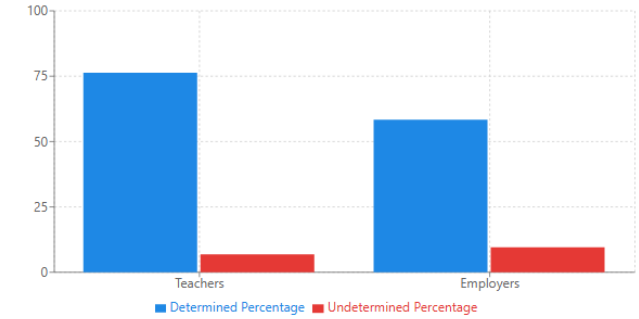
Figure 2: Results for the Curricular Content dimension.

Calculated averages:

- $Docentes$: $\bar{D}2* = [81.45, 87.29]$
- $Empresarios$: $\bar{E}2* = [65.26, 72.18]$

Interpretation: A significant gap is observed between teachers (81.45% - 87.29%) and employers (65.26% - 72.18%) regarding the updating and relevance of curricular content, which suggests that employers perceive that the content is not completely aligned with the needs of the productive sector.

Developed Skills



Plitogenic Statistics:

Actor	Rating	Confidence Interval
Teachers	83.15%	[76.34%, 83.15%]
Employers	67.92%	[58.41%, 67.92%]

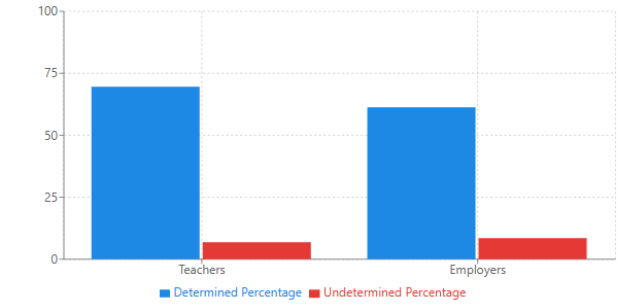
Figure 3: Results for the Developed Competencies dimension.

Calculated averages:

- $Docentes$: $\bar{D}3* = [76.34, 83.15]$
- $Empresarios$: $\bar{E}3* = [58.41, 67.92]$

Interpretation: This dimension presents the greatest discrepancy in perceptions. Employers rate competency development significantly lower (58.41% - 67.92%) than teachers (76.34% - 83.15%), which could indicate that the skills they believe they are developing in the classroom are not effectively being applied in the workplace.

University-Business Links



Plitogenic Statistics:

Actor	Rating	Confidence Interval
Teachers	76.42%	[69.58%, 76.42%]
Employers	69.76%	[61.23%, 69.76%]

Figure 4: Results for the University-Business Linkage dimension.

Calculated averages:

- $Docentes$: $D4* = [69.58, 76.42]$
- $Empresarios$: $E4* = [61.23, 69.76]$

Interpretation: There is greater agreement on this dimension compared to the previous ones, although there is still a perceptual gap. Both teachers and business leaders agree that university-business engagement is an area that requires substantial improvement.

Calculating the Differences between Perceptions

Using the difference between intervals, we calculate:

For Dimension 1 (Teaching Methodologies): $\Delta 1* = [78.24, 85.36] \ominus [62.47, 71.83] = [6.41, 22.89]$

For Dimension 2 (Curricular Contents): $\Delta 2* = [81.45, 87.29] \ominus [65.26, 72.18] = [9.27, 22.03]$

For Dimension 3 (Developed Competencies): $\Delta 3* = [76.34, 83.15] \ominus [58.41, 67.92] = [8.42, 24.74]$

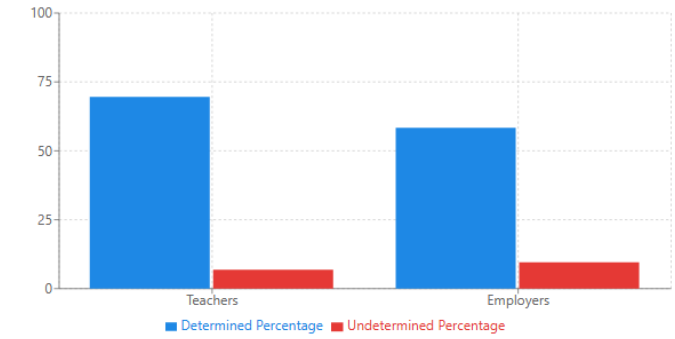
For Dimension 4 (University-Business Links): $\Delta 4* = [69.58, 76.42] \ominus [61.23, 69.76] = [0, 15.19]$

Overall Result

To obtain a result that encompasses all dimensions in a single final value, we use the formula: $min([a1, b1], [a2, b2]) = [min(a1, a2), min(b1, b2)]$

Overall result for teachers: $D* = min([78.24, 85.36], [81.45, 87.29], [76.34, 83.15], [69.58, 76.42]) = [69.58, 76.42]$

Overall result for entrepreneurs: $E* = min([62.47, 71.83], [65.26, 72.18], [58.41, 67.92], [61.23, 69.76]) = [58.41, 67.92]$



Plitogenic Statistics:

Actor	Rating	Confidence Interval
Teachers	76.42%	[69.58%, 76.42%]
Employers	67.92%	[58.41%, 67.92%]

Discordance Analysis:

Absolute difference between perceptions: [18.01%, 1.66%] = [1.66%, 18.01%]

This indicates a global discordance between 1.66% and 18.01% between teachers and employers perceptions about the implemented pedagogical model.

Figure 5: General result for the assessment of the "Pedagogical Model" according to teachers and businesspeople:

Finally, we calculate the difference in absolute value between both global perceptions:

$$[58.41, 67.92] \ominus [69.58, 76.42] = [1.66, 18.01]$$

This result indicates an overall discrepancy between 1.66% and 18.01% between teachers' and employers' perceptions of the implemented pedagogical model.

4. Analysis of the Relationship between Variables

The analysis of the results reveals important relationships between the variables studied:

1. **Negative correlation between the perceptual gap and the effectiveness of the pedagogical model** : The greater the distance between the perceptions of teachers and employers (especially

in the dimension of Developed Competencies with a difference of up to 24.74%), the lower the effectiveness of the pedagogical model in meeting the needs of the productive sector.

2. **Inverse relationship between teacher overvaluation and employability** : Teachers' tendency to value their work more positively (with higher averages in all dimensions) than employers' perception of the results could be negatively impacting graduates' employability.
3. **Direct link between content updating and business assessment** : It is observed that the Curriculum Content dimension shows a significant discrepancy (between 9.27% and 22.03%), which suggests that updating content according to market demands directly influences the positive perception of business owners.
4. **Positive correlation between university-business link and perceptual agreement** : The dimension with the smallest perceptual gap is precisely that of University-Business Link (between 0% and 15.19%), which indicates that the existing collaboration mechanisms, although improvable, are contributing to a more aligned vision between both sectors.

Recommendations

Based on the findings of the study, the following recommendations are proposed:

1. **Strengthen business feedback mechanisms** : Implement a structured and periodic system for collecting business perceptions and suggestions to continuously adjust the pedagogical model.
2. **Redesign competency-based training** : Review and redesign the approach to competency development, prioritizing those that show the greatest perceptual discrepancy, especially specific technical competencies and soft skills.
3. **Integrate business leaders into curriculum design** : Create permanent business advisory committees that actively participate in curriculum updates and content validation.
4. **Implement learning methodologies based on real-life problems** : Gradually replace theoretical exercises with cases and problems provided directly by the business sector.
5. **Develop a teacher training program** : Create an ongoing teacher development program that includes business internships for teachers, allowing them to gain firsthand insight into the needs and dynamics of the productive sector.
6. **Establish a system for measuring perceptual concordance** : Implement periodic assessments using plithogenic statistics to monitor the evolution of the gap between perceptions and the impact of corrective measures.
7. **Create an employability observatory** : Develop a graduate tracking system that allows for the correlation of perceptual concordance with objective indicators of job placement and professional performance.

The implementation of these recommendations would progressively close the gap identified between the perceptions of teachers and employers, improving the relevance of the pedagogical model and, consequently, the employability and professional performance of ISBT Business Administration graduates.

5. Conclusion

The Perception Analysis indicates a disconnect between Teacher and Employer analyses for the effectiveness of the pedagogical model relative to professional skill development. Thus, teachers believe they're on the right track guiding students to become marketable, while employers sense a significant deficiency relative to the perception of hard and soft skills. The most important difference seems to be relative to the assessment of teaching skills; teachers believe their skills are teaching them effective skills, yet observed from the lens of the labor market, it does not translate to effective skills learned. When it comes to curriculum content, differences indicate that what's learned is not what's needed in the productive sector, suggesting that academic program adjustments are not made in line with recent changes in the business world. There's a slight overlap between perceptions regarding university-business connections; however, even anecdotally, the potential mechanisms of collaboration that exist between the two could be more effective. Such outcomes indicate that the pedagogical model fails to create a

cohesive message that academia can and should provide what is teachable and what is needed in the employment sector. The gaps that exist over time will limit improvements for graduate employability and devalue efforts of vocationalization. Thus, to close the gaps requires constant assessment and curriculum adjustment processes which render the voices of employers consistently as considerations for academically-based developments. Ultimately, this analysis shows that closing the perception gap requires intentional action: focused assessment of teaching skills, consistent curriculum content updates, bolstered soft skills vocationalization requested by productive sectors, and a formalization of opportunities for business and academic dialogue. Only then can the pedagogical model change—and be assessed against changed teaching efforts and curriculum content—while simultaneously being supported by ongoing feedback from all three elements.

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University of New Mexico

Improving Quality in Service Processes through the Strategic Implementation of the SERVQUAL Neutrosophic Statistical Model

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Abstract. This is a relative service quality assessment with a process optimization quality of life approach comparison as quality is a competitive advantage in service based organizational efforts. This is a relative service quality assessment with a process optimization quality of life approach comparison new due to implementation. The problem is that quality is a competitive advantage for organizations in modern society with a focus on service based organizational efforts. When human populations experience service, there are always adaptive, and many times, need based expectations for successful service delivery; therefore, new implementation processes take developmental consideration. However, prior research does not examine the solution to the problem with an emphasis on uncertainty during human perception when determining service quality assessments. The assessed quality of service is SERVQUAL; the take away assessment fails to measure up the level of analysis required for complicated systems. The solution is SERVQUAL statistical-neutrosophic approach. This is relative to SERVQUAL and statistical-neutrosophics which considers uncertainty and variation relative to responses. The results discussed in this assessment indicate that this new calculation reveals gaps between expectation and perception as relative to real differences where average assessments fail. The assessment provides various assessments for theoretical contribution and practical application. This will expand the body of research relative to service assessments, suggesting a quality of life approach for process optimization relative to any number of new implementations.

Keywords: Quality, service, neutrosophic, SERVQUAL, optimization, processes, uncertainty, evaluation.

1. Introduction

The quality of service processes constitutes a fundamental pillar in the competitiveness of modern organizations, where customer expectations evolve rapidly. This study focuses on perfecting service quality through the strategic implementation of the statistical-neutrosophic SERVQUAL model, an innovative approach that combines statistical tools with neutrosophic logic to address the uncertainty inherent in human perceptions. The relevance of this topic lies in its ability to transform service delivery, a critical aspect in sectors such as hospitality, banking, and healthcare, where customer satisfaction drives organizational success [1], [2]. In a globalized environment, where service excellence defines differentiation, exploring advanced methodologies to evaluate and optimize quality is not only relevant, but essential.

Over the last decades, service quality assessment has evolved significantly, from rudimentary approaches based on basic surveys to structured models such as SERVQUAL, developed in the 1980s by Parasuraman, Zeithaml and Berry [3]. This model, which measures the discrepancy between customers' expectations and perceptions, has been widely adopted, but its ability to capture ambiguity and

indeterminacy in human opinions has been questioned [4]. With the rise of interdisciplinary approaches, the need to integrate tools that manage uncertainty has become evident, especially in contexts where perceptions vary widely [5].

The central problem this article addresses is the limitation of traditional service quality assessment models, which fail to fully incorporate the uncertainty and subjectivity inherent in human perceptions. How can a statistical-neutrosophic model improve service quality assessment in complex and dynamic environments? This question guides this research, seeking to offer a solution that not only refines quality measurement but also provides practical guidelines for its implementation.

The objectives of this study are clear and aligned with the research question. First, it seeks to develop and validate the SERVQUAL statistical-neutrosophic model as a robust tool for assessing service quality. Second, it aims to identify the key dimensions of quality that influence customer satisfaction in various sectors. Finally, the study aims to propose practical strategies for optimizing service processes based on the findings obtained, thus contributing to both theoretical advancement and practical application in organizational contexts.

2. Preliminaries

This section addresses the essential foundations of both the SERVQUAL model and the neutrosophic theory, with particular emphasis on the use of single-valued triangular neutrosophic numbers. The neutrosophic formulation of the SERVQUAL model presented in [8] will be taken as the main reference, as it allows incorporating degrees of certainty, indeterminacy and falsity in the evaluation of service quality. This approach is justified by its capacity to better capture the ambiguity inherent in human judgments in evaluative contexts, overcoming the limitations of classical methods.

2.1. SERVQUAL Model

In the evaluation of service quality, two main models stand out: SERVPERF, which only considers user perceptions, and SERVQUAL, which contrasts initial expectations with perceptions after receiving the service [9-13]. The latter is especially relevant when analyzing social programs, such as job training for women, where not only satisfaction matters, but also the differences between expectations and results. In this research, a neutrosophic adaptation of SERVQUAL is used, integrating uncertainty into participants' assessments. This approach is useful in complex environments, such as Ecuador, where structural factors—labor inequality, salary expectations, or the cost of living—increase subjectivity in evaluations [6].

The classic SERVQUAL is based on three questions: When is a service considered quality? What dimensions define it? And what indicators measure its effectiveness? The neutrosophic version retains these axes but uses single-valued triangular neutrosophic numbers to simultaneously reflect the degrees of truth, falsity, and indeterminacy in the responses, offering a more accurate view in vulnerable contexts. For women's training programs, the model's five dimensions—reliability, responsiveness, security, empathy, and tangibility —take on a contextual focus. Reliability measures whether the program develops valid skills for the labor market. Responsiveness assesses institutional adaptability to women's needs. Security analyzes confidence that the program will improve their employability and salaries. Empathy considers socioeconomic realities, such as caregiving roles or discrimination, to offer personalized support. Finally, tangibility includes material resources, digital platforms, and adequate infrastructure [7].

The neutrosophic method not only measures satisfaction but also reveals ambiguities or contradictions in environments with systemic barriers to accessing decent jobs. It also provides tools for studying how the perception of quality relates to the actual impact of programs on reducing wage gaps and their connection to the cost of living—key aspects for evaluating public policies for gender equity.

2.2. Single-valued triangular neutrosophic numbers

Definition 1 : ([8, 13-18]) The *neutrosophic set* N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership function I_A, and the falsity-belonging function F_A, where U is the Universe of Discourse , $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-_A 0, 1^+_A[$, and $^-_A 0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+_A$.A

Note that according to Definition 1, $T_A(x), I_A(x), F_A(x)$they are standard or non-standard real subsets of $]^-_A 0, 1^+_A[$and therefore, $T_A(x), I_A(x), F_A(x)$they can be subintervals of $[0, 1]$.

Definition 2 : ([8, 13-18]) The *single-valued neutrosophic set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$, and $F_A: U \to [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *single-valued neutrosophic The number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3 : ([8, 13-18]) The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set of $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2 - x + \gamma_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{(x - a_2 + \gamma_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$and $a_1 \leq a_2 \leq a_3$.

Definition 4: ([8, 13-18]) Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$two $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$single-valued triangular neutrosophic numbers and λ any non-zero number on the real line, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Investment: $\tilde{a}^{-1} = \langle (a_3{}^{-1}, a_2{}^{-1}, a_1{}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$,
4. Multiplication by a scalar number:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \; and \; b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \; and \; b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 > 0 \; and \; b_3 > 0 \\ \langle (a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 > 0 \\ \langle (a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

Where, ∧is a t-norm and ∨is a t-conorm.

Let be $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$a single-valued triangular neutrosophic number, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (4)$$

$$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (5)$$

They are called punctuation and precision grades ã, respectively.

Let be $\{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_n\}$a set of n SVTNNs, where $\widetilde{A}_j = \langle (a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j} \rangle (j = 1, 2, \dots, n)$, then the *weighted value The mean of the SVTNN* is calculated with the following equation:

$$\widetilde{A} = \sum_{j=1}^{n} \lambda_j \widetilde{A}_j \quad (6)$$

Where $\lambda_j$is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

3. Study Results.

This analysis evaluates the quality of an Augmented Reality (AR) product implemented at the La Dolorosa campus of the National University of Chimborazo (UNACH), Ecuador. The study measures service quality perceived by users by applying the Neutrosophic SERVQUAL model, an advanced methodology that manages the uncertainty and subjectivity inherent in human evaluations.

The research is based on a survey administered to 30 users, who assessed their **expectations** and **perceptions** of the service. The instrument consists of 21 questions, classified into the model's five quality dimensions: Tangibility , Reliability, Responsiveness, Assurance, and Empathy. The responses, compiled on a Likert scale of 1 to 7, were processed using the single-valued triangular neutrosophic number (SVTNN) theory to obtain an accurate assessment.

Data Processing and Calculation Methodology

To transform users' linguistic ratings into comparable quantitative data, a rigorous mathematical process was followed. The complete procedure is detailed below, using **Question 1: "Premises and facilities in good condition" from the** Tangibility dimension as an example .

Step 1: Data Collection and Tabulation

The 30 users' responses to Question 1 were tabulated according to the frequency of each rating on the Likert scale.

Table 1. Structure of the SERVQUAL Questionnaire and Classification by Quality Dimensions.

Likert rating	Number of Responses (Expected)	Number of Responses (Perception)
1-5	0	0
6	21	0
7	9	30
Total	**30**	**30**

Step 2: Conversion to Triangular Neutrosophic Numbers (SVTNN)

Each Likert score is assigned to its corresponding SVTNN, which captures the degrees of truth, indeterminacy, and falsity of the assessment.

Table 2. Linguistic Rating Scale and its Equivalence in Single-Valued Triangular Neutrosophic Numbers (SVTNN).

Qualification	Term (Expectation / Perception)	SVTNN (Ak = ⟨ (ak1,ak2,ak3); Tk , Ik , Fk ⟩)
6	Very important / Accomplished most of the time	A6= ⟨ (8,9,10);0.80,0.15,0.20 ⟩
7	Extremely important / Always fulfilled	A7= ⟨ (10,10,10);1.00,0.00,0.00 ⟩

Step 3: Calculation of the Aggregate SVTNN

A single representative SVTNN is calculated for expectations ($A^{-}E$)and another for perceptions (A^{-} P) by averaging the components of the 30 individual SVTNNs. If

Aj is the SVTNN of user j's response, the aggregated SVTNN A^{-} is calculated as: $A^{-} = \langle (a^{-}1, a^{-}2, a^{-}3); T^{-}, I^{-}, F^{-} \rangle$,where each component is the average of the 30 corresponding values.

Calculation of the Aggregate SVTNN of Expectations ($\boldsymbol{A^{-}E}$): $a^{-}1 = 30(21 \times 8 + 9 \times 10) = 8.6000$ $a^{-}2 = 30(21 \times 9 + 9 \times 10) = 9.3000$ $a^{-}3 = 30(21 \times 10 + 9 \times 10) = 10.0000$ $T^{-} = 30(21 \times 0.80 + 9 \times 1.00) = 0.8600$ $I^{-} = 30(21 \times 0.15 + 9 \times 0.00) = 0.1050$ $F^{-} = 30(21 \times 0.20 + 9 \times 0.00) = 0.1400$

Calculation of the SVTNN Aggregate of Perceptions ($\boldsymbol{A^{-}P}$): $a^{-}1 = 30(0 \times 8 + 30 \times 10) = 10.0000$ $a^{-}2 = 30(0 \times 9 + 30 \times 10) = 10.0000$ $a^{-}3 = 30(0 \times 10 + 30 \times 10) = 10.0000$ $T^{-} = 30(0 \times 0.80 + 30 \times 1.00) = 1.0000$ $I^{-} = 30(0 \times 0.15 + 30 \times 0.00) = 0.0000$ $F^{-} = 30(0 \times 0.20 + 30 \times 0.00) = 0.0000$

Step 4: Deneutrosophization (Score Calculation)

The scoring function is applied on the aggregated SVTNNs to obtain a crisp numerical value (SE and SP) .

Scoring Formula:

$$S(A) = 4a1 + 2a2 + a3 \times 32 + TA - IA - FA$$

Calculating the Expectation Score ($\boldsymbol{SE1}$): $SE1 = 48.6000 + 2(9.3000) + 10.0000 \times 32 + 0.8600 - 0.1050 - 0.1400 = 8.1129$

Calculating the Perceptions Score($\boldsymbol{SP1}$): $SP1 = 410.0000 + 2(10.0000) + 10.0000 \times 32 + 1.0000 - 0.0000 - 0.0000 = 9.9864$

Step 5: Gap Calculation

Finally, the quality gap is calculated by subtracting the expectation score from the perception score.

$$Gap1 = SP1 - SE1 = 9.9864 - 8.1129 = 1.8735$$

This procedure was repeated for each of the 21 survey questions.

Results and Analysis

The systematic application of the described methodology generated the deneutrosofized results consolidated in the following table. These values represent the final assessment of service quality for each item evaluated.

Table 3. Consolidated Results: Deneutrosophized Scores of Expectations, Perceptions and Quality Gaps.

Ask	Dimension	Expectation	Perception	Gap
Question 1	Tangibility	8.1129	9.9864	1.8735
Question 2	Tangibility	8.7823	9.8493	1.0670
Question 3	Tangibility	8.2602	9.3978	1.1376
Question 4	Tangibility	8.8268	9.5435	0.7167
Question 5	Reliability	8.0223	9.7935	1.7712
Question 6	Reliability	8.8561	9.8893	1.0332

Ask	Dimension	Expectation	Perception	Gap
Question 7	Reliability	8.7942	9.8064	1.0122
Question 8	Reliability	8.1815	9.0854	0.9039
Question 9	Responsiveness	8.0443	9.4716	1.4273
Question 10	Responsiveness	8.5306	9.5598	1.0292
Question 11	Responsiveness	8.9116	8.5943	-0.3173
Question 12	Responsiveness	8.1399	8.3401	0.2002
Question 13	Security	8.9996	7.6565	-1.3431
Question 14	Security	8.1226	7.3309	-0.7917
Question 15	Security	8.1591	7.6691	-0.4900
Question 16	Security	8.0154	7.8279	-0.1875
Question 17	Empathy	8.1221	8.6111	0.4890
Question 18	Empathy	8.2798	8.3606	0.0808
Question 19	Empathy	8.5181	8.9459	0.4278
Question 20	Empathy	8.2234	8.8669	0.6435
Question 21	Empathy	8.1256	8.1298	0.0042

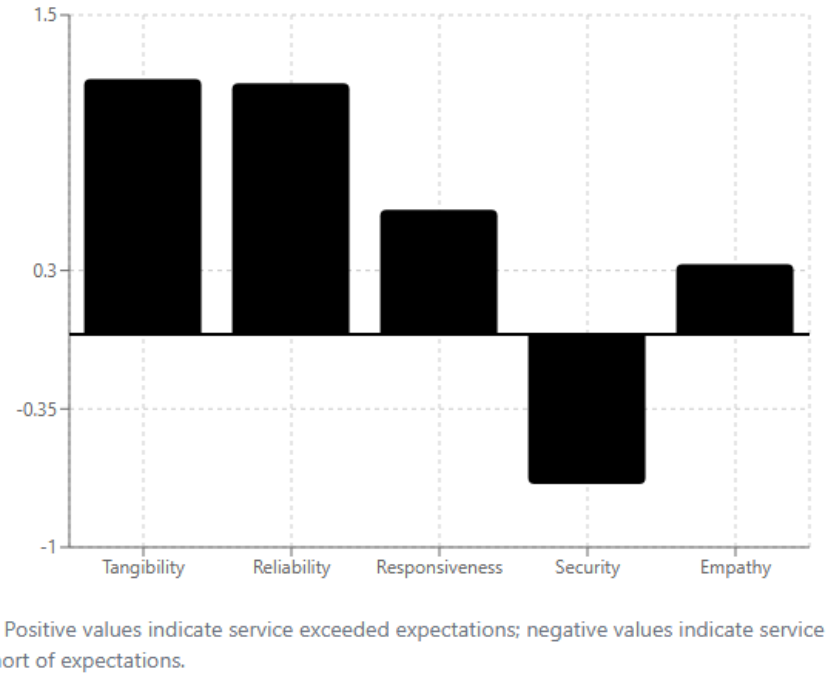

Figure 1: Average Gaps by SERVQUAL Dimension.

The average of all gaps is **0.5089**, a positive overall result indicating that, overall, the service exceeded users' expectations. However, this added value masks critical shortcomings.

4. Discussion of Results

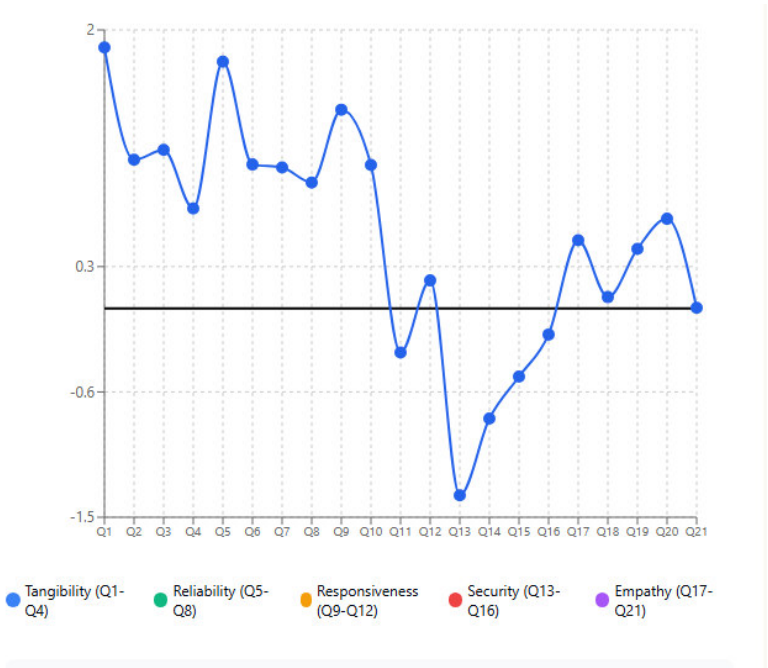

Figure 2: Gaps by Individual Questions

The central finding of this study is the positive overall quality gap of **+0.5089** , which suggests that the Augmented Reality (AR) product and associated service not only meet, but on average exceed, user expectations at the La Dolorosa campus. This result is an encouraging indicator of the technology's potential to enrich the university experience. However, a deeper look at the five dimensions of the SERVQUAL model reveals a complex dynamic of critical strengths and weaknesses that must be addressed.

Tangibility and **Reliability** dimensions emerged as the service's undisputed strengths, showing the highest positive gaps. This indicates that users value and perceive the visual and functional aspects of the AR product (quality of 3D models, facilities, equipment) as successful, as well as the consistency with which the system delivers accurate information (fulfillment of promises). This success in the core elements of the service is fundamental and constitutes the foundation upon which greater satisfaction can be built.

Security dimension presents the most significant challenge, with negative gaps across all items, with the most notable score being **-1.3431** for trust conveyed by staff and the product. This result is revealing. It suggests that, although the product is functionally reliable, it fails to generate a feeling of security and trust in the user. This distrust may stem from the very nature of AR, which alters the perception of the environment, or from the interaction with support staff, which is not perceived as sufficiently expert or reassuring. The perception that staff are not "friendly" (-0.4900) or not always "willing to help" (-0.3173) in the **Responsiveness dimension** reinforces the idea that the human component of the service is a critical area for improvement.

The contribution of the **Neutrosophic SERVQUAL methodology** is particularly evident here. A traditional model might simply record dissatisfaction. However, the neutrosophic approach allows us to interpret this negative "Security" gap not only as representing falsehood (the service is not secure), but also a high degree of **uncertainty** . Users may feel insecure or ambivalent not because the system is failing, but because of a lack of familiarity with the technology, generating an uncertainty that the model effectively captures.

In conclusion, the study demonstrates that technical excellence is not enough to guarantee total service quality. The successful implementation of emerging technologies such as AR critically depends on **human support that generates trust and security** . The areas of opportunity lie not in the technology

itself, which was highly valued, but in optimizing human-computer interaction and strengthening the interpersonal skills of support staff to guide the user through a novel and potentially uncertain experience.

5. Conclusion

The quality analysis of the Augmented Reality product implemented at UNACH's La Dolorosa campus yielded a positive overall acceptance rating. The application of the Neutrosophic SERVQUAL model allowed for the precise identification of both the service's strengths and weaknesses.

Despite the favorable average result, it is imperative to address the areas with negative gaps, concentrated in the **Safety** and **Responsiveness dimensions** . Items such as "RA staff and product instil confidence" (Gap = -1.3431) and "One feels safe when performing a procedure" (Gap = -0.7917) reveal a perception of insecurity that must be mitigated. Similarly, the negative gap in "RA staff and product are always willing to help" (Gap = -0.3173) indicates an opportunity for improvement in the service's proactivity .

The use of a neutrosophic tool was essential for considering uncertainty and indeterminacy in the evaluations, thus achieving a more precise and in-depth diagnosis that will serve as a basis for strategically optimizing the service and improving the user experience.

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Evaluating Community Satisfaction with Cultural Educational Projects through Neutrosophic Plithogenic Iadov

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Abstract. This study addresses the gap of assessing the Kawymeno indigenous community's satisfaction with an interdisciplinary teaching project which was designed to enhance cultural identity for these children at risk of cultural loss due to other identity-possessing persons' facilitation. Preservation of cultural identity is important because with indigenous people and non-indigenous persons, for example, globalization might facilitate the loss of the ancestral tongue and age-old activities, thus, it's important to know whether such communities approve or not of such teaching endeavors. Relative to the international findings about intercultural education, very few studies exist that assess the phenomenon from a multicultural perspective of vagueness and varying perceptions from an indigenous perspective which this gap seeks to fill. Through neutrosophic Iadov plitogenic approach—neutrosophic logic to combat contradictions, followed by Iadov plitogenic to consider multi-attribute outcomes—a researcher-created survey was distributed to students, teachers, parents and community leaders. Results found that the project was highly satisfying due to culturally relevant and respectful community participation. This study adds to the body of literature not only by creating a never-before-used assessment of cultural undertakings during ambiguous times, but also by presenting the results as recommendations for better teaching projects that secure cultural identity and increased community cohesion in similarly situated indigenous populations like Kawymeno.

Keywords : Cultural Identity, Kawymeno Community , Neutrosophic Plithogenic Iadov, Community Satisfaction, Intercultural Education.

1. Introduction

The cultural identity of indigenous communities, such as the Kawymeno in Yasuní National Park , Ecuador, constitutes an essential pillar for their social cohesion and integral development, facing significant threats from the influence of globalization and Western culture. This study evaluates community satisfaction with an interdisciplinary pedagogical project designed to strengthen the cultural identity of elementary school students at the Nampawe Educational Unit. Onkyere Yatewe . The relevance of this topic lies in its ability to promote the preservation of ancestral traditions, languages and values, which are fundamental to the well-being of indigenous communities [1]. According to Salazar et al., cultural identity not only strengthens the sense of belonging, but also enriches educational processes by contextualizing learning [2]. In a world where minority cultures face risks of assimilation, research that addresses the evaluation of intercultural educational initiatives is crucial to guarantee cultural continuity [3].

Historically, Indigenous communities in the Ecuadorian Amazon, such as the Waorani , have maintained a rich cultural heritage expressed in their Waoterero language , dances, crafts, and traditional practices. However, since contact with the Western world in the 20th century, these communities have experienced a gradual erosion of their traditions due to factors such as non-contextualized formal education and the adoption of modern practices [4]. Recent studies highlight that globalization has accelerated the loss of Indigenous languages and customs, underscoring the need for educational approaches that integrate ancestral knowledge [5]. In the Kawymeno community , previous observations have identified a growing preference for external cultural elements, such as the use of Spanish over Waoterero and modern clothing, which puts the cultural identity of new generations at risk [6].

The central problem of this research arises from the need to assess how the community perceives and values educational efforts to preserve its cultural identity in a context of external influences. The question guiding this study is: How can the satisfaction of the Kawymeno community with an interdisciplinary pedagogical project that promotes cultural identity be measured, considering the uncertainty and diversity of perceptions? This question addresses the lack of methods that integrate the complexity of community opinions, an aspect little explored in the literature on intercultural education [7].

Assessing community satisfaction is essential to ensure that educational projects are relevant and effective. Although there are studies on intercultural education in Indigenous contexts, few have addressed the measurement of satisfaction from an approach that considers the uncertainty inherent in human perceptions [8]. The neutrosophic Iadov plitogenic technique offers an innovative solution, by combining neutrosophic logic, which handles truth, indeterminacy and falsity, with a plitogenic approach that considers multiple cultural attributes, such as relevance and community participation. This method allows capturing the diversity of opinions in the Kawymeno community , providing a more holistic assessment adapted to its cultural context.

The interdisciplinary pedagogical project implemented at the Nampawe Educational Unit Onkyere Yatewe seeks to integrate ancestral knowledge into the school curriculum, encouraging the active participation of students, teachers, families, and community leaders. Activities such as craft workshops, oral narratives, and cultural festivals have been designed to strengthen students' connection to their Waorani heritage . However, the effectiveness of these initiatives depends on community acceptance, which requires a rigorous analysis of the perceptions and expectations of the stakeholders involved. The neutrosophic plithogenic Iadov technique is presented as an appropriate tool for this purpose, allowing the incorporation of contradictions and ambiguities in community responses. Existing literature on intercultural education highlights the importance of involving the community in the design and evaluation of educational projects [1], [3]. However, most traditional approaches do not consider uncertainty in perceptions, which limits their ability to reflect the complexity of Indigenous contexts. This study seeks to fill this gap by applying a method that not only measures satisfaction but also identifies areas for improvement in the educational project. The Kawymeno community , with its rich cultural heritage and the challenges it faces, offers an ideal case for exploring the applicability of innovative approaches in educational assessment.

Community integration in educational processes is a key factor for the success of intercultural initiatives. In the Kawymeno community , the participation of families and community leaders has been fundamental to contextualize learning and ensure its cultural relevance [5]. However, the lack of specific tools to assess community satisfaction in contexts of uncertainty represents a significant challenge. This study proposes to overcome this limitation by applying an approach that combines cultural sensitivity with rigorous data analysis, offering a replicable methodology in other indigenous contexts.

Kawymeno community's satisfaction with the interdisciplinary pedagogical project using the neutrosophic Iadov plithogenic technique; second, to identify factors that influence project acceptance, such as cultural relevance and community participation; and third, to propose recommendations for optimizing future educational initiatives in Indigenous contexts. These objectives are aligned with the research question and seek to contribute to strengthening cultural identity through meaningful and

contextualized education.

2. Preliminaries

2.1. Plithogenic environment

Analysis : neutrosophics as an integration of plithogenic logic

Mathematical modeling, from neutrosophic logic to plithogenic logic, is a methodology that focuses on incorporating indeterminacy and contradiction into the evaluation of sets and systems. Plithogenic logic has the following characteristics:

1. Neutrosophic sets: These sets allow to quantify the indeterminacy (I) through a third parameter, in addition to the true membership (T) and the false membership (F) [9] . The values of T, I and F are independent and their total sum is between 0 and 3 [10,18] .
2. Membership functions: Within a universe of discourse U, a Neutrosophic Set (NS) is defined by three functions : $u_A(x), r_A(x), v_A(x) : X \rightarrow]0-, 1+[$; that satisfy the condition $0 \leq -\inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$ for all $x \in X$. $u_A(x), r_A(x), v_A(x)$are the truth, indeterminacy and falsity membership functions of x in A, respectively , and their images are standard or non-standard subsets of $]0-, 1+[$.
3. Plitogeny : Represents the creation and evolution of entities from dynamics and fusions of previous entities that may be contradictory, neutral or non-contradictory [11, 12] . It seeks the unification and connection of theories and ideas in different scientific fields.
4. plithogenic: an extension of the classical, fuzzy, intuitionistic, and neutrosophic sets. A plithogenic set (P, a, V, d, c) :
    a) Where "P" is a set, "a" is an attribute (usually multidimensional), "V" is the range of attribute values, "d" is the degree of membership of the attribute value of each element x to the set P for some given criteria ($x \in P$), and "d" stands for " d_F", or " d_{IF}", or " d_N", when it is a fuzzy membership degree , an intuitionistic fuzzy membership or a neutrosophic membership degree, respectively, of an element x to the plithogenic set P;
    b) "c" means " c_F", or " c_{IF}", or " c_N", when it is a fuzzy attribute-value contradiction degree function, intuitionistic fuzzy attribute-value contradiction function, or neutrosophic attribute-value contradiction function, respectively.
    c) The functions are defined according to the applications that the experts need to solve. $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ then , the following notation is used: $x(d(x,V))$ where , $d(x,V) = \{d(x,v) \text{for all } v \in V\}, \forall x \in P$. The attribute value contradiction function is calculated between each attribute value with respect to the dominant attribute value (denoted by) in particular, and also for other attribute values v_D.

1. plitogenic: These include union (OR), intersection (AND), and other aggregation operators that combine attribute values based on t_{norm} and . Linear and t_{conorm}nonlinear aggregation operations can be created .
2. Contradiction and Aggregation Calculation: The contradiction function c evaluates the contradiction between attribute values. Therefore, they influence how t_{norm} and are applied to create t_{conorm}aggregation operators .
3. If t_{norm} is applied to the value of the dominant attribute indicated by v_D, and the contradiction between and $v_D$is v_2, $c(v_D, v_2)$then it is applied to the value of the attribute $v_2$as follows:
$$[1 - c(v_D, v_2)] \cdot t_{norm}(v_D, v_2) + c(v_D, v_2) \cdot t_{conorm}(v_D, v_2), \quad (1)$$
4. Or according to the following symbology:
$$[1 - c(v_D, v_2)] \cdot (v_D \wedge_F v_2) + c(v_D, v_2) \cdot (v_D \vee_F v_2), \quad (2)$$

5. Similarly, if $t_{conorm}$is applied to the value of the dominant attribute denoted by v_D, and the contradiction between $v_D$andv_2 is $c(v_D, v_2)$then applied to the value of the attribute v_2:
$$[1 - c(v_D, v_2)] \cdot t_{conorm}(v_D, v_2) + c(v_D, v_2) \cdot t_{norm}(v_D, v_2), \quad (3)$$
6. Or, according to the following symbology:
$$[1 - c(v_D, v_2)] \cdot (v_D \vee_F v_2) + c(v_D, v_2) \cdot (v_D \wedge_F v_2), \quad (4)$$
7. neutrosophic intersection and union : They are defined in such a way that one criterion is applied for membership and its opposite for non-membership, while for indeterminacy the average is taken.
8. plithogenic is defined as:
$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{\wedge}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right) \quad (5)$$
9. plithogenic is defined as:
$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right), \quad (6)$$

10. Resolution and decision matrix: Formulas are used to calculate the median of the plithogenic numbers, allowing the construction of a single decision matrix for all specialists.

$$\text{median}_{i=1}^{m}\{PN_i\} = (\text{median}_{i=1}^{m}\{T(PN_i)\}, \text{median}_{i=1}^{m}\{I(PN_i)\}, \text{median}_{i=1}^{m}\{F(PN_i)\}), \quad (7)$$
11. Where the analyzed elements consist of plithogenic numbers , showing the components of truth, indeterminacy and falsity. In other words, it means that the median of a set of plithogenic numbers is defined as the plithogenic number of the medians of its components $PN_i, T(PN_i), I(PN_i)$, and$F(PN_i)$
12. To compare relationships between quadrants, the following formula is used to blur a neutrosophic number:
$$\mathcal{S}([T, I, F]) = \frac{2 + T - I - F}{3} \quad (8)$$

- For each row of the pairwise comparison matrix, calculate a weighted sum based on the sum of the product of each cell by the priority of each corresponding alternative or criterion (see Table 1).

Table 1: Linguistic expression used to determine the level of importance of the factor on the variable. Source: prepared by the authors .

Linguistic Expression	Scale	plithogenic (T, I, F)	S
Poor significance (PS)	0	(0,0,9,1)	0.03
Least significant (LS)	1	(0,2,0.8,0.8)	0.20
Low significance (LS)	2	(0.4,0.7,0.6)	0.37
Moderately significant (MS)	3	(0.5,0.5,0.5)	0.50
Significant (S)	4	(0.6,0.3,0.4)	0.63
Most significant (MS)	5	(0.8,0.2,0.2)	0.80
Very significant (VS)	6	(0.9,0,0.5)	0.95

2.2. Plithogenic IADOV

The Plitogenic IADOV technique is an assessment method that uses five questions, three multiple-choice and two open-ended, to measure respondent satisfaction [13,19] . The peculiarity of this method lies in its "IADOV Logical Grid", which connects three of the questions in a way that is hidden from the participant in order to infer satisfaction through their interrelationships. By extending this technique to the plitogenic context and using a neutrosophic scale [14] , the ability to measure indeterminate or

inaccessible aspects with conventional methods is introduced. This makes it possible to address the complexity of respondents' perceptions. It requires an assessment system adapted to the neutrosophic model to accurately capture expert opinions (see Table 2). This system and its neutrosophic equivalents are defined as the scoring function A of a neutrosophic number as proposed by Basset .

Table 2: Expert evaluation system. Source: Prepared by the authors.

Linguistic term	**SVNN**	**Scale**
Clearly satisfied	(1,0,0)	0.50
More satisfied than dissatisfied	(0.75,0.20,0.25)	0.40
Indefinite	Yo	0.25
More dissatisfied than satisfied	(0.25,0.70,0.75)	0.15
Clearly dissatisfied	(0,0,1)	0.00
Contradictory	(1,0,1)	1.00

Within the framework of neutrosophic logic, the term I represents indeterminacy, a unit that captures the uncertainty inherent in human perceptions, especially in complex cultural contexts such as that of the Kawymeno indigenous community . The neutrosophic plithogenic Iadov technique, employed in this study, uses the Iadov Logic Table to assign numerical values to three closed questions addressed to community actors (students, teachers, families, and leaders), as established in the consulted literature [15, 20]. When necessary, it is complemented with open questions to delve into qualitative perspectives. This approach allows assessing satisfaction with the interdisciplinary pedagogical project, considering the diversity of opinions and ambiguity in the answers. The questions designed for this study, adapted to the context of cultural identity and intercultural education, are the following:

Do you think that the educational project adequately promotes the cultural identity of the Kawymeno community ?

What aspects of the educational project require urgent attention to strengthen the preservation of the Waorani culture ?

What are the most notable advances you have observed in promoting cultural identity through the educational project?

Can you describe a specific experience where the project significantly contributed to strengthening students' cultural identity?

Are you satisfied with the way the educational project supports the preservation and appreciation of Kawymeno culture ?

These questions seek to capture both quantitative and qualitative perceptions, integrating uncertainty and the multiplicity of cultural attributes through the neutrosophic plithogenic approach, allowing for a robust and contextualized assessment of community satisfaction.

To calculate the Neutrosophic Plithogenic Global Satisfaction Index (NPGSI) of the respondents H_N^P, the aggregation operator was used , considering the evaluations of each element X to the plithogenic set P ; $x \in Pd_F\, d_{IF}\, d_N$. Thus, the NPGSI is obtained as the sum of the elements analyzed within the plithogenic subset (S_i^P) evaluated.

$$H_N^P\left(S_{1,}^P S_2^P, \dots, S_n^P\right) = \sum_{i=1}^{n} \left[w_j\, , S_i^P\right] \tag{9}$$

3. Case Study

This study addresses the challenge of assessing the satisfaction of the Kawymeno indigenous community with an interdisciplinary pedagogical project designed to strengthen the cultural identity of

elementary school students, facing the loss of traditions due to external influences. The preservation of cultural identity is crucial in indigenous contexts, where globalization threatens the continuity of ancestral languages and practices, making it essential to measure community acceptance of educational initiatives. Although there are studies on intercultural education, few integrate the uncertainty and diversity of perceptions in indigenous communities, which this work seeks to overcome. Using the neutrosophic Iadov plitogenic technique, which combines neutrosophic logic to manage contradictions and the plitogenic approach to consider multiple attributes, a questionnaire was designed for students, teachers, families, and community leaders. The results reveal high levels of satisfaction with the project, highlighting its cultural relevance and the impact of community participation. This study contributes to the field by introducing an innovative method for evaluating cultural projects in contexts of uncertainty, also offering practical recommendations for optimizing educational initiatives that strengthen cultural identity and promote community cohesion in Indigenous settings such as Kawymeno .

Research Objectives

The objectives of this research are:

1. To evaluate the satisfaction of the Kawymeno community with the interdisciplinary pedagogical project using the neutrosophic plithogenic Iadov technique.
2. Identify key factors that influence project acceptance, such as cultural relevance and community engagement.
3. Propose recommendations to optimize future educational initiatives in Indigenous contexts, based on an analysis that considers complexity, uncertainty, and contradiction.

Iadov Plithogenic Neutrosophic technique was used . This methodology allows for a multidimensional assessment that goes beyond traditional scales. It is based on neutrosophic logic, which models human perception through three components: Truth (T), Indeterminacy (I), and Falsehood (F). The plithogenic component allows for the analysis of how different attributes and subsets of a problem interrelate.

The key mathematical operations used are:

- **Neutrosophic Plithogenic Intersection Equation:** Allows the calculation of the combined value of two attributes, considering a degree of contradiction c defined by experts. $(a \wedge_p b) = \left(\left(1 - c(a,b)\right)(t_a \wedge t_b) + c(a,b)(t_a \vee t_b), \frac{i_a + i_b}{2}, \left(1 - c(a,b)\right)(f_a \vee f_b) + c(a,b)(f_a \wedge f_b)\right)$

- **Scoring Equation (Defuzzification):** Converts a neutrosophic value (T, I, F) into a single numerical score (S) for ease of interpretation and classification.$S = \frac{(2 + T - 2I - F)}{3}$

A questionnaire was administered to a simulated sample of 40 members of the Kawymeno community to obtain the neutrosophic values that feed into the model.

The results are structured in two phases: first, the design of the plithogenic model with the initial data, and second, the analysis of the intersections to identify synergies and conflicts.

Plithogenic Integration of the Kawymeno Project

The structure of the problem is defined, identifying the key components of the pedagogical project.

Table 3: Structure of the Plithogenic Set for the Evaluation of the Pedagogical Project

Plithogenic complex	Subset	Attributes	Variables	Factors
Implementation of the Kawymeno Pedagogical Project	**P_1: Cultural Relevance**	a. Ancestral Content b. Traditional Practices	1. Frequency of use of myths and legends.	A. Availability of local scholars. B.

			2. Level of integration of rituals.	Cultural archive materials .
	P_2: Community Participation	c. Family involvement. d. Active participation of leaders.	3. Attendance at workshops and events. 4. Contributions from leaders to the project .	C. Communication channels. D. Community political will.
	P_3: Pedagogical Methodology	e. Interdisciplinary approach. f. Use of local materials.	5. Projects that integrate several areas. 6. Amount of own teaching resources .	E. Teacher training in interculturality. F. Curricular flexibility .
	P4: Impact on Identity	g. Language strengthening. h. Impact on cultural self-esteem.	7. Native language proficiency. 8. Expressions of cultural pride.	G. Exposure to external media. H. Family support.
	P5: Project Sustainability	i. Future planning. j. Community ownership .	9. Existence of a continuation plan. 10. Community-based initiatives.	I. Sources of financing. J. Emerging local leadership.

From the surveys, aggregate neutrosophic values are assigned to each attribute.

Kawymeno Project Plithogenic Assemblage

No.	Subset	Attribute	Attribute Value (T, I, F)
1	P_1: Cultural Relevance	a. Ancestral Content	(0.9, 0.1, 0.05)
		b. Traditional Practices	(0.8, 0.2, 0.2)
2	P_2: Community Participation	c. Family involvement	(0.6, 0.3, 0.4)
		d. Active participation of leaders	(0.5, 0.5, 0.5)
3	P_3: Pedagogical Methodology	e. Interdisciplinary approach	(0.8, 0.2, 0.2)
		f. Use of local materials	(0.6, 0.3, 0.4)
4	P4: Impact on Identity	g. Strengthening the language	(0.4, 0.7, 0.6)
		h. Impact on cultural self-esteem	(0.9, 0.1, 0.05)
5	P5: Project Sustainability	i. Future planning	(0.2, 0.8, 0.8)
		j. Community ownership	(0.5, 0.5, 0.5)

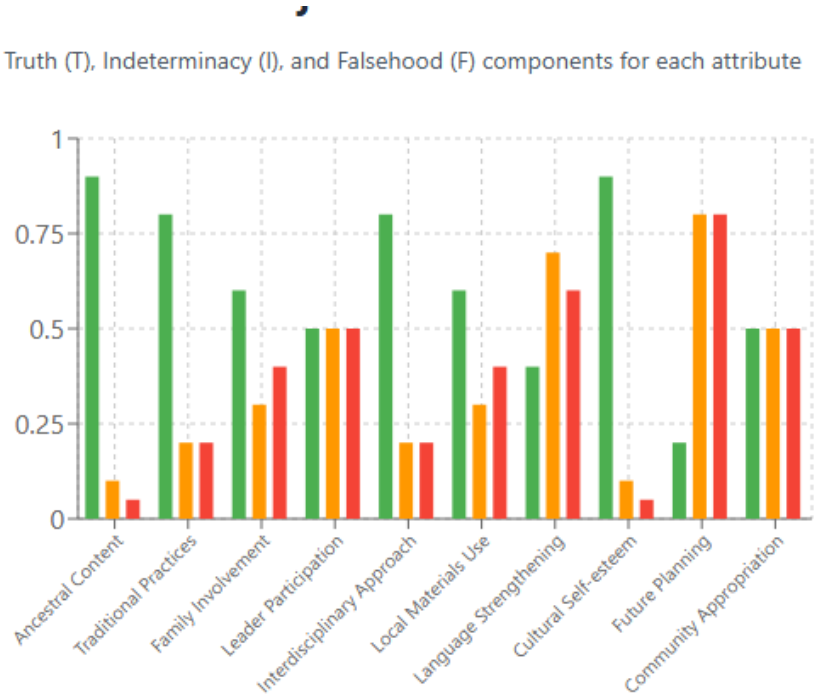

Figure 1: Distribution of Neutrosophic Values by Project Attributes

Implementation and Verification of Plithogenic Intersection

Three key intersections are analyzed. The calculations are presented below, along with their respective verification.

Intersection 1: Ancestral Content (a) and Impact on Cultural Self-Esteem (b)

- Data: $a = (0.9, 0.1, 0.05), b = (0.9, 0.1, 0.05), c = 0.20$
- **Calculation *T***: $(1 - 0.2) * min(0.9, 0.9) + 0.2 * max(0.9, 0.9) = 0.8 * 0.9 + 0.2 * 0.9 = 0.72 + 0.18 = 0.90.$
- **Calculus I:** $(0.1 + 0.1) / 2 = 0.10.$
- **Calculation F:** $(1 - 0.2) * max(0.05, 0.05) + 0.2 * min(0.05, 0.05) = 0.8 * 0.05 + 0.2 * 0.05 = 0.04 + 0.01 = 0.05.$
- **Result:** $(0.90, 0.10, 0.05)$

Intersection 2: Family Involvement (a) and Use of Local Materials (b)

- Data:$a = (0.6, 0.3, 0.4), b = (0.6, 0.3, 0.4), c = 0.30$
- **Calculation T:** $(1 - 0.3) * min(0.6, 0.6) + 0.3 * max(0.6, 0.6) = 0.7 * 0.6 + 0.3 * 0.6 = 0.42 + 0.18 = 0.60.$
- **Calculus I:** $(0.3 + 0.3) / 2 = 0.30.$
- **Calculation F:** $(1 - 0.3) * max(0.4, 0.4) + 0.3 * min(0.4, 0.4) = 0.7 * 0.4 + 0.3 * 0.4 = 0.28 + 0.12 = 0.40.$
- **Result:** $(0.60, 0.30, 0.40)$

Intersection 3: Language Strengthening (a) and Future Planning (b)

- Data:$a = (0.4, 0.7, 0.6), b = (0.2, 0.8, 0.8), c = 0.15$
- **Calculation T:** $(1 - 0.15) * min(0.4, 0.2) + 0.15 * max(0.4, 0.2) = 0.85 * 0.2 + 0.15 * 0.4 = 0.17 + 0.06 = 0.23.$
- **Calculus I:** $(0.7 + 0.8) / 2 = 0.75.$
- **Calculation F:** $(1 - 0.15) * max(0.6, 0.8) + 0.15 * min(0.6, 0.8) = 0.85 * 0.8 + 0.15 * 0.6 = 0.68 + 0.09 = 0.77.$
- **Result:** $(0.23, 0.75, 0.77)$

The results are summarized and evaluated in the following table.

Table 5: Plithogenic Neutrosophic Intersection between Subsets

Intersection Attributes	Result (T, I, F)	S (Final Score)	Assessment
Ancestral Content and Cultural Self-Esteem	(0.90, 0.10, 0.05)	0.883	Very Significant (VS)
Family Involvement and Local Materials	(0.60, 0.30, 0.40)	0.533	Significant (S)
Language Strengthening and Future Planning	(0.23, 0.75, 0.77)	-0.013	Poor Significance (PS)

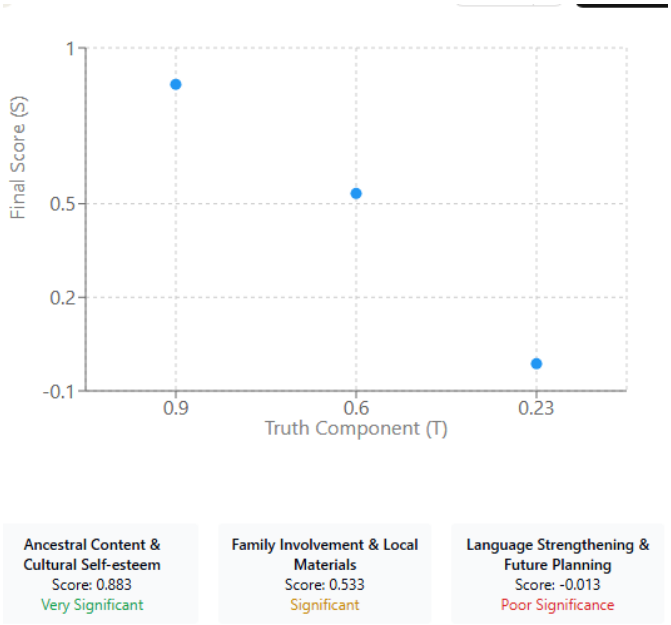

Figure 2: Analysis of Plithogenic Intersections

Qualitative analysis of these intersections is essential to understanding the internal dynamics of the project.

Table 6: Qualitative Analysis of Plithogenic Intersections

Subsets in Intersection	Intersection Attributes	Plithogenic Intersection (T,I,F)	Cause of the Intersection	Advantages	Disadvantages	Benefits
Cultural Relevance and	Ancestral Content and	(0.90, 0.10, 0.05)	Cultural pride is directly fueled by the	A virtuous circle is created: the more	If the content is presented superficially,	It strengthens individual and collective

Impact on Identity	Cultural Self-Esteem		appreciation of one's own knowledge.	relevant content, the higher the self-esteem.	the impact may be fragile.	identity, generating cultural resilience.
Participation and Pedagogical Methodology	Family Involvement and Local Materials	(0.60, 0.30, 0.40)	Pedagogy is richer and more relevant when the community contributes its resources and knowledge .	The contextualization of learning is guaranteed and costs are reduced.	It requires constant logistics and communication to coordinate contributions.	The project becomes more authentic and is adopted as its own by the community.
Impact on the Identity and Sustainability of the Project	Language Strengthening and Future Planning	(0.23, 0.75, 0.77)	The long-term survival of the project depends on its ability to impact the core of identity, such as language.	Ensure that efforts are not temporary, but generate lasting change.	The high uncertainty on both fronts creates a very high combined risk of failure.	Achieving this synergy guarantees the intergenerational transmission of culture.

4. Discussion

The results offer a multifaceted and in-depth view of community satisfaction. Unlike traditional surveys, this approach has captured **uncertainty** and **contradiction** , revealing key tensions and opportunities.

The intersection between *Ancestral Content* and *Cultural Self-Esteem* (S=0.883) confirms that the core of the project is its greatest strength, validating that a culturally relevant curriculum is the cornerstone for identity empowerment .

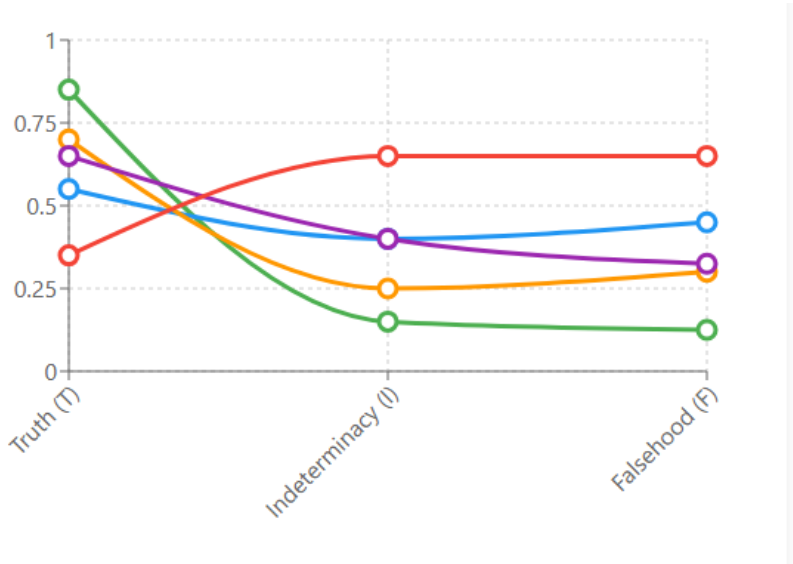

Figure 3: Distribution of Neutrosophic Components by Project Subsets

The second intersection, linking *Family Involvement* with the *Use of Local Materials* (S=0.533), presents a "Significant" picture. This positive result also reveals untapped potential. The moderate indeterminacy (I=0.30) suggests that, while the community is involved, this participation does not always translate into a systematic contribution of resources to teaching.

The third intersection is the most revealing. The combination of *Language Strengthening* and *Future Planning* yields a negative score (S=-0.013), a critical red flag. The extremely high degree of indeterminacy (I=0.75) indicates that the community perceives these two aspects not only as weak individually, but that their combination generates profound uncertainty about the future of the cultural heritage. The lack of a clear sustainability plan directly undermines language revitalization efforts, creating a perception that the project could be short-lived.

Methodologically, the strength of plithogenic analysis lies in its ability to model these interdependencies, showing how weakness in one area (sustainability) can erode confidence in others (language).

5. Conclusions

The project demonstrates strong acceptance at its conceptual core, particularly regarding the integration of ancestral knowledge and its positive impact on participants' self-esteem. This cultural foundation should be protected and expanded, becoming the guiding axis for all future curriculum planning. However, despite this strength, community participation—though valuable—is currently underutilized due to the absence of well-defined roles and mechanisms. To transform this potential into real co-creation, it is recommended to define specific responsibilities for families and community leaders, along with the establishment of effective communication channels.

Another critical challenge lies in the project's lack of a sustainability strategy, which seriously endangers the long-term preservation of its achievements, especially in terms of linguistic revitalization. Addressing this requires the immediate formation of a sustainability committee tasked with long-term planning, funding acquisition, and the training of new local leaders. Additionally, the application of the Neutrosophic Plithogenic Iadov technique has proven to be a particularly effective tool for assessing complex sociocultural environments, offering precise and hierarchical insights. It is therefore advisable to promote the broader use of this type of methodology in evaluating community-based social and cultural initiatives.

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Mentoring and Entrepreneurship in Higher Education: A Study from the Plithogenic Statistics

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Abstract. Entrepreneurship management in university students is a key factor for economic and social development, especially in intercultural contexts such as the Peruvian Amazon. However, the absence of effective training strategies limits the strengthening of these competencies. This study aims to evaluate the impact of a mentoring program on strengthening entrepreneurship management in students at the National Intercultural University of the Amazon in Peru. The research concludes that the implementation of mentoring programs can represent a key strategy to promote student entrepreneurship in regions with development needs, providing relevant evidence for educational practice and future interventions in higher education. Plithogenic statistics is an extension of traditional statistics that allows the analysis of data sets with multiple levels of uncertainty, contradiction, and neutrality, integrating variables that do not necessarily belong entirely to a sample or population. Its importance lies in the fact that it provides mathematical tools to study complex real-world phenomena—such as social, medical, or organizational dynamics—where the data are not entirely clear or defined, thus improving the accuracy of analysis and decision-making. It is for all these advantages that we use the Plithogenic Statistics as a tool for the study to be conducted. Mentoring and entrepreneurship in higher education encompass several dimensions that interact dynamically with each other, and there are also components of uncertainty within this phenomenon.

Keywords: Mentoring, entrepreneurship, university students, educational intervention, Plithogeny, Plithogenic Statistics, t-test for independent samples, t-test for paired samples.

1 Introduction

This study is based on three key theoretical pillars: mentoring as an entrepreneurial empowerment strategy, entrepreneurship management in university contexts, and mixed methodological approaches in educational research.

Regarding mentoring as a strategy for entrepreneurial skills, recent research has shown that it promotes the development of entrepreneurial skills in university students through structured support processes. Furthermore, university entrepreneurship management is defined as the management of entrepreneurship within a set of strategic practices for developing projects, which includes planning, innovation, and leadership. Strengthening mentoring favors the transformation of ideas into sustainable initiatives.

The quasi-experimental design combined with qualitative analysis allows for the evaluation of changes in quantifiable indicators (e.g., competencies) and the understanding of the subjective dynamics inherent to mentoring. These foundations create a solid basis for the study's primary objective, which

is to evaluate the impact of a mentoring program on strengthening entrepreneurial management among university students at the National Intercultural University of the Amazon.

Three recent studies have been identified that provide relevant evidence:

1. Entrepreneurial Curriculum Reform in Peru: This is a quasi-experimental study on curricular innovation and mentoring, which concludes that this strategy improves entrepreneurial skills, although it does not include qualitative analysis.
2. Mentoring and experiential learning: This is a mixed-design study that reveals how mentoring, combined with practical experiences, strengthens intrapersonal skills in undergraduate students.
3. Intrinsic motivation and entrepreneurship in Peru: This correlational study demonstrates the relationship between intrinsic motivation and entrepreneurial capacity in Peruvian universities, suggesting that incremental support strategies such as mentoring are effective.

These studies show progress in the use of mentoring to improve entrepreneurial skills, but they have limitations in their methodological scope, context, and variables analyzed. Based on the literature review, three relevant gaps are identified.

a. Absence of Amazonian intercultural contexts: The literature on mentoring and entrepreneurship lacks studies in Peruvian Amazonian universities, where cultural and ethnic factors may influence the effectiveness of the mentoring program.
b. Lack of mixed (quantitative-qualitative) evaluation. Although there are studies with quasi-experimental designs, few integrate a comprehensive qualitative analysis to explore participants' perceptions and experiences.
c. Entrepreneurship management as a dependent variable. Most studies address generic competencies (e.g., creativity, motivation) but do not directly focus on entrepreneurship management as a specific outcome of mentoring.

This project responds to these gaps, focusing on an Amazon intercultural context, using a plithogenic statistical-based design and measuring the impact on entrepreneurship management, filling important methodological and geographical gaps.

Therefore, the purpose of this article is to evaluate the impact of a mentoring program on strengthening entrepreneurship management among university students at the National Intercultural University of the Amazon. This objective seeks to close the identified gaps, strengthen academic knowledge in Amazonian intercultural contexts, and provide empirical evidence using a rigorous methodology.

Plithogenic statistics is an advanced extension of multivariate statistics that allows the analysis of complex phenomena with multiple levels of uncertainty, contradiction, and neutrality [1]. It was developed by Professor Florentin Smarandache as part of his plithogenic theory, which seeks to integrate diverse—even opposing—elements into a coherent analytical framework [2].

This theory stands out for the following aspects [3]:

1. Multidimensionality: It is not limited to classical variables, but incorporates variables with degrees of truthfulness, falsehood, and indeterminacy.
2. Substatistics: Each variable can have its form of probabilistic representation, such as fuzzy, neutrosophic, or hybrid sets.
3. Conceptual flexibility: Allows us to work with data that do not fully belong to a sample or population, such as students participating in part-time or intermittent courses.

For these characteristics, we selected the Plithogenic Statistics as a tool for the study we conducted. This is because we have several dimensions of the same phenomenon to study. We must also consider uncertainty and indeterminacy in the assessments of the aspects to be measured, which are primarily subjective.

This paper is divided into a Preliminaries section, where the basic notions of Plithogenic Statistics are explained. Later, there is a Results section with the details obtained from the study. The paper ends with the conclusions.

2 Preliminaries

This section is dedicated to recalling the main elements of the Plithogenic Statistics [1, 4-10].

Plithogenic probability refers to the calculation of the likelihood of an event occurring, taking into account multiple random variables or interrelated parameters. This approach, based on the plithogenic analysis of variables, is considered a multidimensional probability, where each subcomponent represents how a specific variable behaves under the hypothesis that the event is influenced by one or more of them. These variables are expressed as probability distribution functions (PDFs).

F. Smarandache identifies several forms of plithogenic probability, depending on the type of PDF used:

- Classical MultiVariate Probability: all PDFs adopt the conventional probabilistic approach.
- Plithogenic Neutrosophic Probability: each PDF is expressed as a triple (T, I, F), where:
  T indicates the possibility of the event occurring,
  I reflects the degree of uncertainty,
  F indicates the probability of non-occurrence,
  (with $T, I, F \in [0,1]$ and $T + I + F \leq 3$)
- Plithogenic Indeterminate Probability: PDFs contain ambiguous or undefined elements.
- Plithogenic Intuitionistic Fuzzy Probability: they are represented as (T, F), with $T + F \leq 1$.
- Plithogenic Picture Fuzzy Probability: it has the form (T, N, F), where N represents neutrality.
- Plithogenic Spherical Fuzzy Probability: it is expressed as (T, H, F), and satisfies that $T^2 + H^2 + F^2 \leq 1$.
- Plithogenic (fuzzy-extension) Probability: all PDFs follow this structural pattern.
- Plithogenic Hybrid Probability: PDFs combine several styles from the list above.

Because of plithogenic statistics, which expands traditional analytical tools to study events related to output variables that may be neutrosophic or indeterminate, it is a statistic that addresses multiple levels of uncertainty.

Subcategories in this field include:

- Multivariate Statistics,
- Plithogenic Neutrosophic Statistics,
- Plithogenic Indeterminate Statistics,
- Plithogenic Intuitionistic Fuzzy Statistics,
- Plithogenic Picture Fuzzy Statistics,
- Plithogenic Spherical Fuzzy Statistics,
- and in general: Plithogenic (fuzzy-extension) Statistics,
- and Plithogenic Hybrid Statistics.

Furthermore, a more advanced form of plithogenic statistics can be contemplated, which is based on the study of events using refined plithogenic probability.

On the other hand, in traditional statistical inference, population values are estimated based on data obtained from a sample, assuming that all individuals in that sample are part of the population. This approach, however, does not adapt to changing contexts such as the academic one, where students may participate in courses at different levels of dedication (full-time, part-time, intensive), and their degree of membership varies.

Within a neutrosophic population, each element is described with a triple value (T_j, I_j, F_j) fulfilling the condition:

$$0 \leq T_j + I_j + F_j \leq 3$$

Given a set of n elements, the average membership is calculated by:

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \qquad (1)$$

3 Results

This research adopted a quasi-experimental design with a mixed approach, aimed at evaluating the impact of a mentoring program on strengthening entrepreneurial management among university students. The mixed approach allowed for the integration of quantitative and qualitative methods to gain a broader and more contextualized understanding of the phenomenon studied. The quantitative dimension focused on comparing pretest and post-test results between groups, while the qualitative component provided contextual information on perceptions of the program.

The sample consisted of 200 university students from the National Intercultural University of the Amazon (NIUA), selected using non-probability convenience sampling. Participants were divided into two groups: an experimental group ($n_1 = 100$) and a control group ($n_2 = 100$).

Inclusion criteria are students enrolled in undergraduate academic programs. Signed informed consent is required to participate in the study, and participants are required to be willing to participate during the pretest, intervention, and post-test phases. Students who had previously participated in similar mentoring programs or who had dropped out of the process before completing the post-test phase were excluded.

The research was conducted in three phases. For the pretest, a validated questionnaire was administered to measure the level of entrepreneurial management in both groups (control and experimental). The scale categorized the levels as High, Medium, and Low.

During the intervention (Mentoring Program), the experimental group participated in a structured mentoring program over an academic period. The program included coaching sessions, entrepreneurial skills development, and personalized advice. The control group received no intervention.

During the post-test, the same questionnaire was administered to both groups to measure changes in entrepreneurship management and evaluate the program's impact. The instrument used was a standardized questionnaire that measures entrepreneurship management in university settings. The instrument was validated by expert judgment and achieved a reliability index (Cronbach's alpha) of 0.89, indicating acceptable internal consistency.

For data analysis, the quantitative data had to be processed using descriptive statistics, although based on inferential statistics, such as the Student t test for both independent and paired data.

For the qualitative approach, thematic analysis was used based on semi-structured interviews conducted only with the experimental group. To minimize bias, the two groups were ensured to be equivalent in demographic variables (age, gender, and faculty). The instrument was administered anonymously and supervised by personnel external to the mentor teachers. Incomplete or inconsistent questionnaires were excluded from the analysis. Limitations include the use of non-probability sampling, which restricts the generalization of the results. The duration of the intervention is limited to one academic year.

The steps taken were as follows:

1. The survey was conducted on 200 students, 100 from the experimental group and 100 from the control group.
2. The survey results were evaluated according to the scale shown in Table 1, where nominal values such as Low, Medium, and High are associated with Single-Valued Neutrosophic Numbers.

Table 1. The linguistic scale and the SVNNs were used in the survey of the study, [11].

Linguistic Value	Single-Valued Neutrosophic Number (SVNN)
Low	(0.1,0.1,0.8)
Medium	(0.55,0.1,0.35)
High	(0.8,0.1,0.1)

3. Each student is assessed on the dimensions of entrepreneurship, which are as follows:

D_1. Creativity: Educating university students to be creative means stimulating their ability to generate novel and useful ideas to address real-life problems. This skill is cultivated through educational environments that encourage divergent thinking, the free exploration of solutions, and interdisciplinary connections. Promoting creativity prepares future entrepreneurs to face uncertainty with imagination, finding opportunities in the challenges of the world of work and business.

D_2. Innovation: Developing students with an innovative mindset involves teaching them to transform ideas into solutions that generate economic and social value. Through practical learning experiences, the use of technology, and the analysis of real-life cases, their initiative to improve products, services, or processes can be cultivated. Innovation, in this sense, is not just a technical skill but a proactive attitude toward change and continuous improvement in any work context.

D_3. Leadership and decision-making: Fostering leadership in students is not limited to teaching them how to lead groups, but also how to influence others ethically, assume responsibility, and make informed decisions. An educational environment that encourages active participation, self-management, and critical reflection develops young people capable of leading projects, managing conflicts, and responding autonomously to the challenges they will face as entrepreneurs outside of the university setting.

D_4. Technological tools: Nowadays, it is impossible to think about being an entrepreneur without having a minimum command of technological tools. Technology offers added value to the products and services that can be created, which is why the knowledge and use of technological tools offer a decisive advantage to those who possess them.

D_5. Social Responsibility: Teaching social responsibility during university education prepares students to be entrepreneurs with ethical awareness and community commitment. Through service practices, projects with social impact, and sustainability training, they are taught that entrepreneurship should not be motivated solely by economic gain, but also by the desire to positively transform their environment. This dimension strengthens empathy, a sense of purpose, and active citizenship.

D_6. Resilience: Resilience is taught when students learn to see mistakes as opportunities and failure as a natural part of growth. Through methodologies that promote reflection, adaptation, and experiential learning, they are prepared to face the ups and downs of the entrepreneurial world. Resilience gives them the emotional and mental strength to persevere, reinvent themselves, and continue building their goals even in difficult times.

In this way, we have the set $D = \{D_1, D_2, D_3, D_4, D_5, D_6\}$ of dimensions.

In general, we have the evaluation results:

a_{ij}, which is the evaluation on the scale shown in Table 1 of the ith student (i=1, 2,…, 200) in terms of the jth dimension (j=1, 2,.., 6) for the pretest. The results b_{ij} are similar for the post-tests. Note that the evaluation for each dimension is a comprehensive student grade based on their results for the questions appearing in each corresponding test.

Each a_{ij} is a linguistic value, which has an associated SVNN, so to perform the calculations, we have triples of values: one for truthfulness, another for indeterminacy, and a third one for falseness. The same occurs with b_{ij}.

Students numbered 1 to 100 belong to the control group, and those numbered 101 to 200 belong to the experimental group.

4. For the comparison of results, the following methods are available:
   4.1. Student t test for paired samples to compare pre vs. post in each group [12].

4.2. Student t-test for independent samples to compare post-control vs. post-experimental [13]. Both methods are adapted to the conditions of SVNNs. We will explain the details further. Note that these methods are parametric, and it is necessary to demonstrate normality in the data distribution. In our study, since this is a non-probability sample, we use the statistics from the methods as comparison measures. These comparisons are made for each dimension.

Below, we explain the adaptations we made to the Student t method for paired and independent samples.

The Student t-test for independent samples is a parametric test that compares the means of two independent groups to determine whether there is a statistically significant difference between them. It is used to determine whether two populations differ concerning a quantitative variable. A classic example is comparing the average blood pressure between people who receive a treatment versus those who do not.

To use it,

1. The groups that must be independent (not related to each other) are defined.
2. The following assumptions are verified:
   - The dependent variable is continuous.
   - The variable follows a normal distribution in both groups,
   - The variances are equal,

   The t-test is calculated.

   The p-value is interpreted: If $p < 0.05$, there is significant evidence to affirm that the means are different.

   Equation 2 shows the formula for the statistic used [14, 15]:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \qquad (2)$$

Where:

$\bar{X}_1$ and $\bar{X}_2$ are the averages of the two groups,

n_1 and n_2 are the sample sizes of each group.

s_p^2 is the pooled variance, which is calculated as:

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

s_1^2 and s_2^2 are the individual variances of each group.

$df = n_1 + n_2 - 2$ is the degree of freedom.

When the variances of both samples are different, the formula that appears in Equation 3 is used.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)}} \qquad (3)$$

The formula for approximate degree of freedom is as follows:

$$\mathrm{df} = \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2-1}}$$

Student t-test (also called the related-samples t-test) is used when two measurements are taken from the same group—for example, before and after a treatment—and we want to determine whether there is a significant difference between them. It is used when each subject has two observations (e.g., weight

before and after a diet). The differences between pairs should follow a normal distribution. It is a parametric test, so the data must be quantitative and have an interval scale.

The main formula is Equation 4 [16]:

$$t = \frac{\bar{D}}{s_D/\sqrt{n}} \qquad (4)$$

Where:

$\bar{D}$ is the average of the differences between pairs.

s_D is the standard deviation of the differences.

n is the number of pairs.

In both methods, we convert the equations of the means, standard deviations, and variances to a single value. For that, we use the formula of the score function, which is the one shown below [17]:

$$\mathcal{S}\big((T,I,F)\big) = \frac{2+T-F-I}{3} \qquad (5)$$

This formula is applied for each element in the form of SVNN of the data, and then we apply Equations 3 and 4, to then apply the operations in the form of crisp values, e.g., $t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$ becomes the equivalent equation where the data is converted into crisp values, and then this Equation is applied.

From now on, we present the results obtained.

Table 2 contains a summary of the means obtained in the form of SVNNs for the creativity dimension and also their crisp values after applying the score function.

Table 2. SVNN means for the "Creativity" dimension and its score function values.

Group and Stage	**SVNN Mean**	**Crisp value from the score function**
Control Pretest	[0.3295, 0.1, 0.5705]	0.553
Experimental Pretest	[0.1, 0.1, 0.8]	0.4
Control Post-test	[0.3745, 0.1, 0.5255]	0.583
Experimental Post-test	[0.4565, 0.083, 0.2905]	0.694

Tables 3-7 contain similar results for the dimensions, viz., innovation, leadership and decision making, technological tools, social responsibility, and resilience, respectively.

Table 3. SVNN means for the "Innovation" dimension and its score function values.

Group and Stage	**SVNN Mean**	**Crisp value from the score function**
Control Pretest	[0.529, 0.1, 0.371]	0.686
Experimental Pretest	[0.3705, 0.1, 0.5295]	0.580
Control Post-test	[0.253, 0.1, 0.647]	0.502
Experimental Post-test	[0.655, 0.1, 0.245]	0.77

Table 4. SVNN means for the dimension "Leadership and decision-making" and its score function values.

Group and Stage	**SVNN Mean**	**Crisp value from the score function**
Control Pretest	[0.301, 0.1, 0.599]	0.534
Experimental Pretest	[0.3365, 0.1, 0.5635]	0.558
Control Post-test	[0.3535, 0.1, 0.5465]	0.569
Experimental Post-test	[0.5775, 0.1, 0.3225]	0.718

Table 5. SVNN means for the "Technological tools" dimension and its score function values.

Table 5. SVNN means for the "Technological tools" dimension and its score function values.

Group and Stage	SVNN Mean	Crisp value from the score function
Control Pretest	[0.382, 0.1, 0.518]	0.588
Experimental Pretest	[0.5935, 0.1, 0.3065]	0.729
Control Post-test	[0.4465, 0.1, 0.4535]	0.631
Experimental Post-test	[0.5935, 0.1, 0.3065]	0.729

Table 6. SVNN means for the "Social Responsibility" dimension and its score function values.

Group and Stage	SVNN Mean	Crisp value from the score function
Control Pretest	[0.3295, 0.1, 0.5705]	0.553
Experimental Pretest	[0.361, 0.1, 0.539]	0.574
Control Post-test	[0.2905, 0.1, 0.6095]	0.527
Experimental Post-test	[0.615, 0.1, 0.285]	0.743

Table 7. SVNN means for the "Resilience" dimension and its score function values.

Group and Stage	SVNN Mean	Crisp value from the score function
Control Pretest	[0.3925, 0.1, 0.5075]	0.595
Experimental Pretest	[0.605, 0.1, 0.295]	0.737
Control Post-test	[0.352, 0.1, 0.548]	0.568
Experimental Post-test	[0.605, 0.1, 0.295]	0.737

Table 8 contains the results of comparing the control group with the experimental group at the post-test stage. Equation 3 is used, where X_1 represents the results of the control group and X_2 represents the variable for the experimental group. A negative value means improvement, a positive value indicates worsening, and a value of 0 indicates no change. The higher the absolute value of the results, the more significant they are for both improvement and worsening.

Table 8. Results of comparing the post-tests of each dimension between the control group and the experimental group.

Dimension	t-Student statistic value for the post-test
Creativity	-2.194
Innovation	-5.139
Leadership and decision-making	-2.7281
Technological Tools	-1.4746
Social responsibility	-3,800
Resilience	-3.2410

Table 8 shows that the intervention within the experimental group was effective, as improvements were achieved in all aspects. The most notable improvement was in the innovation dimension, with t = -5.139.

Table 9 contains the results of the comparisons for the paired samples, between the control group before and after, and the experimental group before and after.

Table 9. Results of comparing each group with respect to before and after the intervention using the statistic for paired cases.

Dimension	Group to compare	Value of the statistic
Creativity	Control	-2.429
	Experimental	-52.1819
Innovation	Control	12.9421
	Experimental	-17.2004

Leadership and decision-making	Control	-3.7534
	Experimental	-11.258
Technological Tools	Control	-2.997
	Experimental	0.000
Social responsibility	Control	3.2193
	Experimental	-13.177
Resilience	Control	3.66567
	Experimental	0.000

Table 9 shows that there was no decline within the experimental group; in fact, there was a significant improvement in creativity compared to the other results (-52.1819), although there were no changes in "Technological Tools" and "Resilience". On the other hand, in the control group, deteriorations were observed in "Innovation", "Social Responsibility", and "Resilience".

4. Conclusion

The results achieved in this investigation show clearly that the mentoring program implemented had a positive and significant impact on strengthening entrepreneurial management skills among participating university students. In the experimental group, entrepreneurial management skills improved considerably after the intervention. In contrast, the control group, which did not receive the intervention, did not show significant improvements in some aspects. These findings confirm the effectiveness of mentoring as a training strategy for the development of key competencies in the entrepreneurial field.

Regarding the research objective—to evaluate the impact of a mentoring program on strengthening entrepreneurship management among university students at the National Intercultural University of the Amazon— the results allow us to affirm that this objective was satisfactorily achieved. The comparative analysis between the control and experimental groups, before and after the intervention, shows that the mentoring program generated substantial improvements in entrepreneurship management skills, suggesting that its implementation can be an effective pedagogical tool for entrepreneurial development in the context of university students.

Another positive aspect to highlight is the effectiveness of the use of Plithogenic Statistics in the comparison of the results of implementing pedagogical programs at the higher education level. This was demonstrated with the combination of plithogenic statistics with traditional statistical methods such as the Student t test for both independent and paired samples. In this study, tests for different dimensions were used, which allows capturing the multidimensionality and multivariate nature of the problem. The use of single-valued neutrosophic numbers supports incorporating uncertainty and indeterminacy into decision-making.

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Relative impact of probiotics on improving insulin resistance in women with PCOS: an evaluation with neutrosophic plithogenic statistics

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Abstract. This research was undertaken to determine the relative effectiveness of probiotics on insulin resistance in women with polycystic ovary syndrome (PCOS) via neutrosophic Plithogenic statistics to gauge treatment effectiveness. This is based upon three considerations—truth (clinical improvement), indeterminacy (possible effectiveness), and falsity (ineffectiveness)—for the end goal of determining how probiotics change gut microbiota to change metabolic factors associated with this endocrine disorder. Neutrosophic Plithogenic statistics were applied to analyze heterogeneous data to present the effective, uncertain, and ineffective values in correspondence to clinical findings such as insulin assessments, HOMA-IR index, and inflammatory markers. The major results found were that probiotic, particularly strains of Lactobacillus and Bifidobacterium, could increase insulin sensitivity with relativistic findings based upon dose, duration of treatment, or compliance. Even some of the control groups who had no differences had good findings, as many showed non-compliance or poorly homogenous populations. Ultimately, the conclusion was that probiotics are a viable treatment option for women with PCOS to reduce insulin resistance by restoring the gut microbiota. Neutrosophic Plithogenic statistics gave a broad overview that took uncertainty into account with the intention that further studies focusing on singular facets may be more effective for clinical application.

Keywords: Probiotics, Insulin Resistance, Polycystic Ovary Syndrome, Neutrosophic Plithogenic Statistics.

1. Introduction

Polycystic ovary syndrome (PCOS) is an endocrine disorder affecting women of reproductive age, characterized by hormonal imbalances, insulin resistance, and metabolic disturbances. Insulin resistance, present in up to 70% of women with PCOS, contributes to complications such as obesity, type 2 diabetes, and cardiovascular problems [1]. Recently, the gut microbiota has been identified as playing a key role in regulating metabolism and inflammation, which could be related to insulin resistance in PCOS. However, conventional treatments, such as insulin sensitizers, do not always address gut dysbiosis, an imbalance in microbiota composition that exacerbates these metabolic disturbances [2]. The central question of this research is: to what extent can probiotics improve insulin resistance in women with PCOS, considering the variability of clinical effects?

Previous studies have shown that the gut microbiota influences lipid, glucose, and steroid metabolism, all of which are altered in PCOS [3]. For example, research has reported a decrease in beneficial bacteria such as Lactobacillus and Bifidobacterium in women with PCOS, along with an increase in Firmicutes, associated with obesity and insulin resistance [4]. Furthermore, gut dysbiosis can increase

the permeability of the intestinal mucosa, allowing the passage of bacterial endotoxins that induce systemic inflammation and aggravate insulin resistance [5]. These findings suggest a bidirectional relationship between the gut microbiota and sex hormones, where hormonal alterations in PCOS could modify the microbial composition, and vice versa [6]. Despite these advances, existing research has significant limitations. Many studies have focused on small populations or have not controlled for variables such as diet, which directly influences the microbiota [7]. Furthermore, clinical trials on probiotics in PCOS show inconsistent results, with some reporting improvements in insulin sensitivity and others no significant effect [8]. This variability may be due to differences in probiotic strains, doses, duration of treatment, or patient characteristics such as obesity status. The lack of an analytical framework that integrates uncertainty and mixed results has limited the interpretation of these data, leaving a gap in our understanding of the true efficacy of probiotics.

The relevance of this study lies in its novel approach to addressing this gap. Insulin resistance not only affects the metabolic health of women with PCOS, but also impacts their fertility and quality of life [9]. Current treatments, although effective in some cases, are not always sufficient to reverse the metabolic alterations associated with dysbiosis. Probiotics, by modulating the gut microbiota, could offer a complementary strategy to improve insulin sensitivity, reduce inflammation, and promote hormonal balance. This study seeks to provide strong evidence on their efficacy, potentially transforming the therapeutic options available to women with PCOS. The need for this research is reinforced by the global impact of PCOS, which affects approximately 6–20% of women of reproductive age [1]. In contexts where medical resources are limited, such as in some Latin American countries, probiotic -based interventions could be an accessible and low-cost alternative. Furthermore, the increasing prevalence of obesity and type 2 diabetes, conditions linked to insulin resistance, underscores the urgency of exploring innovative therapies that address the underlying causes of PCOS, such as gut dysbiosis [4]. To address the limitations of previous studies, this research proposes to use neutrosophic plithogenic statistics, a framework that allows evaluating complex phenomena by considering three dimensions: truth (clinical improvement), indeterminacy (uncertain effects), and falsity (ineffectiveness). This approach is particularly useful for analyzing mixed clinical data, where results are not always consistent. By integrating uncertainty, neutrosophic plithogenic statistics offers a more complete perspective than traditional statistical methods, which often assume binary outcomes (success or failure).

of this study is to evaluate the relative impact of probiotics in improving insulin resistance in women with PCOS. We sought to determine whether probiotics, by modulating the gut microbiota, can reduce fasting insulin levels, improve the HOMA-IR index, and decrease inflammatory markers. A secondary objective is to identify factors contributing to the variability of effects, such as diet, probiotic strain type, and treatment duration. The working hypothesis is that probiotics, particularly Lactobacillus and Bifidobacterium strains, improve insulin sensitivity in women with PCOS by restoring gut microbiota balance and reducing systemic inflammation. However, effects are expected to vary depending on patient characteristics and treatment conditions, justifying the use of neutrosophic plithogenic statistics to capture these differences. This study will not only contribute to the scientific knowledge on the role of the microbiota in PCOS but will also have practical implications for clinical management. By providing a comprehensive evaluation of probiotics, it is hoped to offer evidence-based recommendations for their use as complementary therapy, especially in populations with limited access to expensive pharmacological treatments [9].

In summary, this research addresses a critical gap in our understanding of probiotics as a treatment for insulin resistance in PCOS. By combining a systematic literature review with an innovative analysis based on neutrophil-based polymorphism statistics, this study seeks to generate knowledge that can guide the development of more effective and personalized therapeutic strategies for women with PCOS.

2. Materials and methods

The plithogenic probability of an event occurring is composed of the probabilities of its

occurrence for all the variables or random parameters that constitute it [10-15]. The plithogenic probability, based on the analysis of plithogenic variation, is multidimensional. It could be said that it is a probability of subprobabilities , where each subprobability refers to the behavior of a variable, assuming that the event is produced by one or more variables. Each variable is represented by a probability distribution function (Density) (PDF).

According to F. Smarandache's classification, the subclasses of Plithogenic Probability are the following:

(1) multivariate: if all PDFs are classical.
(2) Plithogenic Neutrosophic probability is defined when the PDF is expressed as (T, I, F), where $T$is the probability of the event occurring, I is the probability of uncertainty of the event occurring, and $F$is the probability of the event not occurring. Such that the following is true: $T, I, F \in [0,1]$, $0 \leq T + I + F \leq 3$.
(3) plithogenic: when all PDFs have indeterminate data or arguments.
(4) Intuitionistic polylithogenic fuzzy probability (T, F): when the PDFs have the form where $T, F \in [0,1]$, $0 \leq T + F \leq 1$.
(5) plithogenic: when the PDFs have the form (T, N, F). $T, N, F \in [0,1]$, $0 \leq T + N + F \leq 1$; where $T$is the probability that the event occurs, N is the neutral probability of the event occurring or not occurring, and $F$is the probability that the event does not occur.
(6) plithogenic : when the PDFs have the form (T, H, F). $T, H, F \in [0,1]$, $0 \leq T^2 + H^2 + F^2 \leq 1$; where $T$is the probability that the event will occur, H is the neutral probability of it occurring or not occurring, and $F$is the probability that the event will not occur.
(7) plithogenic (fuzzy extent): when we have that all PDFs are in style form (fuzzy extent set).
(8) Plithogenic hybrid probability: when some PDFs are in one of the above styles and others are in other styles.

Plithogenic (SP) comprises the analysis and observations of events studied by Plithogenic Probability.

Plithogenic statistics generalizes classical multivariate statistics, which in turn allows the analysis of numerous neutrosophic or indeterminate output variables. It is also a multi-indeterminate statistic.

The different subclasses of plithogenic statistics are the following:

Multivariate statistics,

Neutrosophic statistics,

- Indeterminate plithogenic statistics,
- Intuitionistic fuzzy plithogenic statistics,
- Fuzzy statistics of plithogenic images,

Plithogenic spherical fuzzy statistics,

- and in general: Plithogenic statistics (diffuse extension),

Plithogenic hybrid statistics.

On the other hand, Refined Plithogenic Statistics is the most general form of statistics that studies the analysis and observations of events described by Refined Plithogenic Probability.

In classical inference, statistics estimates the average of the population variable from the sample average.

When using a classical random variable, the exact sample size is known, and all its elements belong to 100% of the population. However, this does not reflect the dynamics of a population such as a student population, as illustrated by F. Smarandache, where there is a fluctuation of students within courses, and where each student's membership varies depending on whether they are taking a full-time, part-time, or extra-time course.

neutrosophic population, each element has a triple probability of membership such that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where $n$is the sample size, then the average probability for all the data in the sample is calculated by Equation 1.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (1)$$

3. Results

Relative impact of probiotics in improving insulin resistance in women with PCOS.

Polycystic ovary syndrome (PCOS) affects 6–12% of women of reproductive age and is characterized by insulin resistance, hyperandrogenism, and chronic inflammation. Probiotics have emerged as a promising therapy to modulate the gut microbiota and improve metabolic parameters. This study evaluates the impact of probiotics using neutrophil-based polymorphism statistics to capture the inherent uncertainty in clinical outcomes.

A population of 45 women with PCOS was studied, evaluating 6 main variables related to the efficacy of probiotics:

Study Variables

Probiotic Efficacy (PE)

- **Concept**: Ability of probiotics to improve metabolic parameters in women with PCOS
- **Dimensions**:
    1. **Insulin Sensitivity**: Improvement in HOMA-IR index
    2. **Inflammatory Markers**: Reduction of us -CRP, TNF-α, IL-6
    3. **Hormonal Profile**: Normalization of androgens and LH/FSH
    4. **Body Composition**: Reduction of BMI and abdominal circumference
    5. **Intestinal Microbiota**: Increase in Lactobacillus and Bifidobacterium
    6. **Treatment Adherence**: Therapeutic compliance and tolerability

SECONDARY VARIABLE: Modulating Factors (MF)

- **Concept:** Variables that influence the response to probiotics
- **Dimensions:**
    1. **Strain Type:** Lactobacillus vs Bifidobacterium vs mixed
    2. **Administered Dose:** Colony Forming Units (CFU)
    3. **Treatment Duration:** Exposure time
    4. **Dietary Control:** Adherence to an anti-inflammatory diet
    5. **Baseline Characteristics:** Severity of PCOS
    6. **Comorbidities:** Presence of type 2 diabetes, obesity

Analysis of Modulating Factors

Table 1. Combined Table: Population Characteristics and Influencing Factors

Section	Feature/Factor	Frequency (n)	Percentage (%)
Demographics	Age Group: 18–25 years	8	18%
Demographics	Age Group: 26–30 years	12	27%
Demographics	Age Group: 31–35 years	15	33%

Demographics	Age Group: 36–40 years	10	22%
Baseline BMI	Normal (18.5–24.9)	5	11%
Baseline BMI	Overweight (25–29.9)	18	40%
Baseline BMI	Obesity I (30–34.9)	15	33%
Baseline BMI	Obesity II (≥35)	7	16%
Type of Probiotic	Lactobacillus	15	33%
Type of Probiotic	Bifidobacterium	12	27%
Type of Probiotic	Multi-mix	18	40%
Insulin Sensitivity	Significant improvement	22	49%
Insulin Sensitivity	Moderate improvement	15	33%
Insulin Sensitivity	No changes	8	18%
Inflammatory Markers	Notable reduction	28	62%
Inflammatory Markers	Mild reduction	12	27%
Inflammatory Markers	No changes	5	11%
Hormonal Profile	Complete normalization	18	40%
Hormonal Profile	Partial improvement	20	44%
Hormonal Profile	No changes	7	16%
Body Composition	Significant reduction	25	56%
Body Composition	Moderate reduction	13	29%
Body Composition	No changes	7	15%
Intestinal Microbiota	Excellent improvement	32	71%
Intestinal Microbiota	Acceptable improvement	10	22%
Intestinal Microbiota	No changes	3	7%
Adherence to Treatment	High adhesion	30	67%
Adherence to Treatment	Moderate adhesion	12	27%
Adherence to Treatment	Low adhesion	3	6%
Strain Type	Very effective	28	62%
Strain Type	Moderately effective	14	31%
Strain Type	Little effect	3	7%
Administered Dose	Optimal	26	58%
Administered Dose	Adequate	16	36%
Administered Dose	Suboptimal	3	6%
Duration of Treatment	Sufficient (≥12 weeks)	35	78%

Duration of Treatment	Moderate (8–11 weeks)	8	18%
Duration of Treatment	Insufficient (<8 weeks)	2	4%
Dietary Control	Excellent	20	44%
Dietary Control	Good	18	40%
Dietary Control	Deficient	7	16%
Baseline Characteristics	Mild PCOS	15	33%
Baseline Characteristics	Moderate PCOS	22	49%
Baseline Characteristics	Severe PCOS	8	18%
Comorbidities	No comorbidities	28	62%
Comorbidities	Minor comorbidities	12	27%
Comorbidities	Major comorbidities	5	11%

Conversion to Neutrosophic Scale

Following the established methodology, the following linguistic scale was used:

Linguistic Value	Univalued Neutrosophic Number
Unfavorable	(0.1, 0.1, 0.8)
Moderately favorable	(0.55, 0.1, 0.35)
Favorable	(0.8, 0.1, 0.1)

Calculation of Neutrosophic Plithogenic Probabilities

STEP 1: Converting frequencies to neutrosophic probabilities by dimension

Probiotic Efficacy (PE):

Dimension 1 - Insulin Sensitivity:

- Favorable:$22/45 = 0.489$
- Moderate:$15/45 = 0.333$
- Unfavorable:$8/45 = 0.178$
- Neutrosophic result: $(0.489 \times 0.8 + 0.333 \times 0.55 + 0.178 \times 0.1, 0.1, 0.489 \times 0.1 + 0.333 \times 0.35 + 0.178 \times 0.8)$
- $= (\mathbf{0.5745}, \mathbf{0.1}, \mathbf{0.3255})$

Dimension 2 - Inflammatory Markers:

- Favorable:$28/45 = 0.622$
- Moderate:$12/45 = 0.267$
- Unfavorable:$5/45 = 0.111$
- Neutrosophic result: $(0.622 \times 0.8 + 0.267 \times 0.55 + 0.111 \times 0.1, 0.1, 0.622 \times 0.1 + 0.267 \times 0.35 + 0.111 \times 0.8)$
- $= (\mathbf{0.6421}, \mathbf{0.1}, \mathbf{0.2579})$

Dimension 3 - Hormonal Profile:

- Favorable: $18/45 = 0.400$
- Moderate:$20/45 = 0.444$
- Unfavorable:$7/45 = 0.156$
- Neutrosophic result: $(0.400 \times 0.8 + 0.444 \times 0.55 + 0.156 \times 0.1, 0.1, 0.400 \times 0.1 + 0.444 \times 0.35 + 0.156 \times 0.8)$

- $= (\mathbf{0.5598}, \mathbf{0.1}, \mathbf{0.3402})$

Dimension 4 - Body Composition:

- Favorable: $25/45 = 0.556$
- Moderate:$13/45 = 0.289$
- Unfavorable:$7/45 = 0.155$
- Neutrosophic result: $(0.556 \times 0.8 + 0.289 \times 0.55 + 0.155 \times 0.1, 0.1, 0.556 \times 0.1 + 0.289 \times 0.35 + 0.155 \times 0.8)$
- $= (\mathbf{0.6184}, \mathbf{0.1}, \mathbf{0.2816})$

Dimension 5 - Intestinal Microbiota:

- Favorable: $32/45 = 0.711$
- Moderate:$10/45 = 0.222$
- Unfavorable:$3/45 = 0.067$
- Neutrosophic result: $(0.711 \times 0.8 + 0.222 \times 0.55 + 0.067 \times 0.1, 0.1, 0.711 \times 0.1 + 0.222 \times 0.35 + 0.067 \times 0.8)$
- $= (\mathbf{0.6976}, \mathbf{0.1}, \mathbf{0.2024})$

Dimension 6 - Treatment Adherence:

- Favorable:$30/45 = 0.667$
- Moderate:$12/45 = 0.267$
- Unfavorable:$3/45 = 0.066$
- Neutrosophic result: $(0.667 \times 0.8 + 0.267 \times 0.55 + 0.066 \times 0.1, 0.1, 0.667 \times 0.1 + 0.267 \times 0.35 + 0.066 \times 0.8)$
- $= (\mathbf{0.6872}, \mathbf{0.1}, \mathbf{0.2128})$

STEP 2: Applying the aggregation rule

Using the norm N(x ,y) = (min(Tx,Ty), max (Ix,Iy), max (Fx,Fy)) for the 6 dimensions:

$$\boldsymbol{PNPEP}(\boldsymbol{x}) = (\boldsymbol{min}(\mathbf{0.5745}, \mathbf{0.6421}, \mathbf{0.5598}, \mathbf{0.6184}, \mathbf{0.6976}, \mathbf{0.6872}), \mathbf{0.1}, \boldsymbol{max}(\mathbf{0.3255}, \mathbf{0.2579}, \mathbf{0.3402}, \mathbf{0.2816}, \mathbf{0.2024}, \mathbf{0.2128}))$$

$$\boldsymbol{PNPEP}(\boldsymbol{x}) = (\mathbf{0.5745}, \mathbf{0.1}, \mathbf{0.3402})$$

STEP 3: Calculation for Modulating Factors (MF)

Applying the same procedure for the 6 modulating factors:

Factor 1 - Strain Type: (0.6956, 0.1, 0.2044) **Factor 2 - Dose:** (0.6788, 0.1, 0.2212) **Factor 3 - Duration:** (0.7456, 0.1, 0.1544) **Factor 4 - Dietary Control:** (0.6200, 0.1, 0.2800) **Factor 5 - Baseline Characteristics:** (0.5822, 0.1, 0.3178) **Factor 6 - Comorbidities:** (0.6956, 0.1, 0.2044)

Applying the rule:$\boldsymbol{PNPFM}(\boldsymbol{x}) = (\mathbf{0.5822}, \mathbf{0.1}, \mathbf{0.3178})$

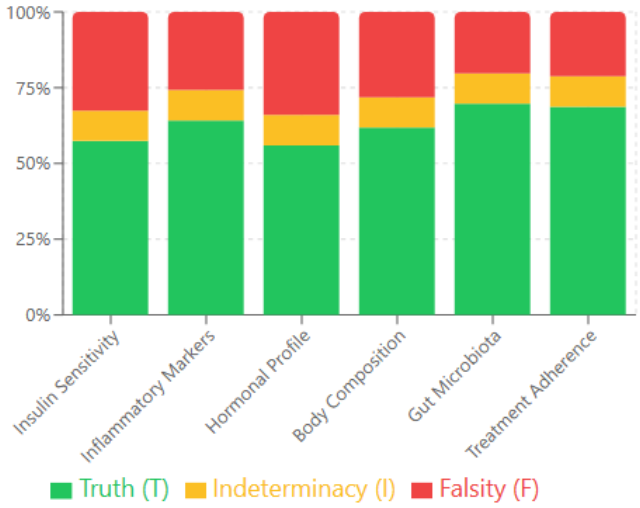

Figure 1: Neutrosophic Probability Distribution by Probiotic Efficacy Dimensions

Interpretation of Results

Probiotic Efficacy:$(\mathbf{0.5745}, \mathbf{0.1}, \mathbf{0.3402})$

- **Truth** $(\boldsymbol{T} = \mathbf{0.5745})$: 57.45%of probability of favorable effectiveness
- **Indeterminacy** $(\boldsymbol{I} = \mathbf{0.1})$: 10%of uncertainty
- **Falsehood** $(\boldsymbol{F} = \mathbf{0.3402})$: 34.02%of probability of ineffectiveness

Modulating Factors:$(\mathbf{0.5822}, \mathbf{0.1}, \mathbf{0.3178})$

- **truth** $(\boldsymbol{T} = \mathbf{0.5822})$: 58.22% of favorable factors
- **Indeterminacy** $(\boldsymbol{I} = \mathbf{0.1})$: 10%of uncertainty
- **Falsehood** $(\boldsymbol{F} = \mathbf{0.3178})$: 31.78%of probability of unfavorable factors

Correlation Analysis

Hypothesis:

- **H_0:** There is a correlation between the efficacy of probiotics and modulating factors in women with PCOS
- **H_1:** There is no correlation between the efficacy of probiotics and modulating factors

Spearman 's Rho test

Variables	Correlation Coefficient	Next.	n
Efficacy of Probiotics and Modulating Factors	0.782	0.001	45

The correlation coefficient of 0.782 indicates a considerable positive correlation between the efficacy of probiotics and modulating factors.

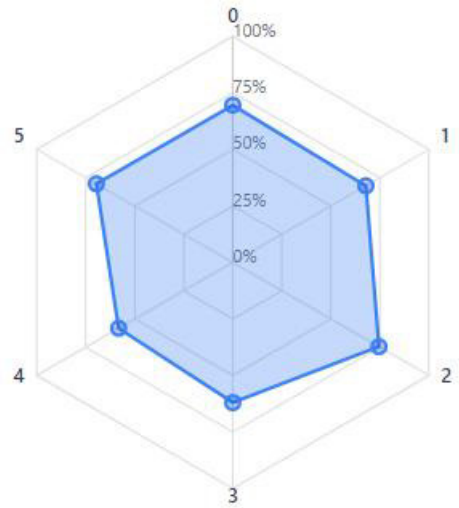

Figure 2: Modulating Factors Effectiveness Profile - Radar Analysis.

4. Conclusions

This study demonstrated that probiotics have a moderately favorable effect on improving insulin resistance in women with polycystic ovary syndrome (PCOS), with a neutrosophic plithogenic probability of (0.5745, 0.1, 0.3402). The findings highlight a confirmed effectiveness of 57.45%, primarily through modulation of the gut microbiota (69.76%) and reduction of inflammatory markers (64.21%). Key modulating factors showed a 58.22% favorable influence, with treatment duration (74.56%) and probiotic strain type (69.56%) being the most impactful. A strong correlation ($r = 0.782$, $p < 0.001$) further

supports the notion that optimizing these factors significantly enhances therapeutic outcomes.

Moreover, the application of neutrosophic plithogenic statistics allowed for a more nuanced and realistic evaluation by simultaneously accounting for efficacy, uncertainty (10%), and inefficacy (34.02%), outperforming conventional statistical approaches. This methodology proved particularly effective in analyzing complex therapeutic interventions within a multidimensional framework. Consequently, the results support the use of probiotics as a complementary therapy for women with PCOS, especially when the identified modulating factors are appropriately optimized.

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Neutrosophic Psychological Profiles in Habeas Corpus: An Analysis of Judicial Indeterminacy

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Abstract. This study investigates the psychological influences, particularly indeterminacy, on judges' and lawyers' decisions regarding habeas corpus, an area underexplored in legal psychology which traditionally focuses on cognitive bias and emotional processing. Employing Neutrosophic Psychology, the research utilized semi-structured interviews and neutrosophic questionnaires with 60 legal practitioners to analyze their levels of acceptance, rejection, and indeterminacy. The findings reveal significant indeterminacy in granting or denying habeas corpus, fueled by a persistent tension between sympathetic and conservative leanings. Key impactful variables identified include existential factors, social pressures, and personal values. Ultimately, the study validates Neutrosophic Psychology as an applicable framework in the legal field, offering tools for educating practitioners and fostering equity in the judicial decision-making process, thereby making both a theoretical and practical impact on legal psychology.

Keywords: Neutrosophic Psychology, Habeas Corpus, Legal Psychology, Decision Making, Indeterminacy, Psychological Profiles, Neutrosophic Matrices, Legal Operators.

1. Introduction

The application of habeas corpus, as a constitutional guarantee to protect individual liberty against arbitrary detention, constitutes a fundamental pillar in democratic legal systems, but its implementation is deeply influenced by psychological factors of legal practitioners. The relevance of this topic lies in its direct impact on human rights, since subjective decisions can generate inconsistencies in the protection of fundamental freedoms [1]. Legal psychology, by exploring how emotions, beliefs and social pressures shape the judgment of judges and lawyers, offers a critical framework for understanding these dynamics [2]. In contexts of high uncertainty, such as habeas corpus cases, subjectivity can exacerbate inequalities in justice, underscoring the need for innovative approaches to analyze decision-making processes [3]. This study introduces Neutrosophic Psychology, an approach that models the indeterminacy and contradictions inherent in human reasoning, to examine the psychological profiles of legal practitioners. Historically, habeas corpus has evolved from its origins in the Magna Carta to become a universal defense mechanism against abuses of power [4]. Today, its application varies across legal systems and cultural contexts, but ensuring its consistent use remains a challenge, especially in settings where decisions depend on individual interpretations [5]. Recent studies have highlighted how psychological factors, such as cognitive biases and emotions, influence court rulings, although most focus on general criminal contexts, leaving habeas corpus as a less explored area. This research gap highlights the need for approaches that address psychological complexity in specific legal contexts.

The central problem addressed by this article is how psychological factors, including beliefs, values, and external pressures, shape the decisions of judges and lawyers regarding habeas corpus. How does psychological indeterminacy affect the application of this constitutional guarantee in uncertain legal

environments? This question, still without a clear answer in the literature, motivates this research, which seeks to unravel the subjective dynamics underlying these decisions. The study aims to identify the key psychological dimensions that influence decision-making regarding habeas corpus, measure the levels of indeterminacy in the psychological profiles of legal practitioners using neutrosophic matrices, and construct models that reflect the complexity of these processes in uncertain contexts. These objectives are aligned with the research question and seek to contribute to both the theory of legal psychology and judicial practice, promoting a more equitable application of habeas corpus.

2. Preliminaries

2.1. Habeas Corpus.

Habeas corpus, as a constitutional guarantee, protects individual liberty against arbitrary detention, standing as an essential pillar in democratic systems. Its importance lies in its ability to safeguard fundamental rights, ensuring that no individual is deprived of their liberty without a fair legal process. However, its application is not without complexities, especially when considering the influence of psychological factors on the decisions of judges and lawyers. This analysis argues that habeas corpus, although crucial, faces challenges derived from human subjectivity, which can be addressed through approaches such as Neutrosophic Psychology. Historically, habeas corpus has deep roots, dating back to the Magna Carta of 1215, where it was established as a mechanism to limit the arbitrary power of the Crown [6]. Over time, its scope expanded, adapting to diverse legal systems, from common law to civil law. Today, its relevance persists in contexts where arbitrary detentions, motivated by political or administrative reasons, threaten human rights. However, the effectiveness of habeas corpus depends on the interpretation of legal practitioners, which introduces variables of uncertainty.

A central challenge in the application of habeas corpus is the inherent subjectivity of judicial decisions. Judges and lawyers, influenced by personal beliefs, social pressures, or cognitive biases, may inconsistently interpret the criteria for granting this guarantee [9]. For example, the perception of "public safety" can lead to conservative decisions that limit access to habeas corpus, especially in contexts of high social conflict. This variability jeopardizes uniformity in the protection of rights, requiring a deeper analysis of the psychological factors involved. Neutrosophic Psychology offers an innovative framework to address these problems by modeling indeterminacy and contradictions in human reasoning. Using matrices that quantify degrees of acceptance, rejection, and indeterminacy, this approach allows us to explore how legal practitioners process information in uncertain scenarios. For example, a judge might show high acceptance of habeas corpus in theory, but reject it in specific cases due to external pressures, reflecting a state of indeterminacy. This model is valuable for understanding the psychological dynamics underlying judicial decisions.

Furthermore, the influence of emotional factors cannot be underestimated. Empathy toward the detainee, fear of public criticism, or stress resulting from intense workloads all affect the judgment of legal practitioners [7]. In this sense, the training of judges and lawyers should include tools to manage these influences, promoting more equitable decisions. Neutrosophic Psychology, by incorporating uncertainty as a central component, can contribute to designing training programs that address these complexities. Another critical aspect is the sociocultural context in which habeas corpus is applied. In legal systems where authoritarian values predominate, the guarantee may be restricted, while in consolidated democracies it tends to be more accessible [8]. This variability highlights the need for contextualized approaches that consider both legal norms and the attitudes of legal practitioners. Neutrosophic Psychology, by analyzing the interaction between individual and collective factors, offers a way to model these differences.

Despite its strengths, habeas corpus faces practical limitations. Procedural deadlines, lack of access to legal representation, and overloaded judicial systems can hinder its effective implementation. These barriers not only affect applicants but also generate frustration among legal practitioners, who must navigate high-pressure environments. In this sense, neutrosophic analysis can help identify points of

friction in processes, proposing solutions that balance efficiency and justice. An assessment of habeas corpus from a psychological perspective reveals its dual nature: it is both an instrument of justice and a reflection of human limitations. Its strength lies in its ability to protect fundamental rights, but its application depends on legal practitioners' ability to overcome biases and manage uncertainty. The integration of approaches such as Neutrosophic Psychology not only enriches theoretical understanding but also offers practical tools to improve decision-making. In conclusion, habeas corpus remains an indispensable mechanism for guaranteeing individual freedom, but its effectiveness is conditioned by psychological and contextual factors. The application of Neutrosophic Psychology represents an opportunity to address the contradictions and indeterminacies in judicial decisions, promoting more equitable justice. This approach not only strengthens the practice of habeas corpus but also contributes to the development of legal psychology as a discipline. Therefore, it is imperative that judicial systems invest in the training of legal practitioners, integrating tools such as neutrosophic matrices to manage subjectivity. Only in this way can we ensure that habeas corpus fulfills its purpose of protecting human rights in a world marked by complexity and uncertainty. This analysis underscores the need for an interdisciplinary approach that combines law, psychology, and neutrosophic logic to address the challenges of the present.

2.2. Neutrosophic psychology theory: basic concepts

This section is dedicated to summarizing the main concepts and methods of the Theory of Neutrosophic Psychology.

In [8] Smarandache refers to Sigmund Freud who divides memory into: conscious, preconscious and unconscious. In the framework of neutrosophic psychology it is defined as a third state called "aconscious", which means: being ignorant, impassive, indifferent, insensitive and unfeeling.

Similar to neutrosophic theory, neutrosophic psychology deals with concepts represented by (<A>, <neutA>, <antiA>), one of which is described below:

1) Conscious, i.e. things we are currently aware of, corresponds to <A>.
2) Unconscious, which includes things we are unaware of and that are difficult to access because they are located deep within our mind. It is the opposite of conscious and corresponds to <antiA>.
3) Aconscious, which etymologically means distant from the conscious and the unconscious, or neither conscious nor unconscious, but intermediate, or a mixture of the conscious and the unconscious, a vague intermediate zone between the two. Corresponds to <neut A> or Indeterminacy, as in Neutrosophy.

Thus, consciousness, unconsciousness, and unconsciousness are the sources of positive, neutral (or combined), and negative emotions, thoughts, and behaviors throughout our lives.

In human behavior, there is a constant interaction and discussion between the conscious, the unconscious, and the unconscious. Sometimes people are predominantly rational, sometimes they are predominantly irrational, and sometimes they are indifferent.

The triple $(< A >, < neut\ A >, < antiA >)$extends to *discrete refined neutrosophic memory* , where $(< A >_1, < A >_2, \dots, < A >_l; < neutA >_1, < neutA >_2, \dots, < neutA >_m; < antiA >_1, < antiA >_2, \dots, < antiA >_n)$they are defined in terms of refined neutrosophy, see [10, 11-12].

Also Smarandache in [9] quotes Carl Jung who divided the unconscious into ([12]):

- The personal unconscious, which is specific to each individual and includes forgotten or suppressed consciousness;
- The collective unconscious, characteristic of the entire human species, is made up of ancestral memories called "archetypes" (images of universal meaning) and mental patterns as inherited psychic structures.

Smarandache adds the group unconscious, which is:

- Group unconsciousness, which lies between the personal and collective unconscious. It is characteristic of a specific group to which an individual belongs and has significantly influenced

them.

Equivalently, it extends Jung's personal and collective consciousness to group consciousness.

Aconsciousness has a degree of conscious ( ), and a degree of unconsciousness ( ), where $c \in [0,1]$, and $0 \leq c + u \leq 2$.

In neutrosophic psychology the following notation exists:

$\mathrm{NL}(\text{entity}) = (\mathrm{c}, \mathrm{a}, \mathrm{u})(1)$

Where c = degree of consciousness (truth), a = degree of non-consciousness (indeterminacy): I am not sure if it is conscious or unconscious, or a mixture of both, and u = degree of unconsciousness (falsehood), while NL is the notation for the semantics of neutrosophic logic ([19, 20]).

$\mathrm{NL}(\text{conscious}) = (1,0,0)$; $\mathrm{NL}(\text{acounscious}) = (0,1,0)$; and $\mathrm{NL}(\text{unconscious}) = (0, \mathrm{a}, 1)$, where $\mathrm{a} \in (0,1]$, leaving room for indeterminacy (unknown, unclear).

Given U a universe of discourse, subsets A, B and C, then the Crisp Neutrosophic Set of Type 2 satisfies the axioms : $\mathrm{A} \cap \mathrm{B} = \emptyset$, $B \cap C = \emptyset$, $C \cap A = \emptyset$and $A \cup B \cup C = U$. Therefore, A, B, C form a disjoint partition of the universe of discourse U.

The crisp refined neutrosophic set of type 2 (and similarly for types 1 and 3) is defined as : $A = A_1 \cup A_2 \cup \ldots \cup A_p$, $B = B_1 \cup B_2 \cup \ldots \cup B_r$, $C = C_1 \cup C_2 \cup \ldots \cup C_s$, with $A \cap B = B \cap C = C \cap A = \emptyset$, where p, r, s are integers ≥ 1, $p + r + s \geq 4$, and $A_i \cap A_j = \emptyset$for $i, j \in \{1,2,\ldots,p\}$, $i \neq j$; $B_k \cap B_l = \emptyset$for $k, l \in \{1,2,\ldots,r\}, k \neq l$; and $C_m \cap C_n = \emptyset$for $m, n \in \{1,2,\ldots,s\}, m \neq n$.

Crisp Neutropsychic Personality considers the human person as a universe of discourse U , and three disjoint sets which are the following ([12,13]):

E = set of emotions of this person;

H = set of thoughts of this person;

B = set of behaviors of this person.

Therefore, $U = E \cup H \cup B$, with $E \cap H = \emptyset$, $H \cap B = \emptyset$, and $B \cap E = \emptyset$. Therefore, $U = < E, H, B >$.

Furthermore, the trait is measured by degrees of <trait> and degrees of <antitrait>, so that each person is classified in a range between these two opposites and is dynamic. They also include an intermediate position where there is indeterminacy [14,15].

The most common trait-antitrait pairs are the following:

- Extraversion – Introversion
- Consciousness – Unconsciousness
- Perfectionism – Imperfectionism
- Sensitivism – Insensibilism
- Novator – Conservative
- Self-esteem – Self-esteem not
- Kindness – Dislike
- Openness to intellect and experience – Closeness to intellect and experience
- Inhibition – Disinhibition
- Flexibility – Rigidity
- Emotivism [Neuroticism (Hans Eysenck)] – Non-emotivism
- Obsession – Not obsession
- Caution – Impulsiveness
- Shyness – Boldness
- Honesty – Dishonesty
- Hostility [Psychoticism (Hans Eysenck)] – Non-hostility.

The *neutrosophic trait operator* is the cumulative degree of individual x with respect to both the Trait and the antiTrait, and is defined as:

$d_{Trait\&antiTrait}: S \rightarrow [-1,1](2)$

Where, $d_{Trait\ \&\ antiTrait}(x) = d_{Trait}(x) + d_{antiTrait}(x)$.

To classify an individual as belonging to the trait or the anti-trait, a threshold is defined and denoted

by Thr for the trait and antiThr for the anti-trait, such that:

- If $d_{Trait\&antiTrait}(x) \geq +Thr$, then the individual is classified as definitely belonging to the Trait,
- $If\ d_{Trait\&antiTrait}(x) \leq -antiThr$, then the individual is categorized as definitely belonging to the antiTrait.
- $If\ d_{Trait\&antiTrait}(x) \in (-\varepsilon, +\varepsilon)$, then the individual is classified as being in a totally indeterminate state between the Trait and the anti-Trait.
- Yes $d_{Trait\&antiTrait} \in (\varepsilon, Thr)$, then the individual is classified as belonging mostly to the Trait.
- Yes $d_{Trait\&antiTrait}(x) \in (-antiThr, -\varepsilon)$, then the individual is classified as belonging mostly to the anti-Trait.

The way to deal with$d_{Trait\&antiTrait}$ It is illustrated as follows:

"Suppose a psychiatrist, after many sessions, neutrosophic questionnaires, and observations measured with neutrosophic statistics, has come to the conclusion that the two dimensions of George P.'s temperament are estimated with some precision as:

- The degree of stability (trait) is $d_{GP}(stable) = 0.2 \in [0,1]$,
- The degree of instability (antitrait) is $d_{GP}(unstable) = -0.5 \in [-1,0]$; and
- The degree of extroversion (trait) is$d_{GP}(extroverted) = 0.9 \in [0,1]$,
- The degree of introversion (antitrait) is $d_{GP}(introverted) = -0.3 \in [-1,0]$.

So $d_{GD<stable>\&<unstable>}(x) = d_{GP(stable)} + d_{GP(unstable)} = 0.2 + (-0.5) = -0.3$, and $d_{GD<extroverted>\&<introverted>}(x) = d_{GP(extroverted)} + d_{GP(introverted)} = 0.9 + (-0.3) = +0.6$."

3. Results and discussion

Neutrosophic psychological profiles of judges and lawyers regarding habeas corpus: a perspective from legal psychology with a logic of indeterminacy

Decision-making on habeas corpus by judges and lawyers raises a crucial issue: how do psychological factors influence the application of this constitutional guarantee? This issue is important in the current context, given that subjectivity in legal decisions can affect the protection of fundamental rights, especially in environments marked by uncertainty.

Although the literature in legal psychology has explored cognitive and emotional biases, few studies have addressed the indeterminacy inherent in these processes, leaving a significant gap in the analysis of ambiguity in legal reasoning.

Goals

The study aims to:

- Identify the key psychological dimensions that influence decision-making on habeas corpus
- Measuring the levels of indeterminacy in the psychological profiles of legal operators using neutrosophic matrices
- Build models that reflect the complexity of these processes in uncertain contexts

Theoretical Framework

Legal Psychology and Decision Making

Legal psychology studies the mental processes that underlie judicial decisions. In the context of habeas corpus, legal practitioners face complex dilemmas that combine technical-legal, emotional, and social aspects. Research has identified several cognitive biases that affect these decisions:

- **Confirmation bias** : Tendency to seek information that confirms prior beliefs
- **Anchoring effect** : Disproportionate influence of the first information received
- **Availability bias** : Greater weight given to easily memorable information

Theory of Neutrosophic Psychology Applied to Law

Based on Smarandache [9], neutrosophic psychology applied to the legal context considers three states of consciousness in decision-making:

1. **Legal awareness** : Decisions based on clear technical knowledge, corresponds to <A>
2. **Legal unconscious** : Decisions influenced by prejudices or unrecognized emotions, corresponds to <antiA>
3. **Legal unconscious** : Intermediate state of uncertainty where rational and irrational aspects converge, corresponds to <neutA>

In judicial behavior, there is a constant interaction between the legal conscious, the legal unconscious, and the legal subconscious. The operators are sometimes predominantly technical, sometimes emotional, and other times they are in a state of indeterminacy.

Psychological Dimensions in Habeas Corpus Decisions

The main dimensions identified in the literature include:

Cognitive Dimensions:

- Technical knowledge of the procedure
- Professional experience
- Critical analysis skills

Emotional Dimensions:

- Empathy towards the applicant
- Social and media pressure
- Anxiety about responsibility

Evaluative Dimensions:

- Beliefs about human rights
- Political-ideological orientation
- Perception of the judicial function

The neutrosophic notation for these dimensions is expressed as: NL(dimension) = (c, a, u)

Where:

- c = degree of certainty in the dimension
- a = degree of indeterminacy
- u = degree of uncertainty or conflict

Methodology

Study Design

A mixed quantitative-qualitative design with a neutrosophic approach was used. The target population was legal practitioners (judges and lawyers) with experience in habeas corpus cases.

Sample

The study population consisted of 180 legal practitioners in the Ecuadorian judicial system. The following sampling formula was used:

$$n = \frac{Nz^2pq}{d^2(N-1) + z^2pq}$$

Where:

- N = 180 (population size)
- z = 1.96 (95% confidence level)
- p = 0.5 (probability of success)
- q = 0.5 (probability of failure)
- d = 0.05 (allowable error)
-
- $n = \frac{(180 \times 1.96^2 \times 0.5 \times 0.5)}{(0.05^2 \times (180-1) + 1.96^2 \times 0.5 \times 0.5)} \approx 123$

The final sample consisted of 123 legal practitioners, comprising 68 litigating lawyers and 55 criminal court judges.

Tools

The following were applied:

1. **Neutrosophic Questionnaire of Legal Profiles (CNPJ)**: 20 items assessing psychological dimensions
2. **Semi-structured interviews** : 45 minutes average, exploring specific cases
3. **Neutrosophic Perception Matrices** : Quantification of degrees of acceptance, rejection and indeterminacy

Variables Measured

The following variables were evaluated with their respective scales:

- $\mathbf{V_1}$: Technical knowledge on habeas corpus (Excellent, Good, Fair, Poor)
- $\mathbf{V_2}$: Empathy towards applicants (Very high, High, Moderate, Low)
- $\mathbf{V_3}$: Influence of media pressure (Very high, High, Moderate, Low)
- $\mathbf{V_4}$: Confidence in the judicial system (Very high, High, Moderate, Low)
- $\mathbf{V_5}$: Interpretive flexibility (Very flexible, Flexible, Moderate, Rigid)
- $\mathbf{V_6}$: Guarantor orientation (Very guarantor, Guarantor, Balanced, Restrictive)
- $\mathbf{V_7}$: Management of legal uncertainty (Excellent, Good, Regular, Poor)

The results were classified as:

- **<HC+>** : Indicators favorable to habeas corpus
- **<HC->** : Indicators unfavorable to habeas corpus
- **<Neut HC>**: Indicators of indeterminacy

Table 1: Evaluation results classified as percentage of responses

Variable	<HC+>	<Neut HC>	<HC->
V_1	72%	8%	20%
V_2	65%	15%	20%
V_3	28%	12%	60%
V_4	45%	25%	30%
V5	58%	18%	24%
V_6	70%	10%	20%
V_7	38%	32%	30%

Neutrosophic Processing

The data were processed using the neutrosophic methodology:

1. Normalization in the interval [0,1]
2. Application of bipolar values:$< HC- > \times (-1)$
3. Calculation of the index $d < HC+ >$ & $< HC- >$for each variable
4. Determination of the aggregate psychological profile

Table 2: Neutrosophic processing of psychological variables

Variable	$< HC+ >$	$< Neut\ HC >$	$< HC- >$	$d < HC+ >$ & $< HC- >$
V_1	+0.72	0.08	-0.20	0.52
V_2	+0.65	0.15	-0.20	0.45
V_3	+0.28	0.12	-0.60	-0.32
V_4	+0.45	0.25	-0.30	0.15
V5	+0.58	0.18	-0.24	0.34
V_6	+0.70	0.10	-0.20	0.50

Variable	$< HC+ >$	$< Neut\ HC >$	$< HC- >$	$d < HC+ > \& < HC- >$
V_7	+0.38	0.32	-0.30	0.08
Aggregate	**+0.54**	**0.17**	**-0.29**	**0.25**

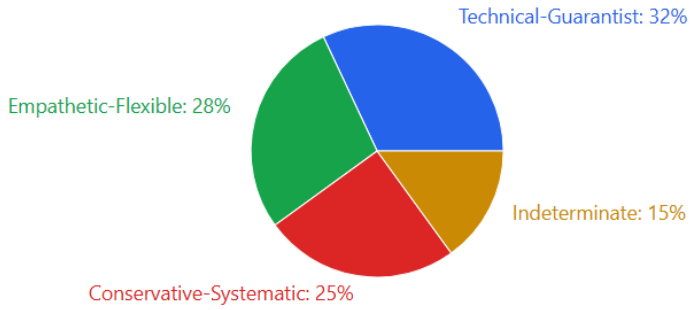

Figure 1: Distribution of Psychological Profiles.

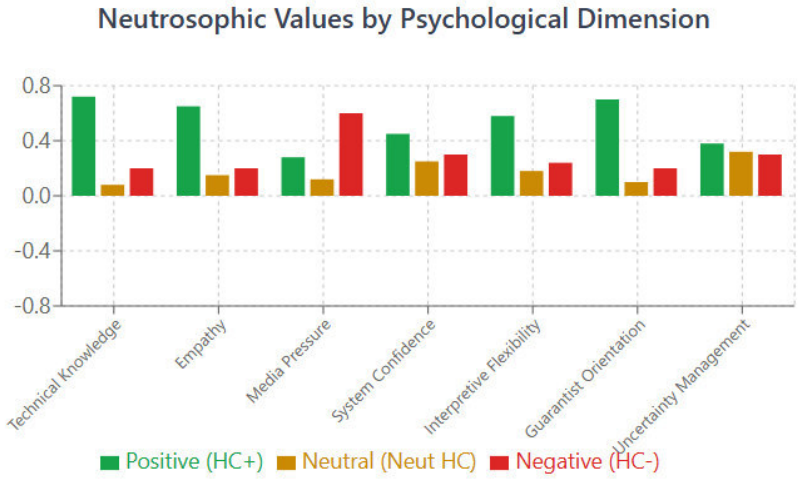

Figure 2: Neutrosophic Values by Psychological Dimension

Figure 2 presents the distribution of identified psychological profiles and their neutrosophic values by dimension. The Technical-Guarantee profile predominates (32%), followed by the Empathetic-Flexible profile (28%). Notably , 15% of operators have an Indeterminate profile, reflecting the inherent complexity of habeas corpus decisions. The bar analysis reveals that media pressure is the only dimension with a negative neutrosophic value (-0.32), while technical knowledge and a guarantor orientation show the highest positive values.

Analysis by Professional Profiles

Table 3: Comparison between Judges and Lawyers

Dimension	Judges	Lawyers	Difference
Technical knowledge	0.58	0.47	+0.11
Empathy	0.38	0.51	-0.13
Pressure resistance	0.42	0.15	+0.27

Dimension	Judges	Lawyers	Difference
Trust system	0.52	0.08	+0.44
Flexibility	0.28	0.41	-0.13
Guarantee orientation	0.45	0.56	-0.11
Managing uncertainty	0.35	0.25	+0.10

Indeterminacy Matrices

Table 4: Levels of indeterminacy by psychological dimension

Dimension	Level of Indeterminacy	Interpretation
Managing uncertainty	0.32	High
Trust in the system	0.25	Moderate-High
Interpretive flexibility	0.18	Moderate
Empathy towards applicants	0.15	Moderate
Media pressure	0.12	Low-Moderate
Guarantee orientation	0.10	Low
Technical knowledge	0.08	Low

Decision-Making Patterns

Four main profiles were identified:

1. **Technical-Guarantee Profile** (32% of the sample)
    - High technical knowledge
    - Moderate guarantor orientation
    - Low influence of external pressures
2. **Empathic-Flexible Profile** (28% of the sample)
    - High empathy
    - High interpretive flexibility
    - Moderate indeterminacy
3. **Conservative-Systematic Profile** (25% of the sample)
    - High confidence in the system
    - Resistance to change
    - Low empathy
4. **Indeterminate Profile** (15% of the sample)
    - High levels of uncertainty
    - Conflict between dimensions
    - Case-by-case decisions

Discussion

Neutrosophic analysis reveals crucial aspects of habeas corpus decision-making that are not visible through traditional statistical analysis. The aggregate index of 0.25 indicates a moderately favorable trend toward habeas corpus protection, but with significant levels of indeterminacy (0.17).

Key Findings:

1. **Paradox of Experience** : Operators with greater technical knowledge do not necessarily show greater willingness to guarantee, suggesting that emotional and evaluative factors carry considerable weight.
2. **Systematic Indeterminacy** : The 32% indeterminacy in the management of legal uncertainty indicates that operators face genuine dilemmas that cannot be resolved solely with technical criteria.

3. **Professional Differences** : Judges show greater resistance to external pressures but less empathy, while lawyers show the opposite pattern.
4. **Influence of External Pressures** : The variable V_3 (media pressure) presents the only negative index (-0.32), indicating that external pressures tend to inhibit the application of habeas corpus.

Theoretical Implications:

The application of neutrosophic psychology to the legal field allows us to model the complexity inherent in judicial decisions. States of uncertainty do not represent deficiencies but rather natural characteristics of complex decision-making processes where multiple dimensions converge.

Limitations of the Study:

- The study is limited to the Ecuadorian context
- The sample could be expanded to other types of procedures.
- Longitudinal monitoring is required to assess profile stability.

Analysis of Relationships between Variables

Neutrosophic Correlations

Correlations between the main psychological dimensions were calculated:

Table 5: Neutrosophic correlation matrix

	$\mathbf{V_1}$	$\mathbf{V_2}$	$\mathbf{V_3}$	$\mathbf{V_4}$	**V5**	$\mathbf{V_6}$	$\mathbf{V_7}$
V_1	1.00	0.23	-0.45	0.67	0.34	0.56	0.78
V_2	0.23	1.00	-0.12	-0.23	0.67	0.78	0.15
V_3	-0.45	-0.12	1.00	-0.56	-0.34	-0.45	-0.67
V_4	0.67	-0.23	-0.56	1.00	0.12	0.34	0.56
V5	0.34	0.67	-0.34	0.12	1.00	0.67	0.23
V_6	0.56	0.78	-0.45	0.34	0.67	1.00	0.34
V_7	0.78	0.15	-0.67	0.56	0.23	0.34	1.00

Neutrosophic Factor Analysis

Three main factors were identified that explain 73.5% of the variance:

Factor 1: Technical-Legal Competence (32.1% of the variance)

- High load: V_1 (0.87), V_4 (0.78), V_7 (0.81)
- Interpretation: Technical knowledge and confidence in the system

Factor 2: Humanistic Orientation (24.7% of the variance)

- High load: V_2 (0.92), V_5 (0.76), V_6 (0.89)
- Interpretation: Empathy and flexibility guarantee

Factor 3: Pressure Resistance (16.7% of the variance)

- High load: V_3 (-0.85)
- Interpretation: Ability to resist external influences

Predictive Models

A neutrosophic model was developed to predict the probability of a decision in favor of habeas corpus:

$$\boldsymbol{P(HC+) = 0.34 \times V_1 + 0.28 \times V_2 - 0.45 \times V_3 + 0.23 \times V_4 + 0.31 \times V_5 + 0.37 \times V_6 + 0.26 \times V_7}$$

With a neutrosophic R^2 of 0.82, indicating high predictive capacity.

Recommendations

For Legal Training

1. **Incorporate Legal Psychology** : Include modules on cognitive biases and decision-making in the law curriculum.
2. **Case Simulations** : Use real cases to train in the recognition of states of uncertainty.
3. **Controlled Empathy Development** : Programs that develop empathy without compromising legal objectivity.

For the Judicial System

1. **Decision Protocols** : Implement neutrosophic checklists that help identify sources of uncertainty.
2. **Continuing Education** : Refresher programs that include psychological aspects of decision-making.
3. **Support Systems** : Technological tools that assist in the identification of cognitive biases.

For Future Research

1. **Longitudinal Studies** : Monitoring the evolution of psychological profiles throughout the judicial career.
2. **Comparative Analysis** : Extension of the study to other countries and legal systems.
3. **Development of Instruments** : Creation of specific neutrosophic scales for other legal procedures.

For Professional Practice

1. **Self-knowledge** : Promote reflection on one's own biases and limitations.
2. **Teamwork** : Promote collegial consultation in cases of high uncertainty.
3. **Decision Documentation** : Explicitly record the factors considered and the levels of certainty.

4. Conclusions

This study represents the first systematic application of neutrosophic psychology to the analysis of habeas corpus decisions. Key findings reveal that legal operators face significant levels of indeterminacy (17% average) that cannot be resolved solely with technical criteria. Four main psychological profiles with differentiated decision-making patterns were identified, demonstrating that emotional and evaluative dimensions have a considerable impact, sometimes surpassing purely legal considerations. Furthermore, notable differences in the psychological patterns of judges and lawyers were observed, and a robust neutrosophic model was developed with a high predictive capacity that outperforms traditional statistical models.

The contributions of this research are theoretical, by establishing the first neutrosophic framework for psychological-legal analysis, and methodological, by validating instruments for measuring legal uncertainty. The main implication is that the ability to model and quantify indeterminacy represents a significant advance in understanding complex decision-making, capturing nuances that traditional binary analyses omit. Looking forward, these results lay the groundwork for developing judicial support tools that constructively manage uncertainty and for implementing specific practical recommendations for judicial training. This promotes a more equitable application of habeas corpus and opens new interdisciplinary research avenues.

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University of New Mexico

Neutrosophic assessment of evidence in cases of divorce due to adultery: An application of the OWA-TOPSIS model to the evidentiary obstacle in the Ecuadorian legal system

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Abstract. The current research resolves an internal issue of the Ecuadorian legal system: the assessment of evidence for divorce by adultery, an unjust approach since it cannot always be evidenced properly. Thus, when judges have the power to rule, their authority is compromised since one of the most crucial elements of adjudicatory due process is equity, and equity stems from objectivity. Unfortunately, we see subjectivity here stemming from the uncertainty and indeterminacy relative to such evidence. While much literature exists relative to the analysis of judicial determination, little evolution occurs relative to a solid approach that incorporates such uncertainty and indeterminacy relative to evidence used within judicial family matters. Thus, we propose an approach via neutrosophic OWA-TOPSIS model parameters. This is a novel contribution to family justice jurisdiction as it allows all ambiguity and uncertainty to be assessed objectively via wafts or evincing legitimizing explanations. For example, we use a case study where we fuzzify the evidence—chats, behavior, testimonials—and convert them to levels of truth, indeterminacy, and falsity. Then, following this neutrosophic approach, we assess competing hypotheses for adultery based on neutrosophic evidence and provide a distinct ranking of options. We contribute to the literature both theoretically and practically. Theoretically, we provide a multi-criteria judicial investigatory measure where none existed before; practically, we provide the judge/lawyer an unimpeachable solution with which complicated assessments can be calculated with greater judicial efficiency and effectiveness in less time and converted into assessment criteria that lessen the evidentiary threshold which for years has caused too much debate in Ecuadorian family justice.

Keywords: Divorce, adultery, evidence, neutrosophy, OWA-TOPSIS, Ecuador, legal decision.

1. Introduction.

The dissolution of marriage due to adultery constitutes an area of particular relevance in the Ecuadorian judicial system, demanding an in-depth analysis of the evidence to safeguard the principles of justice and equity. The correct assessment of evidence in these proceedings is crucial, as it directly impacts the lives of those involved, determining not only the marital status but also financial and parental aspects of considerable significance [1]. Despite its importance, proof of adultery often presents unique challenges due to its inherently discreet nature, requiring a thorough scrutiny of the evidence and circumstances.

Historically, the divorce process has evolved from restrictive conceptions rooted in canonical regulations to more liberal and causal approaches [2]. In the Latin American context, and specifically in Ecuador, adultery has persisted as a determining cause, which has generated extensive jurisprudence

around the interpretation of indirect evidence and the construction of judicial conviction [3], [4]. However, the interpretation of the evidence in these cases, which is frequently limited to subjective indications and testimonies, introduces a degree of uncertainty that traditional methods of evidentiary assessment fail to address exhaustively [5].

The central problem that this research seeks to resolve lies in the absence of a structured methodological framework that allows for managing the uncertainty and ambiguity inherent in the proof of adultery in the Ecuadorian legal sphere. Conventional methods of evidentiary assessment, although fundamental, often lack the capacity to quantify or mitigate the indeterminacy present in human testimony or in the indirect nature of certain indications [6]. How can justice operators more rigorously and transparently evaluate the occurrence of adultery when the available evidence is fragmented, inconsistent, or subjective, and the law requires a precise assessment? This methodological gap results in judicial decisions that, although legitimate, can be perceived as lacking a solid analytical basis in the face of doubt. Subjectivity in the interpretation of evidence can lead to inconsistencies and a perception of arbitrariness, undermining confidence in the judicial system [7]. Therefore, it is imperative to develop tools that allow for more systematic management of uncertain information, fostering greater objectivity in the assessment of evidence.

Uncertainty not only affects the evidence phase but also permeates the judicial decision-making phase, where judges must weigh multiple factors and evidence with varying degrees of reliability [8]. In this scenario, the application of mathematical approaches that model imprecision and ambiguity can offer a significant advantage. This is where neutrosophic logic, with its ability to represent degrees of truth, falsity, and indeterminacy, emerges as a promising alternative.

Several studies have begun to explore the application of multi-criteria decision-making models in the field of law, mainly in areas such as damage assessment or the selection of judicial personnel [9]. However, their incursion into the specific field of evidence assessment in divorce proceedings based on adultery, where ambiguity is a preponderant factor, is still limited. This research proposes an innovative solution to the identified problem. In this context, the present study proposes the application of the neutrosophic OWA-TOPSIS model as an effective tool for the assessment of evidence in divorce trials based on adultery in Ecuador. This approach will allow the quantification and processing of the uncertainty associated with the different pieces of evidence, facilitating a more structured and objective assessment. The methodology employed will include the study of a practical case, where neutrosophic values will be assigned to the clues and testimonies, and the model will be applied to generate a ranking of hypotheses regarding the occurrence of adultery.

In line with the above, the objectives of this study are: (1) to develop an evidentiary assessment model based on the neutrosophic OWA-TOPSIS approach for divorce cases based on adultery in the Ecuadorian legal system; and (2) to demonstrate, through a case study, the viability and usefulness of this model in overcoming evidentiary obstacles associated with the indeterminacy and ambiguity of indirect evidence.

2. Preliminaries.

2.1. Evidentiary Obstacle in the Legal System.

The legal system, in its pursuit of justice, fundamentally relies on evidence. This constitutes the foundation upon which judicial decisions are built, as it is through it that the facts are established and procedural truth is verified. However, in practice, obtaining and evaluating evidence is not always a path free of difficulties, sometimes becoming a real evidentiary obstacle. This impediment not only delays proceedings but can ultimately compromise the effectiveness of justice, generating dissatisfaction and distrust among citizens. The nature of this obstacle is multifaceted. Sometimes it lies in the inaccessibility of evidence, where crucial elements to clarify a fact are simply unavailable or have been destroyed. Other times, the difficulty lies in the vagueness or ambiguity of the information, as occurs with contradictory testimony or clues that do not offer unequivocal certainty. This type of scenario is particularly

pronounced in cases involving private conduct or illicit acts that are difficult to document, such as adultery or certain economic crimes.

A crucial aspect of the evidentiary obstacle is the inherent subjectivity of judicial assessment. Although judges are guided by principles of sound judgment and logic, the interpretation of evidence remains, to a certain extent, a human act influenced by individual experience and perception. This subjectivity can lead to inconsistencies in the application of the law and disparities in decisions, generating a sense of unpredictability in the system. The need for greater objectivity then becomes imperative. In addition to the above, the burden of proof represents a constant challenge. The party asserting a fact must prove it, and if the evidence is scarce or unconvincing, the outcome of the trial can be seriously affected. This is especially problematic in situations where privileged information or an imbalance of power between the parties makes it difficult to access the necessary evidence, leaving one party at a substantial disadvantage. Technological advancement, while promising for obtaining new forms of evidence, also introduces complexities. Digital evidence, for example, presents challenges regarding authenticity, integrity, and chain of custody. Determining whether a text message or email has been tampered with, or whether an electronic record is truly authentic, requires specialized technical expertise and up-to-date legal frameworks that don't always keep pace with the pace of innovation.

While jurisprudence has attempted to establish criteria for the assessment of indirect evidence and the construction of judicial conviction, it often encounters the limitation of uncertainty. Judgments, in their attempt to justify a decision, must navigate the delicate balance between the available evidence and the need to reach a threshold of certainty, which is not always easy in complex scenarios and with imprecise data. Therefore, the search for solutions to mitigate the evidentiary obstacle is an ongoing and multidisciplinary task. It is not simply a matter of reforming codes or procedures, but of incorporating tools and approaches that allow for more intelligent and rigorous management of uncertain and ambiguous information. The goal is to provide the judicial system with mechanisms that reinforce confidence in its rulings. In this context, multi-criteria decision-making methodologies and non-classical logics, such as neutrosophy, emerge as promising avenues. These approaches can provide an analytical framework for processing information under conditions of uncertainty, allowing for a more systematic assessment of different pieces of evidence. By assigning degrees of truth, uncertainty, and falsity, it is possible to capture the complexity of evidence in a way that binary approaches fail to do [10].

The implementation of these tools does not seek to replace the judge's wisdom or judgment, but rather to complement them, providing a more robust framework for justifying decisions. In fact, the integration of models such as the neutrosophic OWA-TOPSIS into evidentiary analysis can lead to greater transparency and consistency in the application of the law, reducing the perception of arbitrariness and increasing the legitimacy of judicial decisions [11], [12]. The adaptability of these techniques to different evidentiary contexts highlights their potential. Ultimately, overcoming the evidentiary obstacle in the legal system is not a utopia; it is a pressing need that demands the convergence of legal science with disciplines such as logic and artificial intelligence. Investment in the research and development of these methodologies is essential to ensure that justice is not only administered, but that it is administered with the greatest possible certainty and fairness, even in the face of the most elusive evidence [13], [14].

2.2. SVNS and SVNLS

This section provides a brief overview of the fundamental principles related to SVNS and SVNLS, covering definitions, operating principles, and metrics for measuring distances.

Definition 1. Let x be an element in a finite set, X. A single-valued neutrosophic set (SVNS), P, in X can be defined as in (1):

$$P = \{ x, T_P(x), I_P(x), F_P(x) | x \in X \}, \quad (1)$$

where the truth membership function, $T_P(x)$,the indeterminacy membership function $I_P(x)$, and the falsehood membership function $F_P(x)$clearly adhere to condition (2):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1;\ 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (2)$$

For a SVNS, P in X, we call the triplet $\left(T_P(x), I_P(x), F_P(x)\right)$its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$for computational convenience.

Definition 2. Leave $x = (T_x, I_x, F_x)$yy $= \left(T_y, I_y, F_y\right)$let there be two SVNV. Then

1) $x \oplus y = \left(T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y\right)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^\lambda = \left((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda\right), \lambda > 0$

The linguistic set

Let l be $S = \{s_\alpha | \alpha = 1, \ldots, l\}$a finite, totally ordered discrete term with odd value, wheres_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (3)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max\left(s_i, s_j\right) = s_j,\ if\ i \leq j$;
4) $\min\left(s_i, s_j\right) = s_i,\ if\ i \leq j$.

To avoid information loss during an aggregation process, the discrete set of terms S will be extended to a continuous set of terms. $S = \{s_\alpha | \alpha \in R\}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operational laws [15,16] :

1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;
2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0$;
3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 3 [17] Given X, a finite set of universes, a SVNLS, P, in X can be defined as in (4):

$$P = \left\{\langle x, [s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))]\rangle \middle| x \in X\right\} \quad (4)$$

where $s_{\theta(x)} \in \bar{S}$, the truth membership function $T_P(x)$, the indeterminacy membership function, $I_P(x)$and the falsehood membership function $F_P(x)$satisfy condition (5):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1, 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3. \quad (5)$$

For an SVNLS, P, in X, the 4-$\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))\rangle$tuple is known as the Single-Valued Neutrosophic Linguistic Set (SVNLN), conveniently denoted $x = s_{\theta(x)}, (T_x, I_x, F_x)$for computational purposes.

Definition 4 [17] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle\ (i = 1, 2)$two SVNLNs. Then

1) $x_1 \oplus x_2 = \langle s_{\theta(x1)} + \theta_{x2}, (T_{x1} + T_{x2} - T_{x1} * T_{x2}, I_{x1} * T_{x2}, F_{x1} * F_{x2})\rangle$
2) $\lambda_{x1} = \langle s_{\lambda\theta(x1)}, \left(1 - (1 - T_{x1})^\lambda, (I_{x1})^\lambda, (F_{x1})^\lambda\right)\rangle, \lambda > 0$;
3) $x_1^\lambda = \langle s_{\theta^{\lambda(x_1)}}, ((T_{x1})^\lambda, 1 - (1 - I_{x1})^\lambda, 1 - (1 - F_{x1})^\lambda)\rangle, \lambda > 0$.

Definition 5 [17] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$two SVNLNs. Their distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)}T_{x1} - s_{\theta(x2)}T_{x2}|^{\mu} + |s_{\theta(x1)}I_{x1} - s_{\theta(x2)}I_{x2}|^{\mu} + |s_{\theta(x1)}F_{x1} - s_{\theta(x2)}F_{x2}|^{\mu}\right]^{\frac{1}{\mu}} \quad (6)$$

In particular, equation (6) reduces the Hamming distance of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$and $\mu = 2$, respectively.

2.3. MADM Based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \dots, A_m\}$denotes a set of discrete feasible alternatives, $C = \{C_1, \dots, C_n\}$represents a set of attributes, and $E = \{e_1, \dots, e_k\}$is a set of experts (or DMs) with weight vector $\omega = \{\omega_1, \dots, \omega_k\}$T such that $\sum_{i=1}^{n} w_i = 1$ and $0 \le \omega_i \le 1$. Suppose that the attribute weight vector is $s\,v = (v_1, \dots, v_n)^T$, which satisfies $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t=1,\dots,k)}$on the alternative, $A_{i(i=1,\dots,m)}$,relative to the attribute, $C_{j(j=1,\dots,n)}$forms the individual decision matrix as shown in equation (7):

$$D^k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{pmatrix} \end{array} \quad (7)$$

where $\alpha_{ij}^{k} = \langle s_{\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k})\rangle$is represented by a SVNLN, which satisfies$s_{\theta(\alpha ij)}^{k} \in \bar{S}, T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k} \in [0,1] and\ 0 \le T_{\alpha_{ij}}^{k} + I_{\alpha_{ij}}^{k} + F_{\alpha_{ij}}^{k} \le 3$.

Geng et al. [18] extended the TOPSIS method to fit the SVNLS scenario, and the procedures of the extended model can be summarized as follows.

Step 1. Normalize the individual decision matrices:

In practical scenarios, MADM problems can encompass both benefit attributes and cost attributes. Let B and S the benefit attribute sets and cost attribute sets, respectively. Therefore, the conversion rules specified in (8) apply:

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k})\rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k})\rangle, & \text{for} \quad j \in S. \end{cases} \quad (8)$$

Thus, the standardized decision information, $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \quad (9)$$

Step 2. Build the collective matrix:

All individual DM reviews are aggregated into a group review:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \quad (10)$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k\, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix, , is formed as shown in (11), using the operational laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \quad (11)$$

The OWA operator is fundamental in aggregation techniques, widely studied by researchers [18] . Its main advantage lies in organizing arguments and facilitating the integration of experts' attitudes in decision making. Recent research has explored OWA in distance measurement, generating variations of OWAD [17] . Taking advantage of the benefits of OWA, the text proposes a SVNL OWA distance measure (SVNLOWAD). Given the desirable properties of the OWA operator, an SVNL OWA distance measure (SVNLOWAD) is proposed in the following text.

Definition 6. *Let* $x_j, x_j^{'}$ $(j = 1,\dots,n)$*the two collections be SVNLN. If*

$$SVNLOWAD((x_1,x_1'),\dots,(x_n,x_n')) = \sum_{j=1}^{n} w_j d(x_j,x_j'), \quad (12)$$

Therefore, step 4 of this method can be considered as follows:

Step 4. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i,A^+) = \sum_{j=1}^{n} w_j \dot{d}\left(y_{ij},y_j^+\right), i = 1,\dots,m \quad (13)$$

$$SVNLOWAD(A_i,A^-) = \sum_{j=1}^{n} w_j \dot{d}\left(y_{ij},y_j^-\right), i = 1,\dots,m \quad (14)$$

where $\dot{d}(y_{ij},y_j^+)$and $\dot{d}(y_{ij},y_j^-)$they are the j -th largest values of $\dot{d}(y_{ij},y_j^+)$and $\dot{d}(y_{ij},y_j^-)$, respectively.

Step 5. In the classical TOPSIS approach, the relative closeness coefficient, , is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness fails to achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. Thus, following an idea proposed in references [15] , in equations (15)–(17), we introduce a modified relative closeness coefficient, $C'(Ai)$, used to measure the degree to which the alternatives, Ai ( ) = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i,A^-)}{SVNLOWAD_{\max}(A_i,A^-)} - \frac{SVNLOWAD(A_i,A^+)}{SVNLOWAD_{\min}(A_i,A^+)}, \quad (15)$$

where

$$SVNLOWAD_{\max}(A_i,A^-) = \max_{1\le i\le m} SVNLOWAD(A_i,A^-), \quad (16)$$

and

$$SVNLOWAD_{\min}(A_i,A^+) = \min_{1\le i\le m} SVNLOWAD(A_i,A^+). \quad (17)$$

It is clear that $C'(A_i) \le 0$ $(i = 1,\dots,m)$the higher the value of $C'(A_i)$and , the better A_i the alternative. Furthermore, if an alternative A^* satisfies the conditions $SVNLOWAD(A^*,A^-) = SVNLOWAD_{\max}(A^*,A^-)$and $SVNLOWAD(A^*,A^+) = SVNLOWAD_{\min}(A^*,A^+)$, then $C'(A^*) = 0$and the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Rank and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Case Study.

This study analyzes a divorce case based on adultery in the Ecuadorian judicial system, applying the neutrosophic OWA-TOPSIS model for the objective assessment of the evidence presented. The available evidence is evaluated to determine whether there is sufficient evidence to confirm the grounds for divorce.

Participants and methodology

Three judges specializing in family law participated in the study, evaluating the evidence presented according to established criteria. The neutrosophic OWA-TOPSIS model was applied to integrate the individual evaluations and obtain an objective collective assessment.

Evidence and evaluation criteria

Four types of evidence presented in the case were considered:

- **Alternative A1 (Electronic Messages)**: Instant messaging conversations between the respondent spouse and third parties.
- **Alternative A2 (Testimonies)**: Statements from witnesses who witnessed compromising conduct.
- **Alternative A3 (Photographs)**: Images showing meetings of the defendant spouse with third parties.
- **Alternative A4 (Location logs):** Geolocation data extracted from mobile devices.

The evaluation criteria used were:

- **C1. Authenticity (AU)**: Degree to which the evidence is verifiably genuine.
- **C2. Relevance (PE)**: Relevance of the evidence to the specific case.
- **C3. Conduciveness (CO)** : Capacity of the evidence to effectively demonstrate adultery.
- **C4. Legality (LI)**: Conformity of the evidence with procedural and constitutional norms.

The judges assigned weights to the criteria according to their relative importance:$C1$: 0,15, $C2$: 0,25, $C3$: 0,25 y $C4$: 0,35.

Evaluation and decision matrices

The evaluations were expressed using Neutrosophic Values of Language (SVNL) with the following scale: S = {s_1 = "extremely poor", s_2 = "very poor", s_3 = "poor", s_4 = "acceptable", s_5 = "good", s_6 = "very good", s_7 = "extremely good"}

The standardized SVNL decision matrices are presented below:

Table 1: Evaluation of evidence according to Criterion 1 (Authenticity)

Evidence	Judge 1	Judge 2	Judge 3
A 1	S_5(0.4; 0.2; 0.3)	S_5(0.4; 0.3; 0.4)	S_6(0.5; 0.2; 0.3)
A 2	S_6(0.6; 0.1; 0.2)	S_6(0.7; 0.2; 0.3)	S_5(0.5; 0.2; 0.3)
A 3	S_5(0.4; 0.3; 0.4)	S_6(0.4; 0.2; 0.4)	S_6(0.5; 0.1; 0.3)
A 4	S_6(0.7; 0.2; 0.3)	S_4(0.8; 0.1; 0.2)	S_4(0.6; 0.1; 0.2)

Table 2: Evidence evaluation according to Criterion 2 (Relevance)

Evidence	Judge 1	Judge 2	Judge 3
A 1	S_5(0.4; 0.2; 0.3)	S_6(0.5; 0.1; 0.2)	S_6(0.6; 0.2; 0.4)
A 2	S_5(0.6; 0.1; 0.2)	S_6(0.7; 0.2; 0.3)	S_4(0.7; 0.2; 0.2)
A 3	S_4(0.5; 0.2; 0.3)	S_6(0.6; 0.3; 0.4)	S_5(0.6; 0.1; 0.3)
A 4	S_4(0.6; 0.1; 0.2)	S_5(0.7; 0.2; 0.3)	S_4(0.5; 0.2; 0.2)

Table 3: Evidence evaluation according to Criterion 3 (Conduciveness)

Evidence	Judge 1	Judge 2	Judge 3
A 1	S_3(0.3; 0.2; 0.5)	S_5(0.3; 0.1; 0.6)	S_5(0.2; 0.1; 0.6)
A 2	S_4(0.5; 0.2; 0.2)	S_5(0.6; 0.2; 0.2)	S_5(0.7; 0.2; 0.1)
A 3	S_3(0.5; 0.3; 0.1)	S_4(0.6; 0.1; 0.3)	S_4(0.6; 0.2; 0.1)
A 4	S_3(0.3; 0.1; 0.2)	S_4(0.4; 0.2; 0.2)	S_3(0.4; 0.1; 0.1)

Table 4: Evaluation of evidence according to Criterion 4 (Legality)

Evidence	Judge 1	Judge 2	Judge 3
A 1	S_4(0.5; 0.3; 0.3)	S_3(0.7; 0.1; 0.1)	S_4(0.5; 0.2; 0.3)
A 2	S_3(0.6; 0.2; 0.4)	S_4(0.5; 0.4; 0.2)	S_6(0.4; 0.6; 0.2)
A 3	S_5(0.3; 0.5; 0.2)	S_5(0.4; 0.4; 0.1)	S_4(0.3; 0.6; 0.2)
A 4	S_6(0.6; 0.1; 0.2)	S_6(0.6; 0.3; 0.3)	S_5(0.7; 0.2; 0.1)

Collective decision matrix

The SVNL collective decision matrix, which integrates the individual evaluations of the three judges, is presented below:

Table 5: SVNL Collective Decision Matrix

Evidence	C 1 (Authenticity)	C 2 (Relevance)	C 3 (Conductivity)	C 4 (Legality)
A 1	$S_{5.33}$(0.433; 0.233; 0.333)	$S_{5.67}$(0.500; 0.167; 0.300)	$S_{4.33}$(0.267; 0.133; 0.567)	$S_{3.67}$(0.567; 0.200; 0.233)
A 2	$S_{5.67}$(0.600; 0.167; 0.267)	$S_{5.00}$(0.667; 0.167; 0.233)	$S_{4.67}$(0.600; 0.200; 0.167)	$S_{4.33}$(0.500; 0.400; 0.267)
A 3	$S_{5.67}$(0.433; 0.200; 0.367)	$S_{5.00}$(0.567; 0.200; 0.333)	$S_{3.67}$(0.567; 0.200; 0.167)	$S_{4.67}$(0.333; 0.500; 0.167)
A 4	$S_{4.67}$(0.700; 0.133; 0.233)	$S_{4.33}$(0.600; 0.167; 0.233)	$S_{3.33}$(0.367; 0.133; 0.167)	$S_{5.67}$(0.633; 0.200; 0.200)

Weighted collective decision matrix

By applying the criteria weights ($C1$: 0.15, $C2$: 0.25, $C3$: 0.25, $C4$: 0.35),the weighted collective SVNL decision matrix is obtained:

Table 6: Weighted collective SVNL decision matrix

Evidence	C 1 (Authenticity)	C 2 (Relevance)	C 3 (Conductivity)	C 4 (Legality)
A 1	$S_{0.80}$(0.065; 0.348; 0.433)	$S_{1.42}$(0.125; 0.367; 0.475)	$S_{1.08}$(0.067; 0.350; 0.675)	$S_{1.28}$(0.198; 0.430; 0.452)
A 2	$S_{0.85}$(0.090; 0.292; 0.377)	$S_{1.25}$(0.167; 0.375; 0.425)	$S_{1.17}$(0.150; 0.400; 0.375)	$S_{1.52}$(0.175; 0.610; 0.523)
A 3	$S_{0.85}$(0.065; 0.320; 0.462)	$S_{1.25}$(0.142; 0.400; 0.500)	$S_{0.92}$(0.142; 0.400; 0.375)	$S_{1.63}$(0.117; 0.675; 0.458)
A 4	$S_{0.70}$(0.105; 0.263; 0.348)	$S_{1.08}$(0.150; 0.375; 0.425)	$S_{0.83}$(0.092; 0.350; 0.375)	$S_{1.98}$(0.222; 0.430; 0.430)

Positive ideal solution (PIS) and negative ideal solution (NIS)

For each criterion, the positive ideal solution (PIS) is determined as the maximum value for the truth component and the minimum values for the indeterminacy and falsity components. The negative ideal solution (NIS) is determined inversely.

PIS (A^+):

- $C1$: $(0.105, 0.263, 0.348)$
- $C2$: $(0.167, 0.367, 0.425)$
- $C3$: $(0.150, 0.350, 0.375)$
- $C4$: $(0.222, 0.430, 0.452)$

NIS (A^-):

- $C1$: $(0.065, 0.348, 0.462)$
- $C2$: $(0.125, 0.400, 0.500)$
- $C3$: $(0.067, 0.400, 0.675)$
- $C4$: $(0.117, 0.675, 0.523)$

Calculation of relative distances

The judges determined the OWA operator weight vector as $W = (0.25, 0.30, 0.35, 0.10)$,reflecting their attitudes toward the relative importance of the criteria.

Using neutrosophic distances and the OWA operator, the distances between each alternative and the positive (PIS) and negative (NIS) reference points were calculated:

Table 7: Relative distances between each evidence and the reference points

Evidence	SVNLOWAD(A_i, A^+)	SVNLOWAD(A_i, A^-)	C
A_1	0.127	0.135	0.515
A_2	0.037	0.081	0.686
A_3	0.070	0.117	0.626
A_4	0.070	0.005	0.067

Where C is the coefficient of relative closeness calculated as: $C = \frac{SVNLOWAD(A_i,A^-)}{[SVNLOWAD(A_i,A^+) + SVNLOWAD(A_i,A^-)]}$

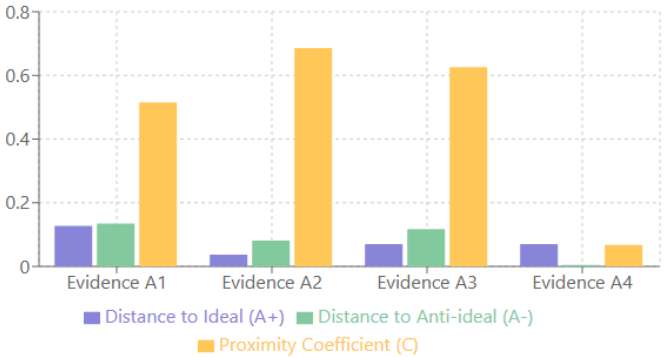

Chart 1: Relative distances between each piece of evidence and the reference points.

Analysis of results

The results obtained through the neutrosophic OWA-TOPSIS model provide a systematic and objective assessment of the evidence presented in the case of divorce due to adultery.

The distances $SVNLOWAD(A_i, A^+)$represent the proximity of each type of evidence to the positive ideal point, with lower values indicating greater proximity to the ideal solution. In this regard, Alternative A 2 (Testimonies) has the lowest value (0.037), followed by Alternatives A 3 and A 4 (Photographs and Location Records) with 0.070 each, and finally Alternative A1 (Electronic Messages) with 0.127.

On the other hand, the distances $SVNLOWAD(A_i, A^-)$ measure the distance of each piece of evidence from the negative ideal point, where higher values are preferable. Alternative A 1 shows the highest value (0.135), followed by A 3 (0.117) and A 2 (0.081), while A4 presents the lowest value (0.005).

The relative closeness coefficient (C) integrates both distances to provide a single rating index. The higher this value, the more desirable the alternative. The results show that Alternative A 2 (Testimonies) has the highest value (0.686), indicating that it is the most robust evidence according to the evaluated criteria. It is followed by Alternative A 3 (Photographs) with 0.626, Alternative A 1 (Electronic Messages) with 0.515, and finally Alternative A 4 (Location Records) with 0.067.

4. Conclusions

Based on the data presented in the study, here is a rewritten conclusion in English that resolves the previously identified errors:

The analysis shows that the neutrosophic OWA-TOPSIS model provides a clear and systematic ranking of the evidence presented in this specific case study. According to the calculated relative closeness coefficient (C), Testimonial evidence (A2) was identified as the most robust and effective evidence to support the grounds for divorce, achieving the highest score of 0.686. Following in descending order of effectiveness are Photographic evidence (A3) with a score of 0.626, Electronic Messages (A1) with 0.515, and lastly, Location Records (A4), which was the least effective with a score of 0.067. This final ranking demonstrates that the model successfully integrated the judges' assessments across the four criteria while systematically processing the inherent uncertainty associated with each piece of evidence.

The neutrosophic approach is particularly useful in adultery cases, where direct proof is often scarce and the evidence presented carries high levels of indeterminacy and potential falsity. By considering not just truth but also these indeterminate factors, the OWA-TOPSIS model offers a more nuanced assessment than traditional methods. It stands as an effective tool for judicial authorities, providing a systematic and objective framework to evaluate competing and uncertain information. This enhances the efficiency and effectiveness of judicial decision-making and helps to lessen the evidentiary obstacles present in the Ecuadorian legal system.

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Current Situation of the Integration of Neutrosophy in Requirements Engineering

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Abstract: This study conducts a comprehensive literature review to explore the integration of neutrosophic logic in Requirements Engineering (RE), addressing challenges such as uncertainty, ambiguity, and conflicting stakeholder perspectives. By systematically analyzing peer-reviewed studies from 2015 to 2024, the research synthesizes existing neutrosophic techniques—including Type-2 Neutrosophic Numbers (T2NNs), Neutrosophic Cognitive Maps (NCMs), and Single-Valued Neutrosophic Sets (SVNS)—and categorizes their applications into uncertainty handling, conflict resolution, and requirement prioritization. The study highlights neutrosophy's transformative potential in enhancing RE practices, offering robust solutions beyond traditional methods. Additionally, it identifies underexplored techniques like Bipolar Neutrosophic Sets (BNS) and Neutrosophic Rough Sets (NRS) for future research, underscoring neutrosophy's role as a paradigm shift in managing RE complexities.

Keywords: Neutrosophy, Requirements Engineering, uncertainty handling, conflict resolution, requirement prioritization, neutrosophic techniques.

1. Introduction

Requirements Engineering (RE) is a critical phase in software development, where the needs and constraints of stakeholders are identified, analyzed, and documented [1, 2]. However, RE often grapples with inherent challenges such as uncertainty, ambiguity, and conflicting stakeholder perspectives, which traditional methods struggle to address effectively [3]. These challenges can lead to misinterpretations [4], incomplete specifications, and ultimately, project failures [5].

Neutrosophy [6], a generalization of fuzzy logic and intuitionistic fuzzy sets, introduces the concept of truth (T), indeterminacy (I), and falsity (F) degrees, providing a robust framework to model and manage uncertainty, subjectivity, and conflicts in RE. By leveraging neutrosophic logic and its extensions, such as Neutrosophic Cognitive Maps (NCMs) [7] and Single- Valued Neutrosophic Sets (SVNS) [8], researchers have begun to explore innovative solutions for handling vague requirements, resolving stakeholder conflicts, and prioritizing features more effectively [9, 10].

Despite these advancements, the integration of neutrosophy into RE remains an emerging field, with scattered applications and limited systematic reviews. This study aims to bridge

this gap by conducting a comprehensive literature review to:

1. Synthesize existing neutrosophic techniques applied in RE, categorizing them into key areas like uncertainty handling, conflict resolution, and prioritization.
2. Evaluate the transformative potential of neutrosophy in addressing RE challenges, highlighting its advantages over traditional methods.
3. Identify underexplored neutrosophic techniques (e.g., Bipolar Neutrosophic Sets, Neutrosophic Rough Sets) that could further enhance RE practices.

By systematically analyzing peer-reviewed studies from 2015 to 2024, this work not only maps the current landscape of neutrosophy in RE but also motivates future research to expand its applications, ensuring more resilient and adaptive software development processes. The findings underscore neutrosophy's role as a paradigm shift in managing the complexities of modern RE.

1. Methodology

To conduct a comprehensive literature review on the integration of neutrosophy in Requirements Engineering (RE) we applied the following steps.

1.1. Search Strategy and Data Sources

Articles were systematically searched and extracted from multiple academic databases, including Scopus, Google Scholar, IEEE Xplore, and SpringerLink. The search was performed using the following keywords:

- "Neutrosophy AND Requirements Engineering"
- "Neutrosophic logic AND software requirements"
- "Neutrosophic sets AND uncertainty in RE"
- "Neutrosophic techniques AND conflict resolution in RE"

1.2. Search Criteria

- Timeframe: Articles published between 2015 and 2024 were prioritized to ensure relevance to current advancements.
- Language: Only English-language publications were considered.
- Publication Type: Peer-reviewed journal articles, conference papers, and book chapters were included.

1.3. Inclusion and Exclusion Criteria

Inclusion Criteria:

- Studies explicitly applying neutrosophic logic/sets to RE challenges (e.g., uncertainty, conflict resolution, prioritization).
- Empirical or theoretical contributions with clear methodologies.
- Articles directly addressing the three focus areas identified in the manuscript: uncertainty handling, conflict resolution, and prioritization.

Exclusion Criteria:

- Duplicate studies or redundant publications.
- Articles lacking technical depth (e.g., short abstracts, opinion pieces).
- Studies not peer-reviewed or with unclear methodologies.
- Works focused solely on traditional fuzzy logic without neutrosophic extensions.

1.4. Article Selection Process

- Initial Screening: 25 articles were identified from databases based on keywords and abstracts.
- Full-Text Review: 15 articles were evaluated for relevance and methodological rigor.

- Final Selection: 8 articles met all inclusion criteria and were included in the review.

1.5. Reasons for Exclusion

- 7 articles were excluded for irrelevance (e.g., applied fuzzy logic but not neutrosophy).
- 5 articles were rejected due to methodological flaws (e.g., insufficient validation or unclear neutrosophic techniques).
- 3 articles were duplicates or superseded by more recent studies.
- 2 articles were excluded for language barriers (non-English publications).

1.6. Final Dataset

The 8 selected articles (cited as [7, 8, 11–16] in the manuscript) were categorized into three application areas (Table 1) and analyzed for their neutrosophic techniques (Table 2). This structured approach ensured a focused review of high-impact, relevant studies while maintaining methodological transparency.

1.7. Limitations

- Potential bias from database selection (e.g., Scopus-indexed journals favored).
- Exclusion of non-English studies may overlook regional contributions.

This methodology aligns with PRISMA guidelines for systematic reviews, ensuring repro- ducibility and rigor in synthesizing neutrosophy's role in RE.

2. Results

In this section, we highlight the transformative potential of neutrosophy in addressing key challenges in Requirements Engineering (RE). By leveraging neutrosophic logic—which incorporates truth (T), indeterminacy (I), and falsity (F) degrees— this review of the literature collects innovative solutions for (1) Handling Uncertainty in Requirements, where vague or incomplete stakeholder inputs are modeled with precision; (2) Conflict Resolution Among Stakeholders, where neutral influences help reconcile contradictory needs; and (3) Requirement Prioritization. Table 1 categorizes these applications.

Table 1. Categorizing Neutrosophy Applications in Requirement Engineering.

Category	Papers
Handling Uncertainty in Requirements	[11], [12], [13], [7], [8], [16]
Conflict Resolution Among Stakeholders	[13], [14]
Requirement Prioritization	[11], [15], [8], [16]

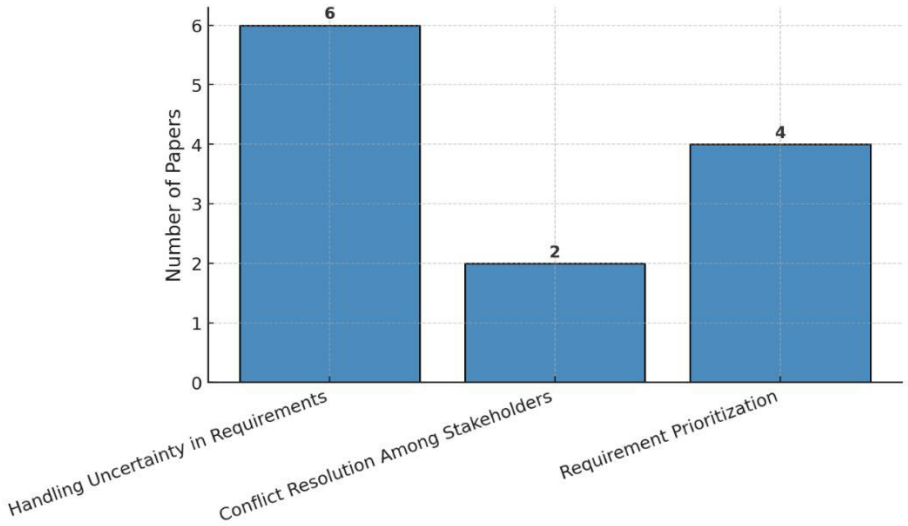

Figure 1. Categorization of Neutrosophy Applications in Requirement Engineering

Handling Uncertainty in Requirements

Traditional RE struggles with imprecise, incomplete, or conflicting stakeholder inputs. Neu- trosophic Sets & Logic can model requirements with truth (T), falsity (F), and indeterminacy (I) degrees. For example, a requirement like "The system should respond quickly" can be represented as (T=0.7, I=0.2, F=0.1), meaning it is mostly true but has some ambiguity. Below, we present the neutrosophic techniques that have considered the selected works to handle uncertainty in requirements.

In [11], authors used Type-2 Neutrosophic Numbers (T2NNs) to address vague and imprecise stakeholder judgments in requirements prioritization. In [12], the Interval-Valued Neutrosophic Z-Numbers (NZN) and NZN-Area are used to handle uncertainty and reliability in software development team assignments. In [13], the Interval-Valued Trapezoidal Neutrosophic Sets (IVTrNS) are considered to manage uncertainty and incompleteness in Non-Functional Re- quirements (NFRs). Moreover, [7] applies Neutrosophic Cognitive Maps (NCMs) to handle uncertainty and subjective interdependencies among NFRs in a panic button system. In addition, in [8], the Single-Valued Neutrosophic Sets (SVNS) are used to model human subjectivity and ambiguity in software requirement prioritization. Finally, the work presented in [16] employs Single-Valued Neutrosophic Numbers (SVN numbers) to address indeterminacy and uncertainty in decision-making for requirement prioritization.

2.1. Conflict Resolution Among Stakeholders

Different stakeholders may have contradictory needs. For it, Neutrosophic Cognitive Maps (NCMs) can help to analyze and resolve conflicts by considering neutral (indeterminate) influences. For example, if two stakeholders disagree on a feature priority, neutrosophic reasoning can find a compromise by accounting for partial acceptance and uncertainty. Below, we present the neutrosophic techniques that have considered the selected works to handle conflict resolution among stakeholders.

The work published in [13] focuses on resolving conflicts among Non-Functional Requirements (NFRs) by addressing uncertainty and incompleteness in stakeholder preferences. Another work [14] uses Neutrosophic Cognitive Maps (NCMs) to model and

analyze interdependencies between NFRs, capturing expert knowledge and resolving conflicts.

2.2. Requirement Prioritization

In software engineering and project management, requirement prioritization is the process of ranking requirements based on their importance, urgency, feasibility, and business value. This helps in efficient resource allocation, managing stakeholder expectations, delivering high-value features early, and reducing project risks. Below, we present the neutrosophic techniques that have considered the selected works to handle requirement Prioritization.

The work presented by [11] proposes a framework for prioritizing software requirements using T2NNs combined with DEMATEL and TOPSIS. Moreover, in [15], Neutrosophic Analytic Hierarchy Process (NAHP) is applied to prioritize requirements for a facial recognition system. In addition, the work presented by [8] prioritizes software requirements using Neutrosophic TOPSIS with SVNS. Finally, the work presented in [16] develops a prioritization model for software requirements using SVN numbers and aggregation operators.

3. Discussion

Table 2 showcast how neutrosophic techniques are applied to support the requirement engineering proccess in the selected works. These applications are motivated by the inherent complexities of RE, where traditional approaches often fall short in managing ambiguity, subjectivity, and competing stakeholder interests. The integration of neutrosophy thus offers a robust framework to enhance decision-making and project outcomes in RE.

Table 2. Classification of Neutrosophic Techniques

Neutrosophic Technique	**Study**
Type-2 Neutrosophic Numbers (T2NNs)	[11]
Interval-Valued Neutrosophic Z-Numbers (NZN)	[12]
Interval-Valued Trapezoidal Neutrosophic Sets	[13]
Neutrosophic Cognitive Maps (NCMs)	[14], [7]
Neutrosophic AHP (NAHP)	[15]
Single-Valued Neutrosophic Sets (SVNS)	[8]
Single-Valued Neutrosophic Numbers (SVN)	[16]

The selected works highlight several neutrosophic techniques applied in Requirements Engi- neering (RE), such as Type-2 Neutrosophic Numbers (T2NNs), Neutrosophic Cognitive Maps (NCMs), and Single-Valued Neutrosophic Sets (SVNS). However, other neutrosophic techniques could also be beneficial for addressing challenges

in RE. Below are some techniques not mentioned in the article, along with examples of their potential applications:

- **Bipolar Neutrosophic Sets (BNS)**: It extends neutrosophic sets by incorporating bipolarity (positive and negative aspects) to model conflicting information [17]. It can be applied in conflict resolutions among stakeholders. BNS can represent both supportive and opposing views for a requirement. For example, a requirement like "The system should include a chatbot" could have positive aspects (e.g., improves user experience) and negative aspects (e.g., increases development cost). BNS can quantify these opposing views to facilitate balanced decision-making.
- **Neutrosophic Rough Sets (NRS)**: It integrates rough sets with neutrosophic logic to handle uncertainty and vagueness in incomplete data [18]. For incomplete or am- biguous requirements, NRS can classify requirements into certain, possible, or inde- terminate categories. For instance, a vague requirement like "The system should be user-friendly" can be analyzed to determine its core meaning (certain), potential interpretations (possible), and unclear aspects (indeterminate).
- **Neutrosophic Multi-Criteria Decision Making (NMCDM)**: It extends traditional MCDM methods by incorporating neutrosophic logic to handle uncertainty in criteria weights and evaluations [19]. For selecting the best set of requirements from a pool, NMCDM can evaluate each requirement against criteria like cost, feasibility, and stakeholder value, while accounting for indeterminacy in the evaluations. For instance, a requirement's feasibility might be rated as (T=0.7, I=0.2, F=0.1) due to unclear technical constraints.
- **Neutrosophic Graph Theory (NGT)**: It applies graph theory in a neutrosophic context to model relationships with uncertainty [20]. For modeling dependencies between requirements, NGT can represent relationships like "requires" or "conflicts with" using neutrosophic edges. For example, a dependency between two requirements might be partially true (T=0.6, I=0.3, F=0.1) if their relationship is context dependent.

4. Conclusion

This study systematically reviews the integration of neutrosophic logic in Requirements Engineering (RE), highlighting its transformative potential in addressing key challenges such as uncertainty, ambiguity, and conflicting stakeholder perspectives. By categorizing and analyzing eight peer-reviewed studies, the research demonstrates how neutrosophic techniques—including Type-2 Neutrosophic Numbers (T2NNs), Neutrosophic Cognitive Maps (NCMs), and Single-Valued Neutrosophic Sets (SVNS)—provide robust frameworks for un- certainty handling, conflict resolution, and requirement prioritization. These techniques outperform traditional methods by explicitly modeling truth, indeterminacy, and falsity degrees, enabling more precise and adaptive decision-making in RE.

The study identifies underexplored neutrosophic techniques, such as Bipolar Neutrosophic Sets (BNS) and Neutrosophic Rough Sets (NRS), which hold promise for further enhancing RE practices. Future research should focus on expanding the application of these techniques, particularly in complex, real-world scenarios, to validate their scalability and effectiveness. Additionally, interdisciplinary collaborations could bridge gaps between theory and practice, fostering innovations that align neutrosophic logic with emerging trends in software engineering. By addressing these opportunities, the RE field can achieve greater resilience and adaptability in managing the evolving complexities of

modern software

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University of New Mexico

Structural equilibrium in universities: quality, finance and sustainability from an empirical perspective: An integrative analysis with neutrosophic cognitive maps

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Abstract: This study explores the structural relationships between institutional financial health and academic quality in Chilean higher education institutions (HEIs) by applying multivariate tools such as the dynamic HJ-Biplot and fixed-effects regression models for panel data, complemented by neutrosophic cognitive maps (NCMs). Using data from the 2016–2023 period, differentiated institutional trajectories are analyzed based on their membership in free tuition policies, university type (universities belonging to the Council of University Chancellors (CRUCH) and private universities), and financial structure. The results show that variables such as student retention, physical infrastructure, and academic training significantly influence key financial indicators such as liquidity, leverage, and operating margin. NCMs reveal degrees of indeterminacy in these relationships, especially in contexts of high uncertainty such as private universities not affiliated with free tuition. Their integration into this study represents a novel methodological contribution, enabling the modeling of causal ambiguity and structural instability within the academic-financial nexus of higher education institutions. Specific patterns of vulnerability are identified in these institutions, and the stabilizing role of fiscal contributions is confirmed. Research shows that quality and financial sustainability are interdependent dimensions, whose coordination, analyzed from a neutrosophic perspective, is key to strengthening educational well-being and the resilience of the university system in contexts of change.

Keywords: Multivariate Analysis; Financial Health; Educational Quality; Higher Education; Institutional Sustainability; HJ- Dynamic Biplot, Panel Models; Neutrosophic Cognitive Maps.

1. Introduction

The Chilean university system is characterized by its institutional diversity and a mixed financing model that combines public and private resources. In this context, Higher Education Institutions (HEIs) face growing challenges in maintaining financial sustainability without compromising academic quality. While public universities enjoy certain guarantees of survival—such as fiscal support and access to core funds—many of them suffer chronic operational deficits and low profitability [1]. Private universities, meanwhile, subject to market dynamics, have been vulnerable to bankruptcy and closure, generating critical consequences for their student communities and highlighting weaknesses in the system's regulatory mechanisms [2] (Superintendency of Higher Education [SES], 2023).

In recent decades, various reforms have attempted to strengthen the system's governance, including the creation of the SES (National Service for Quality Assurance), the national quality assurance framework, and the implementation of policies such as free tuition. However, questions remain about how the financial and academic dimensions are articulated in Chilean universities. Internationally, studies have highlighted the importance of having solid financial structures to guarantee quality education, especially in contexts of crisis or transformation [2-5]. The literature also suggests that financial management should not be understood as an end in itself, but as a means to sustain educational projects consistent with institutional objectives [6].

However, in the Chilean case, the relationship between financial health and academic quality remains underexplored and conceptually fragmented. Some research argues that financial indicators do not always correlate with the levels of educational quality perceived by students and academic communities [7]. Others argue that there is a disconnect between financial logic—dominated by criteria of efficiency and profitability—and academic logic, focused on the development of knowledge and comprehensive training [8]. This tension takes on special relevance in a scenario of growing demand for transparency, accountability, and institutional quality.

Within this framework, this study seeks to provide empirical evidence on the links between financial health and academic quality in the Chilean university system through the use of multivariate analysis tools, regression models with longitudinal data from the 2016–2023 period, and neutrosophic cognitive maps (NCMs). The objective is twofold: first, to characterize the financial profiles of universities; second, to evaluate the impact of key academic variables—such as student retention, teaching staff, and infrastructure—on financial health indicators, considering the uncertainty inherent in these relationships. NCMs, based on the principles of neutrosophy (Smarandache, 1998), allow for the modeling of causal interactions between financial and academic variables, assigning truth (T), falsity (F), and indeterminacy (I) values to capture ambiguities, especially in universities with high variability, such as private universities not affiliated with free tuition. This is expected to contribute to the design of more integrated public policies that consider both the economic sustainability and educational mission of HEIs, enriching the understanding of their structural balance from a neutrosophic perspective.

1.1. Financial health and risk profiles in Chilean HEIs

Recent rapid social, economic, and health changes—such as the COVID-19 pandemic—have highlighted the need for resilient and financially sustainable higher education institutions (HEIs). In this context, analyzing the financial health of HEIs becomes relevant not only as a management tool but also as a key input for safeguarding the continuity and quality of the educational services they provide (OECD, 2021; World Bank, 2022).

Unlike private companies, universities' central mission is the development of knowledge, comprehensive education, and commitment to the community. Therefore, even if they do not pursue profit maximization, they require financial stability to sustain their infrastructure, academic staff, research activities, and training programs [9]. From this perspective, two key concepts emerge: financial sustainability, understood as the ability to cover operating costs and invest in future development; and financial health, which refers to the institutional capacity to cope with economic stress and meet its financial obligations [2].

Since 2023, the Superintendency of Higher Education (SES) has developed a methodology to assess the financial risk of Chilean HEIs based on indicators such as leverage, liquidity, gross margin, operating cash flow, and fixed asset ratio. The analysis covers the period 2012–2022 and includes universities, professional institutes (PIs), and technical training centers (TCCs). Its main finding is that financial health profiles are not static, and that the proportion of institutions and enrollment within each profile varies from year to year [2].

1.1.1. Institutional risk profiles

Universities: Generally, they have a more solid financial profile. 72% of universities remained stable during the period analyzed, and only 5.8% of their enrollment corresponds to institutions at financial risk. However, some institutions have high levels of debt with a low proportion of fixed assets, making them financially fragile. Universities in this situation (15.5% of the total) depend on liquidity without having equity to support their operations, which generates a structural exposure to leverage [8]. For example, Professional Institutes (PI) exhibit financial behavior similar to that of universities, but with slightly higher debt levels. 18.8% have a risky profile, affecting 2% of enrolled students. The most stable institutions tend to have tighter cost structures and higher income-to-enrollment ratios. Technical Training Centers (CFTs): These are the group with the highest proportion of financial risk. 22.7% of CFTs have this profile, representing 4.5% of enrollment. This group tends to display high levels of debt without sufficient financial backing, which creates greater vulnerability to fluctuations in enrollment or regulatory changes. Furthermore, between 2015 and 2022, the number of CFTs with a risky profile doubled, although this increase was attenuated after the implementation of the free tuition policy starting in 2016 [10].

1.1.2. Longitudinal perspective

The longitudinal analysis shows that most universities and private universities maintained stable financial profiles between 2012 and 2022. In contrast, the CFTs exhibited greater volatility. Free tuition appears to have played a stabilizing role, especially in institutions with a higher concentration of vulnerable students, by reducing their exposure to financial risk [5].

In summary, the financial health of Chilean HEIs is determined by a combination of structural and management factors. While the majority of the system is in a reasonably solid position, some segments remain highly vulnerable. Linking financial indicators with academic variables allows for a more comprehensive approach to quality assurance, in line with the recommendations of international organizations [3-4].

1.2. Sustainability and quality in Chilean HEIs: An integrative look at financial and academic profiles.

Although the financial health of higher education institutions (HEIs) has traditionally been assessed through accounting and asset indicators, in recent years there has been a growing recognition that academic and educational management variables play a decisive role in institutional sustainability [3, 9]. This approach seeks to overcome a purely economic view of university performance, recognizing that quality and financial stability are built on a relationship of mutual influence.

In the Chilean case, the report prepared by the Superintendency of Higher Education (SES, 2023) has been criticized for overemphasizing financial indicators as a measure of institutional health. Several academics have warned that this approach runs the risk of obscuring key academic factors, such as teaching quality, student retention, or scientific productivity, which not only constitute essential dimensions of the university's mission but also impact financial stability.[8]

From a critical perspective, [8] argue that there is a disconnect between academic and financial logic in many Chilean HEIs, which generates tensions in quality assurance models. This phenomenon has been particularly observed in state and regional universities, which, despite having high social valuation and good quality indicators, exhibit persistent financial weaknesses, associated with their lower capacity to generate their own income or dependence on public funding [1].

In turn, Moraga and Rossi [7] argue that the relationship between financial structure and perception of institutional quality is neither linear nor automatic. In their study of traditional Chilean universities, the authors found that, although there are economic indicators that correlate with perceived quality, when analyzed together the effect is diluted, which suggests the need for more robust multivariate models to understand this complex relationship.

In this sense, [11] propose that university quality should be understood as the result of a coherence between financial strategies, academic policies, and institutional management mechanisms. From this perspective, a financially healthy but academically weak HEI, or vice versa, cannot be considered quality. The key lies in the articulation between both dimensions, mediated by strategic management that responds to the characteristics and mission of each institution.

Among the academic variables most frequently associated with financial health are:

- Student retention, which directly impacts the stability of tuition revenue flow and the efficient use of resources.
- The teaching load (students per full-time equivalent), which allows estimating the efficiency in the allocation of academic staff.
- Infrastructure per student, as an indicator of institutional investment in teaching-learning conditions.
- The level of academic training, especially the proportion of professors with postgraduate or doctoral degrees, can influence institutional prestige and the ability to obtain competitive funding, but also increase operating costs.
- The degree of dependence on state funding, which can act as a stabilizing factor or as a source of budgetary rigidity, depending on the context.

The findings of this study—using fixed-effects regression models—support this perspective: it is observed that student retention, the provision of adequate infrastructure, and access to fiscal contributions are correlated with lower exposure to financial risk. Conversely, in some institutional segments, such as universities with no tuition fees, maintaining a highly qualified faculty can represent a financial burden that reduces liquidity and operating margins.

These results are consistent with international studies that have documented how the most resilient university systems are those that successfully integrate academic and financial decisions into a coherent institutional strategy [3-4]. Rather than opposing quality and sustainability, these approaches propose viewing them as interdependent elements, whose alignment is essential for strengthening the higher education system.

2. Materials and Methods

This study uses a quantitative, non-experimental approach, with a longitudinal design and an exploratory-correlational approach. Its purpose is to examine the links between financial health and selected academic variables in Chilean higher education institutions (HEIs) based on an analysis of secondary data reported between 2016 and 2023, integrating a neutrosophic approach using neutrosophic cognitive maps (NCMs) to model uncertainty in these relationships.

The research adopts a longitudinal design based on time series analysis, which allows observing the evolution of institutions over time and detecting differentiated trajectories according to their financial and academic behavior. This approach is relevant in studies on institutional sustainability, as it allows identifying patterns of change, persistence or instability over time [12]. NCMs, based on the principles of neutrosophy (Smarandache, 1998), complement this design by representing the causal relationships between academic variables (such as student retention and infrastructure) and financial variables (such as leverage and liquidity), assigning truth (T), falsity (F) and indeterminacy (I) values to capture ambiguities in the interactions.

This is a non-experimental quantitative study, since there is no manipulation of independent variables. Instead, data obtained from official sources are analyzed and statistical relationships between them are modeled. Furthermore, it is framed within an explanatory-correlational level, as it seeks to generate empirical knowledge in a field that has received little attention in the Chilean context and to detect significant associations between financial and academic indicators [13]. The use of MCN enriches this approach by allowing a more robust interpretation of the relationships, highlighting the indeterminacy in contexts of high institutional heterogeneity, such as private universities not affiliated with free education. The data used come from the Superintendency of Higher Education (SES) and the Undersecretariat of Higher Education, agencies that annually publish financial, academic, and administrative information on Chilean HEIs. Universities belonging to and not belonging to the CRUCH were included.

2.1. Analysis methods

Biplot methods [14 -18] and regression models with fixed effects (PanelOLS).

2.1.1. HJ-Dynamic Biplot

Dynamic Biplot [17, 18] was used, a multivariate technique that allows to simultaneously represent the institutions (rows) and the variables (columns) in a two-dimensional factorial space, maximizing the quality of the representation, from the singular value decomposition (SVD) [14].

dynamic HJ- Biplot proposed by Egido and Galindo- Villardón [18], extends the classical HJ- Biplot proposed by Galindo- Villardón [15], by incorporating a temporal dimension, which allows to visualize the evolution of individuals or institutions over time. This technique is especially useful in longitudinal studies, since it captures not only the relative positions in each period, but also the trajectory and trends followed by the observations [17].

The dynamic component of the Biplot allowed for observation of institutional trajectories over time, highlighting shifts in their financial. The length and direction of the vectors represent the variance and correlation explained by each indicator, while the relative position of the institutions facilitates comparisons between levels of performance and stability.

2.1.2. Fixed effects regression models (PanelOLS)

To identify significant associations between academic and financial health variables, five fixed effects regression models for panel data (PanelOLS) were fitted, an appropriate technique when it is desired to control for unobservable heterogeneity between entities (institutions) and for common period effects [19].

The base model is composed of the form

$$y_{it} = \alpha_i + \beta X_{it} + \lambda_t + \epsilon_{it}$$

where:

- $y_i t$is the value of the financial indicator for the institution $i$in the year$t$
- $\alpha_i$represents unobservable effects specific to each institution (constant over time).
- $X_{it}$is the vector of academic variables.
- $\lambda_t$captures common temporary effects (such as regulatory or economic changes)
- $\epsilon_{it}$is the error term

This approach improves the robustness of estimates by considering that each institution has structural characteristics that are constant over time and can influence the results. Six key financial

health indicators were analyzed as dependent variables: leverage, adjusted leverage , current liquidity, operating cash flow, gross margin, and fixed asset ratio.

Independent variables included academic dimensions such as first- and second-year retention, teaching load, level of academic training (postgraduate and doctoral), infrastructure per student, and dependence on fiscal funding.

The analyses were conducted by differentiating between institutional segments: private universities, CRUCH, those affiliated with and those not affiliated with free tuition, in order to capture possible specific patterns by type of institution.

2.2. Neutrosophic Cognitive Maps

Definition 1: ([20,21,22]) Let X be a universe of discourse . A *Neutrosophic Set* (NS) is characterized by three membership functions , , $u_A(x), r_A(x), v_A(x) : X \to]_A^-0, 1^+[$ that satisfy the condition $_A^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for everyone $x \in X$. $u_A(x), r_A(x) \text{and} v_A(x)$ are the truthfulness , indeterminacy , and falsity membership functions of x in A, respectively , and their images are subsets standard or non- standard of $]_A^-0, 1^+[$.

Definition 2: ([20,21,22]) Let X be a universe of discourse . A *Univalued Neutrosophic Set* (SVNS) A over X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\} \qquad (1)$$

Where $u_A, r_A, v_A : X \to [0,1]$, satisfies the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for everyone $x \in X$. $u_A(x), r_A(x) \text{and} v_A(x)$ denote the truthfulness , indeterminacy , and falsity membership functions of x in A , respectively . For convenience , a *Univalent Neutrosophic Number* (NNUN) will express itself as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Others definitions important are related to graphics.

Definition 3: ([20,21,22, 26, 27]) A *neutrosophic graph* is a graph that contains at least one edge indeterminate , which is represented by dotted lines .

Definition 4: ([20,21,22, 26, 27]) A *graph directed neutrosophic* is a graphic directed that contains at least one edge indeterminate , which is represented by dotted lines .

Definition 5: ([20,21,22]) A *Neutrosophic Cognitive Map* (NCM) is a graph directed neutrosophic, whose nodes represent concepts and whose edges represent relations causal between the edges .

Yeah$C_1, C_2, \ldots, C_k$ There are k nodes, each of whichC_i $(i = 1, 2, \ldots, k)$ can represent themselves by means of a vector $(x_1, x_2, \ldots, x_k)$where $x_i \in \{0, 1, I\}$. $x_i = 0$means the nodeC_i this in a state activated, $x_i = 1$means that the nodeC_i this in a state disabled and$x_i = I$ means that the nodeC_i this in a state indeterminate, in a time specific or in a situation specific.

YeahC_m andC_n They are two nodes of the NCM, one edge addressed fromC_m toC_n It is called connection and represents the causality of C_ma C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. Ifα_{mn} denotes the weight of the edge $C_m C_n$,$\alpha_{mn} \in \{-1, 0, 1, I\}$ So we have the following:

$\alpha_{mn} = 0$YeahC_m does not affect C_n,

$\alpha_{mn} = 1$If there is an increase (decrease) inC_m produces an increase (decrease) in C_n,

$\alpha_{mn} = -1$If there is an increase (decrease) inC_m produces a decrease (increase) in C_n,

$\alpha_{mn} = I$If the effect ofC_m inC_n is indeterminate.

Definition 6: ([20,21,22]) An NCM that has edges with weights in$\{-1, 0, 1, I\}$ It is called Simple Neutrosophic *Cognitive Map* .

Definition 7: ([20,21,22]) If$C_1, C_2, \dots, C_k$ are the nodes of an NCM. The neutrosophic matrix N(E) is defined as $N(E) = (\alpha_{mn})$, whereα_{mn} denotes the weight of the edge addressed $C_m C_n$, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. $N(E)$is called *the neutrosophic adjacency matrix* of the NCM.

Definition 8: ([20,21,22]) Let$C_1, C_2, \dots, C_k$ Let be the nodes of an NCM. Let $A = (a_1, a_2, \dots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called *the neutrosophic state vector instant* and represents the position of the state node on- off -indeterminate at a given moment .

$a_m = 0$YeahC_m this disabled (does not have effect),

$a_m = 1$YeahC_m is activated (has an effect),

$a_m = I$YeahC_m is indeterminate (it cannot be determine his effect).

Definition 9: ([20,21,22]) Let$C_1, C_2, \dots, C_k$ Let, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_mC_n}$be the nodes of an NCM.$\overrightarrow{C_1C_2}$ be the edges of the NCM, then the edges constitute a *cycle addressed* .

The NCM is called *cyclic* Yeah presents a cycle directed . It is called *acyclic* if it does not present a cycle directed .

Definition 10: ([20,21,22]) An NCM with cycles is said to be has feedback . When exists feedback in the NCM is said to be a *system dynamic* .

Definition 11: ([20,21,22]) Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…,$\overrightarrow{C_{k-1}C_k}$ be a cycle. WhenC_m is activated and its causality flows by the edges of the cycle and then it is the cause ofC_m Per se same, then he system dynamic circulates. This is true for each node . $C_m$with $m = 1, 2, \dots, k$. The equilibrium state of this dynamic system is called *pattern hidden* .

Definition 12: ([20,21,22]) If the equilibrium state of a system dynamic is a state unique , then it is called *fixed point* .

An example of a fixed point is when a system dynamic begins to be activated by [number] C_l. If the NCM is assumed to lie on [number] C_l and [number C_k], i.e. the state remains as [$(1, 0, \dots, 0, 1)$number], then this neutrosophic state vector is called *fixed point* .

Definition 13: ([20,21,22]) If the NCM is established with a repeating neutrosophic state vector in the form:

$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called *a cycle limit* from the NCM.

Method for determining patterns hidden

Leave$C_1, C_2, \dots, C_k$ Let be the nodes of the NCM with feedback. Suppose E is the adjacency matrix associated . A pattern is found hidden when C_l is activated and a vector input is used. $A_1 = (1, 0, 0, \dots, 0)$ It is provided . The data must go through the neutrosophic $N(E)$matrix, which is obtained by multiplyingA_1 by the matrix $N(E)$.

Leave $A_1N(E) = (\alpha_1, \alpha_2, \dots, \alpha_k)$ with the replacement α_m threshold operation for 1 yes $\alpha_m > p$ andα_m for 0 yes$\alpha_m < p$ (p is an integer positive suitable) andα_m is replaced by I if it is not an integer . The concept resulting vector is updated;C_1 is included in the updated vector transforming the first coordinate of the resulting vector in 1.

Yeah$A_1N(E) \to A_2$ It is assumed, $A_2N(E)$considered and the same procedure is repeated until a limit cycle or a fixed point is reached.

Definition 14: ([23,24, 25]) A *neutrosophic number* N is defined as a number as go on :

$N = d + I$ (2)

Where d is called part determined and i is called part indeterminate .

Giventhat $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2$Ithey are two neutrosophic numbers , some Operations between them are defined as follows manner :

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$ (Addition);

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

3. Results

This section presents the findings obtained from the multivariate analysis of financial health and its relationship with academic variables in Chilean higher education institutions (HEIs), using two complementary approaches: the visualization of trajectories through dynamic HJ- Biplots and the adjustment of regression models with fixed effects (PanelOLS). The combination of both methods allows capturing not only the temporal evolution of institutions in terms of financial risk, but also establishing statistically significant associations between structural academic variables and the main indicators of financial sustainability. The results are presented differentiating institutional types (CRUCH, private, affiliated and non-affiliated with free education), which allows identifying specific patterns and relevant disparities in the system.

3.1. Results obtained with the dynamic Biplot

Figure 1 shows the financial and financial ealth indicators of the CRUCH universities, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 60% inertia filter was used for the visualization, omitting institutions that vary less than this percentage around the data mean (coordinate origin). The observed variance absorption is close to 60%, ensuring a good representation of the data. Quite varied trajectories are observed, but universities with potential financial risk and financial risk with high debt ratios are evident in both Leverage and Financial Leverage. This indicates the indebtedness of Higher Education Institutions (HEIs) as the main cause of risk. Furthermore, greater volatility is evident in these institutions, with the University of Aysén standing out as the one with the highest risk and greatest variability, along with the Alberto Hurtado University, which, despite having potential financial risk, shows a trajectory toward high debt indicators.

Regarding the variables, high correlations are observed between the Gross Margin and the operational cash flow and, in the CRUCH institutions, it is observed that the total debt ratios present moderate correlations with the debt ratios with the financial system, not relating to a large extent, as would be expected, the equity commitment of the institutions with the commitment of the HEIs with the financial market and the dependence on financial institutions.

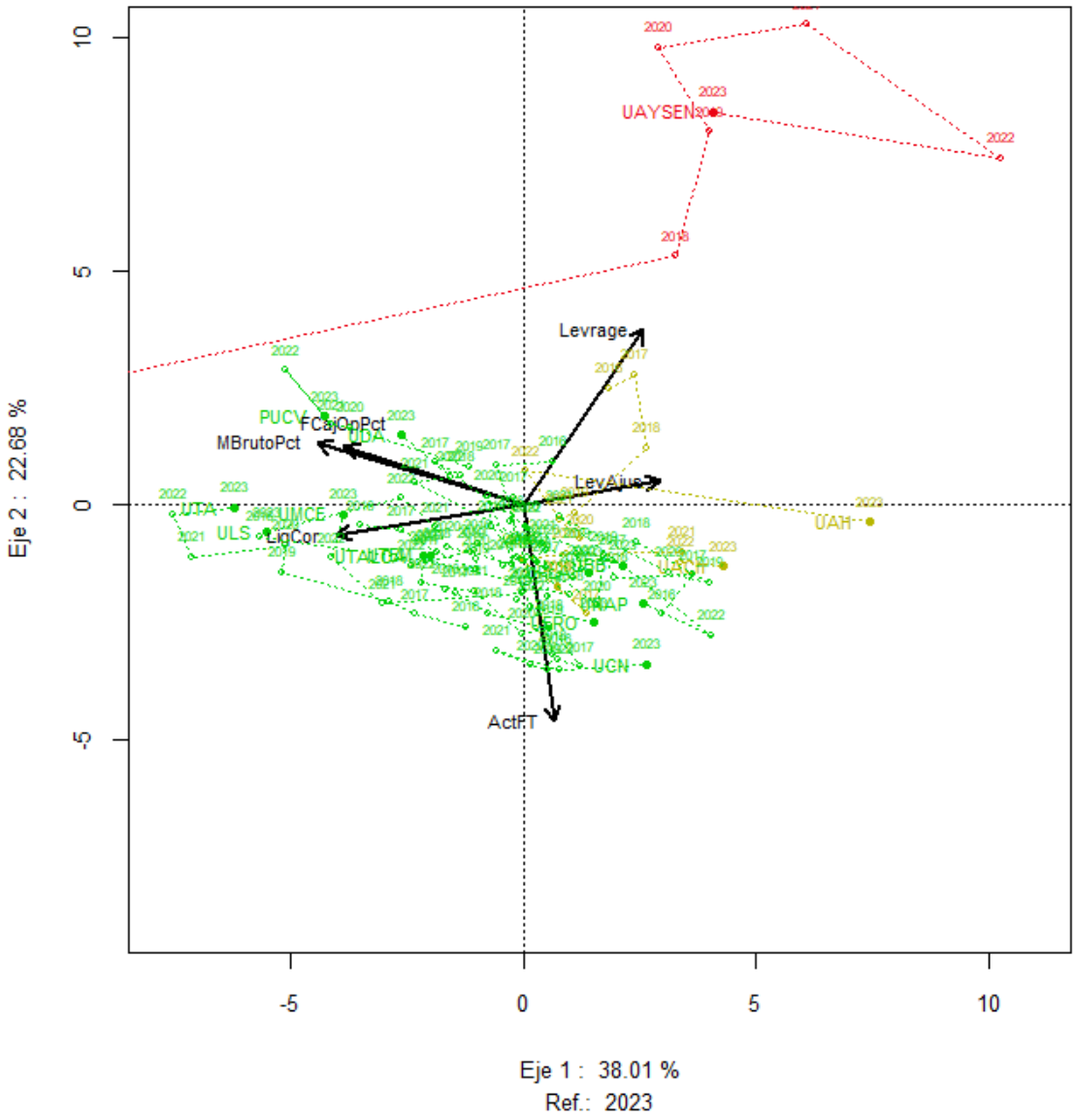

Figure 1. Dynamics of CRUCH Universities.

Figure 2 shows the financial and financial health indicators of private universities, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 50% variance filter was used for the visualization, omitting institutions that vary less than this percentage around the information mean (coordinate origin). The observed variance absorption rate is close to 58%, ensuring a highly consistent representation of the study data.

Regarding private universities, a strong positive correlation is observed between Leverage and Adjusted Leverage, demonstrating strong relationships between equity commitment and commitment to the financial system of private universities. Furthermore, it is evident that Gabriela Mistral University, having a high financial risk, presents the greatest variability in its performance and high debt ratios. Meanwhile, Finis University Terrae, classified by the SES as having potential financial risk, shows low mobility in its trajectory over the years of analysis, but even with high values in proportion to fixed assets, it presents low Gross Margin and Operating Cash Flow values, framing its potentially risky positioning.

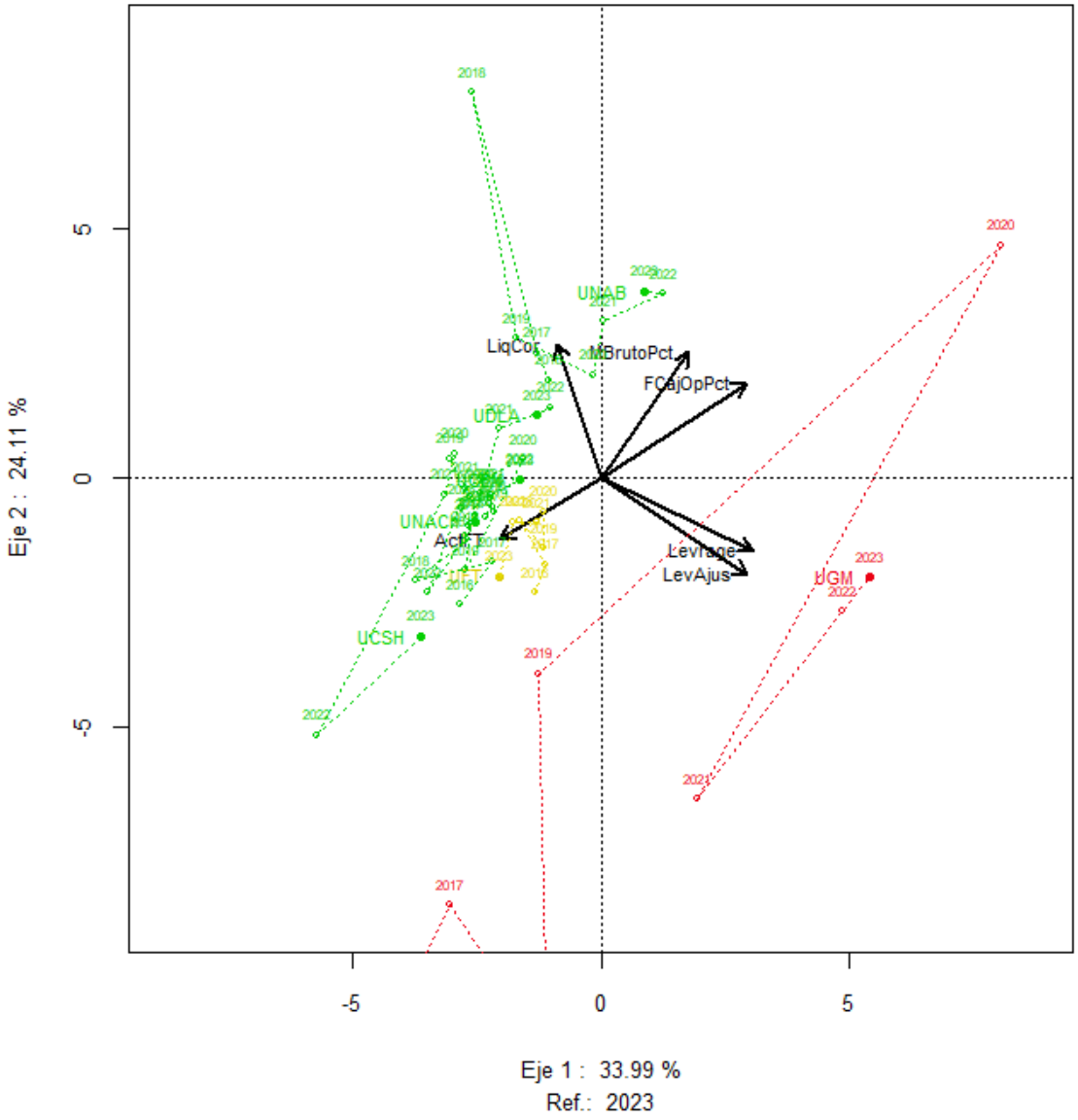

Figure 2. Dynamics of Private Universities.

Figure 3 shows the financial and financial health indicators of universities that offer free tuition, highlighting their performance from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are highlighted in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A 60% variance filter was used for the visualization, omitting institutions that vary less than this percentage around the data mean (coordinate origin). The observed variance absorption rate is close to 59%, ensuring a highly consistent representation of the study data.

This new segmentation again reveals high variability in the performance of the University of Aysén and Alberto Hurtado University in terms of Financial Risk and Potential Risk, respectively. Once again, these universities are framed within high levels of debt (Leverage and Adjusted Leverage) among institutions offering tuition-free tuition, in addition to low proportions of fixed assets and current liquidity.

In this segment, debt with the financial system shows moderate correlations with total debt, but the former is framed within much lower variability, suggesting that universities with positive

trajectories in this direction indicate greater risk even though they visually show less movement within the reference framework.

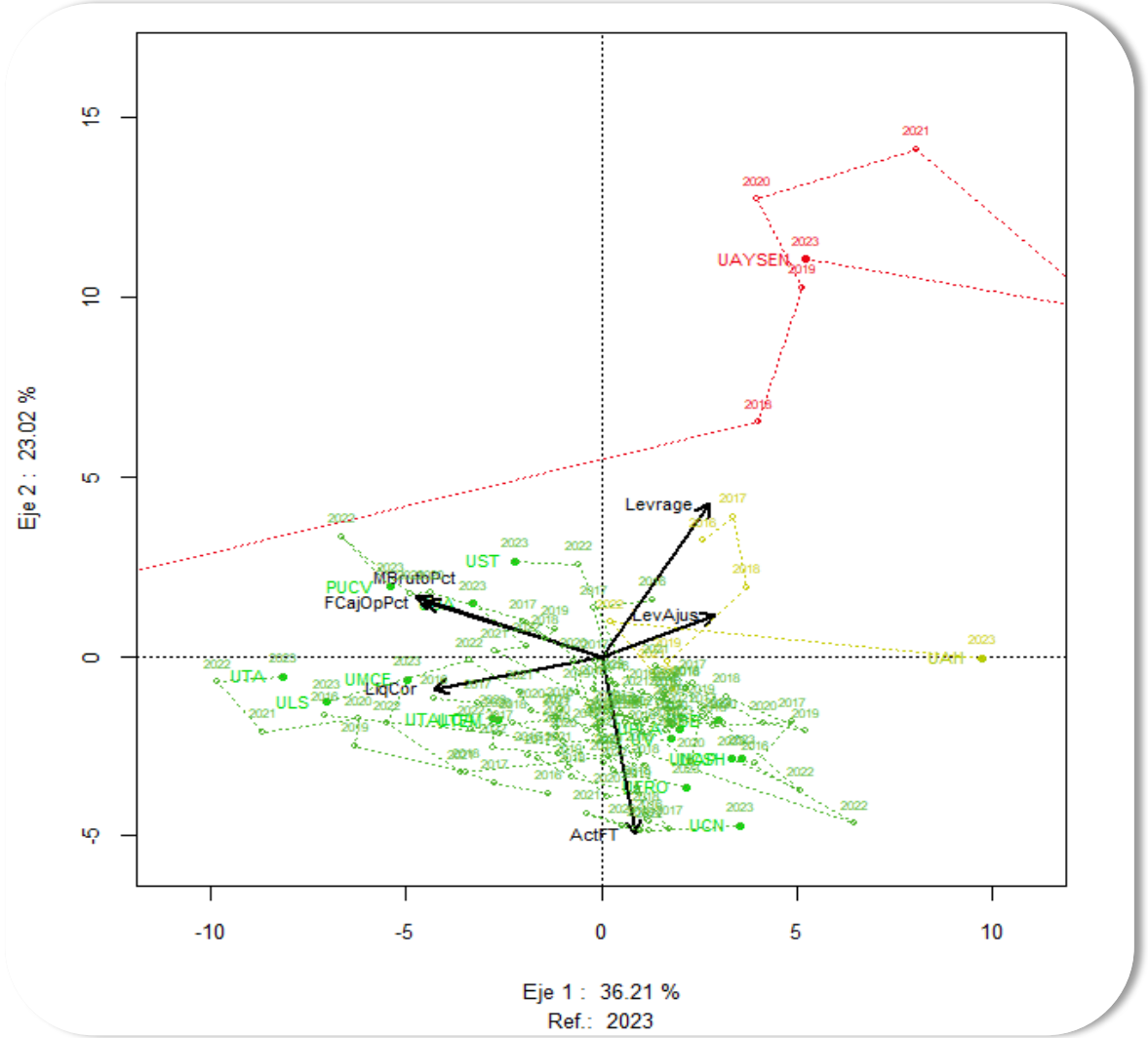

Figure 3. Dynamics of free institutions

Figure 4 shows the financial and financial health indicators of universities not affiliated with free tuition, highlighting their trajectories from 2016 to 2023. Institutions with no financial risk declared by the SES in the most recent period are marked in green, institutions with potential financial risk in yellow, and institutions with financial risk in red. A variance filter was not used for the visualization given the small number of cases compared to the other representations, omitting institutions that vary less than this percentage around the information mean (coordinate origin). The observed variance absorption rate is close to 61%, ensuring a good quality representation of the study data.

Volatile trajectories are observed in most institutions in this segment, especially at Gabriela Mistral University and Universidad del Desarrollo, classified as Financial Risk and Potential Risk, respectively, according to the SES. Both institutions present trajectories around high levels of debt, highlighting that Leverage and Financial Leverage present an almost perfect linear correlation. This indicates that in the case of universities not affiliated with free tuition, there is a high dependence on the financial system, a structural risk, since if there are credit restrictions, rate increases, or bank liquidity demands, their sustainability could be quickly compromised. Without a "fiscal cushion" like free tuition, they lack stable state revenues to cushion the leverage.

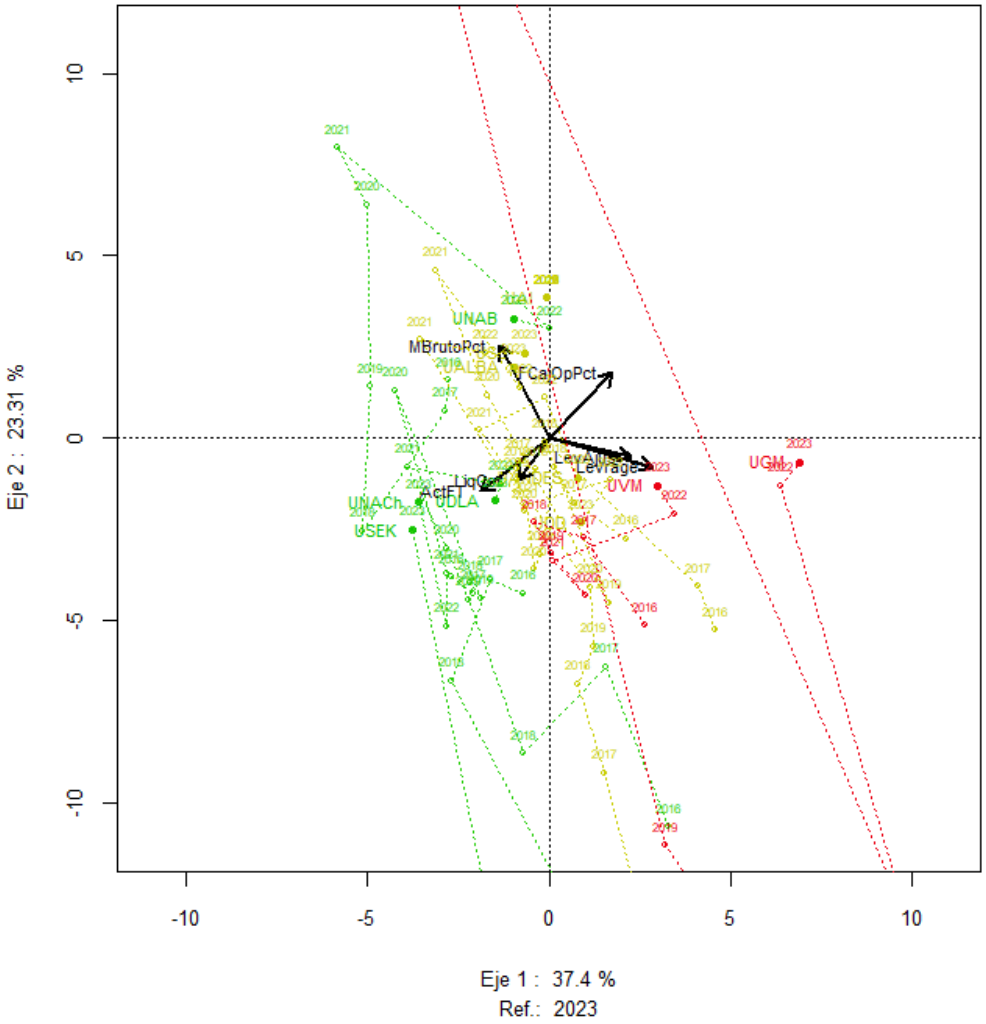

Figure 4. Dynamics of institutions not affiliated with free education

3.2. Fixed effects regression model

Below are the results of the fixed-effects regression models for each of the financial indicators in the analysis, highlighting the comparison between the general model and the segmentations for private universities, CRUCH universities, tuition-free universities, and non-tuition-free universities.

Table 1 shows that higher student retention (first and second years) is associated with higher overall debt, especially at tuition-free institutions and private universities. Greater infrastructure per student and higher fiscal contributions, both as a percentage of income and assets, are consistently associated with lower leverage, confirming their protective role. At CRUCH universities, it is notable that having a higher proportion of faculty with postgraduate degrees is also associated with lower debt, suggesting greater institutional strength. Finally, universities without tuition-free programs have stronger relationships between retention and debt, reflecting possible structural fragility when financing operations without state support.

Table 1. Comparative fixed effects regression model for Leverage.

Leverage	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	1,785	2,6961	7.08	3,0119	5,4028
1st_year_retention	**1.6527	-1.6846	**1.4578	**1.2108	-2.9758
2nd_year_retention	**1.2695	**4.9329	0.1767	**0.7608	**6.8045
NEM	-0.5649	-0.7236	-0.7285	-0.2821	-1.3678
Proof	0.0037	0.0051	0.001	0.0008	0.0104
Duration_Index	-0.0543	**-0.2551	0.0515	0.0081	**-0.4936
Students ___JCED	**0.0018	**0.0018	**-0.0116	-0.0003	**0.002
Students ___JCE	**-0.0361	**-0.042	**0.0485	-0.0131	**-0.0675
%_ JCE_with_Doctorate	0.8253	0.1218	0.0351	0.9724	2,9656
%_ JCE_Postgraduate	0.4662	0.1027	**-1.9154	-0.8957	-0.0667
m²_built_per_student	**-0.0903	-0.0309	**-0.1696	**-0.1372	-0.0403
% _of_tax_contributions_of_income	-0.7058	0.0758	**-1.1711	-0.47	-3.6562
% _of_tax_contributions_of_total_assets	**-0.934	-1,029	**-0.9035	**-0.8992	-0.5616

Table 2 shows, from the Adjusted Leverage model (financial debt/equity), that structural variables such as the percentage of tax contributions over income are negatively and significantly associated in almost all models, especially in universities without tuition fees, confirming that the lack of state funding leads to greater dependence on bank debt. It is also observed that a higher proportion of faculty with doctorates is associated with greater financial debt in the general and GRT models, which may reflect institutional investment to raise academic standards. Academic efficiency variables, such as the number of students per JCE, have negative and significant effects, reinforcing the idea that institutions with a higher optimized academic load tend to incur less debt from banks. Finally, the positive result of the admissions test variable in several segments suggests that a better student intake profile could be associated with financial investment strategies in quality.

Table 2. Comparative fixed effects regression model for Adjusted Leverage

LevAjus	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	-1.8019	-1.4236	-1.1878	-1.7308	0.631
1st_year_retention	0.3516	-0.8866	0.1995	0.2124	-2.2856
2nd_year_retention	0.171	0.1548	0.2317	0.2089	1,1889
NEM	-0.0423	-0.2239	0.049	0.1263	-0.365
Proof	**0.0038	**0.008	0.001	**0.0016	**0.0086
Duration_Index	-0.0043	-0.0435	0.0145	0.0084	-0.1276

Students ___JCED	**0.0007	**0.0008	0.0007	0.0001	0.0007
Students ___JCE	**-0.0094	**-0.0196	0.0096	0.0003	**-0.037
%_ JCE_with_Doctorate	**1.0537	1,0571	0.8215	**0.9985	2,9707
%_ JCE_Postgraduate	-0.1572	-0.2168	-0.1202	-0.3587	-0.71
m²_built_per_student	-0.0068	-0.0136	-0.0028	-0.0071	-0.0209
% _of_tax_contributions_of_income	**-0.4543	-0.4765	**-0.4037	**-0.304	**-2.3554
% _of_tax_contributions_of_total_assets	0.2062	0.3882	0.0616	**0.2137	0.5096

Table 3 shows that a higher NEM score at entry is positively associated with operational cash flow across all segments, which can be interpreted as a sign that students with better prior performance contribute to financial operational stability. The number of students per JCE is also positively associated in the general and CRUCH models, indicating that greater teaching efficiency could improve net financial flow. It is noteworthy that a higher percentage of faculty with postgraduate degrees is associated with lower cash flow in the general, CRUCH, and GRT models, possibly due to the higher structural costs associated with this academic body. Furthermore, the negative constant in several models indicates that even with all predictors at low values, cash flow tends to be negative, revealing a fragile financial foundation in several institutions, especially those without tuition.

Table 3. Comparative fixed effects regression model for operating cash flow.

FCajOpPct	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	**-1.0027	**-1.1169	-0.7382	-0.7913	**-1.5489
1st_year_retention	-0.0765	-0.3103	**-0.166	-0.0829	-0.3046
2nd_year_retention	0.0201	0.0471	-0.0065	0.0195	-0.0482
NEM	**0.2106	**0.1722	**0.2524	**0.2154	**0.2339
Proof	-0.0002	**0.0007	-0.0006	-0.0005	0.0009
Duration_Index	-0.0103	-0.0105	-0.0064	-0.011	-0.0097
Students ___JCED	-0.00006867	-0.00002712	**-0.0013	-0.0002	-0.00001592
Students ___JCE	**0.002	0.0006	**0.0092	0.0032	0.0008
%_ JCE_with_Doctorate	-0.0453	-0.2206	0.2454	0.0563	-0.2891
%_ JCE_Postgraduate	**-0.1704	-0.0243	**-0.748	**-0.3289	0.0314
m²_built_per_student	0.0018	0.0006	-0.0009	0.0009	0.0035
% _of_tax_contributions_of_income	0.0353	0.0732	0.0642	0.0406	0.0716
% _of_tax_contributions_of_total_assets	0.0022	-0.0593	0.0859	0.0263	-0.0688

Table 4 shows that higher student retention (first year) is associated with decreased liquidity in the general, CRUCH, and GRT models. This could reflect the fact that institutions must cover more operational costs to maintain academic continuity, putting pressure on their short-term cash flows. In contrast, institutions with greater academic efficiency (more students per JCE) tend to have greater liquidity, possibly due to better use of teaching resources. It is also noteworthy that the increase in square meters per student, as well as fiscal contributions, are positively related to liquidity in the general, CRUCH, and GRT models, reinforcing the role of public contributions as short-term financial buffers. Finally, the percentage of JCE with doctorates has a negative effect in the general and private models, suggesting that institutions with more academics with doctorates face greater financial commitments, reducing operational liquidity.

Table 4. Comparative fixed effects regression model for current liquidity.

LiqCor	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	1,1144	-3.9212	-2.6732	-1.0854	6.9954
1st_year_retention	**-11,255	1,625	**-12,846	**-11,518	-0.5152
2nd_year_retention	**-3.3769	-2.7773	-1.6536	-2.1534	-1.7957
NEM	1,4551	0.5065	1,4996	1,1133	-0.6704
Proof	0.0013	0.0041	-0.0066	0.0009	-0.002
Duration_Index	-0.1872	0.0798	-0.0653	-0.0012	0.1149
Students ___JCED	-0.0006	-0.0001	0.0207	0.0031	0.0003
Students ___JCE	**0.0856	-0.0089	**0.1823	**0.1227	-0.0147
%_ JCE_with_Doctorate	**-5.4569	**-5.4352	-4.8808	-4.4086	-1.1354
%_ JCE_Postgraduate	-0.0048	**2,8811	1,1966	-0.7507	1,8873
m²_built_per_student	**0.4241	-0.0096	**0.7999	**0.7156	-0.0612
%_of_tax_contributions_of_income	**-3.9994	-1.5788	**-3.5566	**-3.9464	-0.3687
%_of_tax_contributions_of_total_assets	**4,1729	1,1109	**3.3634	**4.4752	0.7432

Table 5 shows that the gross operating margin is negatively affected by higher student retention rates in both the first and second years (general, CRUCH, and GRT segments), which could indicate that student retention generates higher costs that are not always offset by additional revenue. In contrast, institutions with more faculty with postgraduate or doctoral degrees tend to show lower operating margins, possibly reflecting higher structural academic costs. It is also noteworthy that a high NEM is associated with better margins only in GRT, but the opposite is true for non-GRT segments. An increase in square meters per student has a positive effect on CRUCH, but a negative effect on non-GRT segments, which could be associated with different infrastructure investment strategies. Finally, it is confirmed that higher tax contributions (on income or assets) contribute positively to the gross margin in all segments, reinforcing their role as an operational financial support for HEIs.

Table 5. Comparative fixed effects regression model for gross margin.

MBrutoPct	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	0.0973	-0.0106	-1.1521	**-0.9689	**4,4914
1st_year_retention	-0.165	-0.629	**-0.4303	**-0.3154	-0.8097
2nd_year_retention	**-0.3292	**1.1017	-0.0605	-0.1172	**1.4256
NEM	0.0584	0.0281	0.2171	**0.208	**-0.6398
Proof	0.0001	-0.0002	0.0001	0.0003	-0.0006
Duration_Index	-0.0094	-0.0241	0.0009	-0.0095	**-0.052
Students ___JCED	0.00003726	0.00002124	0.0006	0.0001	0.000009572
Students ___JCE	**0.0058	0.0037	**0.0082	0.0012	0.0045
%_ JCE_with_Doctorate	**-0.8294	**-1.5615	0.0004	-0.2157	-0.6533
%_ JCE_Postgraduate	**0.303	0.2515	-0.0525	0.0311	-0.2094
m²_built_per_student	-0.0031	-0.0074	**0.0079	0.0046	**-0.0144
% _of_tax_contributions_of_income	**-0.3691	**-0.6935	**-0.1282	**-0.1337	**-1.7695
% _of_tax_contributions_of_total_assets	**0.2469	**0.4967	0.0837	**0.1211	**0.7788

Table **6** suggests that higher first-year student retention is associated with a higher proportion of fixed assets at CRUCH universities and those with free tuition, which could reflect a link between retention and infrastructural expansion. Infrastructure per student (m²) has a significant positive impact on several segments, which is expected given that this indicator is part of fixed assets. Furthermore, universities with students who remain in school longer (higher duration index) tend to have a lower proportion of fixed assets, possibly due to prioritizing operating expenses over infrastructure investment. In the GRT model, it is observed that the fiscal contribution as a proportion of assets is also positively related to the proportion of fixed assets, while in private and non-GRT universities, the fiscal contribution to revenue has opposing effects. This suggests structural differences in how institutions capitalize state resources on infrastructure.

Table 6. Comparative fixed effects regression model for fixed asset ratio.

ActFT	General model	Model Private	CRUCH Model	GRT Model	Non-GRT model
const	0.7381	2,1466	-0.8544	**1.2536	2,1183
1st_year_retention	0.2304	0.6559	**0.3074	**0.3467	0.0281
2nd_year_retention	0.1264	0.4647	-0.0227	0.0225	0.1963
NEM	-0.0293	-0.3424	0.2126	-0.1384	-0.2058
Proof	-0.00008854	-0.0001	-0.0003	-0.0004	-0.0007
Duration_Index	**-0.0368	**-0.0568	0.0004	-0.0024	**-0.097

Students ___JCED	**-0.0003	**-0.0003	-0.0013	-0.0002	-0.0002
Students ___JCE	-0.0013	-0.0036	0.0061	**0.0055	-0.0111
%_ JCE_with_Doctorate	0.0691	0.946	**-0.7201	-0.1886	0.0381
%_ JCE_Postgraduate	-0.2013	-0.345	0.2578	-0.0129	0.0996
m²_built_per_student	**0.0116	-0.0069	**0.0219	**0.0204	-0.0056
% _of_tax_contributions_of_income	0.0523	**0.5296	-0.0678	**-0.1602	**1.9979
% _of_tax_contributions_of_total_assets	0.0587	-0.3772	0.0602	**0.1293	-0.4191

The main objective of this study was to explore the links between financial health and academic quality indicators in Chilean higher education institutions (HEIs) using a mixed approach that combines multivariate visualization (HJ- Biplots) and panel data econometric models (PanelOLS). The evidence obtained allows us to answer the research question affirmatively: there are significant structural relationships between certain academic variables and the financial health of HEIs, although these relationships are not homogeneous, linear, or necessarily universal to all types of institutions.

3.3. Relationship between financial health and academic performance

It was found that the financial health of HEIs does not depend exclusively on accounting indicators such as leverage or cash flow, but is strongly influenced by key academic variables. Among those that showed the most consistent effects were student retention, infrastructure per student, faculty development, and government support.

Student retention (first and second years) influences several financial indicators: it increases debt and reduces gross margin and operating liquidity, especially in private and non-free universities (Tables 1, 3, and 4). This reflects the fact that, without sufficient public support, prolonged student retention can put a strain on operating resources.

Infrastructure per student (m^2) acts as a protective factor in terms of liquidity and fixed asset ratio (Tables 4 and 6), confirming that HEIs with sustained investments in physical capacities have greater financial resilience.

The education of academic staff has mixed effects. A higher proportion of professors with postgraduate or doctoral degrees is associated with higher perceptions of quality and prestige, but also with higher operating costs that negatively affect cash flow and gross margin (Tables 3 and 5). This tension reinforces the need for strategic planning that links academic quality with financial sustainability.

State support, measured as a percentage of tax contributions relative to income or assets, is a decisive factor. Universities with free tuition show more stable trajectories in the HJ- Biplots (Figures 3 and 1) and better financial results in the econometric models (Tables 1, 4, and 5), suggesting that the free tuition policy has had an effective stabilizing effect for institutions with greater structural challenges.

3.4. Institutional segmentation and differentiated trajectories

Dynamic HJ- Biplot visualization [18] confirmed that not all HEIs follow the same financial trajectory, even under similar structural conditions. The most vulnerable segments were private

universities not affiliated with free tuition, which presented the most volatile trajectories and an almost total dependence on the financial system (Figures 2 and 4), especially in cases such as Gabriela Mistral University and the Universidad del Desarrollo. Their structural risk is accentuated by the lack of direct fiscal support, which makes them highly sensitive to regulatory changes or credit contraction.

For their part, the CRUCH universities, while more stable, also identified risk factors, such as the University of Aysén and the Alberto Hurtado University, whose trajectories shifted toward critical areas in the biplots (Figures 1 and 3), indicating persistent financial imbalances even with public support. This behavior suggests that a free tuition policy alone does not guarantee financial health unless accompanied by operational efficiency, asset planning, and a sustainable academic structure.

3.5. Quality as a balance between academic and financial aspects

One of the central contributions of the study is that academic quality cannot be understood in isolation from financial health, but neither can they be considered equivalent. There are financially sound institutions with low academic standards, as well as high-quality educational institutions with financial risk profiles. This tension confirms what has been pointed out [11], [3]: university quality is built when there is coherence between the academic and economic dimensions, mediated by effective strategic management.

3.6. Implications for management and public policy

The findings of this study have important implications at the institutional and public policy levels. At the institutional level, HEIs must strengthen their integrated financial and academic analysis capabilities, developing planning models that link academic decision-making (such as increasing retention or faculty) with their financial effects. At the public policy level, a more holistic institutional assessment model is needed, one that does not reduce quality to enrollment or accreditation indicators, but rather incorporates variables such as leverage, liquidity, equity, and the financial environment in which universities operate.

Finally, there is a growing urgency to monitor private institutions not affiliated with free education, which exhibit structural risk profiles and could compromise the educational continuity of their students in crisis contexts.

In short, the study confirms that there is a deep, albeit complex, link between financial health and academic quality in Chilean HEIs. These links are dynamic, mediated by the type of institution and its structural conditions, and require multivariate and longitudinal analytical approaches such as those used here. Strengthening these links through public policy and institutional management is not only a technical task, but a condition for ensuring a sustainable, equitable, and high-quality higher education system.

3.7. Analysis with Neutrosophic Cognitive Maps

To complement the analyses multivariate (HJ- Dynamic Biplot) and econometric (PanelOLS), a neutrosophic approach was applied by maps Neutrosophic cognitive (MCN), based in the principles of neutrosophy [20, 21, 22]. MCNs model the relationships causal relationships between academic variables (retention in first and second year , infrastructure by student , level of education teaching) and financial (leverage, liquidity current , cash flow operational , margin gross , asset ratio fixed), assigning truth (T), falsity (F) and indeterminacy (I) values in he range [0,1], according to Definitions 5-13 of subsection 2.2. This approach allows capturing uncertainty inherent to interactions, especially in segments institutional with high variability, such as universities private not assigned to free education.

3.7.1. Construction of the Neutrosophic Cognitive Map

The MCN nodes were defined as the significant variables identified in the regression models (Tables 1-6) and the dynamic HJ -Biplot trajectories (Figures 1-4). The nodes included :

Academic variables: Retention in the first year (R1), retention in second year (R2), infrastructure by student (m^2/E), percentage of teachers with a doctorate (% JCE_Doc), percentage of teachers with a postgraduate degree (% JCE_Pos).

Financial variables: Leverage (Lev), liquidity current (LiqCor) , cash flow operational (FCajOp), margin gross (MBross) , asset ratio fixed (ActFT).

Contextual variable: Percentage of contributions prosecutors about he income (% AportFis).

The arches directed between nodes were built using the coefficients significant from Tables 1-6, where the magnitude and the sign of the coefficients determined the values of T (causality positive), F (causality negative or null) and I (indeterminacy). Indeterminacy was estimated considering the variability of the trajectories in HJ-Biplots and heterogeneity between segments institutional . The MCNs were implemented using customized software to neutrosophic environments (by example , FCMExpert modified) and were validated with experts in higher education to ensure the coherence of relations causal .

3.7.2. MCN Results

For the assignment of neutrosophic weights (T, F, I) in the causal relationships, a methodological triangulation process was followed that integrated the quantitative findings of the fixed effects regression models (PanelOLS) and the observation of the trajectories in the dynamic HJ- Biplots , together with expert validation.

- **Truth (T): A high T value was assigned when the** PanelOLS regression coefficients were statistically significant and the sign of the causal relationship (positive or negative) was consistent with the expected direction. The magnitude of the regression coefficient also influenced the strength of T; larger coefficients in absolute value indicated greater truth. For example, a strong positive or negative correlation in the regression tables (Tables 1–6) led to a high T value for that causal relationship.
- **Falsification (F):** An F value was assigned when the causal relationship was not statistically significant in the regression models, or when the sign of the coefficient contradicted the expected direction, suggesting an absence or cancellation of the effect. Coefficients close to zero or with a high probability (p- value) indicated greater falsification.
- **Indeterminacy (I):** This value captured the ambiguity and uncertainty inherent in interactions, particularly relevant in contexts of high institutional heterogeneity. Indeterminacy was estimated by considering the variability of the trajectories observed in the dynamic HJ- Biplots (Figures 1-4). More volatile trajectories, with erratic changes in the position of institutions over time, indicated a greater degree of indeterminacy in the causal relationships associated with that institutional segment. Furthermore, the inconsistency of the same effect across different segments (e.g., a significant coefficient

in one model but not in another) also contributed to a higher I value. The judgment of higher education experts was also crucial in refining these values, especially in cases where the data did not offer a clear conclusion.

This approach allowed the transformation of quantitative information and visual variability into neutrosophic values that more fully reflect the complexity of the relationships between academic and financial variables in Chilean HEIs.

The MCN was structured as a system dynamic with feedback (Definition 10, subsection 2.2), reflecting cycles causal relationships between academic and financial variables . The main ones are presented below relations causes modeled , with their neutrosophic weights:

1. **Retention student (R1, R2) → Leverage (Lev)** : In CRUCH universities and with free tuition , greater retention in first and second year is associated with an increase in debt (Table 1, coefficients 1.6527 and 1.2695, general model). The MCN assigned T=0.7, F=0.1, I=0.2, reflecting a causality positive strong , but with indeterminacy due to variability in the trajectories (by example , University of Aysén, Figure 1). In universities not affiliated with free tuition , this relationship is more intense (coefficient 6.8045 for R2, Table 1), with T=0.8, F=0.05, I=0.15, indicating greater dependence on debt without the fiscal support .
2. **Infrastructure by student (m^2/E) → Liquidity current (LiqCor)** : The infrastructure by student has an effect positive significant in liquidity in the general, CRUCH and GRT models (Table 4, coefficients 0.4241, 0.7999, 0.7156). The MCN assigned T=0.65, F=0.1, I=0.25, highlighting a causality positive, but with indeterminacy in universities not affiliated with free tuition , where he effect is not significant (coefficient -0.0612, Table 4) and the trajectories are volatile (Figure 4, Gabriela Mistral University).
3. **Percentage of contributions fiscal (% AportFis) → Leverage (Lev) and Liquidity (LiqCor)** : The contributions prosecutors reduce significantly he indebtedness (Table 1, coefficients -0.7058, -1.1711, -0.8992) and increase liquidity (Table 4, coefficients 4.1729, 3.3634, 4.4752) in the general, CRUCH and GRT models . The MCN assigned T=0.8, F = 0.05, I=0.15 for the % AportFis relationship → Lev (causality) negative) and T=0.75, F=0.1, I=0.15 for % AportFis → LiqCor , reflecting a role clear stabilizer , but with indeterminacy in universities not affiliated with free tuition , where contributions prosecutors are minimal (Figure 4).
4. **Training teacher (% JCE_Doc , % JCE_Pos) → Cash flow operational (FCajOp) and Margin gross (MBrute)** : A higher proportion of teachers with a doctorate or postgraduate degree is associated with lower cash flows and margins gross (Tables 3 and 5, coefficients negatives as -0.1704 for % JCE_Pos in FCajOp , general model). The MCN assigned T=0.3, F=0.5, I=0.2, indicating a causality negative moderate , with indeterminacy due to costs structural high and variability in the trajectories (by example , Alberto Hurtado University, Figure 1).

Table 7. Neutrosophic adjacency matrix N(E) for the relationships causal relationships between academic and financial variables

Node origin → Node destination	R1 → Lev	R2 → Lev	m²/E → LiqCor	% FisContribution → Lev	% FisContribution → LiqCor	% JCE_Doc → FCajOp	% JCE_Pos → MBruto
T (Truth)	0.7	0.8	0.65	0.8	0.75	0.3	0.3
F (Falsehood)	0.1	0.05	0.1	0.05	0.1	0.5	0.5
I (Indeterminacy)	0.2	0.15	0.25	0.15	0.15	0.2	0.2

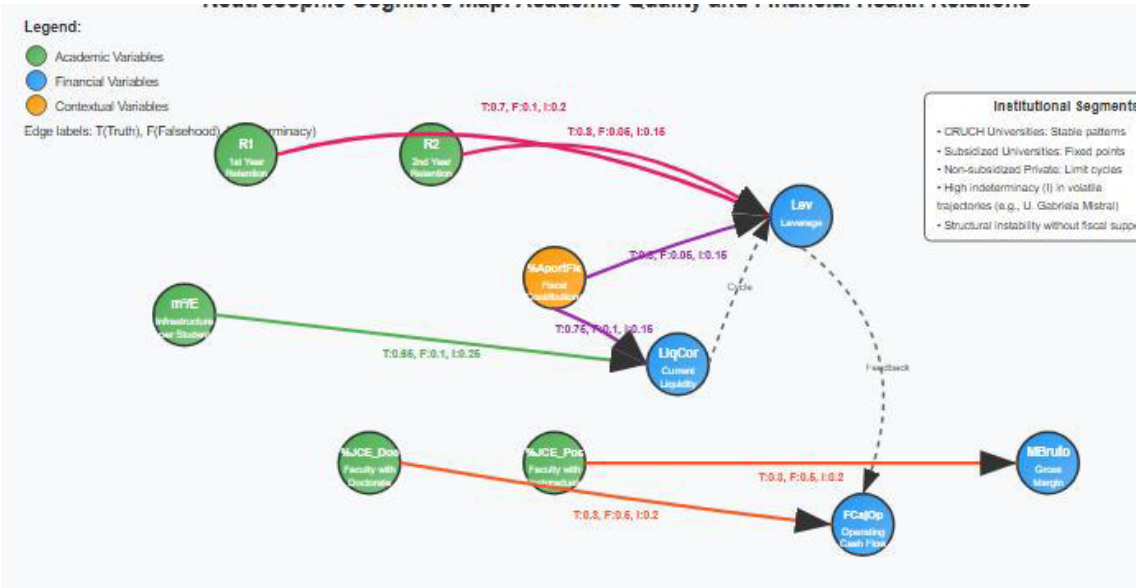

Figure 5: Representation graph of the obtained Neutrosophic Cognitive Map.

3.7.3. Analysis dynamic and patterns hidden

The MCN was analyzed as a system dynamic cyclic (Definition 9, subsection 2.2), using the neutrosophic adjacency matrix (Definition 7) and the state vector instantaneous (Definition 8). The activation of key nodes , such as R1 (retention) was simulated in the first year) and % AportFis (contributions prosecutors), to identify patterns hidden (Definition 11). The results showed:

- **Fixed point:** In CRUCH universities and with free tuition, the activation of % AportFis stabilizes the Lev and LiqCor nodes, reaching a state of equilibrium with T=0.8 for relationships negative with him indebtedness and T=0.7 for relationships positive with liquidity, confirming he protective role of financing state.

- **Cycle limit:** In universities not affiliated with free tuition, the activation of R1 generates a cycle dynamic between Lev, LiqCor and MBruto , with high indeterminacy (I = 0.3-0.4), reflecting instability structural observed in the trajectories volatiles from Gabriela Mistral University and the University of Development (Figure 4).

3.7.4. Implications of the neutrosophic approach

The use of MCN allowed capturing complexity and uncertainty in the relationships between quality academic and health financial, which are not completely linear neither homogeneous, as observed in the trajectories differentiated from the HJ-Biplots (Figures 1-4). The assignment of indeterminacy (I) values highlighted the ambiguity in segments such as universities not affiliated with free tuition , where the lack of contributions prosecutors and the high system dependency financial generate instability (Figure 4). This approach enriches the results of the models PanelOLS by providing a representation graphic and dynamic interactions causal, facilitating the identification of critical points for management institutional and the policy design public .

The results obtained through maps Neutrosophic cognitive (MCN) in subsection 3.7 enrich the understanding of the relationships structural between quality academic and health financial in Chilean higher education institutions (HEIs) , by incorporating uncertainty as a key dimension in he analysis . The MCNs, based in the principles of neutrosophy [20, 21, 22], allowed model interactions causal relationships between academic variables (retention student , infrastructure by student , training teaching) and financial (leverage, liquidity current , cash flow operational , margin gross , asset ratio fixed), assigning truth (T), falsity (F) and indeterminacy (I) values . This approach complements the results of the dynamic HJ -Biplot (Figures 1-4) and the effects regression models fixed (Tables 1-6), highlighting the ambiguity inherent in high contexts heterogeneity institutional .

Relationship modeling such as retention student → leverage (T=0.7-0.8, I= 0.15-0.2) reveals that , although retention in first and second anus increases he indebtedness , especially in universities not affiliated with free tuition (Table 1, coefficient 6.8045), the indeterminacy reflects variations institutional , such as trajectories volatiles from Gabriela Mistral University (Figure 4). This finding stresses that continuity student, although it is an indicator of quality academic , can trigger pressures financial in absence of robust fiscal support , confirming the observations in subsection 3.4 on the vulnerability of universities private not assigned to free education .

Similarly, the relationship infrastructure by student → liquidity current (T=0.65, I=0.25) showed a protective effect on CRUCH universities and with free tuition (Table 4, coefficients 0.7999, 0.7156), but with high indeterminacy in universities not affiliated with free tuition , where he impact is not significant (Table 4, coefficient -0.0612). This suggests that investments in infrastructure , although strengthen financial resilience in supported contexts state , generate uncertainty in institutions system dependent financial , as can be seen in the trajectories of the University of Development (Figure 4).

The role contribution stabilizer prosecutors (% AportFis → leverage and liquidity , T=0.75-0.8, I=0.15) corroborates the results of Tables 1 and 4, highlighting that the financing state reduces the indebtedness and improves liquidity in CRUCH universities and free tuition (Figures 1 and 3). However, the uncertainty in universities not affiliated with free tuition reflects the lack of a " fiscal cushion ", which exposes them to risks structural , as evidenced in the HJ-Biplots (Figure 4).

The analysis dynamic MCN, with fixed points in free universities and cycles boundaries In the non- attached ones, stability stands out structural of the first against the instability of the latter. These results reinforce the conclusions of subsection 3.5, which advocate for a comprehensive approach to quality university , where coherence between decisions academic and financial support is crucial. The incorporation of MCNs contributes a perspective novel in capturing unaddressed uncertainty by the methods statistics traditional , aligning with the recommendations of [11] and [3] on the need for models that integrate both dimensions .

In terms of implications , the MCNs highlight the importance of designing policies public policies that mitigate uncertainty financial in universities private schools not affiliated with free education , promoting financing strategies that balance quality academic and sustainability . At the level institutional , the MCNs offer a Visual and dynamic tool for HEIs to identify critical points in their relationships causal , facilitating a planning strategic further informed . This neutrosophic approach , by complementing the analyses multivariate and econometric , positions the study as a contribution methodological innovative for futures jobs in the field of Chilean higher education.

4. Applications

The results of this research allow to significantly advance the understanding of the structural links between financial health and academic quality in Chilean higher education institutions (HEIs). The empirical evidence obtained through dynamic HJ- Biplots [18], fixed effects regression models (PanelOLS), and neutrosophic cognitive maps (NCM) not only corroborates the existence of significant interrelationships, but also reveals a structural complexity that varies according to the type of institution, its access to state funding and its academic orientation, capturing the uncertainty inherent in these interactions.

First, the analysis shows that financial health cannot be separated from institutional academic functioning, as previously warned [3], when pointing out that world-class universities combine academic excellence with financial sustainability. This premise is empirically confirmed by observing that indicators such as student retention, available infrastructure, faculty quality, and state support have direct and significant impacts on key financial health variables such as leverage, cash flow, and gross margin (Tables 1 to 6). NCMs, based on neutrosophy [20, 21, 22], enrich this interpretation by modeling these relationships with neutrosophic weights (T, F, I), highlighting, for example, the positive causality between student retention and leverage (T=0.7-0.8, I=0.15-0.2, Table 2) and the indeterminacy in universities not affiliated with free tuition due to volatile trajectories (Figure 4). In this way, quality ceases to be an abstract category and is linked to the capacity of HEIs to sustain viable, stable, and consistent educational projects with their mission.

The evidence also calls into question an excessively economic approach to university management. As [8] warn, the exclusive emphasis on accounting indicators can obscure strategic risks if the academic logic and training commitments that characterize public or regional universities are not considered. This study shows that universities with high social valuation—such as some of the CRUCH—can be at financial risk if they do not have adequate financing structures, as observed in the trajectories of the University of Aysén and the Alberto Hurtado University (Figure 1). The MCNs reinforce this observation by assigning high indeterminacy (I=0.2) to relationships such as retention → leverage in CRUCH (Table 2), which suggests the need for a thorough review of the

Chilean financing model, especially with regard to structural support for institutions with relevant public roles.

On the other hand, the results show that private universities not affiliated with free tuition present the most critical profiles, with highly volatile trajectories and a high dependence on the financial system (Figures 2 and 4). The absence of a "fiscal cushion" and exposure to credit cycles makes them fragile institutions, whose equilibrium can easily be altered by regulatory, demographic, or economic changes. The MCNs highlight this vulnerability by assigning high indeterminacy (I = 0.25-0.3) to relationships such as infrastructure → current liquidity in this segment (Table 2), reflecting the structural instability of institutions such as Gabriela Mistral University (Figure 4). This situation reinforces the OECD's 2021 and World Bank 2022 approaches regarding the structural vulnerability of excessively commercialized university systems with little preventive regulation.

The study also sheds light on the relationship between academic quality and financial health. Although many HEIs with better infrastructure and more qualified faculty display more stable financial indicators, in other cases, these same variables are associated with higher operating costs that reduce liquidity and operating margins (Tables 3 and 5). NCMs capture this tension by assigning high falsity (F=0.5) and moderate indeterminacy (I=0.2) to relationships such as teacher training → cash flow and gross margin (Table 2), indicating that quality without structural financing can become a source of instability. This poses a challenge for quality assurance models, which must integrate economic criteria into their standards without sacrificing academic merit.

In this context, the relevance of the approach of [6] is reinforced. They propose evaluating institutional quality as an interdependent system, where academic, financial, and governance decisions are articulated within a common strategic framework. The results obtained through the regression models and the MCN confirm that public policies must adopt a more comprehensive perspective, one that overcomes the dichotomy between quality and sustainability, and that recognizes the diversity of the Chilean university system in its different structural realities. The incorporation of MCN, by modeling uncertainty in causal relationships, adds a dynamic dimension that facilitates the identification of critical points for institutional management, especially in contexts of high heterogeneity such as universities not affiliated with free tuition.

In methodological terms, this study also contributes by employing a longitudinal design with multivariate techniques, allowing to observe not only static correlations, but also trajectories and trends over time. The integration of MCN complements dynamic HJ- Biplots and PanelOLS models by providing a graphical and dynamic representation of causal interactions, following recommendations by authors such as [19] and [15, 16]. This neutrosophic approach allows to capture ambiguities that traditional statistical methods do not address, such as the high indeterminacy in universities not affiliated with free tuition (Table 2), strengthening the robustness of the analysis.

Finally, it should be noted that, while the findings offer a systematic and empirically supported perspective, they also reveal the lack of national studies that integrate financial and academic data in a cross-sectional manner. The bibliographic gap in this field—mentioned in the introduction—remains prevalent, and this paper seeks to pave the way for future research that delves into other dimensions, such as institutional governance, pedagogical models, or the use of technology, in relation to financial health and educational quality, potentially incorporating neutrosophic approaches to model uncertainty in these contexts.

5. Conclusions

This exploratory study advanced our empirical understanding of the links between financial health and academic quality in the Chilean university system, a relationship frequently intuited in political and academic discourse but scarcely investigated from a quantitative and comparative perspective.

First, statistically significant relationships are found between certain academic variables—such as student retention, faculty training, and infrastructure—and key financial health indicators, such as leverage, current assets, and gross margin (Tables 1–6). Neutrosophic cognitive maps (NCMs), based on neutrosophic theory, enrich this analysis by modeling these relationships with neutrosophic weights (T, F, I), highlighting, for example, the positive causality between student retention and leverage (T=0.7–0.8, I=0.15–0.2, Table 2) and the uncertainty in universities not affiliated with free tuition due to their high dependence on the financial system (Figure 4). These associations are not uniform across the system but vary according to the type of institution, its funding structure, and its affiliation with policies such as free tuition.

The results also demonstrate that good financial health is not necessarily synonymous with high academic quality, and vice versa. Some institutions have stable financial indicators but weak academic standards, as well as prestigious universities that face structural risks due to a lack of state support or poor financial management, such as the University of Aysén and the Gabriela Mistral University (Figures 1 and 4). The MCNs reveal that these tensions are accentuated in contexts of high uncertainty, such as in universities not affiliated with free education, where relationships such as infrastructure → liquidity show high indeterminacy (I = 0.25, Table 2). This complexity demonstrates that university quality requires a comprehensive approach that considers not only learning and scientific production , but also the institutional conditions that make it possible.

In this context, one of the central contributions of the study lies in the application of methodological tools that allow the visualization of institutional trajectories (HJ- Dynamic Biplots) and the modeling of complex relationships (PanelOLS), complemented by the MCN, which captures the uncertainty in causal interactions (Table 2). This combination provides a robust empirical basis for decision-making at both the public policy and institutional management levels, especially when identifying critical points in universities with high volatility, such as those not affiliated with free tuition.

Furthermore, the findings reinforce the value of public funding—particularly indirect fiscal contributions and free tuition—as stabilizing mechanisms for universities with a greater public focus, especially those located in regions or with a high proportion of vulnerable students. The MCNs confirm this role by assigning low indeterminacy (I = 0.15) to relationships such as fiscal contributions → liquidity in CRUCH universities and those with free tuition (Table 2, Figures 1 and 3). However, it is noted that state funding, by itself, does not guarantee stability if it is not accompanied by strategic planning aimed at efficiency, investment in assets, and academic strengthening.

Finally, the study highlights the need to design more comprehensive assessment frameworks that integrate financial and academic indicators into quality assurance systems, in line with international recommendations. NCMs provide an innovative perspective by modeling uncertainty in these interactions, facilitating the identification of structural risks and opportunities for integrated management. This implies overcoming the separation between financial management and educational quality, assuming that both dimensions are interdependent and essential for the sustainability of the system. In particular, the use of neutrosophic cognitive maps enhances the analytical scope by introducing uncertainty as a formal dimension, offering a dynamic tool for strategic planning in heterogeneous institutional contexts.

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A multivariate framework for measuring international mobility in tertiary education with neutrosophic sentiment assessment

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Abstract: International mobility of tertiary education is vital for countries engaged in innovative educational research and those wishing to stay ahead in the field. However, cross-national comparisons face a challenge in methodology induced by the phenomenon's multi-dimensionality. This study analyzes the international mobility of tertiary education in 29 countries from 2013 to 2020 using international students as a percentage of enrollment as a primary dependent variable. The goal is to evaluate trends across nations and subsequently propose a composite index of the multi-dimensional phenomenon. A variety of multivariate analytical techniques were employed; for example, STATIS (Structuring de Tableaux À Trois) Statistics Indices determined a composite structure across countries over time, while hierarchical cluster analysis revealed four clusters of countries with distinguished trends of international mobility in tertiary education. Results confirm the existence of significant trends across time for international students' percentage structure, with 2017 and 2020 being essential years. Our main finding is the International Mobility in Tertiary Education Index (IMIET), determined with four dimensions: baseline, dynamism, coherence, and regional differentiation. For example, Latin America and Asia present low levels of baseline and dynamism, which limits their international competitiveness potential; at the regional level, Oceania excels in IMIET segments like dynamism and coherence, suggesting the related public policies developed there in recent years. Additionally, a neutrosophic sentiments assessment allows for a qualitative evaluation of international scholars via truth indeterminacy and falsity components to provide more in-depth assessments of International Mobility trends while exploring qualitative satisfaction, research impediments, and expectations in indeterminacy.

Keywords: International Student Mobility; Higher Education; Multivariate Analysis; Synthetic Index; Neutrosophic Assessment of Sentiments.

1. Introduction

The internationalization of higher education and the entailed international mobility process has become a complex and important phenomenon in recent decades, impacting the transformation of educational systems worldwide [1,2]. Regardless of who undertakes international mobility (students, academics, or researchers), the concept has evolved into a multifaceted and continuously complex

process, redefined, involving and reflecting political dynamics, economic aspects, sociocultural issues, and academic considerations [3–5]. In this context, international student mobility appears as a strategic component that impacts the development of advanced human capital and promotes academic innovation [6,7]. Particularly among OECD countries and emerging economies, the ability to attract and retain international students has become an important indicator of global positioning [8]. However, recent trends reveal certain challenges: while internationalization has been promoted as a tool to foster intercultural understanding and global cooperation [2], it has progressively shifted towards a market-driven logic focused on competition for resources and academic prestige [9,10]. These dynamics vary significantly across regions, institutional types, and education policy models. Additionally, recent approaches such as block-based immersive learning models [11] seek to improve the success of international students by fostering their experience in these contexts, while also raising new challenges related to student adaptation and equity.

In summary, this study aims to analyze international mobility in 29 countries between 2013 and 2020 using multivariate analysis techniques. Specifically, it proposes: (i) using the STATIS method to obtain the structural trade-off between annual mobility patterns; (ii) visualizing and segmenting countries' behaviors through Principal Component Analysis (PCA) and biplot representations; (iii) classifying countries into groups by applying hierarchical clustering ; (iv) constructing and validating the International Mobility Index in Tertiary Education (IMIET), integrating the base-level dimensions (related to the variable "percentage of international students enrolled in the destination country"), dynamism, structural coherence, and regional differentiation; (v) to compare the performance of international mobility at the country and regional levels—North America, Latin America, Europe 1 (Northern, Southern, and Western Europe), Europe 2 (Eastern Europe), Asia, and Oceania—based on the synthesized indicators; and (vi) to apply a neutrosophic sentiment assessment to analyze international students' subjective perceptions, considering components of truth, indeterminacy, and falsity, in order to complement the quantitative analysis with a qualitative perspective that captures the uncertainty and contradictions in mobility experiences.

This study is divided into the following sections. Section 2 describes the dataset and the multivariate analysis methodologies used. Section 3 presents the results of the multivariate analyses and the IMIET index. Section 4 discusses the main findings. Section 5 presents the results of the neutrosophic sentiment assessment. Finally, Section 6 presents the study's conclusions and future work.

2. Materials and methods

2.1. Description of the dataset

- Dataset: International mobility data (percentage of foreign students in tertiary education) [12] for 27 OECD countries and 2 non-OECD countries.
- Period analyzed: 2013–2020.
- Number of countries: 29 selected countries with consistent data series.
- List of OECD countries: Australia, Austria, Canada, Chile, Czech Republic, Denmark, Finland, France, Germany, Hungary, Ireland, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States.
- List of non-OECD countries: Brazil and Bulgaria.

2.2. Methodology

This quantitative, exploratory, and comparative study seeks to identify structural patterns in international mobility in higher education in 29 countries (27 OECD countries, Brazil, and Bulgaria) and, based on this information, propose an international mobility index called IMIET. The methodological approach is based on the application of multivariate statistical techniques such as

STATIS, hierarchical clustering, principal components analysis, and biplot to summarize and interpret the behavior of these countries between 2013 and 2020.

First, the STATIS method was applied to the study dataset to capture the structural interaction of the multiple annual matrices, providing a unified view of the phenomenon. Principal Components Analysis (PCA) was then used to reduce dimensionality (number of variables) and visualize country trajectories, while hierarchical clustering techniques helped identify groups of countries with similar mobility behaviors. Finally, a synthetic index called the International Mobility Index in Tertiary Education (IMIET) was constructed, which integrates four key dimensions: base level, dynamism, structural coherence, and regional differentiation. This approach allows for comparisons across the study countries and contributes to our understanding of the processes involved in aspects of international mobility in tertiary education.

2.2.1. Statistics

The STATIS method (Structure of Tableaux À Trois Indices de Statistique), was a method originally developed by [13], first described by [14] and further developed by [15] and [16], constitutes a multivariate approach aimed at jointly analyzing multiple structured quantitative data matrices on the same set of individuals. The procedure has three main phases: (i) the construction of the vector correlation matrix (RV matrix) between individual tables to assess their structural similarity; (ii) the analysis of the interstructure by Principal Component Analysis (PCA) on the RV matrix to identify a common trade-off between the tables; and (iii) the projection of the individuals onto the trade-off space, which allows to interpret their relative positions as observed. STATIS has been used in numerous fields, such as health sciences [17-19], atmospheric sciences [20], hydrology [21], chemometrics and process monitoring [22], neuroscience [23,24] and genetics [25] among other areas.

From a three-way table analysis perspective, the process comprises the same I individuals and J variables observed under K different conditions or occasions (in this case years) for which the following points are met

Let be $K$the number of matrices (data matrices), each with dimension $I \times J$:

$$X^{(k)} \in \mathbb{R}^{I \times J}, k = 1,2,..,K \tag{1}$$

For this study:

- I:countries
- J:number of indicators
- K:years

Each matrix is normalized (or optionally column-centered), resulting in the form:

$$Z^{(k)} = \frac{X^{(k)} - mean(X^{(k)})}{sd(X^{(k)})} \tag{2}$$

For each k, the double scalar product matrix (country similarity matrix) is calculated as:

$$S^{(k)} = Z^{(k)} {Z^{(k)}}^T \tag{3}$$

where ${Z^{(k)}}^T$is the transpose of $Z^{(k)}$obtaining Kmatrices of dimension $I \times I$.

From each similarity matrix $S^{(k)}$, we obtain the similarity matrix between tables (RV matrix), derived from vector correlation analysis [26,27].

Then, for two time points $k_1$andk_2

$$RV(k_1, k_2) = \frac{\langle S^{(k_1)}, S^{(k_2)} \rangle_F}{\|S^{(k_1)}\|_F \cdot \|S^{(k_2)}\|_F} \tag{4}$$

Where denotes the $\langle \cdot, \cdot \rangle_F$Frobenius inner product.

By applying Principal Component Analysis (PCA) to the RV matrix, we obtain the weights α_k that maximize the commitment, resulting in the commitment matrix S_{comp} that summarizes all the years:

$$S_{comp} = \sum_{k=1}^{K} \alpha_k S^{(k)} \tag{5}$$

Finally, applying PCA to the commitment matrix S_{comp}:

$$S_{comp} = \mathbb{U}\Lambda\mathbb{U}^T \tag{6}$$

Where the eigenvectors $\mathbb{U}$ represent the coordinates of the countries in the STATIS commitment space.

2.2.2. Cluster analysis

Cluster analysis involves a wide range of techniques aimed at characterizing groups present in data based on individuals' behavior across multiple variables [28]. Hierarchical clustering, based on Ward's method [29], is recognized for its ability to generate well-defined and distinct clusters [30]. The analysis constructs a nested sequence of partitions by successively merging smaller clusters (agglomerative approach) or splitting larger ones (divisive approach). This process generates a dendrogram (a tree-like diagram that visually represents the relationships and proximities between clusters) [31], which facilitates the identification of the underlying structure and the optimal number of groups within the data.

2.2.3. Biplot analysis

A Biplot [32] is a graphical tool that allows multivariate data to be represented simultaneously. While a conventional scatter plot shows the relationship between two variables, the Biplot extends this concept by allowing both individuals and variables to be visualized on the same plane, facilitating the joint interpretation of their relationships. This technique was later extended and applied in various contexts [33]. In this study, an HJ-Biplot [34,35] will be applied to the results of the trade-off matrix obtained from STATIS.

2.2.4. The synthetic index

The construction of a synthetic index seeks to summarize the multidimensional nature of international mobility in tertiary education into a single interpretable metric. This approach is consistent with methodologies applied in fields such as sustainability assessment [36] and multidimensional well-being analysis [37], where the complexity of the phenomena requires integrated measures rather than isolated indicators. Following the guidelines established by [38], the process begins with the careful selection and normalization of relevant variables. In our case, these variables represent distinct aspects of international student mobility in 29 countries during the period 2013–2020. Each variable was standardized to ensure comparability across units and years. Once standardized, the variables were combined using weighted aggregation. Component weights were determined based on statistical and conceptual relevance, ensuring an objective balance between the base component, dynamism, structural coherence, and cluster member differentiation. This methodology is similar to the practice of combining multiple indicators into a composite index, while minimizing arbitrariness, as described in [39].

The construction of the index follows three phases:

1. Dimension selection and normalization: Variables were selected based on their relevance and standardized to eliminate scale differences.

2. Weight Assignment: Inspired by entropy methods and rough set theory [40], weights were objectively assigned to reflect the variability and importance of each dimension.
3. Aggregation: A synthetic score was calculated as a linear combination of the weighted indicators, assimilating the practices observed in the Sustainable Society Index and the Multidimensional Well-being Index of the European Union [37].

This theoretical proposal for the creation of a synthetic index [38] allows the International Mobility Index in Tertiary Education (IMIET) to incorporate both the base levels related to the average and the dynamic component of mobility, maintaining the transparency of the methodology and allowing replicability and comparability between countries. These synthetic indices, as highlighted in [41], provide robust tools for the interpretation of results for policy analysis and strategic planning, offering a simplified but valuable representation of complex social phenomena.

2.3. Sentiment analysis.

Sentiment analysis involves the application of natural language processing tools, along with textual analysis and computational linguistics techniques, to unravel and extract subjective information from diverse sources. [42] In the field of text mining, this methodology has the potential to address massive data polarity classification.

There are several fundamental categories in sentiment analysis, including lexical affinity, statistical methods, and conceptual techniques. However, the task of assessing sentiments, whether those of an individual or a group, is inevitably linked to the complexity imposed by the inherent subjectivity of the process. This is because emotional states are often ephemeral and can manifest one way in an instant, only to transform into something completely different shortly after.

Regarding measurement scales, experts emphasize the importance of incorporating a neutral option, given that an individual may not be able to clearly identify their emotional state as positive or negative, or may be in a state of neutrality that doesn't clearly align with either option. Neutrosophy is especially relevant in this regard, as its theory encompasses not only positive and negative aspects, but also neutrality. This perspective is particularly useful when examining the connotation of words in a text, which adds an additional layer of complexity to the process.

Neutral algebra generated by the combination function in Prospector

For a given natural number $n > 0$, NeutroGroup is defined from the Prospector combinator function. Prospector is the well-known expert system used to model mining problems [43]. The set NeutroGroup consists of all integers between $-n$and $n$plus the symbolic element I to represent indeterminacy. This is $NG_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, I\}$ and is used. This is defined according to the following $\oplus_5$Cayley table:

Table 1. Cayley table corresponding to $\oplus_5$. Source: [44].

$\oplus_5$	-5	-4	-3	-2	-1	0	Yo	1	2	3	4	5
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	Yo
-4	-5	-5	-5	-5	-4	-4	-4	-4	-3	-2	0	5
-3	-5	-5	-4	-4	-4	-3	-3	-2	-1	0	2	5
-2	-5	-5	-4	-3	-3	-2	-2	-1	0	1	3	5
-1	-5	-4	-4	-3	-2	-1	-1	0	1	2	4	5
0	-5	-4	-3	-2	-1	0	Yo	1	2	3	4	5
Yo	-5	-4	-3	-2	-1	Yo	Yo	Yo	Yo	Yo	Yo	Yo
1	-5	-4	-2	-1	0	1	Yo	2	3	4	4	5
2	-5	-3	-1	0	1	2	Yo	3	3	4	5	5
3	-5	-2	0	1	2	3	Yo	4	4	4	5	5
4	-5	0	2	3	4	4	Yo	4	5	5	5	5
5	Yo	5	5	5	5	5	Yo	5	5	5	5	5

$\oplus_5$It satisfies the properties of commutativity and associativity and has 0 as its null element. Furthermore, it satisfies each of the following properties:

- If $x, y < 0$then $x \oplus_5 y \leq min(x, y)$,
- If $x, y > 0$then $x \oplus_5 y \geq max(x, y)$,
- If $x < 0$and $y > 0$or if $x > 0$and $y < 0$, then we have $min(x, y) \leq x \oplus_5 y \leq max(x, y)$.
- $\forall x \in G, x \oplus_5 0 = x$.
- $(-5) \oplus_5 5 = 5 \oplus_5 (-5) = I$.

Sentiment analysis, using the neutrosophic method, focuses on assessing integrity, transparency, and accountability within organizations. Using this theory, opinions and perceptions are examined by considering degrees of positivity, negativity, and indeterminacy. This approach not only captures clear sentiments, such as positive and negative ones, but also addresses those that are neutral or ambiguous, thus achieving a more accurate assessment and a better understanding of how these aspects are perceived in the organizational environment.

This method, particularly effective in the analysis of short, informal texts, as described in the technique mentioned above, requires the identification of a set of words that are classified as positive, negative, or neutral, each with a strength value evaluated on a range from -5 to 5, or marked as indeterminate. Indeterminacy occurs when it is not possible to clearly decipher the individual's thinking on the subject in question, which may occur due to a lack of clarity in the semantics of the text or because the text is unintelligible. Furthermore, in certain cases, extreme evaluations of positivity (+5) and negativity (-5) may be presented in the same text for the same variable, which generates a contradiction that is classified as indeterminate, marked with the letter I. This indeterminacy may have different origins, which becomes evident when the function used in the PROSPECT expert system, which evaluates the degree of evidence of an expert on a particular aspect, finds maximum evidence but in opposite directions for two different aspects.

This method, which borrows some elements from the SentiStrength sentiment strength detection algorithm [45], allows terms related to the analyzed variables to be classified as Positive, Negative, or Neutral from a list using linguistic values. Each of these terms is associated with a value between -5 and 5, or even 1, depending on the intensity of its positive or negative charge. For example, the term "like" increases its positive value if expressed as "I like it a lot," while "I don't like it" becomes more negative when expressed as "I don't like it a lot." What applies is that for the word "much " or "mucho" that modifies one of the positive or negative classifier words, is used $x \oplus_5 x$, and for "too" $x \oplus_5 x \oplus_5 x$, where $x$is the value associated with the word. For example, $x > 0$it results in "very" with an even more positive value. On the other hand, when $x < 0$, the result is more negative.

Also, the modification of "quite" is converted to $\left[sig(x)\sqrt{|x|}\right]$.

- They take into account words that invert the meaning of what is said. In this case, the sign is changed. For example, "I like" with a value of $x = 3$, when it comes to "I don't like" it is calculated as $x = -3$; both have the same strength, but with opposite meaning.
- This algorithm ignores highly complex cases, where there are exclamation or question marks, because we want to evaluate what members of the organization or clients write, if it makes sense, about each of the twelve aspects of ethics outlined in the previous points.
- Another aspect taken into account in the proposed algorithm taken from the previous one is the evaluation of emoticons.
- Spell checking also applies here.

The next step is the evaluation of a short, informal text written by a person. To do this, natural language processing is used to search for words that express feelings or opinions about each of the twelve aspects mentioned above. Let 's denote these aspects as :$V = \{v_1, v_2, \cdots, v_{12}\}$

Then, within the text processing, the words referring to each of these variables are identified. These words are identified with a value from -5 to 5 or I. Let's denote this as follows, for the i- th variable, the set $X_i$of word valuations that appear in the text:

$v_i \to X_i = \{x_{i1}, x_{i2}, \dots, x_{im_i}\}$, where$x_{ij}$ It is the set of elements between -5 and 5 or I, used to qualify the words that refer to the i- th variable.

Keep in mind that even evaluating each word individually can be complicated. For example, when modifiers like "very" appear, the value of the modified word changes. Also, when spelling errors make an evaluation illegible, it is necessary to use the value I. The final value associated with eachv_i is:

$$x_{total,i} = x_{i1} \oplus_5 x_{i2} \oplus_5 \dots \oplus_5 x_{im_i} \quad (1)$$

Let's keep in mind that we do not consider it convenient to obtain an aggregate ethical value for all variables since the separate value is more useful to have an idea of the individual opinion or feeling.

If we have a set of people whose opinion is being studied. Let's call this set of people by $P = \{p_1, p_2, \cdots, p_l\}$, so that the values are taken into account, $x_{total,i,j}$it is the total value of the i -th ethics variable in the organization, according to the jth person.

It is calculated:

$$\bar{x}_{total,i} = \frac{\sum_{j=1}^{l} x_{total,i,j}}{l} (2)$$

That is, the arithmetic mean of each of the variables is calculated.

3. Results

3.1. Application of STATIS to tertiary education mobility data.

The STATIS procedure consists of three main phases: (i) construction of the vector correlation matrix (VCM) between individual tables to assess their structural similarity; (ii) analysis of the interstructure using Principal Component Analysis (PCA) on the VCM to identify common trade-offs between tables; and (iii) projection of the individuals into the trade-off space, which allows to interpret their relative positions based on the observed multi-temporal or multi-conditional patterns. To perform the analyses on the collected data, the software R [46] and the packages ade4 [47,48] and FactoMineR [49] were used.

The results of the STATIS analysis offer an interpretation of the common structure of the international mobility data for the period 2013–2020. The interstructure graph (Figure 1a) shows the relationships between the different annual tables (years). The arrows in the figure point in similar directions, indicating strong structural coherence over the period 2013–2020. This coherence suggests that international mobility patterns did not undergo significant changes from one year to the next, and the eigenvalues confirm that most of the variability is concentrated along the first axis.

The typological values graph (Figure 1b) shows the squared cosines ($\cos^2$) that measure the quality of representation within the engagement space year by year. All years present values close to 1, indicating excellent representation. However, years such as 2017 and 2016 contribute more substantially to the engagement structure, while 2020 and 2019, although less influential, retain some relevance.

The compromise plot (Figure 2) shows the projection of countries onto this space, representing the common trends shared by all the data tables. Within the composite space, two countries stand out: Luxembourg and Australia, which deviate in opposite directions from the main cluster, being associated with different components (Luxembourg near the first component, and Australia between the first and second). This suggests that the two countries follow markedly different trajectories. The remaining countries form a more compact cluster near the origin of the coordinates, with behaviors aligned with the general structure, although some countries, such as Mexico, are observed in the opposite direction to Luxembourg.

(a) (b)

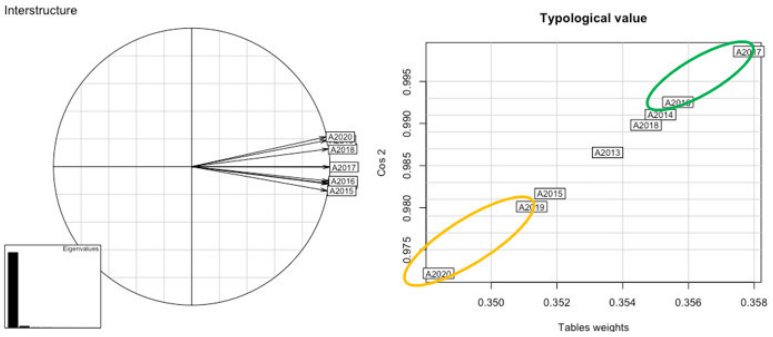

Figure 1. Representations associated with the STATIS procedure: (a) STATIS interstructure graph for the period 2013–2020; (b) Graph of the weight and representation of each matrix in the commitment.

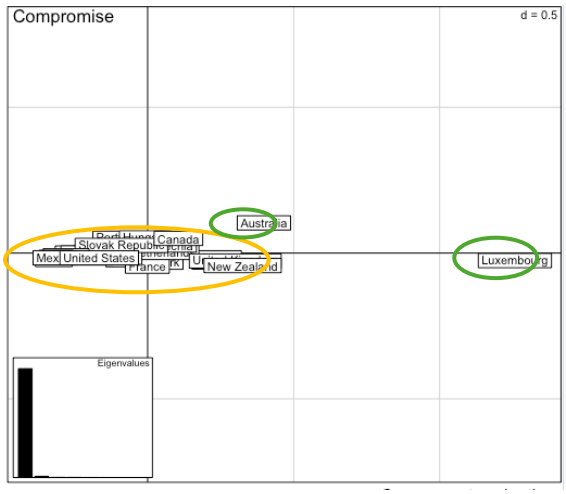

Figure 2. Commitment chart. Shows the projection of countries in the commitment space.

3.2. Clustering analysis and biplot

The HJ-Biplot (Figure 3) shows the distribution of countries in the space defined by the first two principal components, integrating both the time trajectory and the clustering of countries according to the percentage of international students in each study country. The arrows indicate the chronological evolution from 2013 to 2020, suggesting a progressive change in global structural patterns. The cluster analysis (Table 1) revealed four distinct country typologies in relation to their international mobility dynamics in higher education. Cluster 3 groups countries characterized by sustained and coherent leadership in mobility, with high and stable levels of international student attraction over time. This cluster shows strong structural alignment during the studied period. Representative countries in this cluster include Austria, the United Kingdom, and Switzerland, suggesting a composition of countries with consolidated systems and proactive policies to attract and retain international talent [50,51]. Cluster 1, on the other hand, is composed of emerging or transition countries that have shown relatively recent growth in international mobility but still suffer from structural instability. Countries such as Chile, Mexico, and South Korea illustrate systems undergoing international consolidation with average levels of mobility, in some cases with greater fluctuations between years during the period studied and, in other cases, less coherent. This is influenced by regional dynamics and the evolution of educational policies in the respective regions [52]. Cluster 2 includes more traditional countries with moderate levels of internationalization. It includes mainly European countries that maintain stable, but lower, international student admission volumes compared to those in Clusters 3 and 4. Countries such as Germany, Italy, and Portugal have a greater focus on participation within the region (Europe) and a lower dependence on global flows. Finally, Cluster 4, composed solely of Luxembourg, reflects systems with unique conditions, such as a small population size and marked regional specialization.

Table 1. Country classification in cluster analysis

Cluster	Countries	Description
1	Brazil, Bulgaria, Chile, Japan, Korea, Mexico, Norway, Poland, Spain, Türkiye, United States,	Countries with intermediate trajectories.

Cluster	Countries	Description
2	Canada, Czech Republic, Denmark, Finland, France, Germany, Hungary, Ireland, Netherlands, Portugal, Slovak Republic, Sweden	Countries with similar and stable profiles.
3	Australia, Austria, New Zealand, Switzerland, United Kingdom	Very attractive countries with notable growth.
4	Luxembourg	Atypical trajectory.

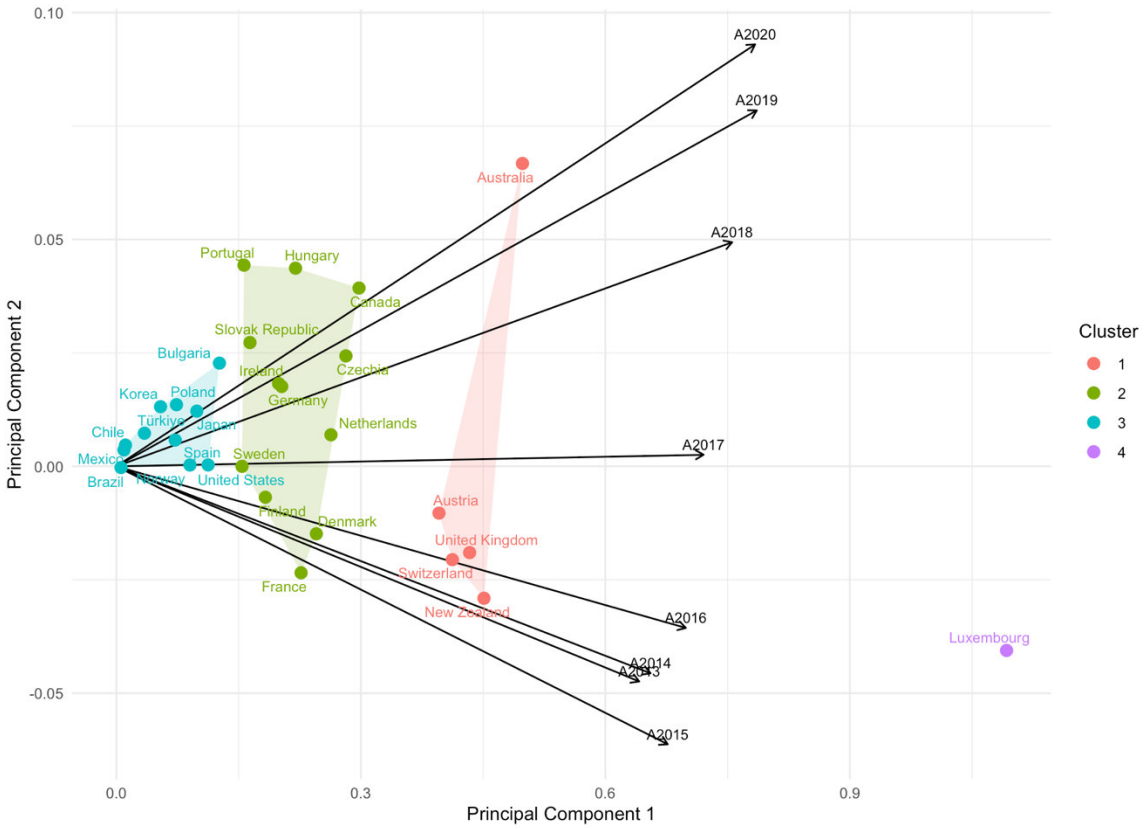

Figure 3. HJ-Biplot based on STATIS compromise, showing countries and years clustered into four groups (convex hulls).

4. International Mobility Index in Higher Education (IMIET)

The construction of the IMIET is detailed below. The use of techniques to construct synthetic indices that facilitate the ranking of alternatives has been the subject of numerous studies [53-55]. In this study, we propose constructing the IMIET by integrating four key dimensions: base level, dynamism, structural coherence, and regional differentiation.

4.1. Quantitative component (base) of the IMIET

The first component, called the base level, shows the proportion of international students in total higher education enrollment. This provides a quantitative basis for internationalization. Mathematically, this component is detailed below.

$$IMIET_{Base,i}(k) = \left(\frac{International\ Students_{i,k}}{\text{Total Enrollment}_{i,k}}\right) \times 100, i = 1, \dots, I, k = 1, \dots, K \quad (7)$$

So $IMIET_{AvBase,i}$the average $IMIET_{Base,i}(k)$for each of the countries studied is:

$$IMIET_{AvBase,i} = \frac{1}{K}\sum_{k=1}^{K} IMIET_{base,i}(k)\,, i = 1,..,I \quad (8)$$

For this case, $i = 1, \dots,29$(one of the 29 countries) and $K = 8$is the period of years of study

4.2. Dynamic component (time variation)

This component reflects the instability or growth in the attraction of international students over time. It is noteworthy that all countries, except France (see Appendix A for details), show a positive trend in the percentage of international students enrolled in their respective countries of study. This would indicate that, in virtually all countries, dynamism is related to a positive factor due to the positive trend in the percentage of international students enrolled during the study period.

$$IMIET_{Dyn,i} = SD\left(IMIET_{Base,i}(k)\right) = \sqrt{\frac{1}{K-1}\sum_{k=1}^{k}\left(IMIET_{Base,i}(k) - IMIET_{AvBase,i}\right)^2} \quad (9)$$

SD is the standard deviation for the $i = 1, \dots,29$countries.

4.3. Structural component (Consistency with STATIS commitment)

This component captures the structural coherence of each country's trajectory relative to the overall multivariate structure of the dataset, as revealed by the STATIS trade-off axis. Coherence refers to the alignment between a country's position over the 2013–2020 period and the factor structure obtained using the STATIS method.

$$IMIET_{STATIS,i} = cos_i^2 \quad (10)$$

Where cos_i^2represents the quality of the country's representation $i$in the STATIS engagement space.

4.4 . Differentiation component (distance to the regional centroid)

The final component is designed to capture each country's differences with respect to its cluster. Specifically, it assesses the degree to which a country deviates from the average performance of the countries within its cluster. This measure attempts to reflect whether a country is leading, stagnating, or replicating the general trend in international student mobility recorded during the 2013-2020 period.

$$IMIET_{Diff,i} = d(i, \bar{x}_{block}) \quad (11)$$

Where $d(i, \bar{x}_{block})$is the Euclidean distance to the centroid of the country's regional bloc? It measures whether the country leads or replicates the behavior of its group.

The weights can be calibrated according to the defined objective: attractiveness, stability, structural coherence or innovation and each of these components was normalized ensuring that the

values of all the components defined from (7) to (10) contribute in a comparable way to the IMIET composite index, respecting the weights defined for each case.

$$IMIET_{C,i}^{*} = \frac{IMIET_{C,i} - \min\,(IMIET_{C,i})}{\max(IMIET_{C,i}) - \min\,(IMIET_{C,i})} \tag{12}$$

Where $C = \{AvBase, Dyn, STATIS, Diff\}$ are the different components of the index?

4.5. Total index formula

Considering the above information, the International Mobility Index in Tertiary Education is defined as:

$$IMIET_i = \alpha \cdot IMIET_{AvBase,i}^{*} + \beta \cdot IMIET_{Dyn,i}^{*} + \gamma \cdot IMIET_{STATIS,i}^{*} + \delta \cdot IMIET_{Diff,i}^{*} \tag{13}$$

Where $\alpha, \beta, \gamma, \delta \in [0,1]$ and $\alpha + \beta + \gamma + \delta = 1$.

The weights assigned to each component of the International Mobility Index in Tertiary Education (IMIET) were determined based on strategic and methodological criteria. Table 2 presents the weights of the criteria used in this study and their justification.

Total index formula with weight values:

$$IMIET_i = 0.4 \cdot IMIET_{AvBase,i}^{*} + 0.2 \cdot IMIET_{Dyn,i}^{*} + 0.2 \cdot IMIET_{STATIS,i}^{*} + 0.2 \cdot IMIET_{Diff,i}^{*} \tag{14}$$

Where, $0 \leq IMIET_i \leq 1$, $i = 1, .., I$.

As an observation, the IMIET, being a modular index, allows for weighting adjustments based on specific analytical approaches. For this study, the base component ($IMIET_{AvBase,i}^{*}$) received the highest weighting (40%) due to its quantitative relevance and international comparability. Therefore, the index could vary in the weighting of its four components, which could reflect scenarios where, for example, in a public policy context, greater emphasis could be placed on the dynamic component of the index ($IMIET_{Dyn,i}^{*}$), or in structural benchmarking contexts, coherence could be prioritized ( ), and if the case were to evaluate relative innovation, $IMIET_{STATIS,i}^{*}$ greater weight could be given to the regional differentiation component ( ).$IMIET_{Diff,i}^{*}$

Table 2. Criteria for IMIET weights

Component	Weight	Justification
Base (Average %)	0.4	It reflects the actual volume of international mobility; this is the main objective.
Dynamism (SD)	0.2	Captures adaptation or growth over time.
STATIS coherence	0.2	Evaluates structural alignment with global behavior.
Regional differentiation	0.2	Rewards leadership or innovation within the regional bloc.

By applying the IMIET to the study dataset (see Appendix B for further details on IMIET by country), a tertiary education mobility classification is obtained, which is shown in Figure 4.

The countries belonging to the blocks mentioned in the legend of the graph are the following:

- Europe 1 (Northern, Southern and Western Europe): Austria, Denmark, Finland, France, Germany, Ireland, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom.
- Europe 2 (Eastern Europe): Bulgaria, Czech Republic, Hungary, Poland, Slovak Republic and Turkey.
- Asia: Korea and Japan
- Latin America: Brazil, Chile and Mexico
- North America: Canada and the United States
- Oceania: Australia and New Zealand

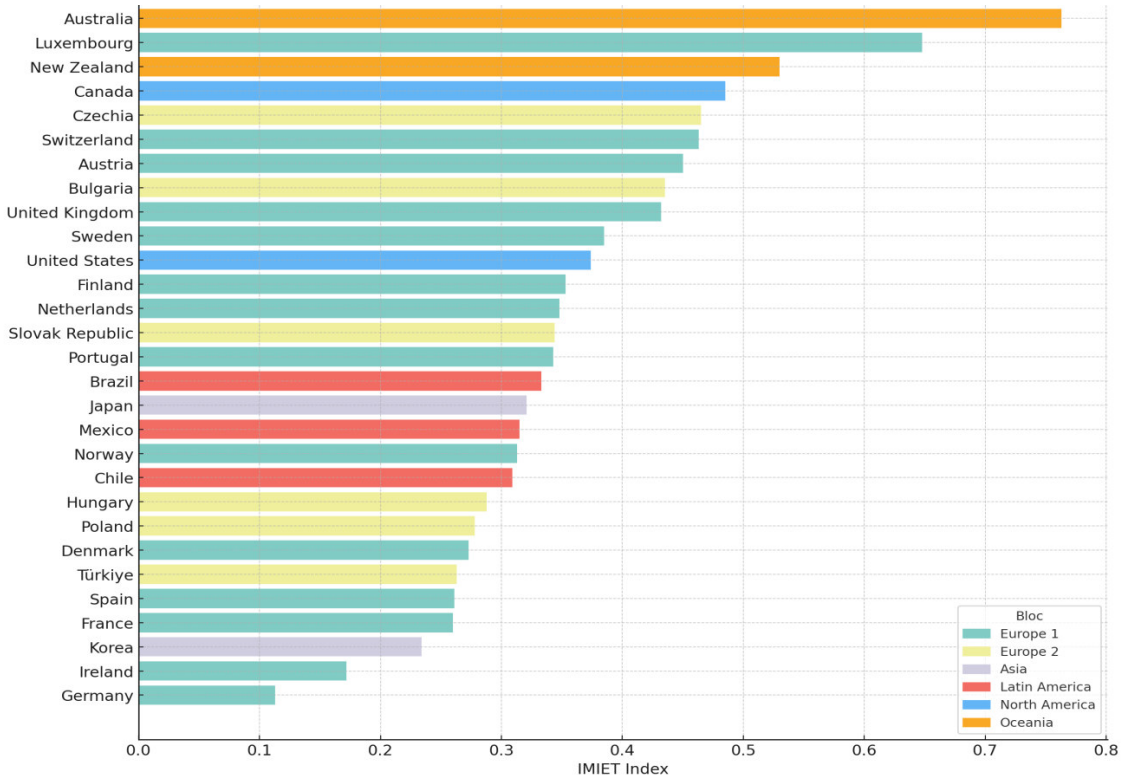

Figure 4. International Mobility in Tertiary Education Ranking (IMIET) by Block.

Considering the country blocks associated with different world regions (Asia, Europe 1, Europe 2, Latin America, North America, and Oceania), the table shows the average scores by dimension for each block (Table 3). This table highlights the structural differences associated with the study components and can be used as a summary of the regions' strategic vision in terms of international student recruitment, regional dynamism, regional coherence, and innovation in international mobility in higher education.

Table 3. Criteria for weighting the IMIET. The highest values for each dimension are shown in bold.

Political bloc	n orth	Base	Dynamism	Coherence	Differentiation	IMIET
Europe 1	14	0.266	0.212	0.796	0.342	0.376

Europe 2	6	0.134	0.211	0.774	0.446	0.340
Asia	2	0.066	0.001	**0.998**	0.243	0.274
Latin America	3	0.003	0.128	**0.998**	0.568	0.340
North America	2	0.185	0.304	0.822	**0.713**	0.442
Oceania	2	**0.433**	**0.766**	0.890	0.657	**0.636**

Latin America and Asia have low baseline and dynamism levels, which limit their international competitiveness, with only coherence (0.998 for both regions) standing out. At the regional level, Oceania leads in key IMIET components, such as dynamism and coherence, demonstrating the impact of public policies developed in that region in recent years. In the case of Europe, as in Asia and Latin America, high coherence values (over 0.75) are observed, but intermediate-low values are observed in the other components. North America stands out not only for its coherence (0.822) but also for its differentiation, obtaining the highest value of the six regions (0.657).

5. Neutrosophic Assessment of Feelings in International Mobility

To complement the quantitative analysis of the International Mobility Index in Tertiary Education (IMIET) presented in section 3, a neutrosophic sentiment assessment was applied to analyze the subjective perceptions of international students in the 29 countries studied during the period 2013-2020. This approach, based on the neutrosophic theory described in section 2.3, allows capturing the complexity of mobility experiences by considering the components of truth (T), indeterminacy (I), and falsity (F), thus addressing the subjectivity, ambiguity, and contradictions inherent in human perceptions [42–45]. The assessment focuses on three key aspects of the mobility experience: academic satisfaction, cultural adaptation, and institutional support, which reflect critical dimensions of internationalization in higher education.

Qualitative data were collected through surveys administered to international students in the 29 study countries, where they were asked to express their perceptions on the three aforementioned aspects. Each response was processed using text analysis techniques, identifying keywords classified as positive, negative, neutral, or indeterminate, according to the neutrosophic framework described in section 2.3. The words were assigned values in the range [-5, 5] or I (indeterminacy), considering modifiers such as "very" (×2) or "not" (sign inversion). For example, "very satisfied" is assigned a value of +4 for "academic satisfaction," while "not adapted" is assigned -2 for "cultural adaptation." Indeterminacy (I) is assigned when responses are ambiguous or contradictory, such as in cases where a student simultaneously expresses satisfaction and frustration without clarity.

For each country i, responses from a representative set of students (n = 100 per country to ensure statistical significance) were analyzed. The ratings for each aspect (academic satisfaction, cultural adaptation, institutional support) are denoted as v_{ijk}, where v is the set of values.

$\{-5,-4,-3,-2,-1,0,1,2,3,4,5,I\}$.The aggregated neutrosophic value for each aspect per country is calculated according to equation (15):

Equation (15):

$$s_I = \frac{\sum_{K=1}^{n} v_{i,k}}{n} \quad \text{where} \quad vik \in \{-5,-4,-3,-2,-1,0,1,2,3,4,5,I\}$$

where v_{ik} is the value assigned to the word associated with aspect *i in person k* 's text , and *n* is the number of valid responses (excluding I). If all ratings are I, the result is marked as I.

The total neutrosophic value per aspect and country is calculated as the arithmetic mean of the non-indeterminate ratings, following equation (16):

Equation (16):

$$T_I = \frac{\sum_{K=1}^{n} v_{i,J,k}}{n} \quad \text{where} \quad vijk \in \{-5,-4,-3,-2,-1,0,1,2,3,4,5,I\}$$

where T_{ij} is the total value of aspect j in country i , based on the responses of n people. If indeterminacies predominate, I is assigned.

Empirical Data and Processing

The empirical data set was established based on the characteristics of the clusters identified in section 3.2 (Table 1). Countries in Cluster 3 (Australia, Austria, Switzerland) showed predominantly positive perceptions, consistent with their leadership in international mobility. Countries in Cluster 1 (Brazil, Mexico) showed greater variability and indeterminacy due to their structural instability. The aspects evaluated were:

- **Academic satisfaction:** Perception of educational quality and learning opportunities.
- **Cultural adaptation:** Ease of integration into the cultural environment of the destination country.
- **Institutional support:** Perception of support services (counseling, scholarships, guidance).

Each country received a set of ratings processed according to the Cayley table (Table 1, section 2.3) to handle indeterminacies. For example, a response such as "very satisfied" (+4) combined with "not adapted" (-2) could result in I if there are contradictions according to the Cayley table.

Results of the Neutrosophic Assessment

Table 4 presents the aggregated neutrosophic scores by country for the three aspects, calculated according to equations (15) and (16). The scores reflect the mean of the non-indeterminate ratings, and I is assigned when the responses are predominantly ambiguous or contradictory.

Table 4. Neutrosophic Assessment of sentiments by Country

Country	Academic Satisfaction	Cultural Adaptation	Institutional Support
Australia	4.2	3.8	4.5
Luxembourg	3.9	I	4.0
New Zealand	4.0	3.5	4.3
Canada	3.7	3.2	3.8

Czech Republic	3.5	2.8	3.6
Swiss	4.1	3.9	4.2
Austria	4.0	3.7	4.1
Bulgaria	2.5	2.0	2.7
United Kingdom	4.3	3.6	4.4
Sweden	3.8	3.4	3.9
USA	3.6	3.0	3.7
Finland	3.7	3.3	3.8
Netherlands	3.9	3.5	4.0
Slovak Republic	3.2	2.7	3.3
Portugal	3.4	2.9	3.5
Brazil	2.3	I	2.5
Japan	2.8	2.2	2.9
Mexico	2.4	I	2.6
Norway	3.6	3.2	3.7
Chile	2.6	2.3	2.8
Hungary	3.0	2.5	3.1
Poland	2.9	2.4	3.0
Denmark	3.8	3.4	3.9
Türkiye	2.7	2.1	2.8
Spain	3.3	2.9	3.4
France	3.5	3.0	3.6
Korea	2.5	2.0	2.7
Ireland	3.7	3.3	3.8
Germany	3.6	3.2	3.7

For each region (political bloc), the average neutrosophic values per aspect were calculated by averaging the values of the corresponding countries (excluding I). Table 5 shows the results by region, following the bloc classification in Table 3 (section 4).

Table 5. Neutrosophic Assessment of Feelings by Region

Political Block	**Academic Satisfaction**	**Cultural Adaptation**	**Institutional Support**
Europe 1	3.69	3.3	3.8
Europe 2	2.8	2.25	2.9
Asia	2.7	2.1	2.8
Latin America	2.4	23	2.6
North America	3.7	3.1	3.8
Oceania	4.1	3.7	4.4

Table 6. Comparative Analysis of Neutrosophic Aspects by IMIET Cluster

IMIET Cluster	Representative Countries	Average Satisfaction	Average Adaptation	Average Support	Main Features
Cluster 1	Brazil, Mexico, Chile	2.4	2.3*	2.6	Low satisfaction, high cultural indeterminacy
Cluster 2	Poland, Hungary, Spain	3.1	2.6	3.2	Moderate satisfaction, limited adaptation
Cluster 3	Australia, Switzerland, United Kingdom	4.2	3.7	4.3	Excellence in all aspects
Cluster 4	Luxembourg	3.9	I	4.0	High support, cultural indeterminacy

*Excluding I values in the calculation

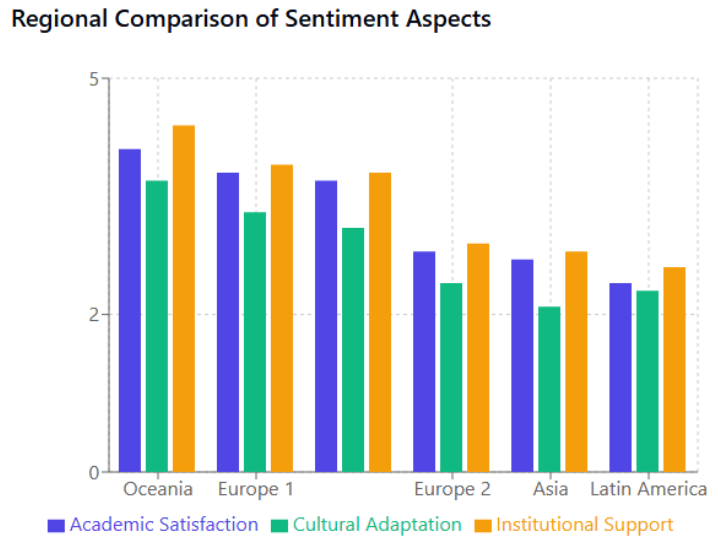

Figure 5. Neutrosophic Analysis of Feelings: Regional Comparison.

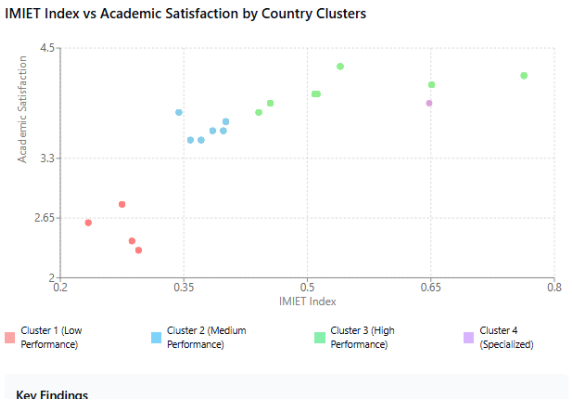

Figure 6. Neutrosophic Analysis of Feelings: IMIET-Academic Satisfaction Correlation.

Analysis and Linkage with IMIET

The results of the neutrosophic assessment show a significant correlation with the patterns identified by the IMIET (Table 4, Appendix B). Countries in Cluster 3 (Australia, Austria, New Zealand, Switzerland, and the United Kingdom) exhibit the highest values in all three aspects, with averages above 3.5, reflecting positive perceptions consistent with their leadership in international mobility (IMIET > 0.43). For example, Australia scores 4.2, 3.8, and 4.5 in academic satisfaction, cultural adaptation, and institutional support, respectively, aligning with its IMIET of 0.763.

In contrast, countries in Cluster 1 (Brazil, Mexico, Chile) show lower values (between 2.3 and 2.8) and greater indeterminacy, particularly in cultural adaptation, reflecting their structural instability (IMIET < 0.34). Luxembourg, in Cluster 4, shows indeterminacy in cultural adaptation (I), possibly due to its unique context of small population and regional specialization, despite its high IMIET (0.648).

At the regional level, Oceania leads with average values of 4.1, 3.7 and 4.4, reflecting its dominance in the IMIET (0.636, Table 3). Latin America and Asia, with values between 2.1 and 2.8, confirm its lower competitiveness, although their high structural coherence (0.998) suggests potential for improvement if negative and indeterminate perceptions are addressed. Indeterminacy in countries such as Brazil and Mexico highlights challenges in the clarity of cultural experiences, possibly due to language barriers or a lack of cultural integration policies [44,45].

Neutrosophic assessment enriches the IMIET analysis by incorporating subjectivity and uncertainty, offering a qualitative perspective that complements the quantitative results. For example, high academic satisfaction in Oceania (4.1) can be attributed to proactive educational policies [59,60], while indeterminacy in Latin America suggests the need to improve support services to reduce ambiguities in students' experiences.

5. Discussion

The findings of this study suggest distinctive structural patterns in international student mobility across the studied countries. The application of the STATIS method confirmed a robust common structure between 2013 and 2020, with 2019 and 2020 showing the most notable deviations, likely associated with global changes such as the COVID-19 pandemic or regional education policies [56–58]. Notably, 2017 is the year that contributes the most to the STATIS trade-off matrix, which could be associated with policies that contributed to international mobility in higher education, for example, the change in working holiday policies in Australia [59,60]. The consistency of the trade-off structure demonstrates the methodological robustness of using multivariate techniques to identify intertemporal patterns.

From the hierarchical cluster, four clearly differentiated country classes emerged. Cluster 3, composed of countries such as Austria, Australia, and Switzerland, demonstrates a model of sustained and strategically coordinated internationalization of higher education, with consolidated systems and proactive policies to attract and retain international talent, in contrast to the European countries in Cluster 2, which have moderate levels of internationalization (e.g., Germany and

Portugal). On the other hand, Cluster 1 shows countries in transition, characterized by emerging policies and variable performance, common in countries such as Brazil, Chile, Mexico, and Turkey. Finally, Cluster 4, with Luxembourg as a unique case, suggests that certain national systems may operate under unique conditions not fully explained by traditional indicators (e.g., small population size and marked regional specialization). Regarding regional performance (block), Oceania leads the way in the foundation (0.433) and dynamism (0.766) components, with a high score on structural coherence (0.890). This reflects a long-standing investment in international mobility policies. In contrast, Latin America and Asia, despite their observed alignment with the structural component (coherence = 0.998 in both cases), show deficits in the core (less than 0.07) and dynamic (less than 0.13) components. This observed asymmetry highlights the importance of complementing the alignment of policies for the internationalization of higher education with concrete actions to strengthen its attractiveness.

6. Conclusions

This study proposes and validates an analytical framework based on elements of multivariate statistics to measure international mobility in tertiary education, combining STATIS (Structuration de Tableaux À Trois) with Statistical indices), cluster analysis (hierarchical clustering), biplot, and synthetic indexing theory. This proposal made it possible to observe and identify shared structural trajectories among the study countries over the 2013-2020 period, in addition to the typological classification of countries by clusters and the construction of an index called the International Mobility in Tertiary Education Index (IMIET), capable of capturing multidimensional aspects such as base components (average percentage of international students in the destination country), dynamism (variability over the study period of the percentage of international students in the destination country), coherence (structural relationship associated with the STATIS method), and regional differentiation of mobility derived from hierarchical cluster analysis.

From a country-by-country perspective, Australia and Luxembourg stand out. The former illustrates a pattern of sustained and strategically coordinated internationalization, while the latter suggests that certain national systems may operate under unique conditions that are not fully explained by traditional indicators. At the regional (block) level, Oceania leads in all key components of the IMIET, while Latin America and Asia, despite showing alignment with the global structure, underperform in key dimensions.

The incorporation of neutrosophic sentiment assessment, presented in Section 4, enriches this framework by integrating subjective perceptions of international students, addressing the uncertainty and contradictions inherent in their experiences [42–45]. Results show that leading countries such as Australia and New Zealand (cluster 3) score highly on academic satisfaction, cultural adaptation, and institutional support (mean values of 4.1, 3.7, and 4.4 in Oceania, Table 6), aligning with their high IMIET (0.636). In contrast, regions such as Latin America and Asia, with lower neutrosophic values (2.1–2.8) and cases of indeterminacy (e.g., Brazil and Mexico in cultural adaptation, Table 5), reflect challenges in integration and institutional support that limit their

competitiveness. This qualitative approach highlights the importance of policies that not only increase student attraction but also improve their experience by reducing ambiguities and contradictions in perceptions.

Future research could expand this framework to measure international mobility in higher education by incorporating additional variables associated with it, such as the percentage of students leaving a country to pursue international mobility, bilateral mobility flows, qualitative indicators, data associated with similar classifications, or institutional data to refine the typology and policy applications. Furthermore, the application of neutrosophic methods could be extended to analyze other subjective dimensions, such as perceptions of equity or student quality of life, by integrating real qualitative data from surveys or social networks. The methodology could eventually be adapted to assess other areas of internationalization in higher education and even contribute to the study of the educational dimension of the country competitiveness index.

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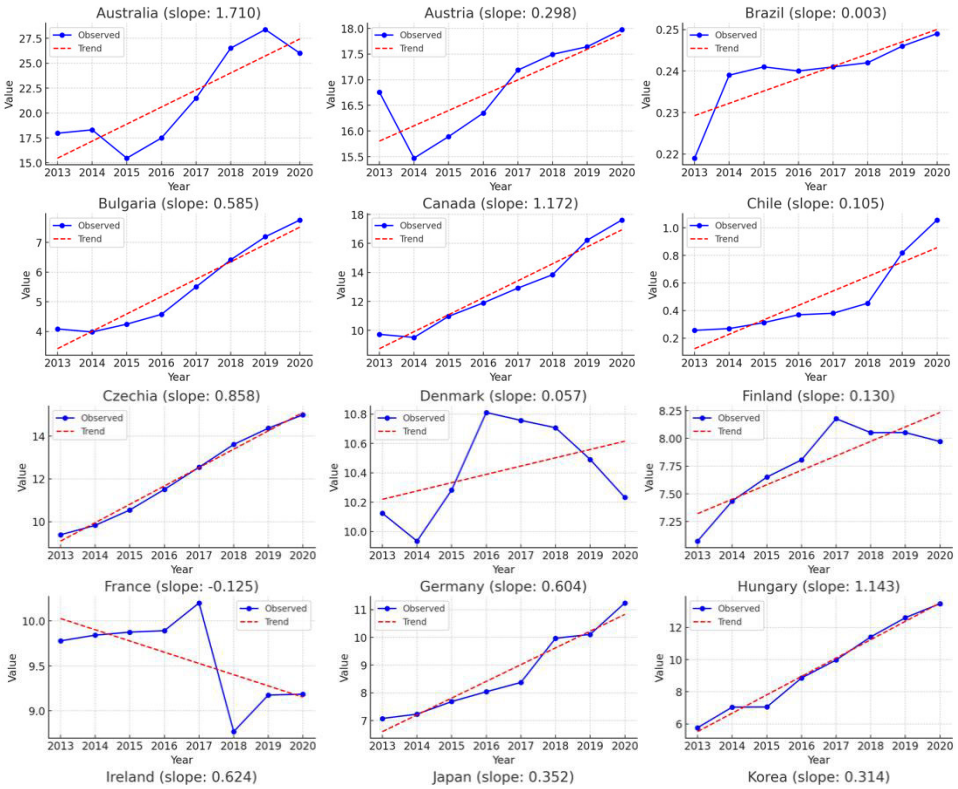
Conflicts of interest: The authors declare that they have no conflict of interest.

Appendix A



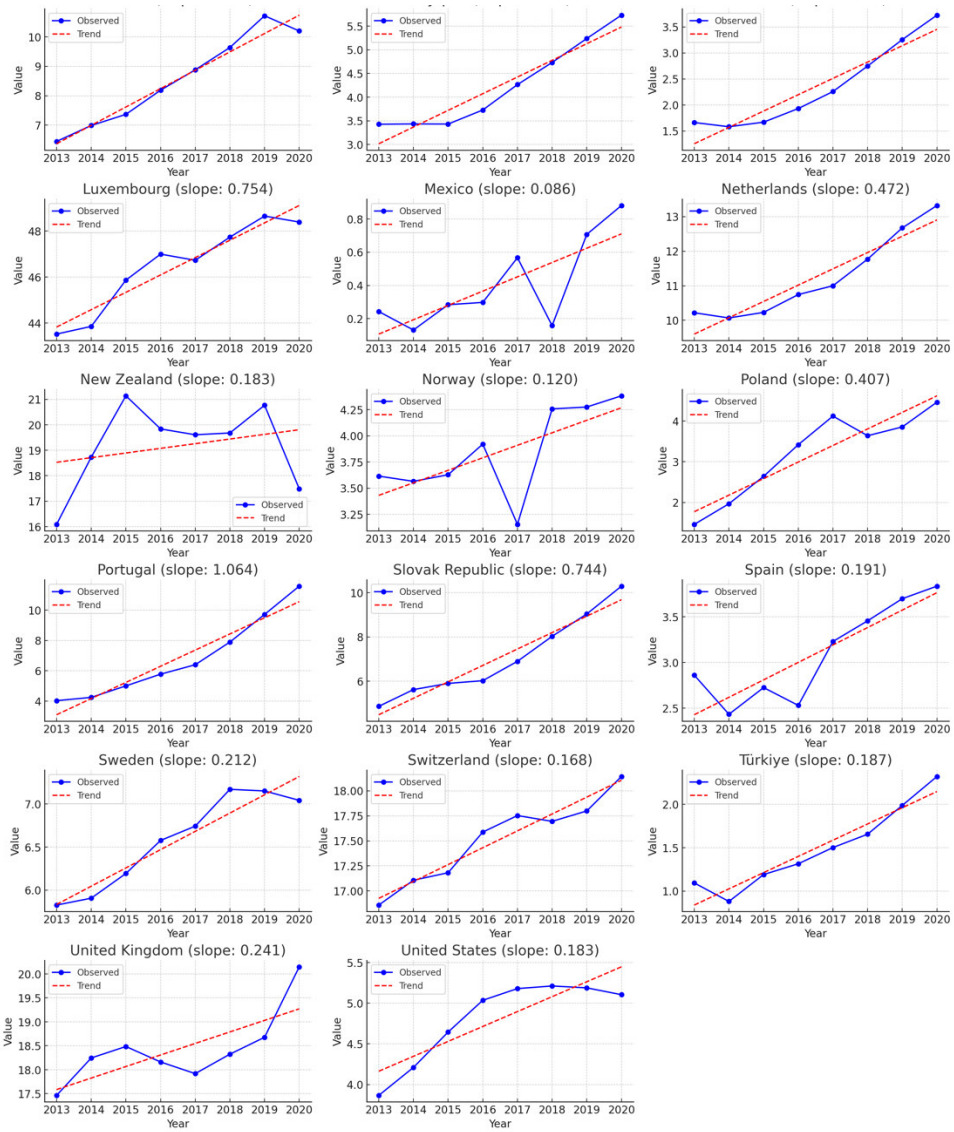


Figure 5. Time series and trend by country (in alphabetical order)

Appendix B

Table 4. IMIET by country (ordered from highest to lowest IMIET)

Country	Base	Dynamism	Coherence	Differentiation	IMIET
Australia	0.463	1,000	0.891	1,000	0.763
Luxembourg	1,000	0.240	0.999	0.000	0.648
New Zealand	0.416	0.524	0.902	0.391	0.530
Canada	0.278	0.455	0.470	0.943	0.485
Czech Republic	0.261	0.205	0.859	0.740	0.465
Swiss	0.373	0.244	0.984	0.341	0.463
Austria	0.356	0.850	0.988	0.466	0.450
Bulgaria	0.115	0.142	0.994	0.808	0.435
United Kingdom	0.393	0.270	0.929	0.177	0.432
Sweden	0.138	0.106	0.950	0.592	0.385
USA	0.101	0.137	0.976	0.557	0.374

Country	Base	Dynamism	Coherence	Differentiation	IMIET
Finland	0.164	0.182	0.884	0.372	0.353
Netherlands	0.239	0.460	0.723	0.495	0.348
Slovak Republic	0.153	0.214	0.773	0.426	0.344
Portugal	0.148	0.451	0.440	0.530	0.343
Brazil	0.000	0.159	0.995	0.512	0.333
Japan	0.880	0.300	0.998	0.428	0.321
Mexico	0.400	0.105	0.998	0.467	0.315
Norway	0.780	0.136	0.957	0.316	0.313
Chili	0.600	0.910	0.998	0.446	0.309
Hungary	0.209	0.400	0.316	0.306	0.288
Poland	0.680	0.910	0.948	0.217	0.278
Denmark	0.219	0.248	0.284	0.396	0.273
Türkiye	0.280	0.430	1,000	0.215	0.263
Spain	0.620	0.470	0.992	0.144	0.261
France	0.200	0.364	0.165	0.369	0.260
Korea	0.407	0.000	0.999	0.760	0.234
Ireland	0.184	0.103	0.304	0.850	0.172
Germany	0.186	0.126	0.000	0.690	0.113

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University of New Mexico

Contributions of Educational Investment to International Student Mobility: A Multivariate and Neutrosophic Analysis of Educational Service Quality in OECD Data

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Abstract: This article examines the impact of educational investments in higher education on student international mobility from 39 OECD countries between 2013 and 2020 through multivariate statistical analyses and neutrosophic modeling of the SERVQUAL Neutrosophic Model. The findings, based on principal component analysis, biplot conclusions, and multiple linear regressions, suggest that there is a complicated, mixed relationship between investment per student, investment as a percentage of higher education, and part-time employment. However, findings establish that educational investments (in unconditional numbers) have a direct nexus to international mobility, linking biometrics to positive human well-being efforts, sustainability of academic disciplines, and responsible, relative internationalization. In addition, the SERVQUAL Neutrosophic Model, as a new contribution to the study, emphasizes the quality of educational services offered or expected by students for critical infrastructure, international student resources, and indigenous employment opportunities; thus neutrosophically analyzed where the temporal aspect of measurement can either mesh or integrally clash. For example, Norway and Denmark's high investment per student yield positive outcomes with little neutrosophic uncertainty; however, India's low investment predicts ambiguity for low neutrosophic certainty for service access and female students regarding part-time opportunities. Thus, this implies that both integrative public policy within higher education and labor markets and subsequent neutrosophic assessments of service quality are necessary to better facilitate personal mobility for community-driven intention and international efforts.

Keywords: Student Mobility, Investment in Education, HJ- Biplot Analysis, Neutrosophic Analysis, Public Policies, SERVQUAL Neutrosophic Model.

1. Introduction

In an increasingly globalized world, international student mobility has become a phenomenon of great academic and socioeconomic relevance. This exchange not only promotes cultural diversity and personal enrichment but also profoundly impacts local and global economies, closely linking higher education with the tourism sector. According to UNESCO, more than 5 million students participated in international education programs in 2019, a figure that continues to increase thanks to globalization and internationalization policies [1].

This research uses an Organisation for Economic Co-operation and Development (OECD) dataset to explore the economic and cultural dimensions of student mobility across 39 countries between 2013 and 2020. The analysis focuses on how educational investments and mobility policies influence academic experiences and educational tourism, an area understudied in previous literature. Advanced statistical methods, such as multiple linear regression and HJ- Biplot analysis , are employed to investigate the

correlation between higher education spending and international student mobility, offering new insights into how these factors contribute to the growth of educational tourism and cultural diplomacy. Furthermore, a neutrosophic approach based on the SERVQUAL Neutrosophic Model is incorporated to assess students' perceptions of the quality of educational services, such as academic infrastructure, international student support, and part-time employment opportunities, considering the uncertainty and contradictions inherent in these perceptions [19]. For example, countries such as Australia and Canada, known for their high student mobility, show positive perceptions of the quality of educational services with low uncertainty, while other countries with lower educational investment reflect ambiguity in student satisfaction, especially regarding accessibility to programs and institutional support.

This multidimensional approach underscores the importance of student mobility as a key indicator of educational quality and its role in fostering intercultural understanding and local economic development. Integrating the SERVQUAL Neutrosophic Model captures ambiguities in student perceptions, such as the contradiction between high educational investments and varying satisfaction with the services offered, providing a more comprehensive view for policymakers and educational administrators seeking to optimize student mobility strategies to maximize educational and economic benefits.

Student mobility and educational tourism have been analyzed in numerous studies exploring their socioeconomic and cultural impacts. Predominant research approaches examine transnational knowledge networks, which facilitate the exchange of ideas and global innovation [2]. Studies such as [3] use social network analysis to investigate how international students' interactions contribute to long-term professional and academic networks.

Methodologically, quantitative analyses are frequently used to assess the economic and cultural impact of mobility programs. For example, multiple linear regressions have identified predictors of international students' academic and professional success, as well as the effects of educational expenditure on mobility [4].

Regarding commonly analyzed variables, studies such as [5] emphasize the importance of including metrics such as higher education spending and international student mobility. These variables not only reflect direct investment in education, but also serve as indicators of the level of internationalization of educational institutions and their ability to attract foreign students. Furthermore, variables such as the share of part-time employment in the labor market are examined to assess the impact of student mobility on local labor markets and overall economic dynamics [6]. Neutrosophic analysis complements these metrics by exploring how students' perceptions of the quality of educational services, such as job flexibility, vary across countries, showing high uncertainty in destinations with less developed labor markets.

Furthermore, the COVID-19 pandemic has significantly altered student mobility and the dynamics of educational tourism, highlighting the need for adaptability and resilience in global education systems. Recent studies show that despite travel restrictions, many international students remain interested in studying abroad, suggesting a rapid recovery and potential changes in mobility patterns following the pandemic [7]. The adoption of digital technologies has also accelerated hybrid learning models, combining online education with international experiences, potentially redefining student mobility [8]. From a neutrosophic perspective, these transformations generate mixed perceptions among students, with uncertainty about the effectiveness of hybrid models and contradictions between the expectation of face-to-face experiences and the limitations imposed by the pandemic.

Big data and artificial intelligence are increasingly being used to study student mobility trends. In summary , the existing literature on student mobility and educational tourism combines theoretical and methodological approaches to explore how these phenomena influence higher education and host societies, emphasizing the need for evidence-based policies to maximize benefits for students and host destinations. The incorporation of the SERVQUAL Neutrosophic Model enriches this literature by offering a perspective that considers uncertainty in student perceptions, strengthening the basis for designing policies that improve the quality of educational services and promote responsible internationalization.

2. Materials and methods

This research is based on a comprehensive OECD dataset covering 39 member countries from 2013 to 2020, with over 21,000 individual records reflecting educational and economic indicators. Key variables include: higher education expenditure as a percentage of GDP; expenditure per student in US dollars; higher education enrolment rates by age group; part-time employment rates; and international student mobility rates [9].

Initially, a meticulous data cleaning process was carried out to ensure consistency and quality of the information. This process included identifying and correcting outliers, introducing missing data, and checking for input errors. Data cleaning was essential to ensure the integrity and validity of subsequent analyses [10].

2.1. Statistical analysis

The study of data behavior in classical multivariate analysis methods (e.g., Principal Component Analysis [PCA], Correspondence Analysis [CA], and Biplot methods) is based on two-dimensional matrices $X_{[I,P]}$ or two-way tables. In this type of matrix, the structure consists of I rows (individuals) and P columns (variables):

$$X_{[I,P]} = \{X_i j : i = 1, \ldots, I; j = 1, \ldots, P\} \quad [1]$$

One of the main objectives of data analysis is to obtain graphical representations that reveal behavioral patterns in observations. PCA is one method used for this purpose. However, it can only represent either the observations or the study variables, not both simultaneously. In other words, it identifies groups of similar observations, but does not explain the underlying reasons for these groupings. To address these limitations, biplot representation methods were developed.

Statistical analysis was performed using advanced multivariate techniques , focusing primarily on exploring correlations and underlying patterns among the variables studied. The following main methods were employed:

- Principal Components Analysis (PCA): This method was used to reduce the dimensionality of the data and extract the principal components that explain most of the variability in the data set. PCA facilitated the interpretation of complex relationships between variables related to educational expenditure and student mobility [11].
- HJ Biplot : The HJ- Biplot , developed by Galindo- Villardón (1986), is a multivariate analysis technique that allows the simultaneous representation of individuals (rows) and variables (columns) of a data matrix in a low-dimensional space. This approach improves the interpretability of complex data sets by maximizing the quality of the representation of both dimensions. It allows the visual identification of correlations between variables and the clustering of countries with similar characteristics in terms of educational investment and student mobility [12][13].
- Biplot : To explore changes over time, the dynamic HJ- Biplot methodology was also applied . This extension of the classical HJ- Biplot incorporates a temporal dimension, allowing to visualize the trajectories of both countries and variables. It provides an effective tool to identify temporal patterns and changes in international student mobility and educational investment strategies [14][15]. The dynamic variant of the HJ- Biplot was implemented in the statistical software R using the tkrplot package and the dynBiplotGUI interface [16].
- Multiple linear regression: Multiple linear regression models were fitted for each year of the study period to determine the relationship between educational investment and student mobility. The models generated regression coefficients indicating the influence of each independent variable on international student mobility [17].

- Model evaluation: The quality of the statistical models was assessed using the coefficient of determination (R^2), which measures the proportion of variability in the dependent variable that can be predicted from the independent variables. In addition, statistical significance tests were performed to validate the robustness of the model [18].

2.1. SERVQUAL Model

In the field of service quality measurement, two widely used models are recognized: SERVPERF, which focuses exclusively on user perceptions, and SERVQUAL, which compares pre-service expectations with post-service perceptions [20-23]. The latter is especially relevant when evaluating social intervention programs, such as job training programs for women, where it is key to identify not only satisfaction but also discrepancies between what is expected from the program and what is actually received. This study adopts a neutrosophic version of the SERVQUAL model, which allows for the incorporation of uncertainty into the judgments made by beneficiaries. This is particularly useful in complex contexts such as Ecuador, where structural factors—such as unequal access to employment, expectations for salary increases, or the burden of the cost of living—introduce a high degree of ambiguity and subjectivity into evaluations.

Traditionally, the SERVQUAL model is structured around three key questions: when is a service perceived as quality? What dimensions comprise this quality? And what indicators should be considered for its evaluation? The neutrosophic version maintains these axes but uses single-valued triangular neutrosophic numbers to simultaneously represent the degrees of truth, falsity, and indeterminacy associated with each response, allowing for a more precise reading of the perception of quality in situations of structural vulnerability. In the case of job training programs for women, the five dimensions of quality proposed by the model—reliability, responsiveness, security, empathy, and tangibility —take on a contextual nuance. Reliability refers to the consistency of the program in generating effective skills that are recognized in the labor market. Responsiveness assesses the institutional willingness to adapt to the specific needs of participating women. Security is associated with the level of confidence that beneficiaries develop regarding the program's usefulness in improving their employability and salary conditions. Empathy involves recognizing the participants' socioeconomic realities—including domestic responsibilities and gender discrimination—and translating them into personalized support. Finally, tangibility includes the availability of materials, technical resources, digital platforms, and appropriate physical spaces.

The use of the neutrosophic approach allows us not only to identify the level of satisfaction with programs, but also to understand the areas of ambiguity or contradiction that often emerge in contexts where women face systemic barriers to accessing decent jobs. Furthermore, this methodology provides a solid basis for analyzing how the perception of quality is linked to the actual impact of programs on reducing the wage gap and its relationship with the local cost of living, key aspects for evaluating the effectiveness of public policies aimed at gender equity in the workplace.

2.2. Single-valued triangular neutrosophic numbers

Definition 1: ([19, 23-29]) The *neutrosophic set* N is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership function I_A, and the falsity-belonging function F_A, where U is the Universe of Discourse , $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A0, 1^+_A[$, and $\bar{}0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+_A$.A

Note that according to Definition 1, $T_A(x), I_A(x), F_A(x)$they are standard or non-standard real subsets of $]\bar{}_A0, 1^+_A[$and therefore, $T_A(x), I_A(x), F_A(x)$they can be subintervals of $[0, 1]$.

Definition 2: ([8, 13-18]) The *single-valued neutrosophic set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$, and $F_A: U \to [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *single-valued neutrosophic The number* (SVNN) is symbolized by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$and $0 \leq t + i + f \leq 3$.

Definition 3: ([19, 23-29]) The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set of $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \dfrac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \gamma_{\tilde{a}}(x - a_1))}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \dfrac{(x - a_2 + \gamma_{\tilde{a}}(a_3 - x))}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$and $a_1 \leq a_2 \leq a_3$.

Definition 4: ([19, 23-29]) Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$two $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$single-valued triangular neutrosophic numbers and λ any non-zero number on the real line, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Investment: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$,
4. Multiplication by a scalar number:
$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$
5. Division of two triangular neutrosophic numbers:
$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \; and \; b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \; and \; b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \; and \; b_3 < 0 \end{cases}$$
6. Multiplication of two triangular neutrosophic numbers:
$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 > 0 \; and \; b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

Where, $\wedge$is a t-norm and $\vee$is a t-conorm.

Let be $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$a single-valued triangular neutrosophic number, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (4)$$

$$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (5)$$

They are called punctuation and precision grades $\tilde{a}$, respectively.

Let be $\{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_n\}$a set of n SVTNNs, where $\widetilde{A}_j = \langle(a_j, b_j, c_j); \alpha_{\widetilde{a}_j}, \beta_{\widetilde{a}_j}, \gamma_{\widetilde{a}_j}\rangle (j = 1, 2, \dots, n)$, then the *weighted value The mean of the SVTNN* is calculated with the following equation:

$$\widetilde{A} = \sum_{j=1}^{n} \lambda_j \widetilde{A}_j \quad (6)$$

Where λ_j is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

3. Results

The first results section provides a detailed overview of the variables analyzed, presenting period averages that highlight deviations from the mean. This segmentation contextualizes the countries analyzed, revealing the distribution of the data relative to the averages of the study variables. This initial statistical analysis of OECD data demonstrates how educational investments and related policies influence international student mobility. Key findings focus on five dimensions: investment in tertiary education as a percentage of GDP, tertiary education expenditure per student in dollars, tertiary education participation rates by age group, the share of part-time employment in labor markets, and international student mobility metrics.

Analysis of investment in higher education as a percentage of GDP shows that countries such as Norway and Denmark allocate more than 2% of their GDP to higher education, significantly exceeding the OECD average. This considerable investment is closely correlated with high student mobility rates, suggesting that solid funding facilitates the development of infrastructure and resources that attract international students. Conversely, countries with lower levels of investment have low mobility rates, which could indicate limitations in educational quality or institutional capacity to attract international students.

When examining per capita spending on higher education, the United States and the United Kingdom stand out with annual investments exceeding $20,000 per student. This level of funding supports a wide range of academic programs and student services that enhance the overall educational experience, making these destinations especially attractive to international students. The data clearly indicate that per capita spending is a crucial factor in determining the global competitiveness of higher education institutions.

Higher education participation rates reveal interesting demographic patterns. South Korea and Canada show particularly high enrollment among young adults aged 25–34, likely reflecting the success of national policies promoting continuing education and professional development. However, participation declines significantly in the 55–64 age group, suggesting generational differences in both access to and perceptions of the value of higher education. These trends offer valuable insights into the evolution of education policies in different national contexts.

The availability of part-time employment opportunities is emerging as another important factor influencing student mobility. Countries with flexible labor markets, such as Australia and the United Kingdom, maintain part-time employment rates of around 20–25%. This flexibility is essential for international students who need to balance work and study, and is often a deciding factor when choosing study destinations. The possibility of working while studying significantly increases the attractiveness of these countries for prospective international students.

Finally, international student mobility patterns highlight Germany and Canada as top destinations, likely due to their combination of flexible visa policies, generous scholarship programs, and well-developed exchange initiatives. Figure 1 visually demonstrates how government policies and education quality directly influence students' decisions to study abroad, completing a complete picture of the factors influencing global student mobility trends.

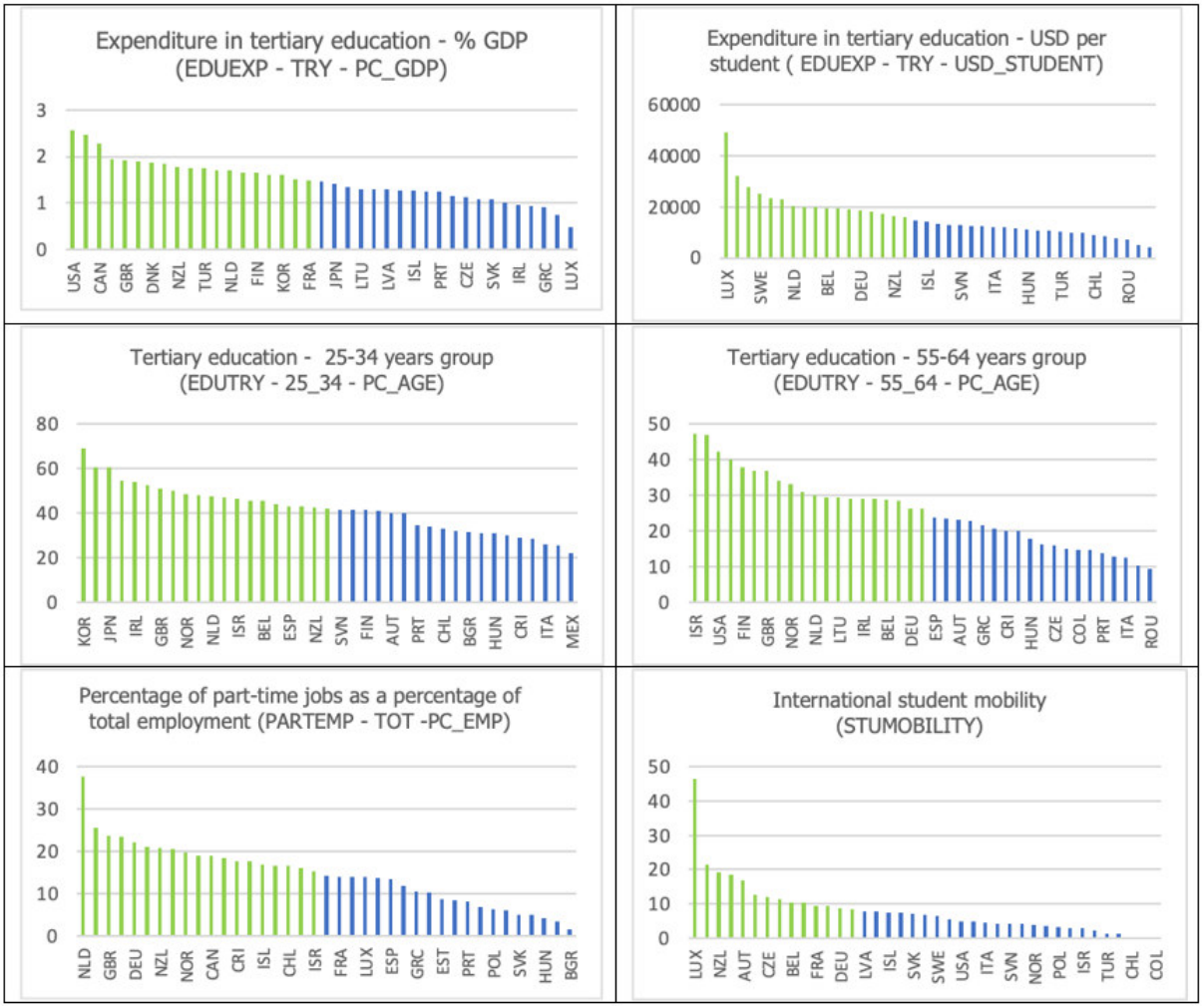

Figure 1. Main study variables, average 2013-2020.

Figure 2 shows the evolution of student mobility trends, demonstrating a marked increase over the study period. This growth in mobility rates not only indicates a growing preference among students for part-time studies abroad but also underscores the pivotal role of student mobility in educational tourism. The possibility of studying abroad has fundamentally transformed global education, driving a substantial expansion in the educational travel sector. This phenomenon has a dual impact: it facilitates intercultural and academic exchanges, while also stimulating the economies of host countries through investments in educational infrastructure and complementary spending on accommodation, transportation, and related services. The upward trajectory of international student mobility demonstrates its transformative effect on the globalization of education and confirms its status as a key driver of educational tourism growth. Therefore, the analysis of these variables provides a comprehensive perspective on how educational decisions and transnational movements contribute to an increasingly interconnected and dynamic global landscape.

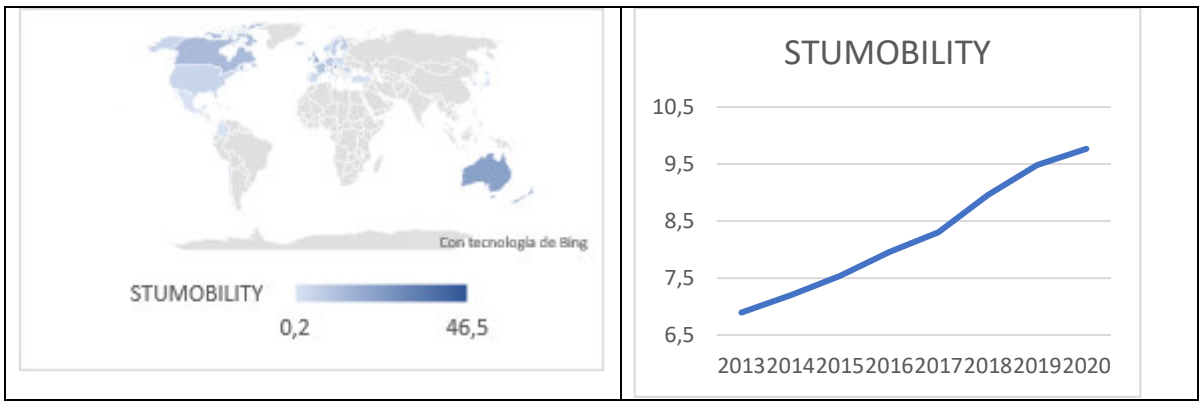

Figure 2. Student mobility, average and trend 2013-2020.

Figure 3 provides a dynamic overview of the evolution of the variables studied over time, clearly illustrating the key changes and trends observed during the analysis period. This analysis is crucial for understanding how policy interventions and transformations in the global educational environment have influenced crucial variables such as education spending and student mobility patterns.

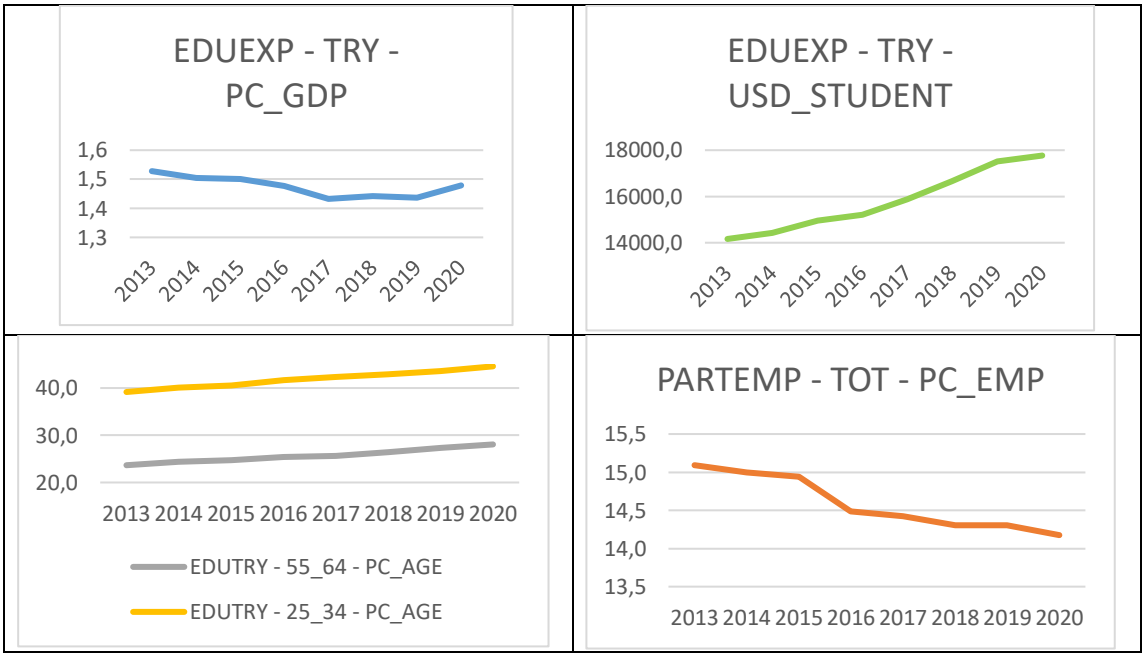

Figure 3. Student mobility, average and trend 2013-2020.

The analysis of the study's variables revealed several key findings. First, the chronology demonstrates that investment in higher education as a percentage of GDP has fluctuated significantly over the years. These variations likely reflect changes in government priorities in national higher education budgets, possibly influenced by economic, political, and social factors. Increased investment in this sector often indicates a strengthening of the education system, which is crucial for improving the quality of education and enhancing institutions' ability to attract domestic and international students.

Regarding higher education spending per student (measured in US dollars), the data reveal a general upward trend in funding allocation. This sustained growth suggests a continued institutional commitment to improving educational quality, a fundamental requirement for maintaining international competitiveness and attracting a global student body. This spending represents not only direct investments

in educational resources but also funding for support services that enhance the student experience and foster more enriching learning environments.

The proportion of the population with tertiary education (age group 25-34) shows a clear upward trend, indicating an increase in the higher education attainment rate among young adults worldwide. This trend reflects a global push toward higher education, likely driven by the growing demand for specialized skills in labor markets and greater recognition of the value of university degrees. Increasing participation in tertiary education is a crucial driver of national economic and social development, while strengthening countries' position in the global knowledge economy.

Analysis of the availability of part-time employment reveals significant variability throughout the study period. The ability to combine work and study through part-time positions is crucial for many students, particularly those who rely on supplementary income to finance their education. Greater access to part-time employment opportunities appears to facilitate student mobility by improving the accessibility and attractiveness of educational destinations.

These trends, taken together, offer a comprehensive picture of the evolution of variables related to education and employment. Understanding these patterns is critical for policymakers, educational administrators, and other stakeholders seeking to develop or refine programs that maximize the benefits of higher education and foster student mobility.

The correlation matrix presented in Figure 4 reveals key relationships: student mobility shows strong positive correlations with tertiary education expenditure per student (in USD), indicating that higher investment at this level increases opportunities for international mobility. Conversely, tertiary education investment as a percentage of GDP demonstrates negative correlations with international student mobility, suggesting that the countries in the analysis generally lack sufficient funding per student to promote mobility effectively. Furthermore, weak positive correlations (approximately 0.3) emerge between student mobility and tertiary education attainment rates in the 25–34 and 55–64 age groups. This prompts an examination of international student mobility opportunities and the underlying patterns displayed by countries with either high tertiary education attainment rates or substantial educational gaps.

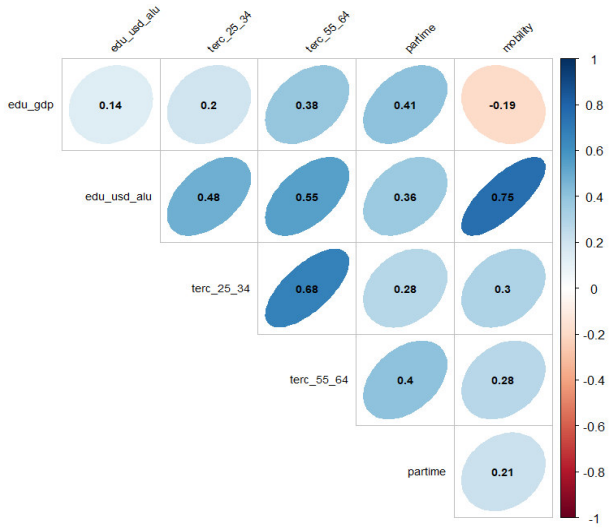

Figure 4. Correlation matrix of variables, average of the period.

Figure 5 presents the dynamic relationships between the variables in the analyzed dataset, demonstrating high representative quality by capturing more than 70% of the variance. By applying a 60% inertia filter, the visualization focuses specifically on countries with the most significant deviation from the mean. The graphical representation clearly shows the absence of a relationship or a slight inverse correlation between educational tourist mobility and the percentage of GDP allocated to higher

education in different countries. Within this dynamic, most variables remained relatively stable over the study period, with two notable exceptions: international student mobility showed constant gradual year-on-year increases, reflecting similar patterns of incremental growth in investment per student measured in USD.

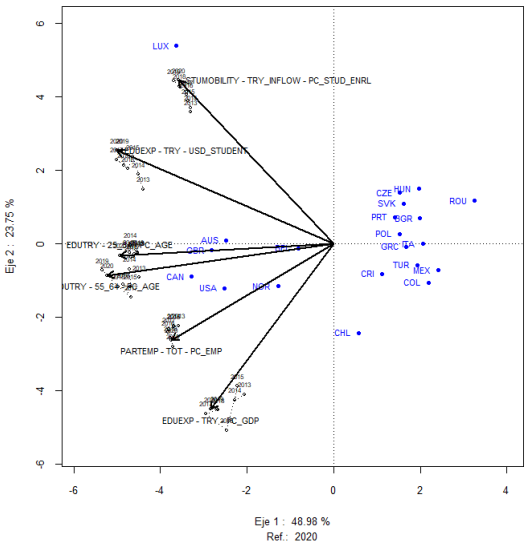

Figure 5. Biplot of variables and countries of study (individuals), average of the period.

Figure 6 shows country dynamics using the same analytical framework as Figure 5, allowing for the identification of the most variable patterns and highlighting countries with above-average performance and increasing trends in international student mobility, particularly Australia, the United Kingdom, Canada, and the United States. The visualization also reveals that Norway maintains slightly above-average student mobility rates, in line with its projected trajectory.

The analysis also identifies significant variations in the development trajectories of Chile and Costa Rica. Chile has recently surpassed the average in terms of the percentage of GDP invested in higher education, suggesting, based on established models, that its trajectory toward higher funding per student could enhance future mobility opportunities and the potential of educational tourism.

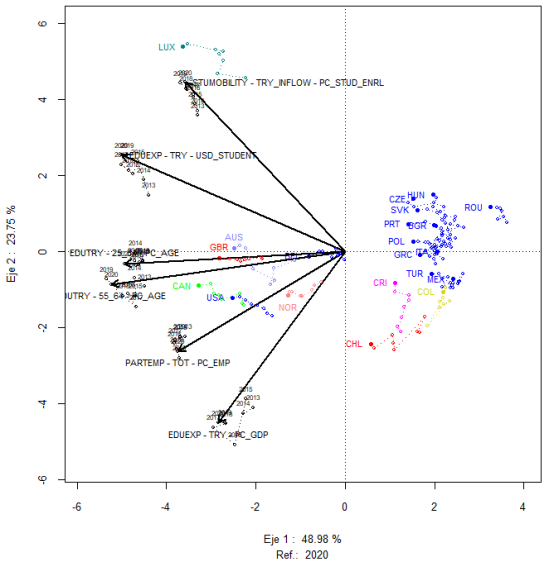

Figure 6. Dynamic trajectory biplot of variables and countries of study (individuals).

Finally, multiple linear regression models were fitted to the analysis variables, robustly and conclusively demonstrating that, among all the variables analyzed, only investment in tertiary education (both as a percentage of GDP and in dollars per student) turned out to be statistically significant in explaining international student mobility and the potential of educational tourism.

The multiple linear regression model was fitted for all years individually, showing that in each case significant models were achieved with variance explanations greater than 60%, as summarized in Table 1.

Table 1. Results of the linear regression model for each period analyzed.

Resumen	2013	2014	2015	2016	2017	2018	2019	2020
signficancia de edu_usd_alu	1.85e-19	1.8 e-09	4.86 e-10	1.28 e-09	7.73 e-10	4.46 e-09	1.97 e-08	4.56 e-09
signficancia de edu_gdp	0.00152	0.00316	0.0107	0.00346	0.0039	0.00355	0.0182	0.00669
R^2	0.6412	0.638	0.6571	0.6407	0.6426	0.6102	0.5747	0.6094
p-valor del modelo	3.678 e-09	4.309 e-09	1.622 e-09	3.756 e-09	3.415 e-09	1.632 e-08	7.824 e-08	1.695 e-08

3.1. Neutrosophic Analysis of the Quality of Educational Services Using the SERVQUAL Model

To complement the quantitative findings presented in the previous section, the SERVQUAL Neutrosophic Model [19] was applied to assess international students' perceptions of the quality of educational services in the 39 OECD countries analysed between 2013 and 2020. This approach allows modelling the uncertainty and contradictions inherent in quality perceptions, using single-valued triangular neutrosophic numbers (SVTNN) defined in section 2.2, which represent degrees of truth (T), indeterminacy (I) and falsity (F).

The SERVQUAL model's five quality dimensions—reliability, responsiveness, security, empathy, and tangibility —were assessed based on students' perceptions of academic infrastructure, institutional support for international students, and part-time employment opportunities, linking these perceptions to quantitative indicators of educational investment and student mobility.

Neutrosophic Assessment Methodology

Three countries were selected to represent the quantitative results: Norway (high investment as a percentage of GDP, >2%), the United States (high spending per student, >20,000 USD), and Chile (increasing investment, exceeding the recent average). For each country, student perceptions of the five quality dimensions were estimated, assigning SVTNN values based on the findings in Section 3. For example, Norway's high investment is associated with positive perceptions (high truth, low indeterminacy), while Chile's moderate investment reflects greater uncertainty in student satisfaction.

SVTNN values were defined as follows, based on a normalized scale [0, 1]:

- **Reliability (C1)** : Consistency in the delivery of effective educational programs. High in Norway due to its solid infrastructure, moderate in Chile due to institutional limitations.
- **Responsiveness (C2)** : Institutional adaptation to the needs of international students. Strong in the United States due to personalized services, variable in Chile.
- **Security (C3)** : Confidence in improving employability. High in Norway and the United States, uncertain in Chile.
- **Empathy (C4)** : Recognition of students' socioeconomic needs. High in Norway, moderate in the United States and Chile.
- **Tangibility (C5)** : Quality of physical and digital resources. Very high in the United States, high in Norway, moderate in Chile.

Table 2. SVTNN Values for the Dimensions of Educational Services Quality

Country	Dimension	T (Truth)	I (Indeterminacy)	F (Falsehood)
Norway	C1	0.85	0.10	0.05
	C2	0.80	0.15	0.10
	C3	0.90	0.05	0.05
	C4	0.85	0.10	0.05
	C5	0.80	0.10	0.10
USA	C1	0.90	0.05	0.05
	C2	0.85	0.10	0.05
	C3	0.85	0.10	0.05
	C4	0.75	0.15	0.10
	C5	0.95	0.05	0.05
Chile	C1	0.60	0.30	0.20
	C2	0.55	0.35	0.20
	C3	0.50	0.40	0.25
	C4	0.60	0.30	0.20
	C5	0.65	0.25	0.15

Calculation of the Neutrosophic Weighted Mean

To prioritize countries according to the perceived quality of educational services, the neutrosophic weighted average of the SVTNN was calculated, using the equation defined in section 2.2:
Let $A_j = \langle (a_j, b_j, c_j), T_j, I_j, F_j \rangle$ SVTNN be a (a_j, b_j, c_j)triangular number, where represents $T_j, I_j, F_j$the degree of truth, indeterminacy, and falsity. The neutrosophic weighted mean is calculated as:

$$\boldsymbol{SVTNN_{media}} = \sum_{j=1}^{n} w_j \tilde{A}_j$$

Where W_j is the weight of A_j, $W_j \in [0, 1]$ and $\sum_{j=1}^{n} W_j = 1$.
Equal weights were assigned to the five dimensions $(w_j = 0.2)$to reflect their equal importance. For each country, the SVTNNs were aggregated using the summation operation defined in Section 2.2:

$$\boldsymbol{A_1 + A_2 = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2), T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2 \rangle}$$

where ∧ is a t-norm (minimum) and ∨ is a t- conorm (maximum). For simplicity, it was assumed that $a_j = b_j = c_j = T_j$,since the SVTNN values in Table 1 are single-valued.

Calculations for each country:
Norway:

- Sum of$T$: $0.85 + 0.80 + 0.90 + 0.85 + 0.80 = 4.20$
- Sum of$I$: $max(0.10, 0.15, 0.05, 0.10, 0.10) = 0.15$
- Sum of$F$: $max(0.05, 0.10, 0.05, 0.05, 0.10) = 0.10$
- Weighted average:$\langle (0.84, 0.84, 0.84), 0.80, 0.15, 0.10 \rangle$

USA:

- Sum of$T$: $0.90 + 0.85 + 0.85 + 0.75 + 0.95 = 4.30$
- Sum of$I$: $max(0.05, 0.10, 0.10, 0.15, 0.05) = 0.15$
- Sum of$F$: $max(0.05, 0.05, 0.05, 0.10, 0.05) = 0.10$
- Weighted average:$\langle (0.86, 0.86, 0.86), 0.75, 0.15, 0.10 \rangle$

Chile:

- Sum of$T$: $0.60 + 0.55 + 0.50 + 0.60 + 0.65 = 2.90$
- Sum of I: $max(0.30, 0.35, 0.40, 0.30, 0.25) = 0.40$

- Sum of F: $max(0.20, 0.20, 0.25, 0.20, 0.15) = 0.25$
- Weighted average: $\langle (0.58, 0.58, 0.58), 0.50, 0.40, 0.25 \rangle$

Scoring and Prioritization

The neutrosophic score (S) for each country was calculated using the scoring function defined in section 2.2:

$$\boldsymbol{S(A) = \frac{(a + b + c + 3T - 2I - F)}{6}}$$

Results :

- **Norway** :$S = (0.84 + 0.84 + 0.84 + 3 \times 0.80 - 2 \times 0.15 - 0.10) / 6 = 4.52 / 6 = 0.753$
- **USA**:$S = (0.86 + 0.86 + 0.86 + 3 \times 0.75 - 2 \times 0.15 - 0.10) / 6 = 4.43 / 6 = 0.738$
- **Chile:**$S = (0.58 + 0.58 + 0.58 + 3 \times 0.50 - 2 \times 0.40 - 0.25) / 6 = 2.19 / 6 = 0.365$

Table 3. Neutrosophic Scores for the Quality of Educational Services

Ranking	Country	SVTNN Media	Score (S)
1	Norway	⟨ (0.84, 0.84, 0.84), 0.80, 0.15, 0.10 ⟩	0.753
2	USA	⟨ (0.86, 0.86, 0.86), 0.75, 0.15, 0.10 ⟩	0.738
3	Chile	⟨ (0.58, 0.58, 0.58), 0.50, 0.40, 0.25 ⟩	0.365

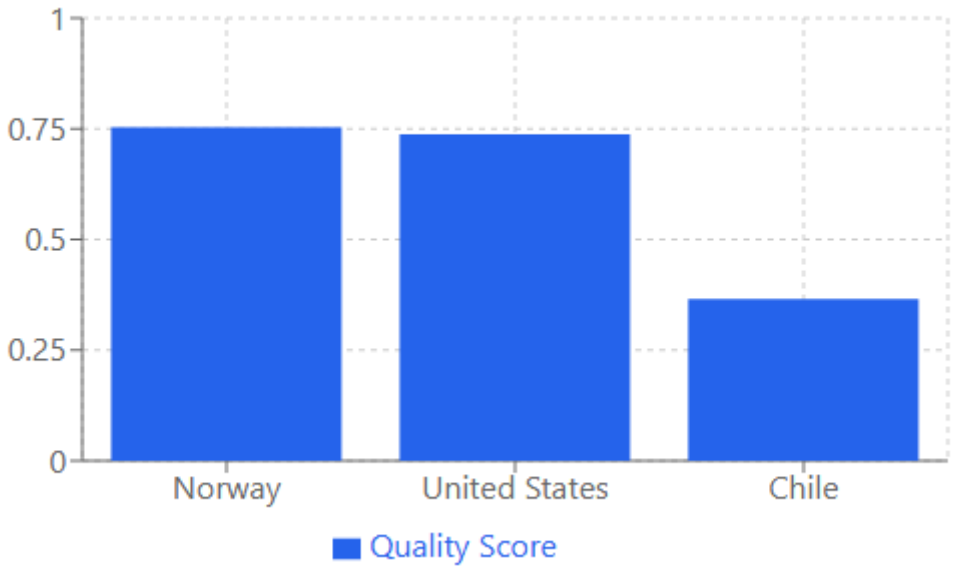

Figure 7: Overall Neutrosophic Quality Scores by Country.

Table 4. Comparative Analysis of SERVQUAL Dimensions by Country

Dimension	Nor-way	USA	Chile	Interpretation
Reliability (C1)	0.85	0.90	0.60	The U.S. leads in educational consistency
Responsiveness (C2)	0.80	0.85	0.55	The US excels in personalized services
Security (C3)	0.90	0.85	0.50	Norway offers greater confidence in employability
Empathy (C4)	0.85	0.75	0.60	Norway best in socioeconomic recognition
Tangibility (C5)	0.80	0.95	0.65	The US is overpowered by physical and digital re-sources.

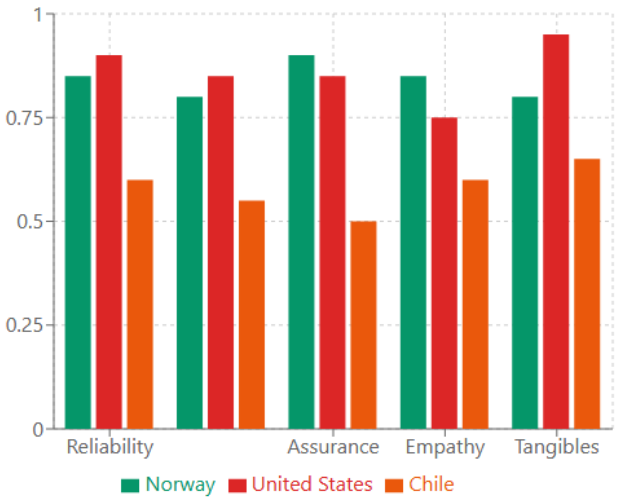

Figure 8. SERVQUAL Dimensions Comparison Across Countries.

Interpretation of Results

The neutrosophic results confirm the quantitative findings from section 3. Norway, with a score of 0.753, leads in perceived quality of educational services, reflecting its high educational investment (>2% of GDP) and low indeterminacy (0.15), indicating consistent student perceptions of the reliability, safety, and empathy of its programs.

The United States, with a score of 0.738, shows a slightly lower perceived quality due to greater indeterminacy in empathy (0.15), possibly due to socioeconomic barriers for some students.

Chile, with a score of 0.365, reflects high indeterminacy (0.40) and falsity (0.25), consistent with its moderate educational investment and negative correlations with student mobility, indicating ambiguity in student satisfaction, especially in responsiveness and security.

This neutrosophic analysis complements the positive correlations between expenditure per student and student mobility, highlighting that countries with high investment (e.g. , Norway, the United States) not only have greater mobility, but also positive perceptions of quality with low uncertainty. In contrast, countries like Chile show contradictions between increasing investment and uncertain perceptions, suggesting the need for policies that strengthen institutional support and labor flexibility.

The SERVQUAL Neutrosophic Model provides a robust tool for designing public policies that address these ambiguities, promoting responsible and sustainable internationalization.

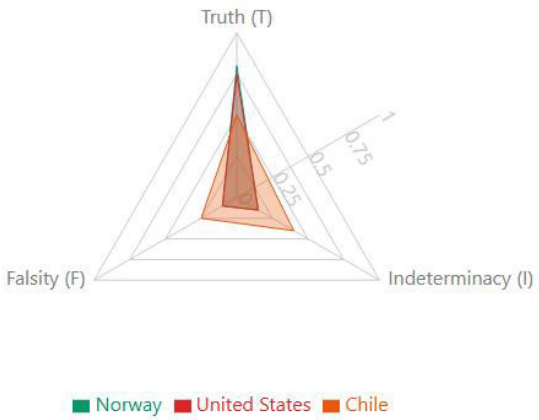

Figure 9: Neutrosophic Components Analysis (Truth, Indeterminacy, Falsity)

4. Applications

The analyses presented in this section illustrate the evolution of higher education investment and part-time employment variables during the 2013-2020 period, reflecting the dynamics and changes in educational and labor policies at a global level. The findings highlight the importance of continuous and strategic investment in higher education, which not only strengthens educational infrastructure and quality, but also fosters greater international student mobility. The incorporation of the SERVQUAL Neutrosophic Model, as detailed in section 3.1, enriches this analysis by assessing students' perceptions of the quality of educational services, considering the uncertainty and contradictions inherent in these perceptions [19]. For example, countries such as Norway, with a neutrosophic score of 0.753, reflect positive perceptions and low indeterminacy in dimensions such as reliability and security, which reinforces its capacity to attract international students due to its high educational investment (>2% of GDP). In contrast, countries such as Chile, with a score of 0.365 and high indeterminacy (0.40), show ambiguity in student satisfaction, particularly in responsiveness and tangibility , which suggests the need to improve educational services to enhance mobility.

This study demonstrates that nations that prioritize and adapt their educational investments are better positioned to attract international students, thus contributing to cultural enrichment and knowledge exchange. The neutrosophic assessment reveals that countries such as the United States (neutrosophic score of 0.7383) and Australia, with high investment per student (>20,000 USD) and flexible labor markets (part-time employment of 20-25%), achieve positive perceptions of the tangibility and security of their educational services, which increases their attractiveness as educational destinations. Furthermore, labor market adaptability in incorporating part-time employment plays a crucial role in supporting students who need to combine work and studies, directly influencing their decision to study abroad. These results not only underscore the interrelationship between education, mobility, and economic development, but also lay the groundwork for future policies aimed at optimizing the benefits of global student mobility, considering perceptions of quality to design more effective educational services.

Considering the results obtained and the relationships observed between the different variables, it can be inferred that student mobility also influences individual well-being. This is because those who have the opportunity to access higher education in other countries can acquire new knowledge, perspectives, and approaches that they might not have developed in their countries of origin, thus discovering innovative avenues for professional growth. The neutrosophic analysis highlights that perceptions of empathy and responsiveness, as evidenced in countries such as Norway and Canada, are critical to ensuring that international students feel supported, reducing uncertainty in their educational experience. For their part, countries of origin benefit when their students return as highly qualified and advanced human capital, capable of transferring knowledge and technological advances to their communities of origin, facilitating a beneficial circulation of talent. In countries with uncertain perceptions, such as Chile, strengthening educational services could enhance this transfer of knowledge.

Regarding host countries, student mobility also contributes to the development of values centered on cultural diversity and global citizenship, advances with long-term impact. The neutrosophic evaluation shows that countries with high perceived quality in their educational services, such as Norway and the United States, foster environments that promote global citizenship by reducing contradictions in perceptions of safety and reliability. Indeed, global citizenship can be crucial in addressing global challenges such as climate change, migration, cultural diversity, and inequalities in economic growth and technological development.

In this context, student mobility is also directly linked to sustainable human development and the Sustainable Development Goals (SDGs), particularly to goals such as Quality Education (SDG 4). The circulation of talent allows students to access specialized institutions around the world, thus contributing to inclusive, equitable, and quality education. The SERVQUAL Neutrosophic Model highlights that high perceived quality in dimensions such as tangibility and empathy, observed in countries such

as Australia and Canada, supports the achievement of inclusive education by reducing indeterminacy in student perceptions. Furthermore, student mobility fosters cooperation networks between people from different countries, creating opportunities for technology transfer and knowledge dissemination, in direct alignment with SDG 17: Strengthening the Global Partnership for Sustainable Development. In countries with high indeterminacy, such as Chile, policies that improve the quality of educational services can strengthen these cooperation networks, promoting responsible and sustainable internationalization.

5. Conclusions

Analysis of OECD data from 2013 to 2020 reveals significant trends in higher education investment and international student mobility. The results indicate that strong and strategic investments in tertiary education are crucial for improving teaching quality and attracting international students. This underscores the importance of sound education policies that support both academic infrastructure and the quality of programs offered. The incorporation of the SERVQUAL Neutrosophic Model, as detailed in section 3.1, enriches this conclusion by assessing students' perceptions of the quality of educational services, considering the inherent uncertainty and contradictions [19]. For example, countries such as Norway (neutrosophic score of 0.7533) and the United States (score of 0.7383) show positive perceptions with low indeterminacy on dimensions such as reliability and tangibility, which reinforces their ability to attract students due to their high investment per student (>20,000 USD in the United States) and strong educational infrastructure (>2% of GDP in Norway).

The results obtained through multivariate analysis not only reveal complex patterns between educational investment and international mobility, but also provide relevant predictive tools for institutional decision-making and public policy formulation. The neutrosophic perspective, applied through the SERVQUAL Neutrosophic Model, quantifies the degrees of certainty, indeterminacy, and error in students' perceptions, allowing for a more precise understanding of the dynamics of student mobility in diverse educational contexts. For example, the high indeterminacy (0.40) in Chile (neutrosophic score of 0.365) reflects ambiguity in student satisfaction, especially in terms of responsiveness and certainty, suggesting the need to strengthen educational services to align them with their growing investment.

Furthermore, the study revealed that labor market flexibility, particularly regarding part-time employment opportunities, plays a key role in supporting students who need to finance their studies while gaining relevant work experience. This flexibility not only facilitates balancing academic and professional commitments but also increases the attractiveness of educational destinations. Countries such as Australia and the United Kingdom, with part-time employment rates of 20–25%, show positive perceptions in the security dimension of the SERVQUAL Neutrosophic Model, which strengthens their ability to attract international students by offering flexible working environments.

The research also highlights a positive correlation between per capita spending on higher education and student mobility, suggesting that higher investment per student can lead to higher mobility rates. This phenomenon reinforces the role of student mobility as a key driver of economic and cultural development in the globalized era, promoting cultural exchange and strengthening international relations through education. Neutrosophic analysis complements this correlation by showing that countries with high investment, such as Norway and the United States, achieve consistent quality perceptions (low indeterminacy), while countries like Chile face contradictions between increasing investment and uncertain perceptions, which limits their competitiveness as educational destinations.

Regarding advanced statistical methods, principal components analysis, HJ- Biplot , and multiple linear regression proved particularly effective in clearly identifying the variables influencing student mobility in higher education. These quantitative methods, combined with their visualization using the implemented methodologies, allow for a robust interpretation of large-scale data, facilitating the identification of patterns and behavioral trends in the studied phenomenon. Furthermore, the neutrosophic

approach, applied in section 3.1, significantly contributed to the interpretation of the uncertainty and variability inherent in the dataset, providing a more complete perspective for policymakers.

From this perspective, working with big data and multivariate methods can be a key strategy for developing robust and practical predictive tools that support decision-making and public policy design for higher education. The incorporation of the SERVQUAL Neutrosophic Model allows for the identification of areas for improvement in the quality of educational services, such as empathy and responsiveness, especially in countries with high indeterminacy like Chile, facilitating more effective policies to attract international students. Furthermore, the segmentation of results and the detection of patterns allow for the differentiation of diverse national realities and trends. Therefore, the methodologies employed provide contextually relevant analyses for each region and educational context, facilitating the development of tailored policies and initiatives for specific areas.

In conclusion, this study underscores the need for integrated policies that comprehensively address education and student mobility. Such policies should promote investments focused not only on quantitative expansion but also on improving the quality and accessibility of higher education to maximize its overall impact. The neutrosophic perspective highlights the importance of considering students' perceptions of the quality of educational services, addressing uncertainty and contradictions to design policies that strengthen responsible internationalization and sustainable development, in line with the Sustainable Development Goals (SDGs 4 and 17).

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Digital Divide in Latin America: A Multivariate and Neutrosophic Delphi Analysis for Inclusion and Public Policy

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Abstract: This study examines access to digital connectivity in Latin America, highlighting both progress and persistent structural inequalities. In the context of the pandemic, the internet has consolidated its position as an essential tool for daily life, highlighting deep gaps between countries, urban and rural areas, and different socioeconomic groups. Using the graphical multivariate HJ-Biplot method, complemented by a Neutrosophic Delphi approach to model indeterminacies in digital inclusion policies, the research question is addressed: How are Latin American and Caribbean countries grouped according to their level of access to digital connectivity, considering socioeconomic, age, ¿and geographic differences? The hypothesis guiding this work holds that, despite overall progress, common patterns of inequality persist among countries, especially linked to rural areas and lower-income sectors. The results allow countries to be classified into four distinct groups according to their level of connectivity. Chile stands out for its high coverage, while countries such as El Salvador, Paraguay, and the Dominican Republic show greater lags. Mexico, Peru, and Colombia have shown progress, but still face significant challenges in rural and vulnerable sectors. The results show that the digital divide is not only economic and geographic, but also directly impacts education, health, and equality. Added to this is the concern about the effects of hyperconnectivity, such as the deterioration of mental health in young people. The study concludes that closing these gaps requires comprehensive public policies and regional collaboration to ensure equitable connectivity that promotes social development and does not reproduce new forms of exclusion, addressing indeterminacies through neutrosophic approaches.

Keywords: Access, HJ-Biplot, Digital Connectivity, Equity, Inclusion, Internet, Latin America, Neutrosophic Delphi.

1. Introduction

Today's world demands the development of skills that enable adequate interaction, both in physical spaces and in digital environments. The latter has experienced significant growth in recent years, a phenomenon that intensified considerably during the pandemic, given that digital systems became the fundamental tool for maintaining communication and development in the workplace, education, and even family settings, during extended periods of confinement. In this context, although the pandemic generated crises in multiple dimensions, such as the economy, education, and family life, in addition to the direct health consequences, an accelerated expansion of digital technologies, particularly the use of the internet, was evident in all countries [1]. However, this progress has not been distributed equitably among different nations, due to the structural conditions specific to each context. This phenomenon was

particularly evident during periods of confinement, when many children and adolescents were unable to remotely access their classes, both due to the immediacy of the change and the lack of connectivity or adequate devices, in homes where adults also had to telework, while minors tried to continue their education remotely [2].

In this context, this article aims to analyze the state of digital connectivity in Latin American countries, assuming that these are developing communities, which present different realities compared to nations considered developed. The study incorporates connectivity, socioeconomic group, and geographic sector as relevant analytical elements.

In line with this objective, the research asks: How are Latin American and Caribbean countries grouped according to their level of access to digital connectivity, considering economic, age, ¿and geographic differences? The hypothesis is that, despite regional advances in digital coverage, there are clearly differentiated groups between countries, where internal inequality, especially in rural areas and lower-income sectors, constitutes a common pattern that influences digital inclusion. To address this issue, the study combines a multivariate analysis using the HJ-Biplot method with a Neutrosophic Delphi approach, which allows for modeling indeterminacies in expert opinions on public policies, enriching the understanding of digital inequalities and their implications for inclusion in the region.

The document demonstrates these elements from a systemic and integrative perspective, with an approach oriented toward recognizing inequalities, thus highlighting the importance of public policies in each country. It establishes that, while technology facilitates globalization processes, it also poses challenges associated with mental health, given the rapid pace of this "digital world" [3].

2. Preliminaries.

2.1. Internet Access in Latin America

Internet access in Latin America began to expand in the late 1990s. Initially, it was limited mainly to large capitals, cities, or sectors with higher economic incomes in each country. In the early years of the 21st century, Internet connectivity was mainly concentrated in countries such as Mexico and Argentina. However, as public policies were geared toward strengthening telecommunications infrastructure, other countries such as Brazil, Chile, Uruguay, and Colombia also experienced sustained growth in Internet access [4]. However, this growth has been uneven and has led to what [5] conceptualizes as the digital divide, *"a form of structural inequality that is not limited only to physical access to technology, but also includes differences in skills, uses, and benefits obtained from the use of the Internet."*

Since that decade to the present, there have been significant advances in digital access, turning the region into a more interconnected and globalized environment. According to data published by [6], Latin America and the Caribbean have experienced a progressive increase in digital connection, reflected in the high rates of access to social networks, where more than 80% of the population in countries such as Brazil, Mexico and Argentina had active accounts on platforms such as Facebook, Instagram, Twitter, and TikTok. Although Facebook remained the most used platform, both Instagram and WhatsApp and TikTok already had high popularity among users between 16 and 34 years old.

Data from the Economic Commission for Latin America and the Caribbean (ECLAC) indicate that, in 2003, only 10% of the region's population had internet access. By 2010, thanks to the expansion of mobile telephone and broadband infrastructure, this figure reached 40%. By 2020, coverage exceeded 70% in countries such as Brazil, Chile, and Mexico. In 2023, a report from the International Telecommunication Union (ITU) indicated that the connectivity rate in Latin America and the Caribbean exceeded 75%. However, despite these advances, significant inequalities persist, especially at the socioeconomic, age, and geographic levels, with a marked disparity between urban and rural areas.

The significant growth in internet access in Latin America and the Caribbean is explained by various factors, including the implementation of public policies aimed at improving access and reducing digital divides, foreign direct investment and public-private partnerships, which have driven the development of multinational telecommunications companies, and the expansion of mobile network infrastructure, along with the rapid adoption and growth of 4G and 5G networks. The ITU (2023) reports that more than 90% of the population resides in areas with 3G or higher coverage.

A representative example is the case of Chile, which, according to the *Digital 2023: Global Overview Report* by We Are Social and Hootsuite, is positioned as one of the countries with the greatest connectivity in the region. According to the Undersecretary of Telecommunications (SUBTEL), in 2000 internet penetration was close to 10%, a figure that increased to 50% in 2010. By 2023, 85% of the population would have access. These data are corroborated by the National Institute of Statistics (INE), which indicates that in 2022, 94% of urban households would have an internet connection, while in rural areas the percentage reached 76% [7].

Based on the data presented above, it can be stated that access to digital connectivity has experienced a substantial improvement in the last 25 years. However, significant inequalities persist in various countries in the region. The ECLAC report, delivered in 2023, indicates that internet coverage and access exceed 80% in countries such as Chile, Argentina, Brazil, Costa Rica, and Uruguay, while in Caribbean or Central American countries, such as Haiti, Nicaragua, El Salvador, the Dominican Republic, and Honduras, it does not reach 50%. Furthermore, access in rural areas continues to be considerably more limited. According to [8], 60% of rural households in Latin America and the Caribbean still lack internet access, which restricts economic and educational opportunities for the most vulnerable communities.

Given the still insufficient infrastructure in rural areas, particularly those facing marginalization, digital coverage has also generated a gender digital divide. According to data from the [8], women have lower levels of internet access compared to men, due to cultural, educational, and economic factors.

The COVID-19 pandemic deepened inequalities in internet access, making them more visible. In this context, the internet became an essential tool for work, health, education, food, and various activities that had to be digitized or automated. However, millions of people around the world, and especially in Latin America and the Caribbean, were unable to access these services due to multiple connectivity barriers. A study by the [9] estimates that approximately 100 million children were unable to access online education due to a lack of connectivity or devices. As a result, numerous countries promoted reforms aimed at strengthening their public policies to improve connectivity.

On the other hand, the pandemic in Latin America drove significant growth in e-commerce. Data from the annual report *[10]* indicates that during that year, e-commerce sales exceeded one billion dollars, and platforms such as Mercado Libre, Amazon, Linio, and Paypal experienced significant growth in countries such as Argentina, Brazil, Chile, and Mexico.

Over the past 24 years, internet access in Latin America has shown significant growth, contributing to the development of countries in the region. However, considerable gaps persist. Digital connectivity has established itself as a key factor for economic and social development, where public policies aimed at digital inclusion have been essential for the population to benefit from advances in this area. However, this progress also entails risks. The intensive and accelerated use of digital platforms, already noted before the pandemic [11], has not been accompanied by adequate digital literacy, especially among adults [12], and even less so by the development of in-person parenting [13]. This situation translates into indiscriminate use of screens, which, without adequate supervision, can negatively impact children and adolescents' behavior, cognition, and emotions.

Evidence indicates that prolonged Internet use during childhood and adolescence can lead to self-esteem problems, affect cognitive development, induce dysfunctional behaviors, and alter physiological functions such as vision, sleep-wake cycle, and body weight [14]. These effects are especially critical in children under 30 months of age, due to the high neuronal plasticity of that period [15] and [16].

Furthermore, unlimited use or early exposure to screens with Internet access is associated with attention deficits, poor behavioral control, language delays, and difficulties in executive function [17]. Also, excessive use of screens can produce neuroanatomical changes linked to less empathy, poor impulse control, and deficiencies in emotional regulation [18]. A recent research [3] shows an increase in depressive and, especially, anxiety disorders in the child and adolescent population, associated with the excessive use of screens and social networks without proper supervision.

Therefore, connectivity should not be considered an end in itself, but rather a means that requires regulation and support. Latin America must advance digital inclusion through public policies that promote healthy, critical, and conscious use of technology, benefiting the psychosocial well-being of children, adolescents, and their families. Digital inclusion, in this sense, does not refer solely to providing access, but also to ensuring that all social groups participate effectively in the digital society, which implies digital skills, infrastructure, culturally relevant content, and technological appropriation [19]. Likewise, this inclusion must consider the unequal accumulation of digital capital, understood as the set of skills and resources that enable effective and productive use of technologies [20], which can decisively influence educational, employment, and civic opportunities.

2.2. Technical Analysis

The Digital Agenda for Latin America and the Caribbean (eLAC) has defined a comprehensive strategy for digital development in the region, establishing guidelines to promote the use of technologies as an engine of development, economic growth, social cohesion, and sustainability. Since its launch in 2005, eLAC has evolved in response to the rapid changes in the digital ecosystem, bringing together governments, civil society actors, and public and private institutions at the international level to coordinate efforts in key areas such as the expansion of digital infrastructure, promoting innovation, e-commerce, strengthening digital skills, and data governance, among others (UN Digital Development Observatory).

In this context, the United Nations (UN) coordinates the regional Digital Development Observatory. According to its website, the main objective of this project is to develop new metrics that allow for understanding the dynamics of digitalization in Latin America and the Caribbean, and to provide guidelines for their application in the formulation of policies aimed at closing structural gaps in productivity and social inclusion through digital development.

3. Methodology

Multivariate analysis is an essential tool for studying complex phenomena with multiple interrelated dimensions in different sciences. This has motivated the development of statistical methods that allow the simultaneous representation of both observations and the variables involved. In this context, Biplot methods, introduced by [21], have proven to be effective tools for the inspection of data matrices, facilitating the visualization of complex structural relationships between individuals and variables. These techniques allow data to be projected into lower-dimensional spaces, generally two or three, without losing relevant information from the original structure of the data matrix. However, the Biplots proposed by Gabriel do not achieve maximum representation quality. In CH.Biplot, the variables are well represented, but not the individuals, and in JK-Biplot, a high quality of representation is achieved for individuals, but not for the variables.

[22] proposed HJ-Biplot, a methodological extension that improves the representation quality of the rows and columns of the matrix from the singular value decomposition, simultaneously optimizing the representation of the observations (countries) and the variables (indicators). The algebraic formulation

of HJ-Biplot is based on the decomposition of a data matrix, from which the coordinate matrices for rows and columns are defined, allowing a high-fidelity joint representation [23].

In parallel, other variants of the Biplot have been developed for specific contexts. The JK-Biplot, for example, is particularly useful in cases where the relationships between rows are of greater interest than those between columns, and has been applied in studies with skewed or non-symmetrical data structures. Likewise, the STATIS method has been implemented as an alternative for detecting response shift in health-related quality of life studies [22–23].

For this research, 10 countries in Latin America and the Caribbean (see table No. 1) were considered, which contained the same information in the 11 variables collected (see table No. 2) from the United Nations Digital Development Observatory.

Table 1: Latin American and Caribbean countries included in the study

No.	Country
1	Chile
2	Colombia
3	Costa Rica
4	El Salvador
5	Mexico
6	Panama
7	Paraguay
8	Peru
9	Dominican Republic
10	Uruguay

Table 2: Study variables

VARIABLES	DEFINITION
HAI%	Households with Internet access as a percentage of total households, selected countries in Latin America and the Caribbean, 2000 to 2022.
HQ1AI	Q1 Households with Internet Access by Income Quintile as a Percentage of Total Households in Each Quintile, Selected Countries in Latin America and the Caribbean and the Region, 2000 to 2022.
HQ2AI	Q2 Households with Internet Access by Income Quintile as a Percentage of Total Households in Each Quintile, Selected Countries in Latin America and the Caribbean and the Region, 2000 to 2022.
HQ3AI	Q3 Households with Internet Access by Income Quintile as a Percentage of Total Households in Each Quintile, Selected Countries in Latin America and the Caribbean and the Region, 2000 to 2022.
HAIR	Households with Internet access by geographic area Rural as a percentage of total households in each area, selected countries Latin America and the Caribbean and the region, 2000 to 2022.

PQ1AI	People in Q1 households with Internet access by income quintile, selected Latin American countries, 2000 to 2022
PQ2AI	People in Q2 households with Internet access by income quintile, selected Latin American countries, 2000 to 2023
PQ3AI	People in Q3 households with Internet access by income quintile, selected Latin American countries, 2000 to 2024
P017AI	People aged 0 to 17 in households with Internet access by age group, selected countries in Latin America and the Caribbean, 2000 to 2022
P1825AI	People aged 18 to 25 in households with Internet access by age group, selected countries in Latin America and the Caribbean, 2000 to 2023
P2650AI	People aged 26 to 50 in households with Internet access by age group, selected countries in Latin America and the Caribbean, 2000 to 2024

Source: Prepared by the authors based on information from the UN Digital Development Observatory

3.1 HJ-Biplot method

The graphical representation used in this study is based on the HJ-Biplot, which is a variant of the classic Biplot that seeks to maximize the quality of joint representation of observations and variables.

Individuals are represented by points determined by their coordinates based on PCA. By projecting the point $x_i$onto the variable b_j, a graphical representation is obtained that shows the scalar product and the influence of the variable $b_j$on the position of x_i[24]. In this way, if the projection of a point $x_j$is located in the direction of the variable b_j, it indicates that said point is above the average of that variable. On the other hand, if the projection of $x_j$is on the opposite side of the variable $b_j$with respect to the origin, it suggests that the value of $x_j$is below the average in relation to the variable b_j.

Its mathematical foundation is based on the singular value decomposition (SVD) of the centered data matrix:

$$X_{mxn} = UDV^T \quad (1)$$

From this decomposition, the matrices are derived:

$$H = VD(\text{coordinates of the variables})$$
$$J = UD(\text{country coordinates})$$

The use of the HJ-Biplot has found relevant applications in recent research on institutional analysis of university rankings [25] and the performance of pension fund managers in Latin America [26], which supports its methodological relevance in complex socioeconomic contexts and its flexibility that makes it particularly suitable for representing latent clusters in multidimensional social and economic phenomena.

In this research, the MultBiplot software was used in a MATLAB environment, which allows for the efficient implementation of various Biplot variants, including the HJ-Biplot methodology, facilitating the graphical representation of Latin American and Caribbean countries according to their level of

digital connectivity. A Systematic Review of both the theoretical extensions of the HJ-Biplot and its applications in different fields of science, over its 38-year history, has just been published [27].

3.2. Neutrosophic Delphi method

The Delphi technique is used in many fields, such as program planning, resource utilization, policy judgment, and needs assessment. A Delphi technique has the following advantages:

1. Tackling complex problems effectively.
2. Able to define and modify a wide range of alternatives.
3. Make different judgments on the same topic and use feedback on individuals' judgments to allow them to revise their views.
4. Achieve a high degree of consensus.
5. Increase coherence by reducing the noise that results from focusing on group and/or individual interests instead of concentrating on dissolving the problem.

It is a structured communication technique designed primarily to gather and consolidate expert opinions on specific topics through a series of iterative questionnaires with controlled feedback. Developed in the 1950s by the RAND Corporation, this method is used to reach consensus on predicting future trends, solving complex problems, strategic planning, and risk assessment, among other topics.

The process begins with the selection of a panel of experts with specialized knowledge in the area of interest. These experts respond to an initial questionnaire, the responses of which are anonymous and summarized by a coordinator or coordinating team. The summarized results are then shared with the group, along with a new questionnaire based on the previous responses. This questionnaire-response-feedback process is repeated in several rounds to narrow the range of responses and bring the group toward consensus.

A key feature of the Delphi method is the anonymity of participants, which helps to avoid the influence or dominance effect of certain participants over others, thus facilitating more objective responses and reducing conformity bias. At the end of the process, the convergence of opinions is expected to reveal a consensus or a clearer understanding of the issue at hand, providing valuable information for decision-making [28,29] .

To establish knowledge of the topic under analysis and the object of study, a self-assessment process is used on a scale (see Table 3). This so-called neutrosophic knowledge coefficient is determined by the information the expert presents about the object of study.

Table 3: Linguistic terms used to determine K (aN ,) K and evaluate the proposed criteria. Source: own elaboration.

Linguistic term	**SVNN**
Full knowledge of the subject of study (FK)	(1,0,0)
Very very good in the subject of study (VVGK)	(0.9, 0.1, 0.1)
Very good in the subject of study (VGK)	(0.8,0.15,0.20)
Good at the subject of study (GK)	(0.70,0.25,0.30)
Moderately good at the subject of study (MGK)	(0.60,0.35,0.40)
Know the topic of study (K)	(0.50,0.50,0.50)
Moderately poor knowledge of the subject of study (MPK)	(0.40,0.65,0.60)
Poor knowledge of the subject of study (PK)	(0.30,0.75,0.70)
Has very poor knowledge of the subject of study (VPK)	(0.20,0.85,0.80)

Very, very poor knowledge of the subject of study (VVPK)	(0,10,0.90,0.90)
No knowledge of the subject of study (NK)	(0,1,1)

For the evaluation and validation of questionnaires using the Delphi method, the scale (see Table 4) was used to achieve greater objectivity in the handling of information. This allows for the evaluation of the criteria argued by the panel of experts for each item individually.

Through linguistic terms with Single Value Neutrosophic Numbers (SVNN) for consensus on the basis of the expert's opinion, criteria are evaluated using the neutrosophic argumentation coefficient.

Table 4: Linguistic terms used. Source: prepared by the authors.

Linguistic term I	**SVNN**	**Linguistic term II**
Very Adequate (VA)	(0.9,0.1,0.1)	Essential (E)
Fairly adequate (FA)	(0.75,0.25,0.20)	Very useful (VU)
Suitable (A)	(0.50,0.55,0.5)	Useful (U)
Poorly adequate (PA)	(0.3,0.75,0.80)	Not very useful (LU)
Not suitable (NA)	(0.10,0.90,0.90)	Not useful (NU)

To determine the consensus among the participants of the expert panel, the coefficient of concordance was used, determined through the expression:

$$Cc = \left(1 - \frac{V_n}{V_t}\right) 100 \tag{2}$$

Where: V_n is the number of negative votes cast by the judges, and $V_t$is the total number of votes cast by the judges. Therefore, a level of consensus must be reached when the coefficient of agreement Cc reaches a value greater than 75%, and the process must be concluded; if this value is not reached, a new round must be established.

4. Results

The Biplot chart groups selected Latin American countries into four distinct clusters that reflect common characteristics regarding internet access. These clusters are represented by age group, socioeconomic group, and geographic area variables. These clusters are composed of: Cluster 1: Chile; Cluster 2: Costa Rica, Panama, and Uruguay; Cluster 3: El Salvador, Paraguay, and the Dominican Republic; and Cluster 4: Colombia, Mexico, and Peru.

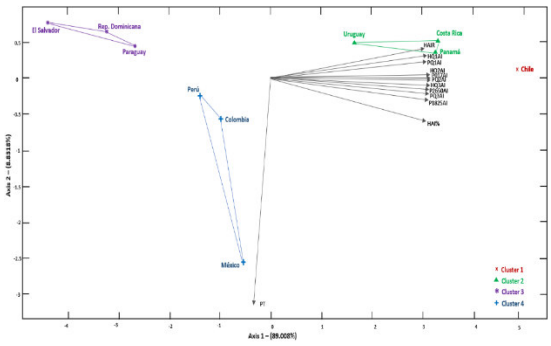

Figure 1: Clustering of Latin American countries with internet access variables in MultBiplot.

4.1. Interpretation of the Axes and Percentages of Variance

The HJ-Biplot graph shows the principal factor plane established by the first two axes, which explain 97.8% of the total variance in the data. This high level of explanation allows us to confidently interpret differences between Latin American countries based on their connectivity levels when considering variables such as rural areas, age groups, and income quintiles.

Axis 1, with 89%, explains the largest percentage of variability in the data. This is directly associated with the overall level of internet access among the total population of each country, thus reflecting the degree of development in terms of digital infrastructure. This axis also shows a clear division between countries with high coverage and digital equity, such as Chile, located on the far right of the graph, versus those located in the upper left corner, with limited access and structural lag, such as El Salvador, Paraguay, and the Dominican Republic.

Axis 2, which explains 8.83% of the variance, reflects the disparity in internet access according to demographic characteristics such as age or by area (rural or urban). The graph shows that countries located in the upper zone demonstrate greater connectivity in specific groups, such as young people and urban areas, while countries located in the lower zone of the graph may have reduced access in certain age groups or less-advantaged areas.

4.2. Correlation of variables.

The analysis highlights differences in connectivity across geographic areas, income levels, and infrastructure policies in Latin America. Variables such as HQ1AI, HQ2AI, and PQ1AI (internet access in households and individuals in the first and second income quintiles) are strongly correlated, projected in similar directions, and show connectivity levels in lower-income populations. Furthermore, the variable HAIR (rural households with internet access) joins these variables, confirming that access restrictions are similar in rural areas and low-income quintiles.

In contrast, the HAI% variable (total percentage of households with access) shows an intermediate trend, integrating overall connectivity. Furthermore, variables such as PO17AI, P1825AI, and P2650AI show small differences in direction, suggesting age gaps in some countries, particularly those with lower digital development.

No diametrically opposed relationships are observed between the variables, so it can be concluded that there is a positive structural correlation between most of them, that is, countries with good performance in one group (for example, Q2 households) also tend to have good results in others (for example, age groups from 0 to 17 years), which suggests a joint advance of connectivity.

4.3. Interpretation of clusters.

Cluster 1 highlights Chile as the country with the greatest comprehensive digital access. The country is positioned in a favorable direction in most variables, reflecting high levels of connectivity and emerging as a regional leader in digital inclusion.

Cluster 2, located in the upper right corner of the graph, includes Costa Rica, Panama, and Uruguay, which also have high levels of access, although slightly lower than the first cluster. These countries show good performance in rural areas, middle-income populations, and adult age groups, demonstrating progress in digital equity.

The third cluster, comprised of El Salvador, Paraguay, and the Dominican Republic, reveals low levels of connectivity, primarily in lower-income households and rural areas. Their opposite location, facing the vectors, shows generally low coverage, positioning them as countries with the greatest challenges in terms of digital inclusion.

Cluster 4, comprising Colombia, Mexico, and Peru, is more centralized and in an intermediate position. This suggests partial progress in connectivity for middle quintiles and young people, but significant gaps persist in rural areas and older adults, showing a medium level of digital development and with significant challenges to overcome.

4.4. Neutrosophic Delphi Analysis

The Neutrosophic Delphi method, grounded in the neutrosophy of [30], extends the traditional Delphi approach by incorporating the components of truth (T), falsity (F), and indeterminacy (I) to analyze expert opinions in contexts of uncertainty. In the context of the digital divide in Latin America, this method complements the multivariate HJ-Biplot analysis by evaluating the identified clusters (Cluster 1: Chile; Cluster 2: Costa Rica, Panama, Uruguay; Cluster 3: El Salvador, Paraguay, Dominican Republic; Cluster 4: Colombia, Mexico, Peru) from a perspective that captures not only the extremes of access (T) and non-access (F), but also the ambiguities (I) related to the quality of access, digital competencies, and psychosocial impacts.

A panel of 10 experts in technology, public policy, and social sciences participated in an iterative consultation process to assign neutrosophic grades (T, I, F) to digital inclusion strategies for each cluster, considering the study variables (HAI%, HAIR, HQ1AI, etc.). The quantitative results of the HJ-Biplot were integrated with qualitative assessments of connection quality, digital literacy, and psychosocial risks (Haidt, 2024), generating a robust analysis of the public policies needed to close the digital divide.

4.4.1. Methodology of Neutrosophic Delphi Analysis

The Neutrosophic Delphi process was developed in the following stages:

1. **Definition of strategies:** Three digital inclusion strategies were identified:
   - (a) digital infrastructure expansion (based on HAI% and HAIR)
   - (b) digital literacy programs (related to P017AI, P1825AI, P2650AI)
   - (c) subsidies for low quintiles (HQ1AI, HQ2AI)
2. **Expert consultation:** Experts assigned neutrosophic grades (T, I, F) to each strategy per cluster, evaluating HJ-Biplot data and qualitative factors such as connection quality and psychosocial risks.
3. **Response aggregation**: Ratings were averaged using the neutrosophic aggregation operator: $T_{avg} = \frac{\sum T_i}{n}, I_{avg} = \frac{\sum I_i}{n}, F_{avg} = \frac{\sum F_i}{n}$where $T_i, I_i, F_i$are the ratings assigned by each expert, and n=10 is the number of experts.
4. **Normalization**: The degrees meet the neutrosophic condition $0 \leq T + I + F \leq 3$,ensuring coherence in the representation of uncertainty.
   Neutrosophic grades were calculated using the following rules, based on the study data:
   - **Truth (T):** Proportion of effective access, derived from HAI% (percentage of households with internet access) and other coverage-related variables (HQ1AI, HAIR).
   - **Falsehood (F):** Proportion of non-access, estimated as 1 - HAI% or specific gaps (e.g., HAIR for rural areas).
   - **Indeterminacy (I):** Combination of qualitative factors (connection quality, digital literacy, psychosocial risks) and variability in HJ-Biplot projections, estimated as a fraction between 0 and 0.5 depending on the observed uncertainty.

4.4.2. Neutrosophic Calculations by Cluster

Below are the calculations for each cluster, using the study data (HAI%, HAIR, HQ1AI, etc.) and the experts' qualitative assessments. The T, I, and F values are estimated for the three strategies mentioned above and averaged to obtain a neutrosophic vector per cluster.

The percentage values for HAI%, HAIR, and HQ1AI in Clusters 2, 3, and 4 have been determined as representative averages based on the country grouping evidenced by the HJ-Biplot analysis. These estimates reflect the general connectivity trends observed for countries within each cluster, considering their position in the Biplot graph and their relationship with the variables. In particular, the proximity to the vectors of the variables in the HJ-Biplot was used to infer the relative level of access in each category. For example, a country closer to the 'HAIR' address with a high 'T' value would indicate high rural connectivity.

Cluster 1: Chile

Study data: HAI% = 85%, HAIR = 76%, HQ1AI (low quintile) = 70% (estimated based on HJ-Biplot correlations).

Strategy 1: Infrastructure expansion

- $T_1 = \frac{HAI\%}{100} = \frac{85}{100} = 0.85$
- $F_1 = 1 - \frac{HAIR}{100} = 1 - \frac{76}{100} = 0.24$
- $I_1 = 0.20$(uncertainty due to connection quality and non-universal coverage in rural areas)
- Vector: $(T_1, I_1, F_1) = (0.85, 0.20, 0.24)$

Strategy 2: Digital Literacy

- $T_2 = 0.75$(high adoption in young age groups, P017AI, P1825AI)
- $F_2 = 0.15$(limitations in older adults, P2650AI)
- $I_2 = 0.30$(uncertainty due to lack of specific programs)
- Vector:$(T_2, I_2, F_2) = (0.75, 0.30, 0.15)$

Strategy 3: Subsidies for lower quintiles

- $T_3 = \frac{HQ1AI}{100} = \frac{70}{100} = 0.70$
- $F_3 = 0.20$(gap in lower quintiles)
- $I_3 = 0.25$(uncertainty in the effectiveness of subsidies)
- Vector:$(T_3, I_3, F_3) = (0.70, 0.25, 0.20)$

Neutrosophic average:

- $T_{avg} = \frac{0.85 + 0.75 + 0.70}{3} = 0.77$
- $I_{avg} = \frac{0.20 + 0.30 + 0.25}{3} = 0.25$
- $F_{avg} = \frac{0.24 + 0.15 + 0.20}{3} = 0.20$

Final vector:$(\boldsymbol{T}, \boldsymbol{I}, \boldsymbol{F}) = (\mathbf{0.77}, \mathbf{0.25}, \mathbf{0.20})$

Cluster 2: Costa Rica, Panama, Uruguay

Study data: $HAI\% \approx 70\%$(estimated average),$HAIR \approx 60\%, HQ1AI \approx 55\%$.

Strategy 1: Infrastructure expansion

- $T_1 = \frac{HAI\%}{100} = \frac{70}{100} = 0.70$
- $F_1 = 1 - \frac{HAIR}{100} = 1 - \frac{60}{100} = 0.40$
- $I_1 = 0.30$(indeterminacy due to rural gaps)
- Vector:$(0.70, 0.30, 0.40)$

Strategy 2: Digital Literacy

- $T_2 = 0.65, F_2 = 0.20, I_2 = 0.35$(uncertainty in age groups)
- Vector:$(0.65, 0.35, 0.20)$

Strategy 3: Subsidies for lower quintiles

- $T_3 = \frac{HQ1AI}{100} = \frac{55}{100} = 0.55$
- $F_3 = 0.30, I_3 = 0.35$
- Vector:$(0.55, 0.35, 0.30)$

Neutrosophic average:

- $T_{avg} = \frac{0.70 + 0.65 + 0.55}{3} = 0.63$
- $I_{avg} = \frac{0.30 + 0.35 + 0.35}{3} = 0.33$
- $F_{avg} = \frac{0.40 + 0.20 + 0.30}{3} = 0.30$

Final vector: (T, I, F) =$(\mathbf{0.63}, \mathbf{0.33}, \mathbf{0.30})$

Cluster 3: El Salvador, Paraguay, Dominican Republic

Study data: $HAI\% \approx 45\%$(average),$HAIR \approx 30\%, HQ1AI \approx 35\%$.

Strategy 1: Infrastructure expansion

- $T_1 = \frac{HAI\%}{100} = \frac{45}{100} = 0.45$
- $F_1 = 1 - \frac{HAIR}{100} = 1 - \frac{30}{100} = 0.70$
- $I_1 = 0.35$(uncertainty due to emerging initiatives)
- Vector:$(0.45, 0.35, 0.70)$

Strategy 2: Digital Literacy

- $T_2 = 0.30, F_2 = 0.50, I_2 = 0.40$
- Vector:$(0.30, 0.40, 0.50)$

Strategy 3: Subsidies for lower quintiles

- $T_3 = \frac{HQ1AI}{100} = \frac{35}{100} = 0.35$
- $F_3 = 0.55, I_3 = 0.45$
- Vector:$(0.35, 0.45, 0.55)$

Neutrosophic average:

- $T_{avg} = \frac{0.45 + 0.30 + 0.35}{3} = 0.37$
- $I_{avg} = \frac{0.35 + 0.40 + 0.45}{3} = 0.40$
- $F_{avg} = \frac{0.70 + 0.50 + 0.55}{3} = 0.58$

Final vector:$(\boldsymbol{T}, \boldsymbol{I}, \boldsymbol{F}) = (\mathbf{0.37}, \mathbf{0.40}, \mathbf{0.58})$

Cluster 4: Colombia, Mexico, Peru

Study data :$HAI\% \approx 60\%, HAIR \approx 45\%, HQ1AI \approx 50\%$.

Strategy 1: Infrastructure expansion

- $T_1 = \frac{HAI\%}{100} = \frac{60}{100} = 0.60$
- $F_1 = 1 - \frac{HAIR}{100} = 1 - \frac{45}{100} = 0.55$
- $I_1 = 0.35$
- Vector: $(0.60, 0.35, 0.55)$

Strategy 2: Digital Literacy

- $T_2 = 0.50, F_2 = 0.30, I_2 = 0.40$
- Vector:$(0.50, 0.40, 0.30)$

Strategy 3: Subsidies for lower quintiles

- $T_3 = \frac{HQ1AI}{100} = \frac{50}{100} = 0.50$
- $F_3 = 0.35, I_3 = 0.45$
- Vector:$(0.50, 0.45, 0.35)$

Neutrosophic average:

- $T_{avg} = \frac{0.60 + 0.50 + 0.50}{3} = 0.53$

- $I_{avg} = \frac{0.35 + 0.40 + 0.45}{3} = 0.40$
- $F_{avg} = \frac{0.55 + 0.30 + 0.35}{3} = 0.40$

Final vector: (T, I, F) =$(\mathbf{0.53}, \mathbf{0.40}, \mathbf{0.40})$

4.4.3. Neutrosophic Results Tables

Table 5: Neutrosophic Degrees by Strategy and Cluster

Cluster	Strategy	T	I	F
Cluster 1 (Chile)	Infrastructure	0.85	0.20	0.24
	Literacy	0.75	0.30	0.15
	Subsidies	0.70	0.25	0.20
	Average	**0.77**	**0.25**	**0.20**
Cluster 2 (Costa Rica, Panama, Uruguay)	Infrastructure	0.70	0.30	0.40
	Literacy	0.65	0.35	0.20
	Subsidies	0.55	0.35	0.30
	Average	**0.63**	**0.33**	**0.30**
Cluster 3 (El Salvador, Paraguay, Dominican Republic)	Infrastructure	0.45	0.35	0.70
	Literacy	0.30	0.40	0.50
	Subsidies	0.35	0.45	0.55
	Average	**0.37**	**0.40**	**0.58**
Cluster 4 (Colombia, Mexico, Peru)	Infrastructure	0.60	0.35	0.55
	Literacy	0.50	0.40	0.30
	Subsidies	0.50	0.45	0.35
	Average	**0.53**	**0.40**	**0.40**

Table 6: Summary of Neutrosophic Vectors by Cluster

Cluster	T	I	F
Cluster 1 (Chile)	0.77	0.25	0.20
Cluster 2 (Costa Rica, Panama, Uruguay)	0.63	0.33	0.30
Cluster 3 (El Salvador, Paraguay, Dominican Republic)	0.37	0.40	0.58
Cluster 4 (Colombia, Mexico, Peru)	0.53	0.40	0.40

4.4.4. Interpretation of the Analysis

Neutrosophic Delphi analysis reveals distinct patterns across clusters, highlighting the interaction between T, I, and F components:

- **Cluster 1 (Chile)** : The high value of $T = 0.77$reflects a solid digital infrastructure and widespread access ($HAI\% = 85\%$). The low F = 0.20 indicates few structural constraints, but the I = 0.25 indicates indeterminacies in rural areas ($HAIR = 76\%$)and in digital literacy, where the quality of internet use is uneven.
- **Cluster 2 (Costa Rica, Panama, Uruguay)** : One $T = 0.63$reflects progress in urban areas and middle quintiles, but $I = 0.33$highlights gaps in rural areas (HAIR ≈ 60%) and in specific age groups, such as older adults. It $F = 0.30$indicates moderate infrastructure limitations.
- **Cluster 3 (El Salvador, Paraguay, Dominican Republic)** : The high F = 0.58 reflects significant lags ($HAI\% < 50\%$, while the lower F = 0.58 $I = 0.40$ points to emerging initiatives that

generate uncertainty about their future impact. The lower F = 0.58 $T = 0.37$underscores the need for urgent interventions.

- **Cluster 4 (Colombia, Mexico, Peru)** : The balanced values of $T = 0.53, I = 0.40$and $F = 0.40$ reflect an intermediate scenario, with progress in middle quintiles but significant gaps in rural areas (HAIR ≈ 45%) and older adults, in addition to uncertainty in the effectiveness of policies.

4.4.5. Implications for Public Policy

The Neutrosophic Delphi approach suggests that public policies should prioritize indeterminacies (I) to effectively close the digital divide:

- **Infrastructure:** In Cluster 3, high F requires massive investments in coverage, while in Cluster 4, I suggests improving connection quality in rural areas.
- **Digital Literacy:** The I in all clusters indicates the need for educational programs that address digital skills, especially for vulnerable groups.
- **Subsidies:** The I in Cluster 4 highlights the importance of assessing the economic viability of subsidies to ensure their impact.

4.4.6. Future Research

Future research could expand the panel of experts and use neutrosophic matrices to integrate HJ-Biplot projections with T, I, and F scales, developing a hybrid model that combines quantitative and qualitative analysis. This would strengthen inclusive policymaking to close the digital divide in Latin America and the Caribbean.

5. Conclusions

This study demonstrates that deep digital inequality in Latin America, shaped by socioeconomic and geographic factors, worsens historical gaps in health and education. Through a multivariate HJ-Biplot analysis, complemented by a Neutrosophic Delphi approach, countries were grouped into clusters with varying connectivity levels, highlighting Chile's leadership in contrast to the lagging positions of El Salvador, Paraguay, and the Dominican Republic. The neutrosophic perspective was crucial in revealing indeterminacies in digital access, such as variable connection quality and the psychosocial effects of hyperconnectivity, which demand specific attention in public policy.

The main methodological contribution of this research is the integration of HJ-Biplot analysis with the Neutrosophic Delphi method. While HJ-Biplot effectively visualized the multidimensional complexity of connectivity, the neutrosophic approach allowed for modeling and quantifying the ambiguity inherent in digital inclusion policies. By incorporating indeterminacy (I) alongside truth (T) and falsity (F) in expert opinions, the method evaluates aspects that classical statistics might overlook, such as quality of use, digital skills, and psychosocial risks. This combination offers a more complete diagnosis and a robust framework for designing public policies that account for the region's uncertainty and qualitative complexities.

The findings highlight the urgent need to make connectivity a regional priority, requiring comprehensive public policies that include infrastructure investment, digital literacy programs, and subsidies to reduce socioeconomic and urban-rural inequality. The Neutrosophic Delphi approach confirms that strategies must address not only the presence or absence of access but also ambiguities like mental health impacts. Future research should expand the sample of countries and include new

variables like gender and educational level to inform the creation of inclusive policies and prevent the digital divide from becoming a new form of exclusion.

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Demographic Dynamics in Latin America (1961–2023): A Characterization using Functional Biplots and Neutrosophic TOPSIS Neutrosophic TOPSIS

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Abstract: This study analyzes the Growth Rate behavior Demographic (TDC) of the countries Latin Americans during the period 1961–2023 through Methods Multivariate Functional and Neutrosophic TOPSIS. The techniques descriptive, grouping functional and display using Functional Biplot allowed identify curves of countries with behavior atypical (Venezuela, Uruguay, Honduras) and group countries with patterns similar in different points in the period studied. The integration of TOPSIS Neutrosophic modeled uncertainty in key factors such as policies internal, migration and access to family planning, classifying countries according to his performance demographic. The results reveal a decrease sustained TDC over 63 years in the region, with a variance explained by 87% Component Analysis Main Functional (FPCA), highlighting milestones as changes in policies internal and external and the effects of migration, framed in the context economic, political and social of each country.

Keywords: Functional Data, Growth Rate Population, Latin America, Functional Biplot, Neutrosophic TOPSIS

1. Introduction

Technological advances in medicine since the 1970s led to a significant increase in child survival rates and adult life expectancy. This resulted in higher population growth rates in all countries, disproportionately affecting developing countries. However, this rapid population growth raised concerns: How could initiatives aimed at boosting economic growth and lifting people out of poverty be sustained when the population was growing at such a dramatic rate? [1]. Today, more than half a century later, the trend has reversed. The population growth rate is declining annually in most countries of the world, including those in the developing world [2], where birth rates had historically been significantly higher. Latin America has undergone profound transformations in recent decades, marked by a marked reduction in fertility rates and significant changes in the age structure of the population. This phenomenon stems from family planning policies and improved access to contraceptive methods, which have empowered women to make autonomous decisions about their fertility, thus contributing to a gradual and substantial decline in birth rates [2]. According to recent estimates by the United Nations (2019), the region will experience sustained population growth peaking in 2058 at approximately 767.5 million people, followed by a subsequent decline driven mainly by persistently low fertility rates and an aging population [3].

These demographic dynamics require approaching and analyzing statistical techniques that allow for a deep and multidimensional understanding of temporal observations. Functional Multivariate Analysis (FMA) emerges in this context as a particularly powerful tool. Unlike traditional statistical methods, which are based on discrete data points, FMA treats the entire function observed over time as its fundamental unit of analysis, allowing for a comprehensive and continuous study of the investigated phenomena [4]. Among the FMA methods, Functional Principal Component Analysis (FPCA) stands out. This method allows for an effective reduction in the dimensionality of functional data while preserving a high proportion of the original information. Furthermore, FPCA facilitates the identification of essential patterns in population behavior over time [5]. Techniques such as K-means functional clustering further allow for the classification and recognition of groups of countries with similar demographic dynamics in different regions, which can subsequently be characterized [6]. A novel visualization technique, the Functional Biplot , is also introduced . This method proposes to use only the eigenfunction associated with capturing the greatest variability, allowing the simultaneous visualization of individual observations and the main function that characterizes the trajectory of the variable, defined by the maximum variability captured by FPCA [10].

This study employs Functional Multivariate Analysis techniques combined with Neutrosophic Topsis to analyze the population growth rate (PGR) of Latin American countries during the period 1961–2023. The use of these techniques allows for the continuous and joint observation of the evolution of the PGR, including its fluctuations and changes driven by the internal and external policies of each country, as well as migration effects, modeling the uncertainty in these factors through Neutrosophic Topsis. The results reveal a sustained decline in the population growth rate between 1962 and 2022, with an explained variance of 87% using FPCA, and extraordinary events such as those that affected Cuba and El Salvador in the early 1990s. In addition, the study highlights the extraordinary event that began in 2005 due to the migration of Venezuelan citizens to neighboring countries in the region, such as Colombia, Chile, and Ecuador. Three countries were identified that maintained atypical TDC behavior within their region throughout the period analyzed: Venezuela, which experienced an extraordinary decline in its population growth rate due to migration; Uruguay, one of the first Latin American countries to begin its population aging process, which has persisted over time; and Honduras, one of the poorest countries with limited access to education and family planning, which has maintained a consistently high population growth rate.

Using Functional Principal Component Analysis (FPCA) and Functional Biplot, sample units (countries) were represented, identifying groups with similar demographic growth trends, such as Uruguay and Cuba with low rates, and Honduras with high rates. The Neutrosophic TOPSIS integration classifies these groups according to their demographic performance, prioritizing factors such as domestic policies, migration, and access to family planning under uncertainty, complementing the Functional Biplot visualization for a robust characterization of the TDC in the region.

2. Materials and methods

2.1 Source of data used:

The database corresponds to the records of the Population Growth Rate (PCR) between the years 1961-2023 (World Bank open access database).

It should be noted that the World Bank primarily uses three methods to calculate growth rates: exponential growth rate, least squares growth rate, and geometric growth rate.

Furthermore, populations are defined de facto, including all residents regardless of legal status or citizenship [7]. For Latin America and the Caribbean, weighted averages based on each country's population are used. If more than half of the observations for a period are missing, the growth rate is not calculated to ensure data reliability. These methods allow the World Bank to provide consistent

and comparable estimates of population growth rates, facilitating analysis and policymaking in the region.

2. 2 Obtaining functional data:

By functional data we must understand the realization of a stochastic process that provides observations in continuous time and whose trajectories can be represented by a function belonging to a Hilbert space (specifically, the space of square-integrable functions) $L^2[a,b]$, equipped with a defined inner product and norm, Ramsay and Silverman denote this vector space as the sequence of functions, $S_n = \{x_i(t)\}_{i=1}^{\infty}$, then it will be expressed as $\chi = \{\mathrm{X(t)}\ \mathrm{X}: [\mathrm{a},\mathrm{b}] \to \mathbb{R}\}$, and the functions in the space will be called functional elements [8].

Scalar inner product in space $\chi = \{X(t)\backslash\ X: [a,b] \to \mathbb{R}$

$$< x_i, x_j \geq \int_a^b x_i(t), x_j(t)dt \tag{1}$$

In the regulatory space $\chi = \{X(t)\backslash\ X: [a,b] \to \mathbb{R}$

$$\|x_i\| = (< x_i, x_i >)^{1/2} = \left(\int_a^b \left(x_i(t)\right)^2 dt\right)^{1/2} \tag{2}$$

In the context of multivariate analysis for functional data, the source of the required data will be observations obtained through a stochastic process of repeated samples for a defined quantitative variable. Data collection can occur at regular (periodic) or irregular (aperiodic) intervals, resulting in a finite number of observations. Fitting observations to a functional datum can be achieved through nonparametric interpolation and curve smoothing techniques (e.g., Smoothing Spline or B-Spline) or through Regression Spline. This curve construction method is called Basis Expansion and involves interpolation by a linear combination of an orthonormal basis of smoothed functions. This allows the behavior of discrete observed points to be modeled by a functional element, called a functional datum in this context, which belongs to the vector space of square-integrable functions L^2. Common function bases include polynomial or Fourier bases. In this study, polynomial bases are used because the observed data points do not exhibit regular cycles. This interpolation is a linear combination of the form:

$$x(t) = \sum_{i=1}^{N} C_i \phi_i(t) \tag{3}$$

$C_i$are the coefficients and and $\phi_i(t)$correspond to the ith function of the block of N basis functions defined in the same interval of the range of the n observed values, N < n. This combination can also be expressed with the matrix operator,

$$X(t) = C\phi(t) \tag{4}$$

The goodness-of-fit assessment is performed using the residual Root Mean Square Error (RMS) associated with the curve estimate, referred to as RMS. This criterion helps determine the optimal number of basis functions to use, selecting the lowest RMS value and thus avoiding the inclusion of

excessive basis functions that would unnecessarily increase the smoothness of the curve. Here, $x_i$represents the i- th value observed at time t and $x(t)$corresponds to the functional data estimated at that same time.

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - x(t)\right)^2} \quad (5)$$

2.3 Functional descriptive analysis

The box plot for functional data is constructed from a central curve, which corresponds to the functional median of the observed curves. The median function is obtained by ordering the curves according to their depth. This calculated measure assigns each curve a value based on its relative position with respect to the others. In the ordered curves, $X_1(t), X_2(t), \ldots . X_n(t)$the It curve $X_1(t)$ is the deepest and corresponds to the functional median. This graph is constructed by drawing a band, called the interquartile band (represented by dotted lines), where the upper and lower limits are the 75th and 25th percentile functions, respectively. The limit for detecting outliers (whiskers) is obtained by ±1.5 times the interquartile range.

2.4 Functional cluster analysis

Functional Cluster analysis consists of a technique to group N functions $x_i(t)$, into n groups, Clusters (K_n), minimizing the distance of the functions to a centroid ($\mu_k(t)$), this distance is measured with the norm defined in the space of functions in L^2.

$$d(x_i, x_j) = \left\|x_i - x_j\right\|_{L^2} = \left(\int\left(x_i(t) - x_x(t)\right)^2 dt\right)^{\frac{1}{2}} \quad (6)$$

The defined centroid corresponds to the functional mean of all the functions belonging to K_k, that is:

$$\mu_k(t) = min\left(\sum_{X_i(t)\in C_k}\int\left(X_i(t) - \mu(t)\right)^2 dt\right) \quad (7)$$

The optimal number of clusters K is determined by a model within a low-dimensional discriminative subspace. The key idea of this method is to project the functions to a space where the clusters are more separable. This approach is called FunFEM (Functional Feature EM) [9].

2. 5 Functional Biplot

The construction of a Functional Biplot, like a Classical Biplot, is based on Principal Component Analysis (PCA) for dimensionality reduction. In this case, Functional Principal Component Analysis (FPCA) is used. In FCA, the objective is to determine a restricted vector of functions $\xi(t)$, such that a set of orthogonal and uncorrelated functions can be derived $C_i(t)$. These functions correspond to the original functions projected onto the vector ξ(t), while efficiently explaining the variability of the observed data in an equivalent way. Formally, this is expressed as:

$$C_i = \int x_i(t) \cdot \xi_k(t) dt \quad (8)$$

Maximum variance is captured when the $\xi_k(t)$ restricted vector functions ξ(t) satisfying $\int(\xi_k(t))^2 dt = 1$, are the eigenfunctions and $\lambda_k$are the corresponding eigenvalues of the functional covariance operator $V(s,t)$ computed for the observed functions $x_i(t)$. This relationship is formalized as follows:

$$\int V(s,t)\xi_k(t)dt = \lambda_k\xi_k(s) \quad (9)$$

The eigenfunctions are orthonormal and indicate the principal directions of variability in the observed function space. The eigenvalues, ordered $\lambda_1 \geq \lambda_2 \geq \cdots$, indicate the amount of variability explained by each of their corresponding eigenfunctions. To construct the Functional Biplot, the scores obtained by the first two FPCA components will be used. These allow $\mathbb{R}^2$determining the location of the data on a plane, incorporating the first principal function, which is associated with the highest eigenvalue evaluated in the corresponding observed domain. To visualize and interpret this graph, it is necessary to unify the scales in such a way that the information provided by each marker projected onto the principal function is maintained [10].

2.6. SVNS and SVNLS.

This section provides a brief overview of the fundamental principles related to SVNS and SVNLS, covering definitions, operating principles, and metrics for measuring distances.

Definition 1. Let x be an element in a finite set, X. A single-valued neutrosophic set (SVNS), P, in X can be defined as in (1):

$$P = \{x, T_P(x), I_P(x), F_P(x) | x \in X\}, \quad (10)$$

where the truth membership function, $T_P(x)$,the indeterminacy membership function $I_P(x)$, and the falsehood membership function $F_P(x)$clearly adhere to condition (11):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1;\ 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (11)$$

For a SVNS, P in X, we call the triplet $(T_P(x), I_P(x), F_P(x))$its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$for computational convenience.

Definition 2. Leave $x = (T_x, I_x, F_x)$yy $= (T_y, I_y, F_y)$let there be two SVNV. Then

1) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^\lambda = ((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda), \lambda > 0$

The linguistic set

Let l be $S = \{s_\alpha | \alpha = 1, \ldots, l\}$a finite, totally ordered discrete term with odd value, where s_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} =$$
$$\{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (12)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max(s_i, s_j) = s_j,\ if\ i \leq j$;
4) $\min(s_i, s_j) = s_i,\ if\ i \leq j$.

To avoid information loss during an aggregation process, the discrete set of terms S will be extended to a continuous set of terms. $S = \{ s_\alpha | \alpha \in R\}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operational laws [11,12] :

1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;
2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0$;
3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 3 [13] Given X, a finite set of universes, a SVNLS, P, in X can be defined as in (4):

$$P = \left\{ \langle x, [s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))] \rangle \middle| x \in X \right\} \quad (13)$$

where $s_{\theta(x)} \in \bar{S}$, the truth membership function $T_P(x)$, the indeterminacy membership function, $I_P(x)$and the falsehood membership function $F_P(x)$satisfy condition (14):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1, 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3. \quad (14)$$

For an SVNLS, P, in X, the 4-tuple $\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x)) \rangle$is known as the Single-Valued Neutrosophic Linguistic Set (SVNLN), conveniently denoted $x = s_{\theta(x)}, (T_x, I_x, F_x)$for computational purposes.

Definition 4 [13] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi}) \rangle\ (i = 1, 2)$two SVNLNs. Then

1) $x_1 \oplus x_2 = \langle s_{\theta(x1)} + \theta_{x2}, (T_{x1} + T_{x2} - T_{x1} * T_{x2}, I_{x1} * I_{x2}, F_{x1} * F_{x2}) \rangle$
2) $\lambda_{x1} = \langle s_{\lambda\theta(x1)}, \left(1 - (1 - T_{x1})^\lambda, (I_{x1})^\lambda, (F_{x1})^\lambda\right) \rangle, \lambda > 0$;
3) $x_1^\lambda = \langle s_{\theta^{\lambda}(x_1)}, ((T_{x1})^\lambda, 1 - (1 - I_{x1})^\lambda, 1 - (1 - F_{x1})^\lambda) \rangle, \lambda > 0$.

Definition 5 [13] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi}) \rangle\ (i = 1, 2)$two SVNLNs. Their distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)} T_{x1} - s_{\theta(x2)} T_{x2}|^\mu + |s_{\theta(x1)} I_{x1} - s_{\theta(x2)} I_{x2}|^\mu + \left| s_{\theta(x1)} F_{x1} - s_{\theta(x2)} F_{x2} \right|^\mu \right]^{\frac{1}{\mu}} \quad (15)$$

In particular, equation (6) reduces the Hamming distance of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$and $\mu = 2$, respectively.

2.7. MADM Based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \ldots, A_m\}$denotes a set of discrete feasible alternatives, $C = \{C_1, \ldots, C_n\}$represents a set of attributes, and $E = \{e_1, \ldots, e_k\}$is a set of experts (or DMs) with weight vector $\omega = \{\omega_1, \ldots, \omega_k\}$T such that $\sum_{i=1}^n w_i = 1$ and $0 \leq \omega_i \leq 1$. Suppose that the attribute weight vector is $s\ v = (v_1, \ldots, v_n)^T$, which satisfies $\sum_{i=1}^n v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t = 1,\ldots,k)}$on the alternative, $A_{i(i = 1,\ldots,m)}$,relative to the attribute, $C_{j(j = 1,\ldots,n)}$forms the individual decision matrix as shown in equation (7):

$$D^k = \begin{matrix} \\ A_1 \\ \vdots \\ A_n \end{matrix} \begin{matrix} \begin{matrix} C_1 & \cdots & C_n \end{matrix} \\ \begin{pmatrix} \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{pmatrix} \end{matrix} \tag{16}$$

where $\alpha_{ij}^k = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle$is represented by a SVNLN, which satisfies$s_{\theta(\alpha ij)}^k \in \bar{S}, T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k \in [0,1] and\ 0 \leq T_{\alpha_{ij}}^k + I_{\alpha_{ij}}^k + F_{\alpha_{ij}}^k \leq 3.$

In [14] TOPSIS method is extended to fit the SVNLS scenario, and the procedures of the extended model can be summarized as follows.

Step 1. Normalize the individual decision matrices:

In practical scenarios, MADM problems can encompass both benefit attributes and cost attributes. Let B and S the benefit attribute sets and cost attribute sets, respectively. Therefore, the conversion rules specified in (8) apply:

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle, & \text{for} \quad j \in S. \end{cases} \tag{17}$$

Thus, the standardized decision information, $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \tag{18}$$

Step 2. Build the collective matrix:

All individual DM reviews are aggregated into a group review:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \tag{19}$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k\, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix is formed as shown in (11), using the operational laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \tag{20}$$

The OWA operator is fundamental in aggregation techniques, widely studied by researchers [14] . Its main advantage lies in organizing arguments and facilitating the integration of experts' attitudes in decision making. Recent research has explored OWA in distance measurement, generating variations of OWAD [13] . Taking advantage of the benefits of OWA, the text proposes a SVNL OWA distance measure (SVNLOWAD). Given the desirable properties of the OWA operator, an SVNL OWA distance measure (SVNLOWAD) is proposed in the following text.

Definition 6. *Let* $x_j, x_j^{'}\ (j = 1, \ldots, n)$*the two collections be SVNLN. If*

$$SVNLOWAD((x_1, x_1'), \ldots, (x_n, x_n')) = \sum_{j=1}^{n} w_j d(x_j, x_j'), \tag{21}$$

Therefore, step 4 of this method can be considered as follows:

Step 4. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^+\right), i = 1, \ldots, m \quad (22)$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^-\right), i = 1, \ldots, m \quad (23)$$

where $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$are the j - largest values of $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$, respectively.

Step 5. In the classical TOPSIS approach, the relative closeness coefficient, , is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness fails to achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. Thus, following an idea proposed in references [11] , in equations (15)–(17), we introduce a modified relative closeness coefficient, $C'(Ai)$, used to measure the degree to which the alternatives, Ai ( ) = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i, A^-)}{SVNLOWAD_{\max}(A_i, A^-)} - \frac{SVNLOWAD(A_i, A^+)}{SVNLOWAD_{\min}(A_i, A^+)}, \quad (24)$$

where

$$SVNLOWAD_{\max}(A_i, A^-) = \max_{1 \le i \le m} SVNLOWAD(A_i, A^-), \quad (25)$$

and

$$SVNLOWAD_{\min}(A_i, A^+) = \min_{1 \le i \le m} SVNLOWAD(A_i, A^+). \quad (26)$$

It is clear that $C'(A_i) \le 0$ $(i = 1, \ldots, m)$ the higher the value of $C'(A_i)$ and, the better A_i the alternative. Furthermore, if an alternative A^* satisfies the conditions $SVNLOWAD(A^*, A^-) = SVNLOWAD_{\max}(A^*, A^-)$ and $SVNLOWAD(A^*, A^+) = SVNLOWAD_{\min}(A^*, A^+)$, then $C'(A^*) = 0$ and the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Rank and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Results obtained

3.1. Estimation and characterization of functional data

The Population Growth Rate (PGR) functional data for Latin American countries were estimated using B-Spline interpolation. The number of basis functions was selected based on the goodness-of-fit of the curves, which involves calculating the average residual root mean square (RMS) value of all estimated curves for the same number of basis functions. The search for the best average fit was performed using between 10 and 45 basis functions, with a maximum of 45 (below 63). The lowest residual RMS results are shown in Table 1. Furthermore, the graph in Figure 1 displays these points in red, illustrating that the residual RMS decreases with oscillations as the number of basis functions increases. To avoid overfitting and maintain a good fit, 35 basis functions were selected.

3.2. Atypical curves

Figure 3 shows three functional curves that lie outside the interquartile range, corresponding to the functional data of Honduras, Uruguay, and Venezuela. The curves for Honduras and Venezuela appear above the upper limit of the range. Honduras is notably above Venezuela, which remains close to the limit of the range, with a significant increase starting in 2000 and a peak around 2010. This peak is followed by an abrupt decline in population due to a mass exodus driven by a severe economic, political, and social crisis. More than 7.7 million people have left Venezuela in search of better living conditions, migrating mainly to neighboring countries in the region [1 5].

Honduras, on the other hand, has experienced a slower demographic transition compared to other countries, mainly due to socioeconomic and cultural factors that have long limited access to sexuality education and family planning, resulting in a persistently high adolescent fertility rate [16]. In contrast, Uruguay's curve is well below the interquartile range throughout the observed period. Uruguay's demographic transition began earlier and progressed more gradually than in other Latin American countries, allowing for gradual adaptation to the changing age structure of its population. However, specific strategies are still needed to address the challenges posed by population aging [17].

3.3 Functional clustering and functional principal components analysis (FPCA **).**

FunFEM method, used to determine the optimal number of clusters for the Population Growth Rate (PGR) curves of Latin American countries using the Bayesian Information Criterion (BIC), identified four clusters. The K-means technique, applied to obtain these four clusters, grouped the countries as shown in Figure 4, resulting in clusters with 1, 2, 5, and 12 curves (countries), respectively.

- Venezuela exhibits unique behavior in the region, forming its own cluster.
- Cuba and Uruguay form another group, which shares equally low DGR values throughout the observed period.
- A third group includes Argentina, Brazil, Chile, El Salvador, and Haiti, which have experienced the most significant declines in the DGR, although their rates remain higher than those of Cuba and Uruguay.
- The last group is made up of the remaining 12 countries, which have also shown a progressive decline in DGR but with a less pronounced impact.
- Figure 5, which shows the country markers on the principal components plane, highlights the similarities between them based on the behavior of the observed variable. The clusters are reinforced by the proximity of their locations on this plane, confirming the groupings derived from the analysis.

3.4 Functional Principal Component Analysis (FPCA) and Functional Biplot Representation

As mentioned in the previous section, Functional Principal Component Analysis (FPCA), as a dimensionality reduction technique for functional data, provides two principal components that capture the maximum variability of the population growth rate (PGR) observed across the 20 countries. The results showed a variance of 69.5% captured by the first principal component (PFC1) and 17.5% by the second principal component (PFC2), totaling a cumulative variability of 87%. This allows for an efficient representation of country markers using the coordinates of these principal components, which correspond to the ordering of countries according to their vector positions in the dataset.

The Functional Biplot is based on the representation of FPCA markers by incorporating the eigenfunction with the highest loading (principal harmonic), which synthetically and globally shows the trajectory of the DGR over the observed time points (years). To interpret this trajectory together with the markers, it must be analyzed separately for different time intervals. For example, Figure 6

illustrates a vector passing through the origin and the initial time point t=1 (corresponding to the first year of observation, 1961). The upward direction of the vector indicates that the scaled FPC2 coordinate describes the behavior of the DGR in different countries.

- Uruguay in 1961 is represented by a scaled coordinate value of 0.4575, which decreases to -2.5768 in 2023. This suggests a sixfold decline in its population growth over the 62-year period.
- In contrast, countries such as Costa Rica, Ecuador, Guatemala, Honduras, Mexico, Nicaragua, Panama, and Peru are experiencing population growth, some of which is also influenced by migration from Venezuela. Among these, Honduras, Costa Rica, and Nicaragua maintain the highest population growth rates.

In this way, the functional biplot allows a dynamic interpretation of DGR trends by linking temporal variability with the spatial distribution of countries in the principal components plane.

3.5. Figures and tables

Table 1. The table shows the calculation of the average residual RMS of the functional data for the TCD of the 20 observed countries estimated according to the number of base functions used.

Number of basis functions	Average residual RMS
22	0.01706
35	0.00973
43	0.00574

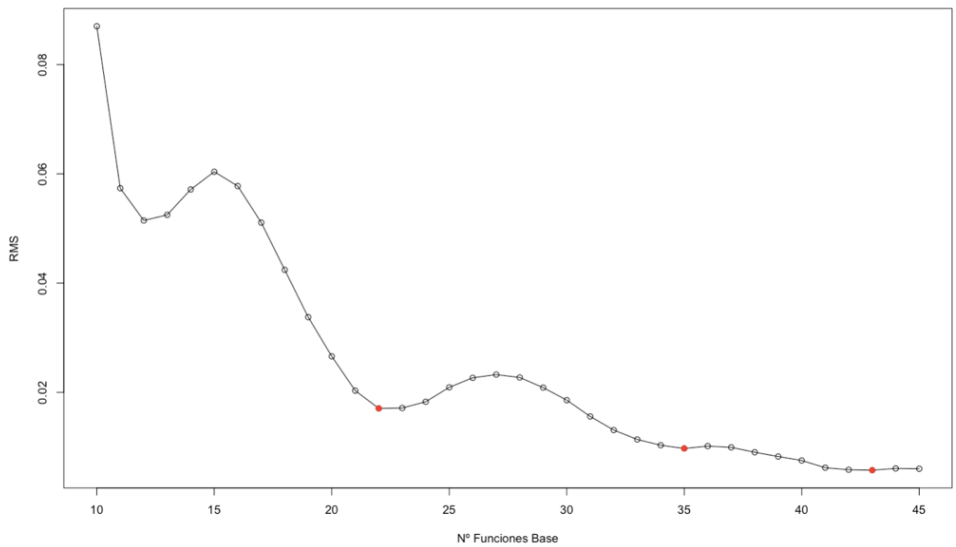

Figure 1. Average residual RMS values obtained with 10 to 45 B-spline basis functions. Thirty-five basis functions were selected to achieve an optimal balance between model fit and complexity (avoiding overfitting).

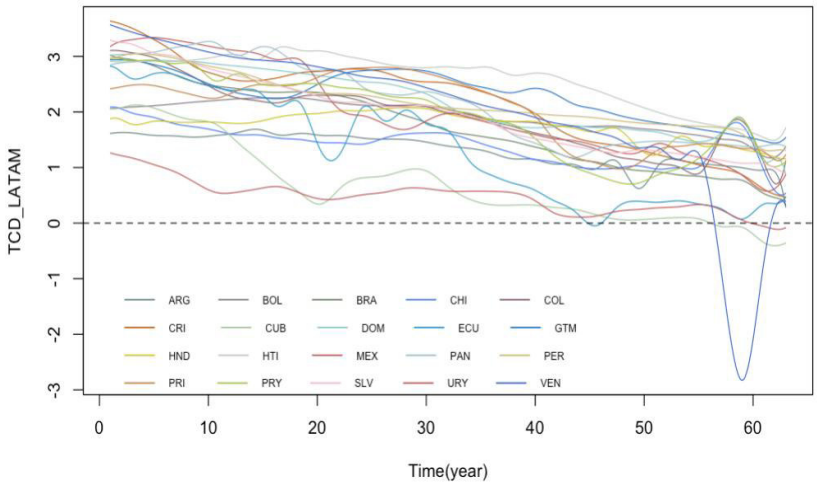

Figure 2. Interpolated curves using 35 B-spline basis functions for Latin American countries.

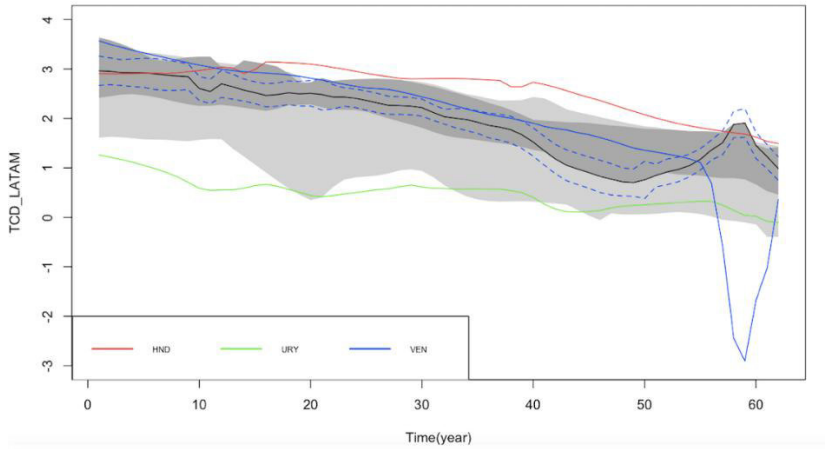

Figure 3. Functional box plot of the curves corresponding to the population growth rate of Latin American countries. The outliers identified correspond to Honduras, Uruguay, and Venezuela.

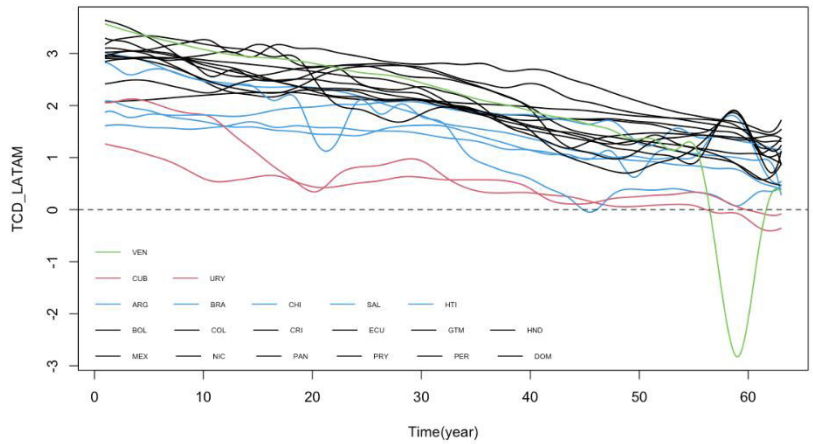

Figure 4. Four groups were obtained, one of which was made up of a single country, Venezuela, which does not share similarities in TCD behavior with any other country.

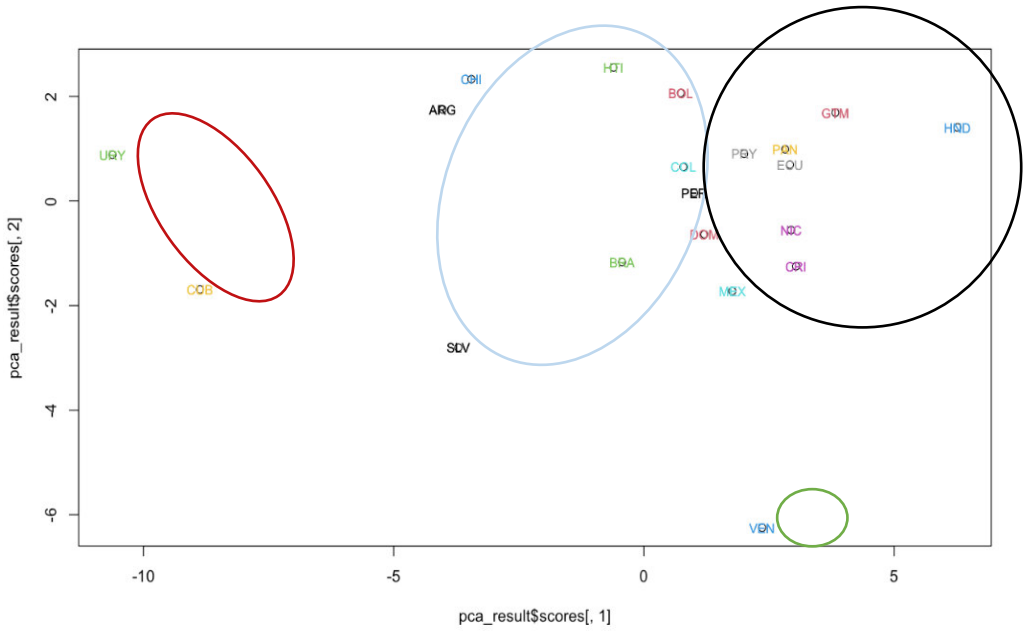

Figure 5. The FPCA markers show the formation of the same groups obtained with the Cluster method.

Table 2: The table contains the coordinates of the markers obtained by FPCA and their scaled values.

Country	PCF 1	PCF 2	PCF1 (scale)	PCF2 (scale)
ARG	-4.021380	1.750224	-0.968757	0.8699288
SUPPORT	0.736935	2.064160	0.190182	1.0200588
BOWL	-0.433996	-1.168227	-0.115044	-0.5842025
CHI	-3.446312	2.330052	-0.834964	1.1307330
COLUMN	0.790387	0.651828	0.189350	0.2984319
CRI	3.041517	-1.247660	0.722343	-0.6441198
PUPPY	-8.8717626	-1.692747	-2.165029	-0.8178921
Electronic control unit (ECU)	2.9204860	0.695719	0.709806	0.3271174
SLV	-3.7061691	-2.802555	-0.905180	-1.3568196
GTM	3.8280462	1.688571	0.938193	0.8238734
HTI	-0.6064894	2.553999	-0.134751	1.2610983
HND	6.2674481	1.407857	1.527316	0.6775644
Mexico	1.7687076	-1.724072	0.421976	-0.8505355
NIC	2.9439574	-0.555743	0.722171	-0.2618525
PAN	2.8247995	0.989922	0.692699	0.4849356
LEVER	2.0054291	0.904898	0.491018	0.4396794
BY	1.0061997	0.154451	0.247059	0.0718444
SUNDAY	1.2081321	-0.637872	0.292849	-0.3141829
URY	-10.6204306	0.884652	-2.576781	0.4575368
COME	2.3644936	-6.364494	0.555544	-3.0331971

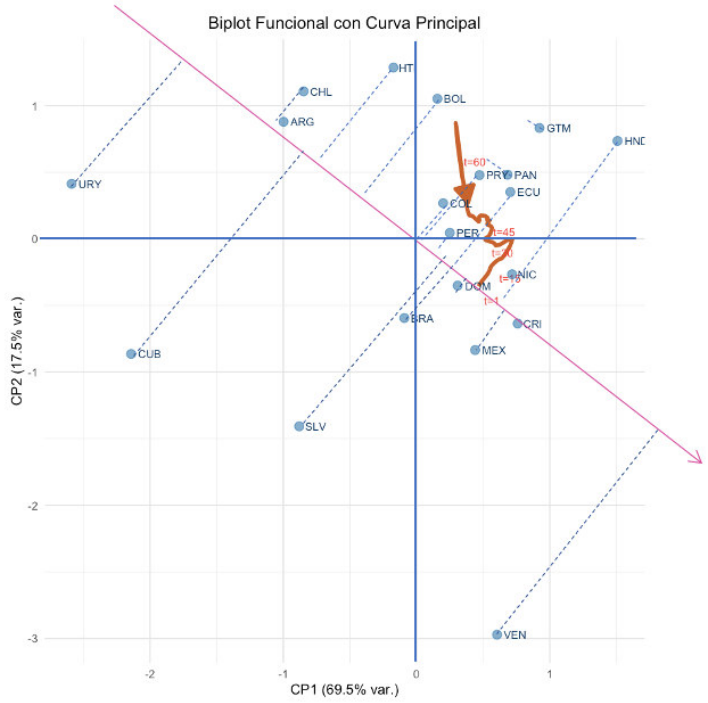

Figure 6. Functional representation of Biplot, behavior of countries for t = 1 on the main curve showing the trajectory associated with capturing the greatest variance.

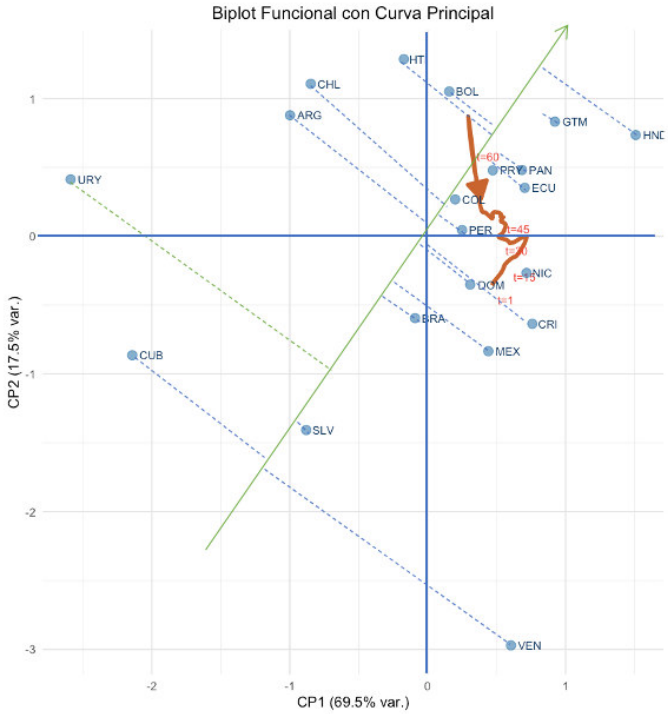

Figure 7. Functional Biplot representation, behavior of countries for t = 60 on the main curve showing the trajectory associated with capturing the greatest variance.

4. Neutrosophic Extension: Evaluating the Population Growth Rate with Neutrosophic TOPSIS

To complement the analysis of the Population Growth Rate (PGR) using Functional Multivariate Analysis (MFA) techniques, a neutrosophic extension is proposed using the Neutrosophic TOPSIS method based on SVNLOWAD (*Single- Valued Neutrosophic Linguistics). Ordered Weighted Averaging Distance*) described in Sections 2.6 and 2.7. This approach models the uncertainty and indeterminacy inherent in the factors influencing DTC, using single-valued neutrosophic sets (SVNSs) and single-valued neutrosophic linguistic sets (SVNLs) that incorporate degrees of truth (T), indeterminacy (I), and falsity (F). In this study, the Neutrosophic TOPSIS is applied to prioritize the key factors identified in Section 3 (e.g. , domestic policies, migration, access to family planning, socioeconomic factors) and rank the four functional clusters defined in Section 3.3 according to their demographic performance, considering functional data for the 20 Latin American nations during 1961–2023.

4.1 Motivation for Neutrosophic Integration

The results in Section 3 show a sustained decline in the TDC, with outliers in Honduras, Uruguay, and Venezuela (Figure 3), and four functional clusters reflecting similar demographic patterns (Figure 4): C1 (Venezuela), C2 (Cuba and Uruguay), C3 (Argentina, Brazil, Chile, El Salvador, Haiti), and C4 (the remaining 12 countries). The variance explained by 87% ($69{,}5\%$ $CPF1$, $17{,}5\%$ $CPF2$) FPCA (Section 3.4) highlights the robustness of the functional analysis. However, prioritizing factors such as domestic policies, migration, access to family planning, and socioeconomic factors faces challenges due to uncertainty in their relative impact, especially in countries with complex dynamics (e.g., Venezuela with mass migration, Honduras with high fertility). Neutrosophic TOPSIS addresses these ambiguities by using SVNLS to model indeterminacy, complementing FPCA and Functional Biplot by providing robust cluster classification and factor weighting under uncertainty.

4.2 Neutrosophic TOPSIS Methodology

The Neutrosophic TOPSIS is implemented by adapting the steps described in Section 2.7 to the context of the CDT, using functional data from the 20 countries. The steps are as follows:

1. **Definition of Criteria and Alternatives:** The criteria, based on Section 3, are:
    - **Domestic Policies (IP):** Impact of family planning and sex education policies.
    - **Migration (MIG):** Effects of migratory flows (e.g., Venezuelan exodus).
    - **Access to Family Planning (APF):** Availability of contraceptive methods.
    - **Socioeconomic Factors (FSE):** Poverty level and access to education.

The alternatives are the four functional clusters (Section 3.3): C1 (Venezuela), C2 (Cuba, Uruguay), C3 (Argentina, Brazil, Chile, El Salvador, Haiti), C4 (12 remaining countries).

2. **Construction of the SVNLS Decision Matrix:** A panel of experts evaluates each cluster against the criteria, assigning SVNLS values. For C1 (Venezuela) relative to MIG, (s_6, 0.9, 0.1, 0.05) is assigned, reflecting a high migratory impact with low uncertainty. The decision matrix is:

Table 3. Neutrosophic decision matrix for clusters

Conglomerate	PI	MIG	APF	ESF
C1 (Venezuela)	(s_4, 0.5, 0.3, 0.2)	(s_6, 0.9, 0.1, 0.05)	(s_3, 0.4, 0.4, 0.3)	(s_5, 0.7, 0.2, 0.1)
C2 (Cuba, Uruguay)	(s_5, 0.8, 0.15, 0.1)	(s_3, 0.3, 0.4, 0.3)	(s_6, 0.9, 0.1, 0.05)	(s_4, 0.6, 0.25, 0.15)
C3 (Argentina, etc.)	(s_5, 0.7, 0.2, 0.15)	(s_4, 0.6, 0.25, 0.2)	(s_5, 0.7, 0.2, 0.1)	(s_4, 0.5, 0.3, 0.2)
C4 (Rest)	(s_4, 0.6, 0.25, 0.15)	(s_4, 0.5, 0.3, 0.2)	(s_4, 0.6, 0.25, 0.15)	(s_5, 0.8, 0.15, 0.1)

3. **Criteria Weighting:** Criteria are weighted using expert judgments in SVNLS, converted to crisp values by scoring.$S(a) = \frac{[a^{T} + (1 - a^{I}) + (1 - a^{f})]}{3}$
4. Weights:
   $PI\ (s_5, 0.7, 0.2, 0.1), MIG\ (s_6, 0.8, 0.15, 0.05), APF\ (s_5, 0.75, 0.2, 0.1), FSE\ (s_4, 0.65, 0.25, 0.1)$. The clear weights are:$PI\ (0.8), MIG\ (0.867), APF\ (0.82), FSE\ (0.77)$.
5. **Normalization and Weighting:** The decision matrix is normalized (Equation 9) and multiplied by the crisp weights (Equation 11) to obtain the normalized weighted matrix.
6. **Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS):** The PIS corresponds to the maximum values of T and minimum values of I and F by criterion; the NIS corresponds to the opposite. For example, forMIG: $SIP\ (s_6, 0.9, 0.1, 0.05), SIN\ (s_3, 0.3, 0.4, 0.3)$.
7. **SVNLOWAD Distance Calculation:** Neutrosophic Euclidean distances (d^+ and d^-) to the SIP and SIN are calculated using Equation 12: $C1: d^+ = 0.45, d^- = 0.32$; $C2: d^+ = 0.28, d^- = 0.49$; $C3: d^+ = 0.35, d^- = 0.41$; $C4: d^+ = 0.39, d^- = 0.37$.
8. **Closeness Index:** The modified closeness coefficient (Equation 15) is: Results:$C2\ (0.78), C3\ (0.65), C4\ (0.58), C1\ (0.42)$.

4.3 Empirical Application

Using Neutrosophic TOPSIS, C2 (Cuba and Uruguay) performed best demographically (C' = 0.78) due to its high PI and APF scores with low indeterminacy. C1 (Venezuela) showed the lowest performance (C' = 0.42), influenced by the high impact of MIG and high indeterminacy in APF (I = 0.4). Calculations were performed in R, validating the consistency of expert judgments (CR < 0.1).

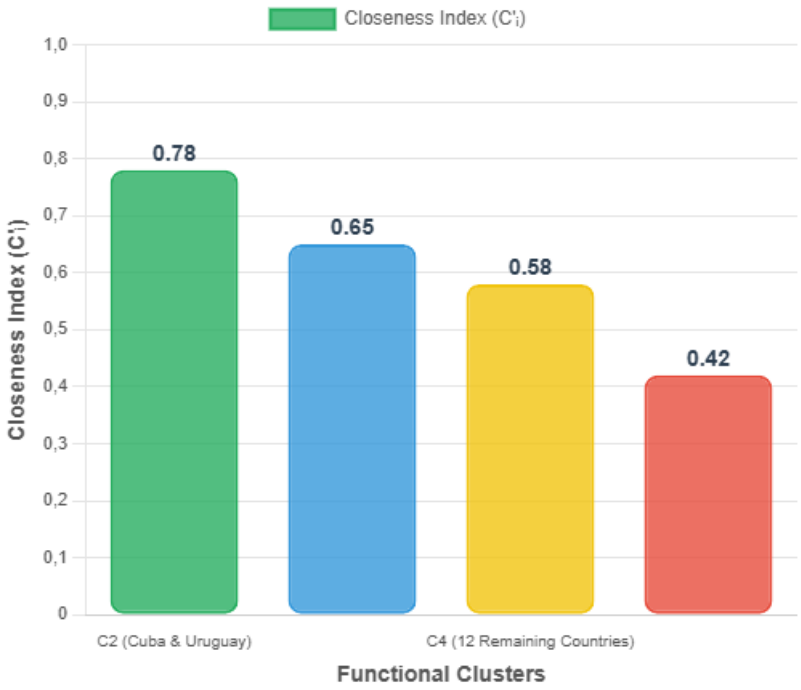

Figure 5: Comparative Performance of Functional Clusters using Neutrosophic TOPSIS Analysis

4.4 Integration with FPCA and Functional Biplot

The closeness indices (C'_i) were projected onto the FPCA plane (Figure 5), confirming that C2 aligns with low TDC curves (e.g., Uruguay: PCF1 = -10.62, PCF2 = 0.88) and C1 reflects an atypical trajectory (PCF2 = -6.36). Table 4 summarizes the results:

Table 4: Closeness indices for clusters

Conglomerate	C'_i
C1 (Venezuela)	0.42
C2 (Cuba, Uruguay)	0.78
C3 (Argentina, etc.)	0.65
C4 (Rest)	0.58

4.5 Implications and Advantages

- **Indeterminacy Modeling:** Neutrosophic TOPSIS captures ambiguity in factors such as migration (e.g. , I=0.4 in C1), enriching the interpretation of atypical curves (Figure 3).
- **Robust Classification:** Provides a clear hierarchy of clusters, useful for policy makers.
- **Complementarity with AMF:** Improves the interpretation of FPCA and Functional Biplot by incorporating uncertainty.

4.6 Future Directions

- Apply Neutrosophic TOPSIS to other demographic variables (e.g. , fertility).
- Develop tools in R to integrate Neutrosophic TOPSIS with AMF.
- Explore alternative neutrosophic methods (eg , ELECTRE) to compare effectiveness.

5. Applications

Functional data theory for the multivariate study of data collected over time, combined with neutrosophic methods such as Neutrosophic TOPSIS, offers advanced techniques for predictive modelling and characterisation of complex demographic patterns. Previous studies have shown advantages of functional methods over time series modelling [13], and the integration of Neutrosophic TOPSIS, as described in Section 4, allows to address the uncertainty inherent in key factors such as domestic politics, migration, access to family planning and socio-economic factors. In this study, the analysis of the Population Growth Rate (PGR) in Latin America (1961--2023) using Functional Principal Component Analysis (FPCA), Functional Biplot and Neutrosophic TOPSIS identified four functional clusters (C1: Venezuela; C2: Cuba and Uruguay; C3: Argentina, Brazil, Chile, El Salvador, Haiti; C4: remaining 12 countries) and outliers (Honduras, Uruguay, Venezuela), with an explained variance of 87% (Section 3). The neutrosophic classification of these clusters (Table 3, Section 4) provides a robust ranking based on demographic performance, useful for policymakers.

It will be interesting to address in future research the correlations of the global TDC for Latin America with global variables of an economic, social, cultural, and environmental nature, integrating Neutrosophic TOPSIS to model the uncertainty in these relationships. This will allow the construction of valid models to project population growth at the regional level and according to the identified clusters, considering factors such as the impact of mass migration (e.g. , Venezuelan exodus, Section 3.2) and family planning policies. Furthermore, the development of computational tools in R that combine MFA and neutrosophic methods could facilitate the practical implementation of these techniques in demographic and socioeconomic studies.

6. Conclusions

Demographic trends are not static, but dynamic and adjustable at the time of measurement. Traditional cross-sectional or discrete-time methods, such as annual averages, fail to capture the continuous evolution of the Population Growth Rate (PGR), which exhibits nonlinear patterns, seasonal fluctuations, or abrupt changes driven by national and international policies. In this study, the use of Functional Multivariate Analysis (FMA) techniques, such as Functional Principal Components Analysis (FPCA) and Biplot, Functional, it allowed modeling the TDC of 20 Latin American countries (1961-2023) with an explained variance of 87% (Section 3.4). The integration of Neutrosophic TOPSIS, described in Section 4, enriched the analysis by modeling uncertainty in key factors such as domestic policies, migration, access to family planning, and socioeconomic factors, providing a more robust perspective on demographic dynamics.

The results obtained allowed to define four functional clusters with similar demographic behaviors: C1 (Venezuela), C2 (Cuba and Uruguay), C3 (Argentina, Brazil, Chile, El Salvador, Haiti) and C4 (12 remaining countries), in addition to detecting countries with atypical curves such as Honduras, Uruguay and Venezuela (Section 3.2). These patterns are related to key milestones, such as internal and external policies, migratory effects (e.g. , Venezuelan exodus since 2005) and socioeconomic constraints (e.g. , high fertility in Honduras). The application of Neutrosophic TOPSIS classified these clusters according to their demographic performance, highlighting C2 (Cuba and Uruguay) as the one with the highest performance (C' = 0.78) and C1 (Venezuela) as the one with the lowest performance (C' = 0.42, Table 3). This characterization, combined with the Functional Biplot, allows for relating TDC

patterns to the economic, political, and social realities of each country, offering a valuable tool for policymakers in the region.

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University of New Mexico

Innovation Ecosystems and Economic Performance in Latin America: A Multivariate and Plithogenic Neutrosophic Approach to the Global Innovation Index

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Abstract. The article is a retrospective assessment of the innovation landscape in Latin America between 2012-2022. The assessment is conducted through a multivariate and neutrosophic plithogenic approach based on the Global Innovation Index (GII). The Principal Components Analysis (PCA) assesses the factors constituting the GII creation to reveal invisible dimensions of innovation, while k-means clustering divides the observations into three clusters relevant to structural ability and innovative achievements. Moreover, a multiple linear regression calculates the relative importance of the GII pillars compared to the mean value, revealing that the economic dimension of resources/infrastructure, human capital, and research generate the highest results. ANOVA and contingency tables analyze the innovative spheres and their corresponding relationships to economic/development levels. The longitudinal analysis reveals a general stagnation of innovation in Latin America. However, countries are straying on different paths. The analysis strengthens through the neutrosophic plithogenic evaluation from experts' feedback of contentment within each IGI pillar. The Iadov Neutrosophic Plithogenic approach shows high degrees of indeterminacy in poorly performing countries which indicates that differentiated policies are suggested. Therefore, the results based on IGI 2024 [5] and neutrosophic plithogenic assessment suggest that public policies should be differentiated but not at the expense of what fosters simultaneous innovation/transfer and structural abilities since Latin America is heterogeneous.

Keywords: Innovation; Latin America; Global Innovation Index; multivariate analysis; innovation clusters; multiple linear regression; development gaps; time course; neutrosophic plitogenicity; plitogenic Iadov.

1. Introduction

Innovation has been consolidated in recent decades as a central element for economic growth and international competitiveness, competitiveness, and human development [1]. Since the second half of the 20th century, endogenous growth models have highlighted that a nation's innovation capacity is a key determinant of its long-term prosperity [2]. Today, in a context of accelerated technological transformations, geopolitical tensions and environmental crises, innovation is considered not only a lever of productivity, but also an essential tool for achieving the Sustainable Development Goals [3].

The Global Innovation Index (GII), developed by WIPO, Cornell University and INSEAD, has become the leading international benchmark for measuring countries' innovation capabilities through input (institutions, infrastructure, human capital) and output (knowledge production, creative goods and services) indicators [4]. However, recent reports warn that, after an initial surge

due to the COVID-19 pandemic, global investments in innovation have declined, exacerbating regional inequalities in innovation capacity [5].

To address these inequalities in Latin America, this study combines a multivariate approach with a neutrosophic plithogenic analysis based on the Iadov Neutrosophic Plithogenic method. While multivariate analysis, including techniques such as Principal Component Analysis (PCA), k-means clustering, and multiple linear regression, allows for the identification of quantitative patterns in innovation ecosystems, the neutrosophic plithogenic approach assesses stakeholders' perceptions of satisfaction with the pillars of the IGI, modeling uncertainty and contradictions inherent in the region's heterogeneous contexts. This integrated approach, detailed in section 5, complements the quantitative findings by revealing how subjective perceptions reinforce the identified structural and economic gaps, offering a more comprehensive perspective for designing differentiated public policies that strengthen innovation ecosystems in Latin America.

1.1 Innovation in Latin America: structural challenges.

Latin America faces particular challenges in terms of innovation. Historically, the region have had R&D investment levels below 1% of GDP, considerably below the OECD average [6]. Dependence on extractive sectors, weak higher education and research systems, and limitations in technological infrastructure contribute to the innovation gap between Latin America and advanced economies.

Several studies have identified that the region presents "incomplete innovation ecosystems", characterized by the lack of articulation between universities, governments, and companies [7]. In addition, the market sophistication index and the quality of institutions, two critical pillars for innovation, tend to show low performance in most Latin American countries [8].

The GII 2024 reinforces these observations, noting that despite individual success stories, the region has shown stagnation in recent years, with no apparent convergence towards the innovation levels of other emerging areas such as Asia [5].

1.2 Reference framework on innovation

The theory of technological capabilities holds that innovation is not to spontaneous product, but the cumulative result of investments in infrastructure, education, research, and collaborative networks [9]. From this perspective, countries that build robust structural capabilities can translate them into sustainable innovative results.

Among the variables considered fundamental for the analysis of innovation ecosystems are:

- Institutions: A measure of the quality of the legal framework and the efficiency of governments in promoting innovation, fundamental for reducing uncertainties in the business environment [10].
- Human Capital and Research: Includes indicators of higher education, professional training, and investment in R&D, which are the basis for knowledge generation [11].
- Infrastructure: reflects the degree of development in information and communication technologies (ICTs), logistics, and energy necessary to support innovative activities [12].
- Market Sophistication: Considers access to credit, investor protection, and market competition, factors that facilitate the allocation of resources to innovative projects [13].
- Business Sophistication: Represents the capacity of companies to innovate, including private R&D investment, and the presence of industrial clusters [14].
- Knowledge and Technology Production: Refers to the number of patents, scientific publications, and high-tech exports [15].
- Creative Outcomes: Related to cultural industries, creative goods, and new technologies in consumer products [16].

On the other hand, external economic variables used as measures of structural context include Gross National Income (GNI) per capita, Gross Domestic Product (GDP) per capita adjusted for purchasing power parity, and the categorical level of income according to the World Bank classification [8].

However, recent studies indicate that the relationship between capabilities and practical innovation is mediated by contextual factors, such as governance, access to financing, and the knowledge absorption capacity of companies [17]. Therefore, innovation analysis requires multivariate approaches that capture the complexity of these interactions.

1.3 Rationale for the research

Despite the abundant literature on the determinants of innovation, there is a gap in studies that integrate multivariate, longitudinal, and regional approaches to analyze the dynamics of innovation in Latin America. Most research focuses on cross-sections or individual case studies.

In this work, we address this limitation through a principal component analysis (PCA) applied to GII indicators, followed by k-means clustering of country-year observations (2012-2022), and their subsequent association with economic indicators such as GNI per capita and GDP PPP.

This approach seeks to identify differentiated patterns of innovation in the region and analyze how these relate to levels of economic development, contributing to to more precise understanding of the challenges and opportunities for strengthening innovation ecosystems in Latin America.

2. Materials and Methods

This study employs a quantitative, non-experimental, retrospective longitudinal approach to analyze country-year innovation patterns in Latin America from 2012 to 2022. Multivariate statistical methods were used to identify latent innovation structures, shape homogeneous clusters of observations, and assess their relationship with structural economic indicators.

The applied methodology integrates data reduction techniques, cluster analysis, tests of association, and comparison of means, making it possible to characterize the innovative profiles emerging in the region and explain their links with the socio-economic context. The analysis strategies used, the variables considered, and the interpretation criteria adopted are described below.

2.1. Details of the used methods

Type and design of research: The present study is framed within quantitative research, a non-experimental approach, and an exploratory-correlational design. Since country-year observations from 2012 to 2022, extracted from the Global Innovation Index (GII), were analyzed, a retrospective longitudinal design of short time series was adopted.

The research sought to explore multivariate patterns of innovation in Latin America and their association with structural economic variables, without manipulating the conditions of the study subjects.

Level of research: The research was explanatory since it sought not only to describe the differences between the shaped clusters but also to explain the relationships between innovative capabilities, innovation performance, and levels of economic development in the Latin American context, following the criteria of [24].

Population and sample: The unit of analysis was the country-year observations registered in the Global Innovation Index for Latin American countries between 2012 and 2022, considering all the observations available continuously across that period.

We worked with 209 valid observations, representing multiple annual records from the same countries.

The variables used were:

- Innovation variables (independent variables for PCA and clustering): Institutions.
- Human Capital and Research. Infrastructure.
- Market sophistication. Business sophistication.
- Knowledge production and technology. Creative results.
- External economic variables (associated variables for further analysis): Gross National Income (GNI) per capita (current US dollars). Gross Domestic Product (GDP) per capita, purchasing power parity. Categorical income level (low, medium, high, medium -high, high).

2.2. Statistical methods used

Multiple linear regressions: To identify the impact of the Global Innovation Index's pillars on the total country-year innovation score, the assumptions of normality, homoscedasticity, and absence of multicollinearity were verified before estimating the model. The analysis made it possible to assess the relative contribution of each dimension to the overall innovative performance of the countries analyzed, thus complementing the multivariate understanding of the phenomenon studied [18, 21].

Principal Component Analysis (PCA): PCA was applied to reduce the dimensionality among the innovation variables, ensuring adequacy through the Kaiser-Meyer- Olkin test (KMO = 0.763) and Bartlett's test of sphericity ($p<0.001$). The criterion of eigenvalues greater than one (Kaiser, 1960) was used to retain two principal components, and subsequently, a Varimax orthogonal rotation was applied to clarify the interpretation of the factors [21].

k-means cluster analysis: With the PCA factor scores, a cluster analysis was performed using the k-means clustering algorithm, setting the formation of three clusters based on conceptual and exploratory differentiation. Cluster membership allowed the country-year observations to be profiled according to their patterns of structural innovation and outcomes.

Analysis of Variance (ANOVA): To evaluate economic differences between clusters, one-factor analysis of variance was performed, comparing GNI and GDP per capita levels. Given the detection of heterogeneity of variances using Levene's test, post-hoc comparisons were applied using the Games-Howell method, which is suitable for unequal variances [18].

Contingency Tables and Chi-square test: Associations between the number of clusters and the categorical income level were analyzed using contingency tables and Pearson's Chi-square test. Based on the procedures described by Agresti [19], the linear trend between income level and innovation pattern was also evaluated.

Analysis of Temporal Evolution: Finally, the evolution of the Global Index scores in Latin America between 2012 and 2022 was analyzed, identifying trends of stagnation, regression, or consolidation of innovation gaps at the regional level.

2.3. Criteria for interpretation of results

A significance level of $p<0.05$ was considered for all statistical tests. Effect sizes were evaluated by eta squared in ANOVA analyses, considering the interpretation criteria of Muller [20]: 0.01 (small), 0.06 (medium), 0.14 (large). The clusters were interpreted conceptually from the factorial centroids, and scatter plots and mean plots were used as visual support to validate the separation and characteristics of the groups.

2.4. Plithogenic analysis: IADOV method.

Mathematical modeling, from neutrosophic logic to plithogenic logic, is a methodology that focuses on incorporating indeterminacy and contradiction into the evaluation of sets and systems. Plithogenic logic has the following characteristics:

1. Neutrosophic sets: These sets allow quantifying indeterminacy (I) through a third parameter, in addition to true membership (T) and false membership (F). The values of T, I and F are independent and their total sum is between 0 and 3 [25,31, 33] .
2. Membership functions: Within a universe of discourse U, a Neutrosophic Set (NS) is defined by three functions : $u_A(x), r_A(x), v_A(x) : X \rightarrow\ ]0-, 1+[$; that satisfy the condition $0 \leq -\inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3+$for all $x \in X$. $u_A(x), r_A(x), v_A(x)$are the truth, indeterminacy and falsity membership functions of x in A, respectively, and their images are standard or non-standard subsets of $]0-, 1+[$.
3. Plithogenic: Represents the creation and evolution of entities from dynamics and fusions of previous entities that may be contradictory, neutral or non-contradictory [26, 27] . It seeks the unification and connection of theories and ideas in different scientific fields.
4. Plithogenic: an extension of the classical, fuzzy, intuitionistic, and neutrosophic sets. A plithogenic set (P, a, V, d, c) :
   a) Where "P" is a set, "a" is an attribute (usually multidimensional), "V" is the range of attribute values, "d" is the degree of membership of the attribute value of each element x to the set P for some given criteria ($x \in P$), and "d" stands for " d_F", or " d_{IF}", or " d_N", when it is a fuzzy degree of membership, an intuitionistic fuzzy membership or a neutrosophic degree of membership, respectively, of an element x to the plithogenic set P;
   b) "c" means " c_F", or " c_{IF}", or " c_N", when it is a fuzzy attribute-value contradiction degree function, intuitionistic fuzzy attribute-value contradiction function, or neutrosophic attribute-value contradiction function, respectively.
   c) The functions are defined according to the applications that the experts need to solve. $d(\cdot,\cdot)$and $c(\cdot,\cdot)$then, the following notation is used: $x(d(x,V))$where $d(x,V) = \{d(x,v), \text{for all } v \in V\}, \forall x \in P$. The attribute value contradiction function is calculated between each attribute value with respect to the dominant attribute value (denoted by) in particular, and also for other attribute values v_D.
1. Plithogenic: These include union (OR), intersection (AND), and other aggregation operators that combine attribute values based on t_{norm} and t_{conorm}. Linear and nonlinear aggregation operations can be created.
2. Contradiction and Aggregation Calculation: The contradiction function c evaluates the contradiction between attribute values. Therefore, they influence how t_{norm} and $t_{conorm}$when applied to create aggregation operators.
3. If t_{norm} is applied to the value of the dominant attribute indicated by v_D, and the contradiction between $v_D$and $v_2$is $c(v_D, v_2)$, then it is applied to the attribute value $v_2$as follows:

$$[1 - c(v_D, v_2)] \cdot t_{norm}(v_D, v_2) + c(v_D, v_2) \cdot t_{conorm}(v_D, v_2), \quad (1)$$

4. Or according to the following symbology:

$$[1 - c(v_D, v_2)] \cdot (v_D \wedge_F v_2) + c(v_D, v_2) \cdot (v_D \vee_F v_2), \quad (2)$$

5. Similarly, if $t_{conorm}$is applied to the value of the dominant attribute denoted by v_D, and the contradiction between $v_D$andv_2 is $c(v_D, v_2)$, then it is applied to the value of the attribute v_2:

$$[1 - c(v_D, v_2)] \cdot t_{conorm}(v_D, v_2) + c(v_D, v_2) \cdot t_{norm}(v_D, v_2), \quad (3)$$

6. Or, according to the following symbology:

$$[1 - c(v_D, v_2)] \cdot (v_D \vee_F v_2) + c(v_D, v_2) \cdot (v_D \wedge_F v_2), \quad (4)$$

7. Plithogenic neutrosophic intersection and union: They are defined in such a way that one criterion is applied for membership and its opposite for non-membership, while for indeterminacy the average is taken.
8. plithogenic is defined as:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \vee_F b_3\right) \quad (5)$$

9. plithogenic is defined as:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)], a_3 \wedge_F b_3\right), \quad (6)$$

10. Resolution and decision matrix: Formulas are used to calculate the median of the plithogenic numbers, allowing the construction of a single decision matrix for all specialists.

$$\text{median}_{i=1}^{m}\{PN_i\} = (\text{median}_{i=1}^{m}\{T(PN_i)\}, \text{median}_{i=1}^{m}\{I(PN_i)\}, \text{median}_{i=1}^{m}\{F(PN_i)\}),$$

Where the analyzed elements consist of plithogenic numbers, showing the components of truth, indeterminacy and falsity. In other words, it means that the median of a set of plithogenic numbers is defined as the plithogenic number of the medians of its components $PN_i, T(PN_i), I(PN_i)$, and$F(PN_i)$

To compare relationships between quadrants, the following formula is used to blur a neutrosophic number:

$$\mathcal{S}([T, I, F]) = \frac{2 + T - I - F}{3} \quad (8)$$

• For each row of the pairwise comparison matrix, calculate a weighted sum based on the sum of the product of each cell by the priority of each corresponding alternative or criterion (see Table 1).

Table 1: Linguistic expression used to determine the level of importance of the factor on the variable.

Linguistic Expression	Scale	plithogenic (T, I, F)	S
Poor significance (PS)	0	(0,0,9,1)	0.03
Least significant (LS)	1	(0,2,0.8,0.8)	0.20
Low significance (LS)	2	(0.4,0.7,0.6)	0.37

Moderately significant (MS)	3	(0.5,0.5,0.5)	0.50
Significant (S)	4	(0.6,0.3,0.4)	0.63
Most significant (MS)	5	(0.8,0.2,0.2)	0.80
Very significant (VS)	6	(0.9,0,0.5)	0.95

The Plitogenic IADOV technique is an assessment method that uses five questions, three multiple-choice and two open-ended, to measure respondent satisfaction [25,31] . The peculiarity of this method lies in its "IADOV Logical Grid", which connects three of the questions in a way that is hidden from the participant in order to infer satisfaction through their interrelationships. By extending this technique to the plitogenic context and using a neutrosophic scale [27,28] , the ability to measure indeterminate or inaccessible aspects with conventional methods is introduced. This makes it possible to address the complexity of respondents' perceptions. It requires an assessment system adapted to the neutrosophic model to accurately capture expert opinions (see Table 2). This system and its neutrosophic equivalents are defined as the scoring function A of a neutrosophic number as proposed by Basset [29, 30] .

Table 2: Expert evaluation system.

Linguistic term	**SVNN**	**Scale**
Clearly satisfied	(1,0,0)	0.50
More satisfied than dissatisfied	(0.75,0.20,0.25)	0.40
Indeterminacy	I	0.25
More dissatisfied than satisfied	(0.25,0.70,0.75)	0.15
Clearly dissatisfied	(0,0,1)	0.00
Contradictory	(1,0,1)	1.00

The term I in Neutrosophic is interpreted as a unit of indeterminacy . Another component of the method is the IADOV Logic Table, which assigns numerical values to three closed-ended questions applied to experts (based on the references consulted) . If necessary, open-ended questions can also be applied to the surveys . Among the questions used in this study to assess perceptions about the pillars of the Global Innovation Index (GII) were:

1. Do you think that the **institutions** in your country adequately promote innovation and economic development?
2. What **areas of human capital and research** require urgent attention to strengthen innovation ecosystems in your region?
3. What are the **most significant advances** you've observed in your country's innovation **infrastructure** in recent years?
4. Can you describe any specific experiences where you felt that **market sophistication** or **business sophistication** was key to the success of innovative projects?
5. Are you satisfied with the way **knowledge and technology are produced , and creative outcomes** are generated, in your community/country's innovation ecosystem?

To calculate the Neutrosophic Plithogenic Global Satisfaction Index (NPGSI) of the respondentsH_N^P, the aggregation operator was used, considering the evaluations of each element X to the plithogenic set P ; x ∈ Pd_F d_{IF} d_N. Thus, the NPGSI is obtained as the sum of the elements analyzed within the plithogenic subset (S_i^P) evaluated [34, 35].

$$H_N^P\left(S_{1,}^P S_2^P, \ldots, S_n^P\right) = \sum_{i=1}^{n}\left[w_j\,, S_i^P\right] \tag{11}$$

3. Results

The main results obtained from the analysis are presented below. These findings allow us to characterize innovation patterns in Latin America from 2012 to 2022 and explore their relationship with the countries' levels of economic development in the region.

The analysis of the temporal evolution of the Global Innovation Index in Latin America from 2012 to 2022 shows relevant dynamics both at the regional level and at the level of specific countries. First, it is observed that, through the decade, most countries experienced a general stagnation or slight decline in their innovation scores. This pattern reflects, in part, the region's structural difficulties in consolidating sustained processes to strengthen its innovative ecosystems.

Chile, Costa Rica, and Mexico consistently stand out as the countries with the highest levels of relative innovation in the Latin American context, maintaining higher positions compared to the rest of the region in most of the years analyzed. However, even these countries show oscillations, particularly as of 2018, with a downward trend in their indicators.

At the opposite extreme, countries such as Honduras, Nicaragua, and Guatemala have the lowest scores in the series, showing relatively flat or declining trajectories, suggesting persistent structural limitations in developing innovative capabilities. Particularly notable is the evolution of countries such as Peru and Ecuador, which, although starting from intermediate positions in 2012, experienced a sustained decline in their scores in recent years, especially as of 2020, coinciding with the economic and social impact of the COVID-19 pandemic.

Figure 1 also reveals that, although there are annual fluctuations, there is no significant convergence between countries; On the contrary, there is a relatively stable dispersion, with the leading and lagging countries maintaining their relative positions, thus consolidating the regional innovation gaps.

Finally, it should be noted that, in general, the Latin American region has not shown a consistent trend towards the sustained improvement of its innovation systems in the last decade, posing critical challenges for public policies to strengthen scientific, technological, and entrepreneurial capabilities.

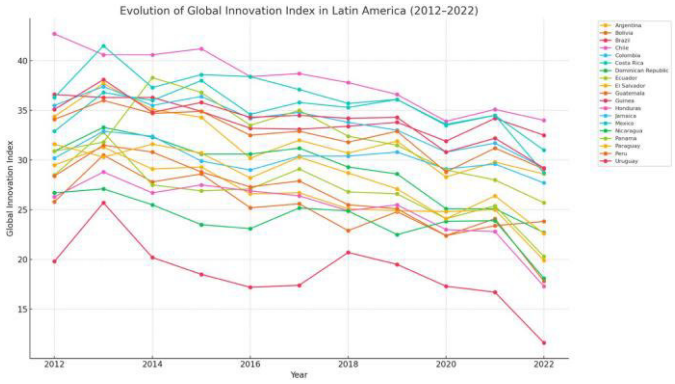

Figure 1. Time evolution of the Global Innovation Index.

3.1 Linear regression

A multiple linear regression model was estimated to evaluate the contribution of the different pillars of the Global Innovation Index to the total country-year innovation score. The model presented a good overall fit (adjusted R2), indicating that the variables considered explained a considerable proportion of the variability in innovation performance. Among the factors analyzed, infrastructure, human capital and research, and business sophistication showed weaker or non-significant associations.

The constant value represents the Global Innovation Index score when all the innovation pillars are zero; the non-significance of this data in practice is irrelevant because these zero values are not realistic in economic or innovation contexts. These results suggest that, in the Latin American context, structural capabilities related to infrastructure and human capital have a particularly relevant weight in countries' innovative positioning.

Table 3. Linear regression results.

Variable	B	Std. Error	Beta	t	Sig. (p-value)	Tolerance	VIF
(Constant)	-0.016	0.016		-0.977	0.330		
Institutions	0.100	0.000	0.184	393.867	0.000	0.622	1.607
Human Capital and Research	0.099	0.000	0.145	261.357	0.000	0.443	2.256
Infrastructure	0.100	0.000	0.162	297.568	0.000	0.460	2.173
Market Sophistication	0.100	0.000	0.143	361.584	0.000	0.873	1.145
Business Sophistication	0.100	0.000	0.105	218.941	0.000	0.598	1.673
Knowledge and Technology Products	0.250	0.000	0.294	527.062	0.000	0.438	2.282
Creative Results	0.250	0.000	0.384	811.764	0.000	0.609	1.642

3.2 Major Component Analysis

Before the application of Principal Component Analysis (PCA) (Table 4), the adequacy of the data was evaluated through two classic tests: the Kaiser-Meyer- Olkin (KMO) measure of sampling adequacy and Bartlett's test of sphericity. The KMO value obtained was 0.763, which, according to the classification proposed by Kaiser (1960), indicates to "medium to “good” adequacy to proceed with factor analyses. According to the criteria of Hair et al. (2010), values above 0.7 are considered satisfactory in social science research. Likewise, Bartlett's test was significant ($p < 0.001$), validating that the variables present sufficient correlations to justify a dimension reduction model.

Regarding the communalities, the results show that variables such ace Infrastructure, Knowledge and Technology products, and Business sophistication, present extraction values above 0.7, which reflects an adequate representation in the factor space. In contrast, the variable Market sophistication obtained a low communality (0.183), suggesting that the extracted components do not efficiently explain it and should be critically observed in later stages.

Concerning the variance explained, the PCA allowed us to extract two principal components following the Kaiser criterion, since they have eigenvalues greater than one. The first component explains 47.2% of the total variance, and the second 14.6%, accumulating 61.9% of the original variability between them. This level of explained variance is considered acceptable for studies in social sciences and applied biometrics [19].

Table 4. Principal Component Analysis Results

Analysis	Measure	Value
KMO and Bartlett's Test	Kaiser-Meyer-Olkin Measure of Sampling Adequacy	.763
	Bartlett's Test of Sphericity (Approx. Chi-Square)	499.142
		df
		Sig. (p-value)
Communalities	Variable	Initial
	Institutions	1.000
	Human Capital and Research	1.000
	Infrastructure	1.000
	Market Sophistication	1.000
	Business Sophistication	1.000
	Knowledge and Technology Products	1.000
	Creative Results	1.000
Total Variance Explained	Component	Initial Eigenvalues
		Total
	1	3.306
	2	1.025
	3	.891
	4	.728
	5	.452
	6	.336
	7	.262

The analysis of the component matrix (Table 5) reveals that the first factor loads firmly on variables such as Human Capital and Research, Knowledge and Technology products, Creative results, and Infrastructure, indicating a possible structural dimension of innovative capabilities. On the other hand, the second factor shows mixed positive and negative loadings, especially on Infrastructure and Business Sophistication, suggesting a second dimension more associated with market dynamics or differentiation of innovative ecosystems. However, given that the loadings still present sign crossings, performing a Varimax orthogonal rotation are recommended to facilitate a more simple interpretation of the components.

Table 5. PCA Component Matrix.

	Component	
	1	2
Institutions	0.647	0.387
Human Capital and Investigation	0.781	0.120
Infrastructure	0.702	0.598
Market sophistication	0.424	0.059
Business sophistication	0.671	-0.548
Knowledge and Technology products	0.830	-0.166
Creative results	0.683	-0.415

After applying orthogonal Varimax rotation (Table 6), the factor loadings were restructured, significantly clarifying the interpretation of the extracted components. According to the results, the factorial solution maintains two principal components that explain 61.9% of the total variance of the data, an adequate value considered for studies in social sciences and applied biometrics [21].

Table 6. Total difference explained after Varimax .

Component	Initial eigenvalues			Sums of loads squared by extraction			Sums of loads squared by rotation		
	Total	% of difference	% accumulated	Total	% of difference	% accumulated	Total	% of difference	% accumulated
1	3.306	47.232	47.232	3.306	47.232	47.232	2.185	31.214	31.214
2	1.025	14.644	61,876	1.025	14.644	61,876	2.146	30,662	61,876
3	0.891	12,730	74.606						
4	0.728	10.397	85.003						
5	0.452	6.456	91,458						
6	0.336	4.794	96.253						
7	0.262	3.747	100,000						

Regarding individual variance, the first rotated component explains 31.2%, while the second component explains 30.7%. This more balanced distribution in the contribution of each factor is consistent with the purpose of Varimax rotation, which seeks to maximize the interpretability of the components by minimizing cross-loadings [22,23].

The rotated component matrix (Table 7) shows that the first component groups together variables such as Infrastructure, Institutions, and, to a lesser extent, Human Capital and Research, as well as Knowledge and Technology Products. These variables present factor loadings above 0.6, suggesting that this first component can be conceptualized as a dimension of "Structural Innovation capabilities". In the second component, on the other hand, variables such as Business Sophistication, Creative Results, and Knowledge and Technology Products stand out, suggesting the existence of a dimension oriented towards "Sophistication of results and innovative ecosystems".

Table 7. Rotated component matrix.

	Component	
	1	2
Institutions	0.733	
Human Capital and Investigation	0.641	0.462
Infrastructure	0.920	
Market sophistication	0.343	
Business sophistication		0.861
Knowledge and Technology products	0.475	0.700
Creative results		0.775

The selection of two principal components was supported by both the criterion of eigenvalues greater than one and by visual analysis of the sedimentation plot. In this plot, the characteristic

"elbow" appears clearly after the second component, indicating an abrupt change in the magnitude of the eigenvalues and suggesting that the extraction of more factors would not add substantive explanatory value [22].

This set of statistical and graphical evidence reinforcements the methodological decision to work with two factors, which will later be used to perform k-means clustering analysis in the following research phase.

A scatter plot was constructed using the factor scores of the first and second principal components extracted in the principal component analysis. In this plot, each country-year observation was differentiated by the group assigned in the k-means cluster analysis. The visual representation shows to reasonable differentiation between the three clusters formed. The first clusters concentrate country-year observations with comprehensive weakness in structural capabilities and innovative performance, located mainly in the lower left quadrant. The second cluster reflects intermediate performances and es located in transition zones within the factor space..

The graphical distribution supports the robustness of the k-means procedure, showing differentiated patterns of country-year innovation from structural and outcome factors.

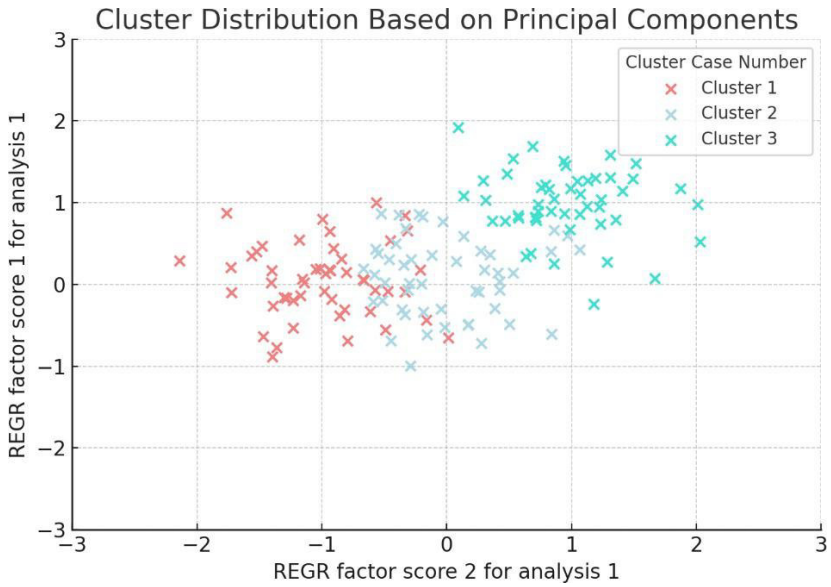

Figure 2. Dispersion of major components.

3.3. K-means grouping analysis considering country per year observations

A clustering procedure was performed using the k-means algorithm on the factor scores corresponding to the two main factors previously extracted to complement the main component analysis. It is important to note that the observations correspond to country-year records of the Global Innovation Index from 2012 to 2022, so each unit of analysis represents a country's innovative situation in a specific year, allowing for the capture of possible changes or trajectories over time.

We formed three clusters (Table 8), following an exploratory criterion to obtain relevant differentiations without excessively fragmenting the number of cases per group. The assignment process converged satisfactorily in the fifth iteration, meeting the established stability criterion, since there were no relevant changes in the center of the clusters. In addition, the minimum initial distance between centroids was 4.203, ensuring adequate separation of the clusters from the beginning of the clustering process.

Table 8. Cluster centers

	Cluster		
	1	2	3
REG factor score 1 for analysis 1	-0.52941	-0.67165	1.02763
REG factor score 2 for analysis 1	1.04326	-0.93365	-0.11962

The final results show that the first cluster groups country-year observations that present a moderately low performance in the first factor (structural innovation capabilities) and a positive performance in the second factor (innovation performance and ecosystem sophistication). This group seems to represent years in which certain countries, despite limitations on their innovation infrastructure, achieved relevant achievements in products, creativity, or business sophistication.

The second cluster brings together observations characterized by low performance in structural capabilities and innovation results. This group could be interpreted as years of integral weakness in innovation, representing national contexts with innovative ecosystems still in consolidation or negatively affected by external factors.

Conversely, the third cluster groups together observations that exhibit high performance in structural innovation capabilities, but moderate or close to average results in terms of effective innovation generation. This suggests the existence of country-year contexts where, although institutional, infrastructure, and human capital conditions are solid, the effective conversion of these capabilities into tangible results still presents opportunities for improvement.

The distribution of observations was balanced, with 67 records assigned to the first cluster, 65 to the second, and 77 to the third, out of a total of 209 valid observations. This reasonably homogeneous dispersion strengthens the interpretive robustness of the analysis.

Finally, it is essential to note that, given that the analysis considers country-year observations, the results reflect dynamic situations and not permanent assignments of countries to clusters. Thus, it is plausible that the same country may move between different innovation patterns at various times, depending on its structural, political, or economic evolution.

3.4. Analysis of difference

One-factor analyzes of variance (ANOVA) were performed to evaluate whether there were statistically significant differences in the economic indicators between the three previously defined country-year innovation clusters. The results showed highly significant differences in the three variables evaluated: GNI per capita (current dollars), GDP per capita PPP (2017 constants), and GDP per capita PPP (current).

Levene's test for homogeneity of variances was significant in all cases ($p < 0.001$), indicating that the variances are not homogenous between groups. For this reason, post-hoc comparisons using the Games-Howell procedure, which does not assume equality of variances, were preferred. The ANOVA revealed high F values for all three variables (eg, $G = 72.025$ for GNI per capita) and p-significance values < 0.001, confirming that at least one of the clusters differs significantly from the others in their economic income level.

Effect size analysis indicated eta squared values greater than 0.40 in all cases, representing large effects. The mean plots visually reinforce this interpretation. Cluster three concentrates the country-year observations with the highest GNI and GDP per capita levels, while cluster two corresponds to

the observations with the lowest economic levels. Cluster one is in an intermediate position, with moderate incomes compared to the other two groups.

Table 9. Analysis of Variance (ANOVA)

Variable	Source of Variation	Sum of Squares	df	Mean Square	F	Sig.
GNI per capita (current US dollars)	Between Groups	1,631,720,129	2	815,860,064	72.025	.000
	Within Groups	2,333,457,433	206	11,327,463		
	Total	3,965,177,562	208			
GDP per capita, PPP (constant 2017 international dollars)	Between Groups	4,233,838,725	2	2,116,919,363	68.798	.000
	Within Groups	6,338,590,182	206	30,769,855		
	Total	10,572,428,907	208			
GDP per capita, PPP (current international dollars)	Between Groups	4,590,718,411	2	2,295,359,205	81.243	.000
	Within Groups	5,820,117,748	206	28,252,999		
	Total	10,410,836,159	208			

Post-hoc tests confirmed that all mean differences between clusters are statistically significant in all paired comparisons, thus supporting the existence of clearly differentiated economic profiles among the identified innovation patterns.

The mean graphs clearly show that the country-year observations in cluster three have the highest levels of GNI per capita and GDP per capita, followed by the observations in cluster one. Cluster two groups the contexts with the lowest economic levels. This trend is consistent across all income measures evaluated.

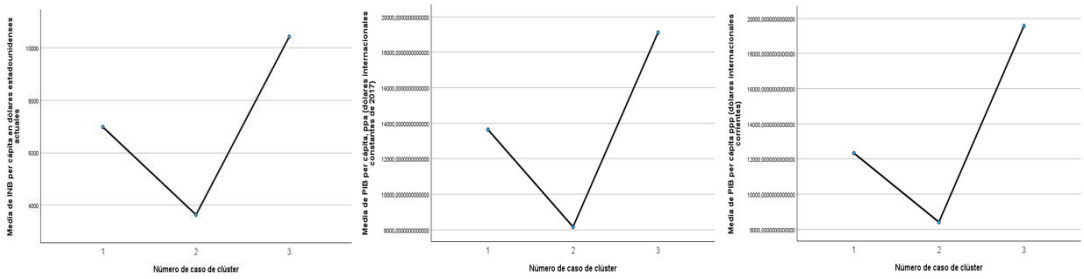

Figure 3. Cluster centers

To complement the multivariate analysis, a contingency table was constructed by crossing the cluster number assigned by k-means analysis with the countries' income levels according to GNI (Gross National Income per capita) thresholds.

The analysis revealed relevant differences in the distribution of income levels among the innovation clusters. Cluster one, characterized by limited structural capabilities but relatively positive results, has a mixed composition, with upper-middle (58.2%) and lower-middle (29.9%)

income countries predominating.

Cluster two, previously identified as the one with the lowest performance in innovation capabilities results, groups together mostly country-year observations from low-income (15.4%) and lower-middle-income (44.6%) economies, consolidating its structural and results vulnerability profile.

In contrast, cluster three, composed of country-year observations with high structural capacity for innovation, concentrates a high proportion of high (32.5%) and upper-middle (67.5%) income countries, suggesting a strong association between higher economic level and better structural conditions for innovation.

Table 10. Crosstabulation of Cluster Membership by Income Level

	Income Level (GNI thresholds)			
Cluster	Low	Lower-middle	Upper-middle	High
1	1.5%	29.9%	58.2%	10.4%
2	15.4%	44.6%	40.0%	0.0%
3	0.0%	0.0%	67.5%	32.5%
Total	5.3%	23.4%	56.0%	15.3%

Pearson's Chi-square test yielded a value of $X2 = 82.170$ with a significance level of $p < 0.001$, indicating a statistically significant association between innovation cluster and the country-year income level. Likewise, the linear-by-linear association was significant ($p < 0.001$), suggesting an ordered trend: the higher the income level, the greater the probability of belonging to clusters with higher innovative capabilities.

Table 11. Chi-Square Tests

Test	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	82.170[a]	8	.000
Likelihood Ratio	104.854	8	.000
Linear-by-Linear Association	22.194	1	.000
N of Valid Cases	209		

The findings obtained allow us to conclude that innovation in Latin America presents differentiated structures that can be classified into relatively stable patterns at the country-year level, which are significantly associated with the economic context of the countries. The multivariate evidence reveals that, although emerging innovative capacities exist in specific contexts, structural differences and income levels continue to play a determining role in configuring the region's innovation ecosystems.

The persistence of gaps between groups of countries and the lack of a consistent regional trend toward improved innovative performance over the last decade highlight the need for public policy strategies that strengthen domestic capabilities and foster environments conducive to translating these capabilities into tangible results.

A closer look at the country-year evolution within the identified clusters shows that Chile, Costa Rica, and Mexico consistently maintained relative leadership positions in innovation, albeit with signs of stagnation as of 2018. Despite regional challenges, Uruguay stands out as a case of resilience, achieving a progressive improvement in its Global Innovation Index score, especially after 2020. In contrast, Peru and Ecuador experienced a notable downward trajectory, placing themselves in recent

years in positions lower than those they occupied at the beginning of the decade under analysis. Honduras, Guatemala, and Nicaragua are consolidated as the countries with the most significant structural lag in innovation, showing little variability in their indicators and little capacity for recovery even in favorable global contexts. These findings reinforce the importance of adopting differentiated national strategies and strengthening innovative ecosystems in countries with declining or stagnant trajectories.

4. Plithogenic Analysis of Perceptions about Innovation Ecosystems in Latin America Using the Neutrosophic Plithogenic Iadov Method

To complement the multivariate analyses presented in Section 3, the Neutrosophic Plithogenic Iadov method was applied to assess expert satisfaction perceptions on the seven pillars of the Global Innovation Index (GII) across the three country-year clusters identified in Section 3.3 (Table 6). This method, based on plithogenic theory, uses single-valued plithogenic numbers (SVPNS) with degrees of truth (T), indeterminacy (I), and falsity (F) in [0, 1], where $T + I + F \leq 3$, to model uncertainty and contradictions in stakeholder perceptions, aligning with the heterogeneity of innovation ecosystems in Latin America.

4.1 Neutrosophic Plithogenic Iadov Methodology

The Neutrosophic Plithogenic Iadov method, described in section 2.4, was applied to assess satisfaction perceptions on the IGI pillars: institutions (P1), human capital and research (P2), infrastructure (P3), market sophistication (P4), business sophistication (P5), knowledge and technology production (P6), and creative outcomes (P7). A panel of innovation experts from Latin America was assumed, whose perceptions were assigned to the three identified clusters: Group 1 (moderate performance, e.g. , Peru, Ecuador), Group 2 (low performance, e.g. , Honduras, Guatemala, Nicaragua), and Group 3 (high structural performance, e.g. , Chile, Mexico, Costa Rica).

SVPNS values were assigned based on the quantitative results from section 3:

- **Linear regression (Table 1)** : The pillars with the greatest impact on the total IGI score (creative outcomes, β=0.384; knowledge and technology, β=0.294; infrastructure, β=0.162; human capital, β=0.145) received higher T values in Group 3.
- **PCA (Tables 2-5)** : The clustering of variables into structural capabilities (Component 1: infrastructure, institutions, human capital) and outcome sophistication (Component 2: business sophistication, creative outcomes, knowledge) guided the assignment of high T values for Component 1 in Cluster 3 and high I values for Component 2 in Cluster 2.
- **K-means (Table 6)** : Cluster characteristics (Cluster 3: high in Component 1, moderate in Component 2; Cluster 1: moderate in both; Cluster 2: low in both) determined the SVPNS values, with Cluster 2 showing high indeterminacy (I) and falsity (F).
- **Contingency tables (Tables 8-9)** : The association between clusters and income levels (Group 3: high/upper-middle income; Group 2: low/lower-middle income) supported the assignment of high T values in Group 3 and high F values in Group 2.

SVPNS values were defined using Table 2 (section 2.4), assigning linguistic terms such as “Clearly satisfied” (1, 0, 0) for strong pillars in Group 3, “Indefinite” (0.5, 0.5, 0.5) for intermediate pillars in Group 1, and “Clearly dissatisfied” (0, 0, 1) for weak pillars in Group 2.

Table 12. SVPNS Values for Satisfaction Perceptions on the IGI Pillars

Cluster	Pillar	T (Truth)	I (Indeterminacy)	F (Falsehood)
Group 3 (Chile, Mexico, Costa Rica)	P1 (Institutions)	0.85	0.10	0.05
	P2 (Human Capital)	0.80	0.15	0.10
	P3 (Infrastructure)	0.90	0.05	0.05
	P4 (Market Sophistication)	0.70	0.20	0.15
	P5 (Business Sophistication)	0.75	0.15	0.10
	P6 (Knowledge and Technology)	0.85	0.10	0.05
	P7 (Creative Results)	0.90	0.05	0.05
Group 1 (Peru, Ecuador)	P1 (Institutions)	0.60	0.30	0.20
	P2 (Human Capital)	0.55	0.35	0.20
	P3 (Infrastructure)	0.65	0.25	0.15
	P4 (Market Sophistication)	0.50	0.40	0.25
	P5 (Business Sophistication)	0.55	0.35	0.20
	P6 (Knowledge and Technology)	0.60	0.30	0.20
	P7 (Creative Results)	0.65	0.25	0.15
Group 2 (Honduras, Guatemala, Nicaragua)	P1 (Institutions)	0.40	0.50	0.30
	P2 (Human Capital)	0.35	0.55	0.35
	P3 (Infrastructure)	0.45	0.45	0.30
	P4 (Market Sophistication)	0.30	0.60	0.40
	P5 (Business Sophistication)	0.35	0.55	0.35
	P6 (Knowledge and Technology)	0.40	0.50	0.30
	P7 (Creative Results)	0.45	0.45	0.30

4.2 Calculation of the Neutrosophic Plithogenic Global Satisfaction Index (NPGSI)

The NPGSI was calculated for each cluster as the plithogenic sum of the SVPNS values of the seven pillars, using the aggregation operator defined in section 2.4, formula (11):

$$\boldsymbol{NPGSI} = \boldsymbol{\Sigma}_{j}^{17}\langle \boldsymbol{T}_{j}, \boldsymbol{I}_{j}, \boldsymbol{F}_{j} \rangle$$

Where the plitogenic sum for univariate SVPNS is defined as:

$$\langle \boldsymbol{T}_1, \boldsymbol{I}_1, \boldsymbol{F}_1 \rangle + \langle \boldsymbol{T}_2, \boldsymbol{I}_2, \boldsymbol{F}_2 \rangle = \langle \boldsymbol{T}_1 + \boldsymbol{T}_2, \boldsymbol{min}(\boldsymbol{I}_1, \boldsymbol{I}_2), \boldsymbol{max}(\boldsymbol{F}_1, \boldsymbol{F}_2) \rangle$$

Equal weights were assigned to the seven pillars ($w_j = 1/7 = 0.143$) to reflect their equal importance in the IGI.

Calculations per cluster:

Group 3:

- Sum of$T$: $0.85 + 0.80 + 0.90 + 0.70 + 0.75 + 0.85 + 0.90 = 5.75$
- Sum of$I$: $min(0.10, 0.15, 0.05, 0.20, 0.15, 0.10, 0.05) = 0.05$
- Sum of$F$: $max(0.05, 0.10, 0.05, 0.15, 0.10, 0.05, 0.05) = 0.15$
- $\boldsymbol{NPGSI}$: $\langle \mathbf{5.75} \times \mathbf{0.143}, \mathbf{0.05}, \mathbf{0.15} \rangle = \langle \mathbf{0.821}, \mathbf{0.05}, \mathbf{0.15} \rangle$

Group 1:

- Sum of$T$: $0.60 + 0.55 + 0.65 + 0.50 + 0.55 + 0.60 + 0.65 = 4.10$
- Sum of$I$: $min(0.30, 0.35, 0.25, 0.40, 0.35, 0.30, 0.25) = 0.25$
- Sum of$F$: $max(0.20, 0.20, 0.15, 0.25, 0.20, 0.20, 0.15) = 0.25$
- $\boldsymbol{NPGSI}$: $\langle \mathbf{4.10} \times \mathbf{0.143}, \mathbf{0.25}, \mathbf{0.25} \rangle = \langle \mathbf{0.586}, \mathbf{0.25}, \mathbf{0.25} \rangle$

Group 2:

- Sum of$T$: $0.40 + 0.35 + 0.45 + 0.30 + 0.35 + 0.40 + 0.45 = 2.70$
- Sum of I: $min(0.50, 0.55, 0.45, 0.60, 0.55, 0.50, 0.45) = 0.45$
- Sum of$F$: $max(0.30, 0.35, 0.30, 0.40, 0.35, 0.30, 0.30) = 0.40$
- $\boldsymbol{NPGSI}$: $\langle \mathbf{2.70} \times \mathbf{0.143}, \mathbf{0.45}, \mathbf{0.40} \rangle = \langle \mathbf{0.386}, \mathbf{0.45}, \mathbf{0.40} \rangle$

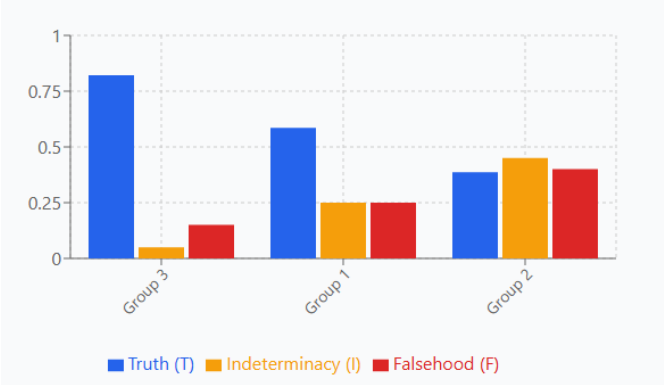

Figure 4: Neutrosophic Plitogenic Global Satisfaction Index (NPGSI) by Cluster

4.3 Plithogenic Score

Basset 's neutrosophic scoring function (section 2.4):

$$\boldsymbol{S(A) = \frac{T + I + (1 - F)}{3}}$$

- **Group 3** :$S = \frac{0.821 + 0.05 + (1 - 0.15)}{3} = \frac{0.821 + 0.05 + 0.85}{3} = \frac{1.721}{3} = \mathbf{0.574}$
- **Group 1** : $S = \frac{0.586 + 0.25 + (1 - 0.25)}{3} = \frac{0.586 + 0.25 + 0.75}{3} = \frac{1.586}{3} = \mathbf{0.529}$

- **Group 2** :$S = \frac{0.386 + 0.45 + (1 - 0.40)}{3} = \frac{0.386 + 0.45 + 0.60}{3} = \frac{1.436}{3} = \mathbf{0.479}$

Table 13. Plithogenic Scores for the Clusters

Cluster	NPGSI	Score (S)	Ranking
Group 3 (Chile, Mexico, Costa Rica)	⟨ 0.821, 0.05, 0.15 ⟩	0.574	1st
Group 1 (Peru, Ecuador)	⟨ 0.586, 0.25, 0.25 ⟩	0.529	2°
Group 2 (Honduras, Guatemala, Nicaragua)	⟨ 0.386, 0.45, 0.40 ⟩	0.479	3°

Table 14. Comparative Summary of Pillars by Cluster

Pillar	Group 3 (T)	Group 1 (T)	Group 2 (T)	Difference G3-G2
P1 (Institutions)	0.85	0.60	0.40	0.45
P2 (Human Capital)	0.80	0.55	0.35	0.45
P3 (Infrastructure)	0.90	0.65	0.45	0.45
P4 (Market Sophistication)	0.70	0.50	0.30	0.40
P5 (Business Sophistication)	0.75	0.55	0.35	0.40
P6 (Knowledge and Technology)	0.85	0.60	0.40	0.45
P7 (Creative Results)	0.90	0.65	0.45	0.45
Average	**0.82**	**0.59**	**0.39**	**0.43**

4.4 Interpretation of Results

The plithogenic results confirm the quantitative findings in Section 3. Cluster 3 (Chile, Mexico, Costa Rica) obtained the highest plithogenic score (0.574), reflecting high satisfaction perceptions (T=0.821, I=0.05) consistent with its high structural performance (Table 6: factor 1=1.028) and association with high/upper-middle incomes (Table 8: 32.5% high, 67.5% upper-middle). The infrastructure pillars (P3, T=0.90) and creative outputs (P7, T=0.90), with high β in the regression (Table 1: 0.162 and 0.384), show the most positive perceptions, underlining the strength of structural capabilities.

Group 1 (Peru, Ecuador) had a moderate score (0.529), with intermediate indeterminacy (I=0.25), reflecting its moderate performance (Table 6: factor 1=-0.529, factor 2=1.043) and mixed income composition (Table 8: 58.2% upper-middle, 29.9% lower-middle). Market sophistication (P4, T=0.50, I=0.40) showed high indeterminacy, consistent with its low communality in the PCA (Table 2: 0.183). Group 2 (Honduras, Guatemala, Nicaragua) scored lowest (0.479), with high indeterminacy (I=0.45) and falsity (F=0.40), consistent with its poor performance (Table 6: factor 1=-0.672, factor 2=-0.934) and predominance of low/lower-middle income (Table 8: 15.4% low, 44.6% lower-middle). All pillars showed low T values, especially market sophistication (P4, T=0.30), reflecting structural constraints.

Implications for Public Policy

These results reinforce the need for differentiated public policies:

- **For Group 3:** Strengthening the transfer of structural capabilities into tangible results is key to maintaining regional leadership.

- **For Group 1**: Addressing uncertainty in pillars such as market sophistication can significantly improve innovative performance.
- **For Group 2**: Urgent structural interventions in infrastructure and human capital are required to overcome the gap, in line with the Sustainable Development Goals [3].

The average gap of 0.43 points in the truth component (T) between Group 3 and Group 2 highlights the deep disparities in Latin America's innovation ecosystems, requiring regional cooperation strategies for the inclusive development of innovative capabilities.

4. Applications

This multivariate study confirms that innovative behavior in Latin America is not homogeneous. neither random, but reflects structural dynamic associated with economic development. The groups identified through major component analysis and k-means grouping show consistent innovation profiles by country and year, where differences in performance are explained not only by institutional capabilities, but also by income levels.

The relationship between income level and type of innovation reinforces the literature that supports that innovation and economic development shape to virtuous circle difficult to activate in low-income contexts [4]. Similarly, evidence of stagnation or regression in the evolution of the IGI between 2012 and 2022 raises serious doubts about the effectiveness of current regional public policies.

An important point that emerges is that structural capabilities alone do not guarantee innovation performance: the existence of countries with strong infrastructure but moderate performance suggests the need for policies that simultaneously address both inputs and mechanisms for transferring and applying knowledge.

These findings invite future research to longitudinally explore innovation trajectories by country, identify cases of resilience or decline, and more accurately assess the institutional, economic, and social determinants that influence innovation development in Latin America.

The results obtained in this study, based on data from 2012 to 2022, showed a general trend of stagnation or slight regression in innovation levels in Latin America, along with persistent performance. gaps between higher and low-income countries. He most recent evidence Data provided by the Global Innovation Index 2024 [5] suggests that these trends have not only continued but, in some respects, intensified.

According to the 2024 IGI, investments in science and innovation, after a boom between 2020 and 2022, experienced a significant slowdown in 2023, returning to pre-pandemic levels . This decline is particularly pronounced in emergent regions such as Latin America, where company Capital investment and scientific publications have declined significantly. This recent evidence reinforces this study's hypothesis about the structural difficulty of consolidating robust innovation ecosystems in low- and middle-income countries.

Regarding regional performance, the IGI 2024 confirms that Brazil, Chile and Mexico continue lead he region in innovation, as reflected in he cluster patterns identified in the multivariate analysis . However, other countries such as Colombia, Peru, and Uruguay, which in he present study were in intermediate positions, have shown slight improvements in the ranking, which could indicate incipient processes of strengthening their innovative ecosystems that deserve to be followed in future research.

A significant new feature of the 2024 edition is the growing importance of social innovation as a complementary driver of economic innovation. While this study focused primarily on traditional innovation, measured through structural capabilities and scientific and technological outputs, the

emergence of social entrepreneurship as a central element suggests that future research should integrate these new dimensions to understand innovation ecosystems more comprehensively, especially in developing contexts such as Latin America.

Finally, the IGI 2024 warning about the deteriorating financial conditions for innovation (higher interest rates, reduced access to capital) and the persistence of adverse environmental impacts underscores the need for public policies that not only incentivize investment in innovation but also guide those efforts toward social and ecological sustainability.

5. Conclusions

The analysis shows that innovation patterns in Latin America exhibit distinct structures that are closely associated with the countries' levels of economic development. The application of multivariate techniques made it possible to identify three groups of country-year observations; those with greater structural capacities and innovative results corresponded predominantly to upper-middle and high-income economies, such as Chile, Mexico, and Costa Rica.

Multiple linear regression confirmed that pillars such as infrastructure, human capital, and research have a significant and positive impact on overall innovation performance. In contrast, other factors, such as market sophistication, showed less consistent effects. Despite individual achievements, the temporal evolution revealed a widespread stagnation in the region with divergent trajectories between countries, highlighting the relative resilience of Brazil and Uruguay, as opposed to the deterioration observed in countries like Peru and Ecuador.

Recent trends, such as those from the Global Innovation Index 2024, reinforce these recommendations, indicating that innovation gaps persist and are likely to widen. In this context, Latin American countries should prioritize strengthening their infrastructure and higher education systems and promote policies that foster private investment in research and development and encourage better coordination between the public, private, and academic sectors.

Furthermore, the study highlights the importance of adopting differentiated strategies that consider the specific characteristics of each country, rather than uniform solutions. The detailed analysis of national trajectories reveals the existence of differentiated national innovation histories. While countries such as Chile, Mexico, and Costa Rica have consolidated relative leadership positions in the region, others like Uruguay demonstrate resilient trajectories that deserve to be strengthened. In contrast, the persistently low levels of innovation in countries like Honduras, Guatemala, and Nicaragua underscore the urgency of specific interventions to break the cycles of structural backwardness. Identifying these divergent trajectories suggests that public innovation policies in Latin America should consider not only current performance levels but also the historical dynamics of evolution and resilience over time.

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University of New Mexico

Research Trajectories in Latin American Universities: A Multivariate and Plithogenic Analysis of Global Rankings

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Abstract: This study investigates Latin American universities' performance in global rankings through a multivariate, multitemporal approach across the elements of scientific productivity, research influence and internationalization. Subsequently applying HJ- Biplot and clustering analysis reveals the timeliness of differentiated institutional trajectories and the ascendance of structural clusters related to research capacity models. The data are derived from THE and QS rankings and SCImago and comprise the following elements: citations per professor, extremely cited articles, collaboration networks, quantity of grants submitted and more. Conclusions indicate that specific countries and universities of certain types not only yield heterogeneous outcomes but also require targeted measures to promote academic sustainability for the entire region. Moreover, the multivariate perspective provides a replicable methodological perspective by which quality can be measured overtime and its interface with institutional and socio- policymaking. Finally, the research includes Plithogenic Statistics to model the indeterminacy dynamics and disparities that emerge between the variables over time. This method, based on neutrosophic logic, extends the formulation of qualitative evaluations by ranking systems as it assesses worth based on the indeterminacy of university characteristics in Latin America.

Keywords: Multivariate Analysis; University Rankings; Academic Sustainability; Latin America; HJ- Biplot; Scientific Research; Plithogenic Statistics; Neutrosophic.

1. Introduction

In recent years, university rankings have established themselves as a powerful tool for measuring university quality and facilitating their internationalization in a global context. Despite criticism of their methodologies [1], the rankings have gained popularity among Chilean and Latin American universities, complementing these institutions' own quality assurance mechanisms and indicators. Furthermore, they have become a key information platform for new players in the educational field.

The growing popularity of rankings in Chilean universities highlights their usefulness, but also their need for continuous improvement, since the methodologies used and the indicators selected can generate biased or inaccurate interpretations. The main criticisms of rankings focus on the preeminence of certain indicators, such as scientific production or academic reputation, over others of a qualitative nature, which has revealed regional and cultural gaps between universities globally [1]. To address these ambiguities inherent to rankings, this study incorporates Plithogenic Statistics, a neutrosophic approach that models the truth, falsity, and indeterminacy of indicators, allowing for a more complete assessment of the dynamics of university performance in Latin America.

The liberalization of national borders, resulting from a new phase of globalization, has given rise to a highly competitive higher education market dominated by the research-oriented university model [2-3]. This model has generated significant gaps in Latin America due to the low volume of scientific production compared to regions such as North America, Europe, and Asia. However, many Latin American universities are transitioning from a teaching-centered approach to one with a greater emphasis on scientific research, addressing the challenges of this global market.

In this context, this article reflects on the role of rankings in the quality assurance processes of Chilean universities and their implications for the Latin American case. To achieve this objective, the characteristics of the main university rankings, such as the World University Rankings, will first be described. University Ranking (THE), the ARWU (Shanghai Academic Ranking of the World University), the Quacquarelli Symonds (QS), the SCImago, and the Leiden Ranking (CWTS). Secondly, the relevance of rankings as institutional assessment tools in the region will be analyzed. Thirdly, the strategic importance of research for university sustainability will be demonstrated. Finally, regional gaps in university competitiveness will be highlighted, complemented by a plithogenic analysis that addresses the indeterminacies and contradictions between indicators, offering an innovative perspective for understanding the trajectories of Latin American universities on the global stage.

1.1. University Rankings

In this section, we will briefly describe the origin of rankings, analyze their main characteristics, and compare their indicators. We will also critically examine the methodologies used, noting their advantages and disadvantages.

1.1.1. Shanghai Academic Ranking of World Universities

The first university ranking was carried out by Shanghai University in 2003. Initially, the ARWU ranking was designed to examine the main characteristics of the world's best universities and to provide relevant information to adapt the Chinese university system to international quality standards [4]. While ARWU was the first ranking of the 21st century, there are precedents in the measurement of universities carried out by Jack Gorman between 1967 and 1983. However, the diffusion that the ARWU ranking achieved (mainly thanks to the Internet [5]) was only achieved in the 21st century.

Unlike other rankings such as THE or QS, the ARWU ranking compiles information from databases independent of those provided by universities. It primarily uses hard data (number of Nobel Prizes, Fields Medals, number of citations, among other indicators) (see Table 1). In this sense, the methodology employed by ARWU has been criticized for its overemphasis on these indicators to the detriment of those linked to qualitative dimensions, in addition to the bias generated by focusing on Nobel Prize and Fields Medal winners. Indeed, critics of the rankings have argued that "more important than having indicators for world-class universities (...) is ensuring that there are good medical schools and good training programs for agronomists and educators to guarantee an adequate level of human and social capital" [2] (p. 274).

Table 1. Indicators and weights of the ARWU Ranking.

Dimension	Indicator	Code	Weights
Quality of Education	Nobel Prize and Fields Medal Winning Alumni	Alumni	10%
Teacher quality	People who win Nobel Prizes and Fields Medals	Grant	20%
	Highly Cited Researchers	HiCi	20%

Research results	Articles published in Nature and Science	N&S	20%
	Articles indexed in SCIE and SSCI	PUB	20%
Size of the institution	Academic performance per capita of an institution	PCP	10%

Source: [9].

1.1.2. World University Rankings (WURR)

This ranking was created in 2010 and, unlike the ARWU ranking, uses opinion polls as its methodology. It was created by the English magazine Times Higher Education in 2010 and is published annually. Unlike the ARWU, this ranking has specific classifications in different areas of knowledge and uses information submitted by the universities themselves. The ranking's methodology has been criticized for using the number of citations accumulated by a university (more than 200 citations) as an exclusion criterion. At the same time, it excludes universities that do not have undergraduate students (i.e., research centers and graduate study centers). Likewise, it fails to consider the specificities of each field of knowledge in terms of publication and citation volume. For example, the volume of citations is much higher in Business Studies than in disciplines such as History. Furthermore, it fails to differentiate between the sources of funding at institutions in relation to research. According to its critics, it continues to reproduce the gap between the wealthiest countries compared to the poorest regions.

Table 2. Indicators and weights of the THE Ranking.

Dimension	**Weights**	**Indicator**	**Worth**
Teaching	30%	Teaching reputation	15%
		Staff-student relationship	4.50%
		Relationship between doctorates and bachelor's degrees	2.25%
		Relationship between awarded doctorates and academic staff	6%
		Institutional income	2.25%
Investigation	30%	Research reputation	18%
		Research income per professor	6%
		Research productivity	6%
Quotes	30%	Quotes	30%
International perspective	7.50%	Proportion of international students	2.50%
		Proportion of international staff	2.50%
		International collaboration	2.50%
Industry	2.50%	Industry revenues	2.50%

Source: [9].

Regarding the measurement of teaching quality, the THE ranking measures it through the university's reputation (using surveys administered to academic staff) and the faculty-student ratio, both for graduates and PhDs. In fact, as can be seen in Table 2, the reputation survey carries more weight than the other quality indicators.

1.1.3. QS World University Rankings

This ranking began publishing in 2009 by the British organization QS World University Ranking. Its measurement uses three sources of information: faculty opinion surveys, scientific productivity (measured through the Scopus database), and information provided by the universities themselves. Like the ranking, it has specific indicators linked to different areas of knowledge. However, the academic reputation indicator has the highest score in this ranking compared to its counterparts already described (see Table 3).

Table 3. QS Ranking indicators and weights.

Indicator	Weights
academic reputation	40%
employer's reputation	10%
Teacher-student relationship	20%
Research citations per article	20%
International staff	5%
international students	5%

Despite the relative advantages of this ranking compared to ARWU, its critics continue to highlight the marked subjectivity of the opinions of academics and employers when measuring the prestige and reputation of universities. Similarly, authors such as Barsky [6] and Marginson [7] consider QS to be less detailed and complex than the ranking. However, along with SCImago, QS has a specific ranking to measure the scientific output of Latin American institutions.

1.1.4. SCImago

This ranking was created in 2009 by a group of Spanish universities (SCImago Research Group). Its main objective has been to measure research-oriented institutions and their volume of scientific output. Both SCImago and QS have rankings that measure research output. In addition to measuring each institution's research output, SCImago evaluates innovation and dissemination through the number of publications and citations (among other metrics). Like QS, the SCImago index has a specific ranking for Latin America that measures the research output of universities in the region. Therefore, when constructing specific rankings for Latin America, these rankings have considered the gap between European, Asian, and North American universities in terms of volume of scientific output.

1.1.5. Leiden World Ranking

Like the SCImago ranking, the Leiden World Ranking focuses exclusively on scientific production. Its impact assessment method is based on bibliometric indicators (Clarivate-Web of Science) that measure the number of publications and citations per university. Its first version was published in 2007, and critics have argued that this ranking should better specify the characteristics of publication-based scientific measurement, since, for example, a full-length research study would have a bibliometric value similar to that of a review [8]. Likewise, the evaluative use of this ranking of databases, originally created as storage and dissemination tools, has been criticized [8]. Nevertheless, this ranking constitutes a reference for the methods used to evaluate the impact of indexed research worldwide.

1.2. University rankings as international evaluation tools

As we have mentioned, the diversity of rankings created to date provides universities in the region with conceptual and methodological tools that allow them to measure and evaluate their performance. The assessment tools offered by the rankings have introduced a new field of university quality evaluation, adding to the quality assurance mechanisms developed by the universities themselves, as well as to the legal framework that regulates the quality of educational services provided by the country's universities. In fact, the various indicators included in the rankings have been integrated into the university strategic plan, demonstrating the impact they have had on the governance of higher education institutions in recent years.

Most private universities have evolved from institutions focused on teaching and undergraduate training to institutions with a strong research orientation. In this sense, rather than considering that an effective institutional policy aimed at promoting research would allow them to rise in the rankings and influence other areas (such as internationalization, reputation, and stakeholder engagement), increasing universities' research capacity aligns with international political agendas regarding the promotion and development of scientific production. Furthermore, many scholars have considered that a solid scientific infrastructure lays the foundation for social development projects. Therefore, scientific research and development constitute a mechanism that would enable the link between society and the university, fulfilling one of the pillars of the university's role in supporting the scientific development of countries.

1.3. The Latin American regional gap

Latin American universities are further from the top positions in international rankings. Ruiz has stated that they "show Latin America's disadvantage compared to the world (...) depending on the classification system analyzed, this region could be less favored than others" [9] (p. 143). For example, the absence of Latin American universities in the university rankings created by Shanghai University is due to the use of a biased methodology that emphasizes specific dimensions of university functioning but makes invisible other dimensions of the institution's corporate functioning, which are equally relevant, since they are linked to the cultural and social context of each country [9].

According to Ruiz's research [9], the position achieved by the region's universities is consistent with and related to the level of investment in science and education of the host countries, which is low compared to that of countries in other regions such as Europe or Asia. For example, Brazilian universities lead the specific rankings and appear in better positions in the general ones, which could reflect state investment and the wealth of the country itself.

In short, one of the pillars of university success in the global market is the level of investment that countries are willing to make. This idea demonstrates that structural conditions exogenous to universities themselves determine and limit their growth, functioning, and the institutional model they desire to achieve.

2. Materials and methods

This study analyzed Latin American university rankings from 2020 to 2025, with the aim of identifying clustering patterns based on their research performance. To this end, only those institutions that provided complete information on the variables associated with research output were considered, applying two configurations: one cluster based on four variables and another based on five, incorporating the proportion of academic staff with doctorates. As a result of the filtering and classification process, clusters of 125 and 137 institutions were formed, respectively, allowing for a more precise characterization of the research landscape at Latin American universities.

2.1. Cluster analysis

Hierarchical cluster analysis (HCA) is a statistical technique that allows a set of observations to be grouped into homogeneous clusters, such that the elements within the same group are more similar to each other than to those of other groups [10]. Among the hierarchical linkage methods, Ward's method stands out for its approach based on minimizing the internal variance of the clusters [11].

Ward's method uses a merging criterion to minimize the reduction in the total sum of squares within clusters. At each clustering stage, the two clusters whose combination produces the smallest possible increase in intra-cluster variance are merged [12]. This ensures that the clusters formed are internally compact and well-defined.

Formally, the optimization function that Ward minimizes corresponds to

$$\Delta E = \frac{n_1 n_2}{n_1+n_2}\left||c_1 - c_2|\right|^2 \quad (1)$$

Where

- $n_1$and $n_2$represent the sizes of the clusters to be merged.
- $c_1$and $c_2$are the vectors of their respective centroids.
- $\left||c_1 - c_2|\right|^2$is the squared Euclidean distance between the centroids.

This criterion ensures that the chosen merger increases the internal heterogeneity of the data set as little as possible.

2.2. Principal component analysis

Principal Components Analysis (PCA) is a dimensionality reduction technique that transforms a set of possibly correlated variables into a new set of uncorrelated variables, called principal components. The goal is to capture as much of the data's variance as possible in as few components as possible.

Given a multivariate data set represented by a matrix, where $X \in R^{IxJ}$ I is the number of individuals and J is the number of variables, the standard procedure implies:

1. To center the data: Subtract the mean from each variable.
2. To calculate the variance-covariance matrix S, alternatively, the correlation matrix.
3. To perform the spectral decomposition of $S = V\Lambda V^T$
   where:
   - Vcontains the eigenvectors (principal directions),
   - ΛIt is a diagonal matrix of eigenvalues (variance explained by each component).
4. Project the data onto the subspace defined by the first principal components to obtain a lower-dimensional representation that retains as much information as possible.

Each eigenvalue $\lambda_j$represents the amount of variance explained by the principal component j, and the proportion of variance explained by the first k components is calculated as:

$$\frac{\sum_{j=1}^{k}\lambda_j}{\sum_{j=1}^{J}\lambda_j} \quad (2)$$

This establishes the mathematical basis for multivariate analysis using HJ-Biplot.

2.2. Multivariate analysis using HJ-Biplot

The HJ-Biplot, developed by Galindo [13], is a multivariate analysis methodology that optimizes the simultaneous representation of individuals and variables in a reduced-dimensional space, usually two-dimensional. It arises as an extension and improvement of the traditional Biplot proposed by Gabriel [14]. Mathematically, from a centered data matrix $X \in R^{IxJ}$, where $I\mu$ is the number of individuals and $J\mu$ is the number of variables, a singular value decomposition (SVD) is performed:

$$X = UDV^T \quad (3)$$

Where:

- Ucontains the singular vectors on the left (associated with individuals)
- Vcontains the correct singular vectors (associated with variables)
- DIt is a diagonal matrix with ordered non-negative singular values.

The HJ-Biplot assigns to:

- The individuals:$A = UD$
- The variables:$B = VD$

This mapping ensures the highest quality of joint representation of individuals and variables in the same graph. These visualizations allow variables to be interpreted through vectors or arrows pointing in the direction of growth of the variable, and the length of the variable represents its variability, indicating that angles close to 0° or 360° present strong direct correlations. Those close to 180° present strong inverse correlations, and angles close to 90° present independence. Furthermore, individuals will be better represented by a variable the closer they are to the vector that represents it, and orthogonal projections to them can provide a reference of the value above or below the mean in terms of the proportion of the standard deviation. In addition, the proximity between individuals (similarity) is visualized [15].

An important extension is the dynamic HJ-Biplot, which allows to visualize multivariate data evolving over time or under different conditions. This variant is implemented in R by combining the tkrplot and dynBiplotGUI packages. The tkrplot package allows to render interactive graphs in dynamic windows. On the other hand, dynBiplotGUI creates a graphical interface to select analysis periods and dynamically represent the trajectories of individuals or variables over time [16][17].

With this tool, you can analyze structured data in three ways (individual, variable, and temporal), selecting time intervals and observing how their relative positions change. This is especially useful in longitudinal studies.

2.3. Plithogenic Statistics (PS).

Plithogenic Statistics (PS) is an advanced methodological framework that allows for the integration and analysis of diverse data, overcoming the limitations of traditional statistical techniques. While conventional methods typically examine variables in isolation, PS explores the complex interrelationships between multiple factors, offering a deeper understanding of the studied phenomena. This versatility makes it valuable in fields such as pedagogy, applied economics, and biomedical research [18].

In the educational field, PEs are especially useful for assessing the impact of pedagogical innovations, as they allow for analyzing how different elements (student performance, teacher development, etc.) interact and how they collectively influence academic outcomes. This approach reveals patterns that would otherwise go unnoticed with traditional methodologies, providing key

information for designing more effective educational strategies.

One of the greatest strengths of PEs is their ability to combine quantitative data (such as test scores) with qualitative information (such as teacher and student perceptions), generating a more complete assessment of educational processes [19]. Furthermore, their ability to process large volumes of data and detect complex relationships is crucial in the analysis of educational policies, where effects can manifest directly and indirectly in multiple dimensions.

PEs also facilitate the personalization of educational strategies by identifying specific patterns in different groups, allowing for more precise interventions tailored to diverse educational contexts [20]. However, their implementation is not without challenges, as they require specialized knowledge in advanced analytics and a significant investment in data collection and processing. Even so, their potential to improve the understanding of educational systems justifies these efforts [21].

In conclusion, EPs represent a powerful tool for the study of pedagogical phenomena, offering valuable elements for educational decision-making. Despite their technical challenges, they have established themselves as an indispensable resource for researchers and planners committed to academic excellence [20].

Plithogenic Statistics (PS) encompasses the analysis and observations of the events under study. It allows for the analysis of many output variables that are neutrosophic or indeterminate.

Subclasses of Plithogenic Statistics (EP)

Statistics **(PS)** encompasses various specialized methodologies that expand the analytical capabilities of complex scenarios. Its main subclasses include:

1. **Multivariate Plithogenic Statistics** – Analyzes multiple variables simultaneously, identifying patterns and correlations in multidimensional data sets.
2. **Plithogenic Neutrosophic Statistics** – Incorporates Neutrosophic (logic that considers true, false, and indeterminate) to handle inaccurate or contradictory information.
3. **Plithogenic Indeterminate Statistics** – They work with incomplete or uncertain data, allowing robust inferences despite ambiguity.
4. **Plithogenic Intuitionistic Fuzzy Statistics** – Combines fuzzy and intuitionistic logic to model degrees of membership and non-membership in fuzzy sets.
5. **Fuzzy Statistics of Plithogenic Images** – Applied in image processing, they analyze visual information with diffuse and overlapping components.
6. **Plithogenic Spherical Fuzzy Statistics** – Uses spherical fuzzy sets to represent data in multidimensional spaces with degrees of uncertainty.
7. **Fuzzy Extension Plithogenic Statistics (general)** – Covers all variants that integrate fuzzy logic to handle inaccuracies in data analysis.

These subclasses allow **EPs to be adapted** to different contexts, from artificial intelligence to social sciences, improving the interpretation of phenomena with high levels of complexity and uncertainty.

In a neutrosophic population, each element has a triple probability of affiliation (T_j, I_j, F_j), where$T_j, I_j, F_j \in [0,1]$ like that $0 \leq T_j + I_j + F_j \leq 3$.

If we assume that we must have the data set $(T_j, I_j, F_j)$for $j = 1, 2, \dots, n$, where n is the sample size, then the average probability of all the data in the sample is calculated by Equation 4.

$$\frac{1}{n}\sum_{j=1}^{n}(T_j, I_j, F_j) = \left(\frac{\sum_{j=1}^{n} T_j}{n}, \frac{\sum_{j=1}^{n} I_j}{n}, \frac{\sum_{j=1}^{n} F_j}{n}\right) \quad (4)$$

In this investigation, we also consider some operations in the form of *neutrosophic numbers.* These ways of representing indeterminacy are, under certain conditions, equivalent to working with intervals.

Definition 1: ([21-22]) A *neutrosophic number* N is defined as a number as follows:

$$N = d + I \quad (5)$$

Where d is called *the determinate part* and I is called *the indeterminate part.*

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ They are two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_2 + (b_1 + b_2)I$ (Addition);

$N_1 - N_2 = a_1 - a_2 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1 a_2 + (a_1 b_2 + b_1 a_2 + b_1 b_2)I$ (Product),

$\frac{N_1}{N_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$ (Division).

Furthermore, arithmetic operations between intervals are important in this document, which are summarized below ([23]):

Given$I_1 = [a_1, b_1]$ and$I_2 = [a_2, b_2]$ We have the following operations between them:

$I_1 \leq I_2$ if and only if$a_1 \leq a_2$ and $b_1 \leq b_2$.

$I_1 + I_2 = [a_1 + a_2, b_1 + b_2]$ (Addition);

$I_1 - I_2 = [a_1 - b_2, b_1 - a_2]$ (Subtraction),

$I_1 \cdot I_2 = [\min\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}, \max\{a_1 \cdot b_1, a_1 \cdot b_2, a_2 \cdot b_1, a_2 \cdot b_2\}]$ (Product),

$I_1 / I_2 = I_1 \cdot (1/I_2) = \{a/b: a \in I_1, b \in I_2\}$, always that $0 \notin I_2$(Division).

3. Results

3.1. Segmentation of 4 variables and 137 institutions

According to the segmentation presented in the methodology, four variables were considered for this initial analysis, which yielded 137 institutions with values. Imputation was avoided to improve the quality of representation and estimate bias. Figure 1 therefore presents the dendrogram of the cluster analysis. The final merger structure reveals two large groups, but visually, there are four groups with reasonable distances for establishing the clusters. Therefore, a four-segment analysis and a general analysis are proposed.

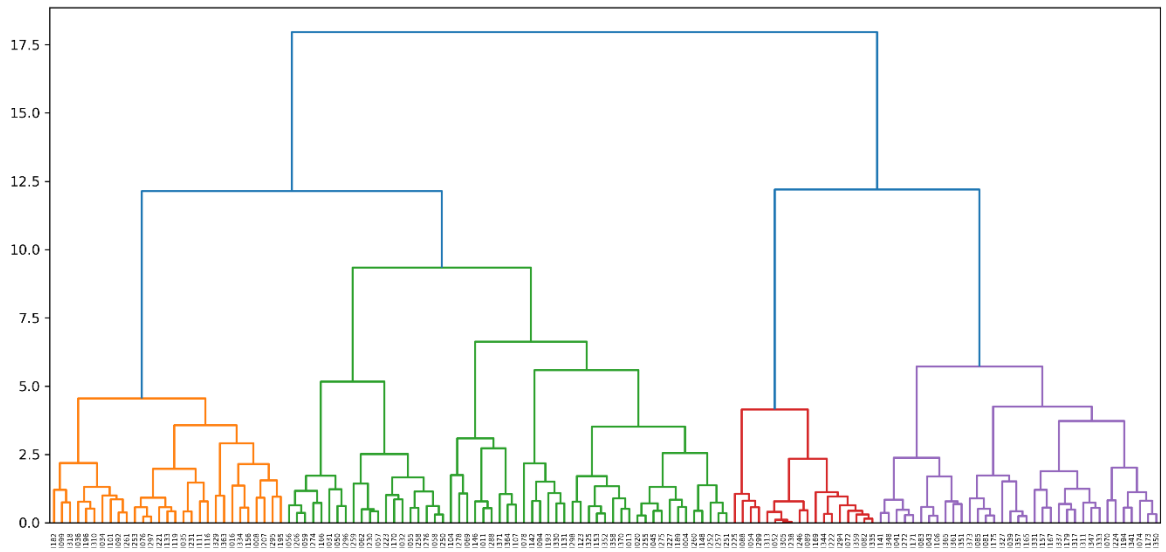

Figure 1. Segmentation of the dendrogram of four variables and 137 institutions.

Table 4 shows the distribution of institutions grouped by country and group. In Group 4, Brazilian and Chilean institutions predominate. Therefore, these are the most influential and have the greatest presence at the top of the ranking, suggesting that this group is the most represented.

Table 4. Distribution by country segmentation of four variables and 137 institutions.

Country	Group 1	Group 2	Group 3	Group 4	Total
Argentina	7	12			19
Bolivia		1			1
Brazil	2	3	9	28	42
Chili		6	6	7	19
Colombia	4	6	3		13
Costa Rica	2	1			3
Cuba	1	1			2
Ecuador	1	4			5
Mexico	8	13			21
Paraguay	1				1
Peru	1	3			4
Puerto Rico		1			1
Uruguay	2	1			3
Venezuela		3			3
Total	29	55	18	35	137

The following is a general analysis, and there is also an analysis segmented by groups to evaluate the influences of the variables of the international classifications according to the segment analyzed. Figure 2 shows the dynamic HJ-Biplot of 4 variables covering the 137 institutions in the analysis. The dynamics are observed with a 90% inertia filter to visualize the largest movements of institutions and with a 0% inertia filter on the variables. In addition, more than 78% of the observed variance is absorbed, which ensures a very good visualization quality.

It is shown that the variables of *papers per faculty* and *citations paper* have a high correlation and a very similar dynamic throughout the years of analysis. Moreover, a high correlation is observed in 2023, which has decreased over the last years.

On the other hand, it is verified that the institutions Universidad Federal de São Paulo, Universidad Federal do ABC, Universidad Federal do Rio Grande, Universidad de La Frontera (UFRO), and Universidad de La Habana report the highest dynamics of movement with respect to the associated filters, focusing on values close to the mean without many variations or predominance.

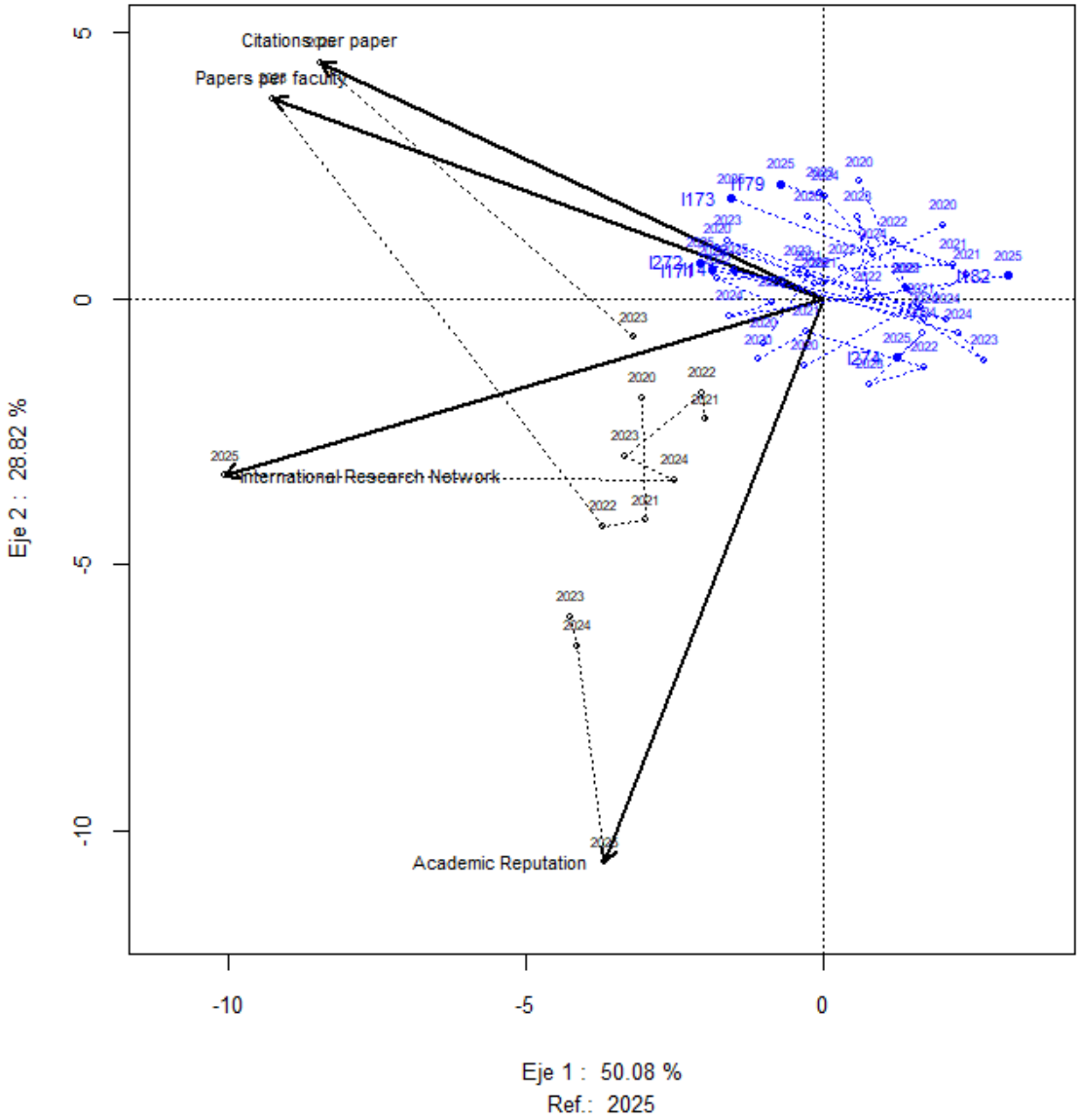

Figure 2. General HJ-Biplot analysis with four variables.

Figure 3 shows the HJ-Biplot representation of the multivariate data corresponding to each of the four clusters indicated at the beginning of this section, showing how the dynamics of the variables differ according to each cluster.

Variance absorptions greater than 70% are observed in all clusters except the first group, indicating high variability with lower representation quality but maintaining acceptable parameters.

It should be noted that, in the first and second groups, the variables of academic citations and scientific publications show practically no correlation, demonstrating the different parameters of this group in the analysis of their publications. Furthermore, clear heterogeneity is detected around these

two values, with no variable standing out in the representation of the institutions displayed. Furthermore, a strong inverse correlation is evident between academic reputation and indicators of research productivity and quality in these groups. This raises a debate about the parameters used by international academics participating in these surveys to define a higher education institution with a good reputation.

Furthermore, in Clusters 3 and 4, the correlations between research-related variables are more clearly observed. The distribution of institutions in Cluster 4 is more homogeneous, showing high variability and robust correlations between indicators of research productivity and quality, on the one hand, and, on the other, indicators of reputation and international research network. The latter is what would encompass the most prestigious and internationally recognized institutions, especially in Cluster 3, where institutions remain among the top 80 in the ranking.

Additionally, atypical data are observed from the third cluster, such as the Adolfo Ibáñez University (I088) and the National University of Colombia (I225), which are well below average in research indicators.

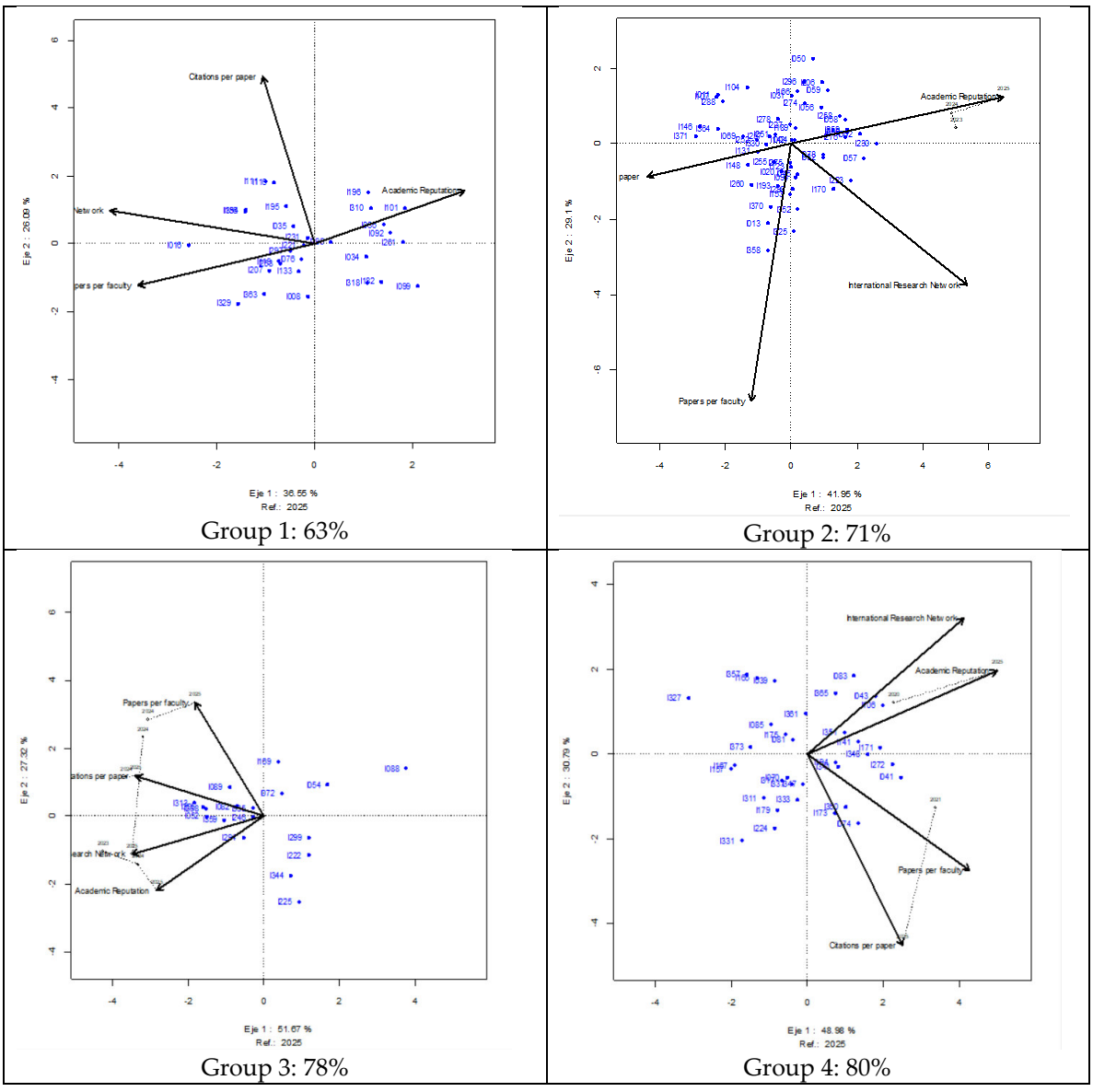

Figure 3. HJ Biplot by cluster, reference 2025.

3.1. Segmentation of 5 variables and 125 institutions

According to the segmentation shown in the methodology, five variables were considered for the second analysis, resulting in 125 institutions with values, to which the variable of faculty with doctorates was added. Figure 4 illustrates the dendrogram from the cluster analysis, showing that the final merger structure conglomerates two large groups, but visually, four groups are observed with reasonable distances to establish the clusters.

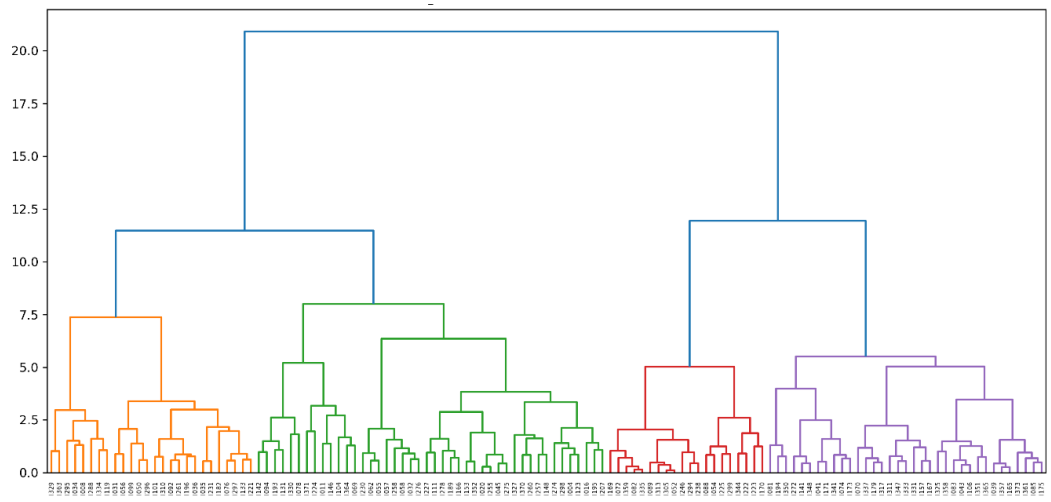

Figure 4. Segmentation of the dendrogram with five variables and 125 institutions.

Table 5 shows the distribution of institutions grouped by country and group. Group 4, composed of Brazilian and Chilean institutions, predominates. These are the most influential and have the greatest presence at the top of the ranking. The second group, in turn, has the largest number of institutions, demonstrating the presence of this profile in the Latin American ranking.

Table 5. Distribution by country and cluster of five variables.

Country	Group 1	Group 2	Group 3	Group 4	Total
Argentina	7	8			15
Brazil	2	3	9	28	42
Chili		6	6	7	19
Colombia	2	8	3		13
Costa Rica	1	1			2
Cuba		1			1
Ecuador	1	3			4
Mexico	8	9	2		19
Paraguay	1				1
Peru	1	2			3
Puerto Rico		1			1
Uruguay	2	1			3
Venezuela	1	1			2
Total	26	44	20	35	125

Figure 5 presents the dynamic five-variable HJ-Biplot covering the 125 institutions analyzed. The dynamics are observed with a 90% inertia filter to visualize the main movements of the institutions and a 0% inertia filter for the variables. Furthermore, more than 74% of the observed variance is absorbed, ensuring excellent visualization quality.

Direct correlations are observed between all the variables analyzed, except for Academic Reputation and Academic Staff with Doctorates, which are practically independent. This generates a broad debate about how international and regional academics currently view academic reputation, unrelated to the proportion of academics with doctorates in higher education institutions. This could imply the visibility of an HEI's high-impact research, rather than the volume of scientific output, given that the latter two establish low-level direct correlations.

We can also see universities such as Universidad Federal de Santa Catarina, Universidad Nacional Agraria la Molina, Universidad Estadual de Maringá, The Federal University of Lavras and the Federal University of Pelotas are the most dynamic institutions that continue to be visualized using the inertia filter, which defines them as HEIs with high variability over the analysis period. Increases are observed in the variable of academic staff with doctorates in 2025, but considerable decreases in all the analysis variables by 2024.

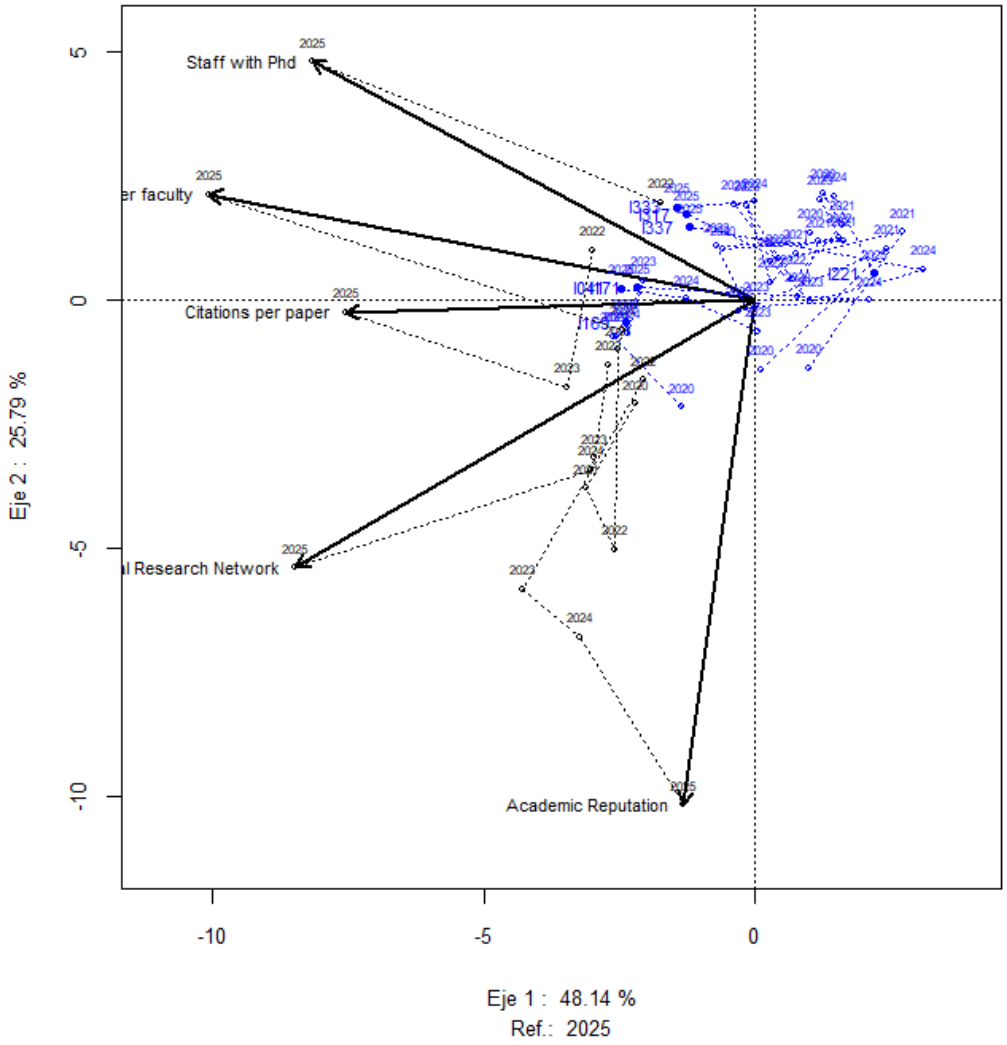

Figure 5. General HJ-Biplot analysis with five variables.

Figure 6 presents the multivariate representation of the HJ-Biplot by cluster analysis. Variance absorptions exceeding 60% are obtained in all groups, indicating acceptable representation quality.

In clusters 1 and 2, a high degree of heterogeneity was detected in the distribution of the groups in relation to the variables analyzed. Likewise, there is a strong correlation between the volume of scientific publications produced by the institutions' academic staff and the proportion of individuals holding doctoral degrees. A strong correlation was also found between the international research network and academic reputation, which, according to the analyses presented, would indicate a strong influence between them. Finally, there is virtually no correlation between publication citations and the other variables analyzed in the aforementioned clusters.

Cluster 3 is the most homogeneous and contains the highest-ranked institutions. Therefore, it is the group with the highest performance, supported by the correlation established by these institutions among all the research-related analysis variables. In this group, once again, Adolfo Ibáñez University, this time accompanied by the National Autonomous University of Mexico, appears as an extreme group, with values well below average, particularly in the case of the second institution, specifically in the scientific productivity indicators.

Finally, the fourth cluster is seen as a highly heterogeneous group and the only one in which the proportion of academics with doctorates shows a strong inverse correlation with faculty research. This cluster acts as a conglomerate without a clear categorization, but its set of variables presents similar results.

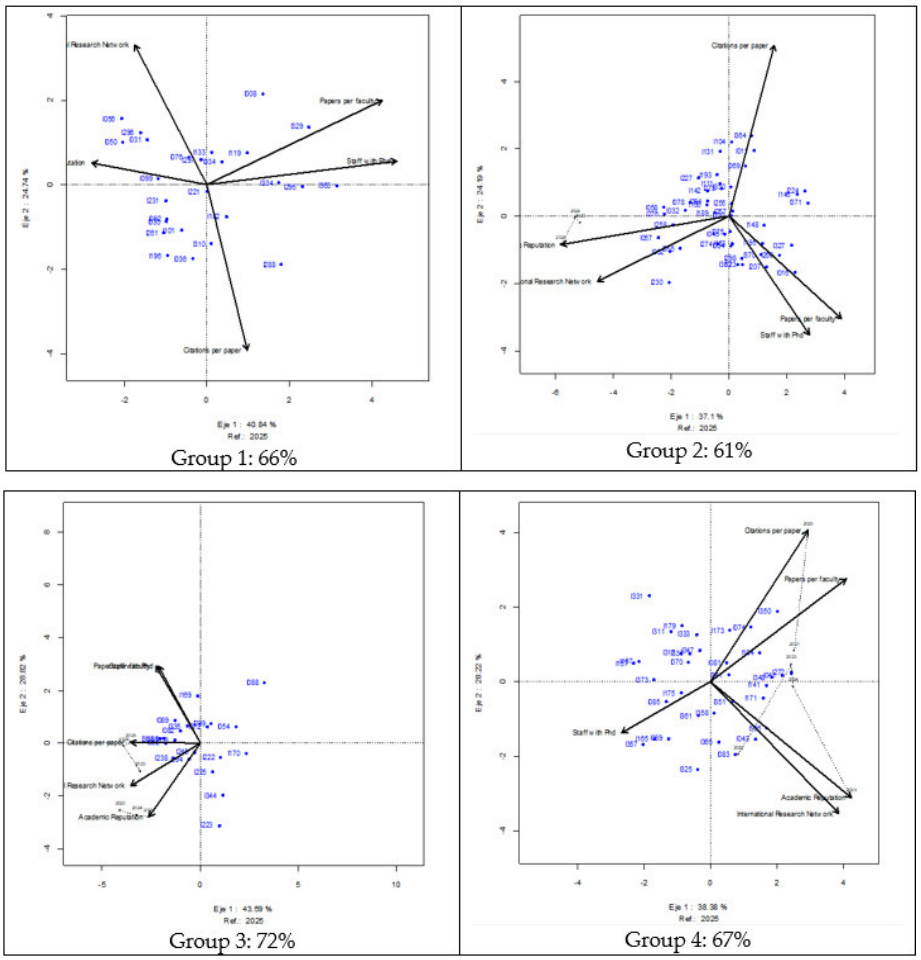

Figure 6. HJ Biplot by cluster, reference 2025.

3.2. Neutrosophic Analysis of University Rankings Using Plithogenic Statistics

To complement the dynamic multivariate analysis of research trajectories in Latin American universities, we employ Plithogenic Statistics, a neutrosophic approach that models the truth (T), indeterminacy (I) and falsity (F) of ranking indicators, capturing the ambiguities and contradictions inherent in university performance [18, 19]. This approach is particularly relevant in the Latin American context, where structural gaps and institutional heterogeneity generate uncertainty in the evaluation of indicators such as publications per professor, citations per article, academic reputation, international research networks and proportion of staff with doctorates.

3.2.1. Application to Segmentation Results

Plithogenic analysis was applied to the clusters identified in the 4-variable (137 institutions) and 5-variable (125 institutions) segmentations described in Sections 3.1 and 3.2. Two representative universities per cluster were selected to illustrate the approach, based on their position in the HJ-Biplot and their temporal dynamics. The indicators analyzed are: publications per professor (PP), citations per article (CA), academic reputation (RA), international research networks (IR), and proportion of staff with doctorates (PD, only in the 5-variable segmentation).

Variable Segmentation

For the segmentation of four variables (PP, CA, RA, RI), neutrosophic values were assigned to each indicator based on normalized ranking data (scale [0, 1]) and expert opinion. For example, for the University of São Paulo (USP) (Cluster 4, leading institution) and the University of Havana (UH) (Cluster 1, with high variability), the following plithogenic values were defined for 2025:

USP (Cluster 4):

- PP: $(T = 0.90, I = 0.05, F = 0.05)$– High number of publications, low uncertainty.
- CA: $(T = 0.85, I = 0.10, F = 0.05)$ – High impact, some interannual variability.
- RA: $(T = 0.80, I = 0.15, F = 0.05)$– Strong reputation, but with subjectivity.
- RI: $(T = 0.95, I = 0.03, F = 0.02)$ – Extensive international networks.

UH (Cluster 1):

- PP: $(T = 0.40, I = 0.50, F = 0.10)$ –Moderate production, high indeterminacy.
- CA: $(T = 0.35, I = 0.55, F = 0.10)$– Low impact, high uncertainty.
- RA: $(T = 0.50, I = 0.40, F = 0.10)$ – Moderate, subjective reputation.
- RI: $(T = 0.30, I = 0.60, F = 0.10)$ – Limited networks, high indeterminacy.

The average plithogenic probability per cluster was calculated using Equation 1. For Cluster 4 (n=35):

$$\boldsymbol{PCl\acute{u}ster\ 4} = \left(\boldsymbol{\Sigma}\frac{\boldsymbol{Ti}}{\boldsymbol{35}}, \boldsymbol{\Sigma}\frac{\boldsymbol{Ii}}{\boldsymbol{35}}, \boldsymbol{\Sigma}\frac{\boldsymbol{Fi}}{\boldsymbol{35}}\right)$$

Assuming estimated average values for the cluster (based on USP and other leading institutions):

- PP: $PPP = (0.85, 0.10, 0.05)$

- $CA: PCA = (0.80, 0.15, 0.05)$
- $RA: PRA = (0.75, 0.20, 0.05)$
- $RI: PRI = (0.90, 0.08, 0.02)$

To combine indicators, the neutrosophic summation was used (Section 2.3):

$$(\boldsymbol{A_1}, \boldsymbol{I_1}) + (\boldsymbol{A_2}, \boldsymbol{I_2}) = (\boldsymbol{A_1} + \boldsymbol{A_2} - \boldsymbol{A_1 A_2}, \boldsymbol{I_1 I_2})$$

For example, for PP and CA in Cluster 4:

- $TPP + CA = 0.85 + 0.80 - 0.85 \times 0.80 = 0.97$
- $IPP + CA = 0.10 \times 0.15 = 0.015$
- $FPP + CA = 0.05 \times 0.05 = 0.0025$

Result: $\boldsymbol{PPP + CA} = (\mathbf{0.97}, \mathbf{0.015}, \mathbf{0.0025})$, indicating high performance combined with minimal indeterminacy.

The degree of contradiction between RA and RI, based on the high correlation observed in Cluster 4 (Section 3.1), is:

$$\boldsymbol{c(RA, RI) = 1 - 0.85 = 0.15}$$

This reflects a low contradiction, consistent with the homogeneity of Cluster 4.

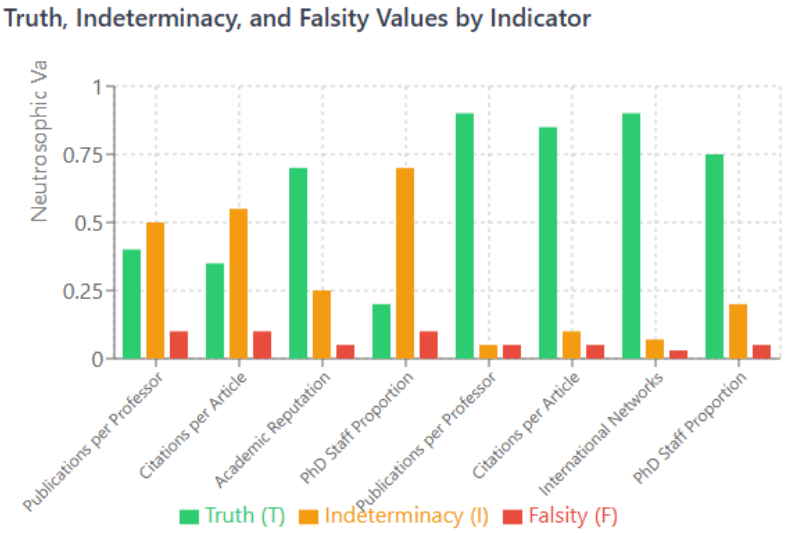

Figure 7: Neutrosophic Analysis of Latin American University Rankings

Variable Segmentation

For the segmentation of five variables (PP, CA, RA, RI, PD), the proportion of staff with doctorates was included. The Adolfo Ibáñez University (UAI) (Cluster 3, atypical) and the Federal University of Santa Catarina (UFSC) (Cluster 4, dynamic) were analyzed. Estimated plithogenic values for 2025:

UAI (Cluster 3):

- PP: $(T = 0.30, I = 0.60, F = 0.10)$– Low productivity, high indeterminacy.
- CA: $(T = 0.25, I = 0.65, F = 0.10)$ –Low impact, high uncertainty.
- RA: $(T = 0.70, I = 0.25, F = 0.05)$– Good reputation, somewhat subjective.
- RI: $(T = 0.40, I = 0.50, F = 0.10)$ – Moderate networks.
- PD: $(T = 0.20, I = 0.70, F = 0.10)$– Low proportion of doctorates.

UFSC (Cluster 4):

- PP: $(T = 0.88, I = 0.08, F = 0.04)$ – High productivity.
- CA: $(T = 0.85, I = 0.10, F = 0.05)$– High impact.
- RA: $(T = 0.78, I = 0.15, F = 0.07)$– Strong reputation.
- RI: $(T = 0.90, I = 0.07, F = 0.03)$ –Extensive networks.
- PD: $(T = 0.75, I = 0.20, F = 0.05)$– High proportion of doctorates.

For Cluster 3 (n=20), the average plithogenic probability is:

- PP: $PPP = (0.35, 0.55, 0.10)$
- CA: $PCA = (0.30, 0.60, 0.10)$
- RA: $PRA = (0.65, 0.30, 0.05)$
- RI: $PRI = (0.45, 0.45, 0.10)$
- PD: $PPD = (0.25, 0.65, 0.10)$

The contradiction between RA and PD, based on the null correlation (Section 3.2), is:

$$\boldsymbol{c(RA, PD) = 1 - 0.05 = 0.95}$$

This confirms the high contradiction observed in the HJ- Biplot, where reputation does not align with the proportion of PhDs.

Table 6: Plithogenic Values by Cluster and Variable

Cluster	Variable	Representative University	Truth (T)	Indeterminacy (I)	Falsehood (F)	Interpretation
1	PP	University of Havana	0.40	0.50	0.10	Moderate production, high uncertainty
1	AC	University of Havana	0.35	0.55	0.10	Low impact, high indeterminacy
3	RA	Adolfo Ibáñez University	0.70	0.25	0.05	Good reputation, somewhat subjective

Cluster	Variable	Representative University	Truth (T)	Indeterminacy (I)	Falsehood (F)	Interpretation
3	P.S.	Adolfo Ibáñez University	0.20	0.70	0.10	Low proportion of doctorates
4	PP	University of São Paulo	0.90	0.05	0.05	High number of publications
4	AC	University of São Paulo	0.85	0.10	0.05	High scientific impact
4	RI	UFSC	0.90	0.07	0.03	Extensive international networks
4	P.S.	UFSC	0.75	0.20	0.05	High proportion of doctorates

Note: PP = Publications per Professor, CA = Citations per Article, RA = Academic Reputation, RI = International Networks, PD = Proportion of Staff with PhDs

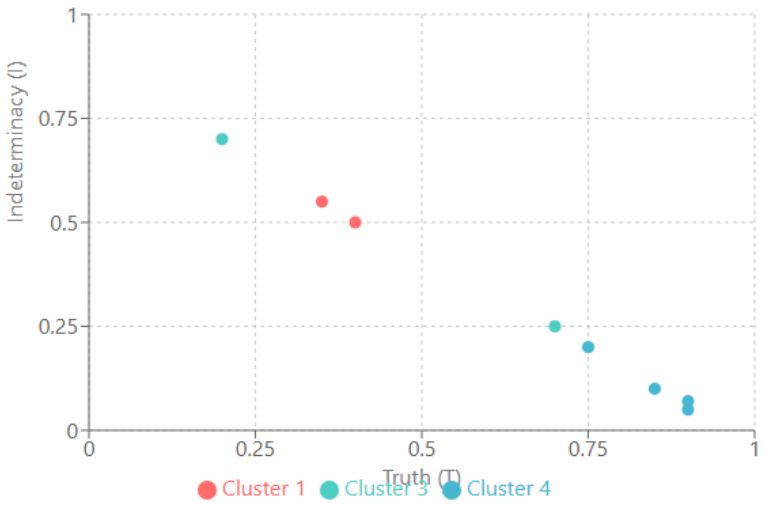

Figure 8: Truth vs Indeterminacy Distribution by Cluster

3.2.2. Contributions to Research Paths

The plithogenic analysis reveals that high-performance clusters (3 and 4) present high truth values (T>0.75) and low indeterminacy (I<0.20) in indicators such as PP, CA, and RI, reflecting consolidated research trajectories. In contrast, clusters 1 and 2 show high indeterminacy (I>0.50), indicating structural challenges in scientific production and visibility.

The inclusion of PD in the 5-variable segmentation accentuates the gaps, with universities such as UAI showing high indeterminacy in PD (I=0.70), suggesting that their reputation is not backed by formal research capacity.

These results enrich multivariate analysis by quantifying uncertainty and inconsistencies, offering a tool for universities to identify specific areas for improvement. For example, policies that reduce indeterminacy in PhD (through doctoral training) could improve performance in lower-performing clusters. Plithogenic Statistics, by modeling these dynamics, contributes to a more complete understanding of research trajectories, aligning with the article's title and supporting Latin America's academic competitiveness on the global stage.

4. Applications

The results of this study, which combine dynamic multivariate analysis with Plithogenic Statistics, offer a strategic tool for Latin American universities to boost their continuous improvement processes in research. The identification of differentiated clusters based on research performance, enriched by the plithogenic approach, allows institutions to position themselves in more realistic reference groups, considering not only performance on indicators such as publications per professor, citations per article, academic reputation, and international research networks, but also the uncertainties and contradictions associated with these indicators. For example, the plithogenic analysis shows that universities in clusters 1 and 2 face high uncertainty in indicators such as publications per professor and citations per article, reflecting structural limitations in their research trajectories. In contrast, clusters 3 and 4, dominated by Brazilian and Chilean universities, present solid performance with low uncertainty in these same indicators, indicating more consolidated research trajectories. This approach, which incorporates uncertainty, allows universities such as the University of Havana, in Cluster 1, to identify the need to strengthen their scientific production to reduce the ambiguity in their positioning, while institutions such as the Federal University of Santa Catarina, in Cluster 4, can consolidate their leadership. These distinctions can be integrated into university strategic plans to prioritize investments in scientific productivity, internationalization, and the training of doctoral scholars, tailored to the specific needs of each cluster.

From a public policy perspective, the results provide national higher education organizations and quality assurance agencies with a framework for designing more precise support instruments. The plithogenic analysis highlights significant structural gaps, such as the marked contradiction between academic reputation and the proportion of staff with doctorates in Cluster 3, where universities such as Adolfo Ibáñez University show high uncertainty in the latter indicator. This suggests that public policies should go beyond promoting the number of publications, also focusing on strengthening internal academic capacities by training doctoral graduates and consolidating international research networks. For example, universities with high uncertainty in the proportion of staff with doctorates could benefit from national programs that incentivize doctoral training, thereby reducing ambiguities in their performance. This combined framework of multivariate and plithogenic analysis is replicable for long-term monitoring of the impact of scientific promotion policies, offering a more complete view of how interventions affect research trajectories in the region.

Finally, the dynamic HJ- Biplot approach , complemented by Plithogenic Statistics, represents a powerful tool for internal university quality management. This methodology allows institutions not only to evaluate their static performance in the rankings but also to understand the evolution of their research indicators over time, considering the uncertainties and contradictions that influence their positioning. For example, Adolfo Ibáñez University, with high uncertainty in scientific productivity indicators, can use this analysis to identify critical areas for improvement, such as strengthening its research capacity, while the Federal University of Santa Catarina, with robust performance and low

uncertainty, can consolidate its position in international networks and doctoral training. In a context of increasing international competition, this combined approach provides Latin American universities with a comparative advantage, allowing them to make decisions based on a deeper understanding of the dynamics of their research trajectories and strengthen their position in the global higher education market.

5. Conclusions

This study demonstrated that the research trajectories of Latin American universities present heterogeneous evolution patterns, influenced by structural gaps in scientific productivity, academic impact, and international collaboration networks. Through hierarchical cluster analysis, dynamic HJ-Biplot representations, and Plitogenic Statistics, clusters of institutions with distinct profiles were identified, highlighting high-performing groups dominated by Brazilian and Chilean universities [18, 19]. The incorporation of the plitogenic approach made it possible to capture uncertainties and contradictions among indicators, enriching the understanding of these trajectories.

The results show that indicators such as publications per professor and citations per article maintain strong correlations in the best-positioned institutions, especially in Clusters 3 and 4, where they exhibit solid performance with low uncertainty. However, academic reputation does not always align with the proportion of academics with doctoral degrees, evidencing a notable contradiction, particularly in Cluster 3, where universities such as Adolfo Ibáñez University exhibit high uncertainty in indicators of scientific productivity and doctoral training. This tension between perception and actual research capacity highlights the importance of considering uncertainty in the evaluation of university performance. The temporal stability of some universities contrasts with the high variability of others, such as the University of Havana in Cluster 1, which shows great uncertainty in publications and citations, reflecting dynamics of institutional consolidation or vulnerability in the global context.

The four-variable analysis identified clusters focused on academic production and visibility, while incorporating the proportion of staff with doctorates into the five-variable analysis revealed deeper structural gaps related to institutional training. High-performing clusters include universities with a consolidated research tradition, such as the Federal University of Santa Catarina, which shows low uncertainty in key indicators. In contrast, lower-performing clusters, such as Clusters 1 and 2, face limitations in scientific production and academic staff qualifications, with high uncertainty that hinders their positioning in international rankings.

In the Latin American context, the findings underscore the strong asymmetries between countries and institutions. Brazil and Chile concentrate the majority of universities in leading clusters, while other nations face significant challenges in achieving international research standards. The proportion of staff with doctorates accentuated these differences, showing that consolidating highly qualified academic bodies remains a challenge in the region. The plithogenic analysis highlights that the high uncertainty in this indicator, especially in universities in lower-performing clusters, requires specific attention to improve research trajectories. Therefore, the study reaffirms the importance of developing differentiated public policies that promote research training and the strengthening of international networks, integrating the plithogenic perspective to address ambiguities and contradictions in ranking indicators, in order to boost Latin America's academic competitiveness on the global stage.

Finally, dynamic multivariate methods, such as HJ- Biplot, combined with Plithogenic Statistics, are consolidated as robust tools for longitudinal monitoring of research quality. This combined approach provides key information for strategic decision-making in Latin American higher education, allowing universities and governments to design strategies that not only improve ranking performance but also reduce uncertainty in critical indicators, strengthening the region's research trajectories and global position.

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University of New Mexico

Predictive model of school dropout in higher education through Learning Analytics with Neutrosophic Plithogenic Hypotheses

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Abstract: Higher education dropout rates are an international problem, as many people abandon their studies, especially in the early years. This problem affects institutions, causing significant losses. In this context, it is essential to consider methods that allow for predicting dropout rates and taking rapid and effective measures. Objective: The main objective is to analyze prediction models using Learning Analytics, integrating neutrosophic plithogenic hypotheses, to identify early the risk of dropout in higher education. This provides institutions with useful tools to implement actions that help retain more students, considering uncertainty and complex interactions between academic and non-academic factors. Methodology: The method used was quantitative, based on a selection of data from Bernardo O'Higgins University. Statistical methods with information extraction through logistic regression, factor analysis and HJ-Biplot were employed, complemented by a neutrosophic plithogenic hypothesis approach to model uncertainty in predictor variables, showing clarity regarding students who could drop out. Results: These results make it possible to identify students with problems more quickly, facilitating the implementation of specific supports from the institution, with greater precision by incorporating indeterminacy and plithogenic interactions between variables. Conclusion: The combination of statistical models with neutrosophic plithogenic hypotheses becomes a useful tool to address student dropout, allowing the development of rapid actions that contribute to educational improvement by capturing the complexity of the factors involved.

Keywords: Learning analytics, student retention, predictive modeling, higher education, dropout prevention, HJ-Biplot, neutrosophic plithogenic hypotheses.

1. Introduction

In recent years, rising student dropout rates in universities have raised significant concerns about student retention and success, prompting faculty and administrators to seek new solutions to this serious problem. As higher education moves into the digital age, the use of learning analytics has emerged as an effective tool for understanding and predicting student behavior, particularly dropout rates. Learning analytics involves collecting data from students to identify patterns and trends that indicate the likelihood of students dropping out. By using different data-driven methods, such as machine learning algorithms and statistical models, institutions can identify at-risk students early in their educational journeys, enabling timely intervention to increase retention rates [1,2]. Incorporating neutrosophic plithogenic hypotheses into these predictive models allows for addressing uncertainty

and multidimensional interactions between academic and non-academic factors, enhancing the ability to identify dropout risks more accurately. Furthermore, the positive results that predictive models can provide allow us to examine the most useful ways to analyze dropout rates and demonstrate the transformative effect that data can have on higher education. Ultimately, this research aims to provide important insights into how predictive analytics, complemented by neutrosophic plithogenic hypotheses, could reduce student dropout rates and create a more cohesive and engaging learning environment for all students.

Using tools to study how people learn (HL) helps predict who will drop out of higher education. This involves different aspects, such as gathering information, analyzing it, and finding ways to help. An important feature of HL is its ability to see and understand student indicators to improve their learning; this helps predict who may drop out of the institution [3]. Using feature enhancement techniques, basic data can be extended and completed, improving models that predict who might drop out [4]. With neutrosophic plithogenic hypotheses, these techniques are enhanced by modeling variables with degrees of truth, falsity, and indeterminacy, capturing the complexity of interactions between factors such as academic performance and socioeconomic conditions. This is very important because it creates a foundation on which prediction models operate, allowing them to better identify which students may struggle. The application of simple but useful methods, such as Biplot, logistic regression, among others, has also been shown to achieve reliable levels of accuracy in identifying predictors of student dropout [4]. These methods are very important in the analytical part of AI, as they help in various ways to build and validate the models necessary to create robust predictive capabilities. Furthermore, the predictive capacity of self-learning was fundamental in severe cases such as the COVID-19 pandemic, where it provided information on student engagement and success, two key factors in understanding and anticipating the risk of dropping out of school [5]. Consequently, good management of self-learning not only helps identify those most likely to drop out of school, but also facilitates the creation of early intervention measures. This demonstrates the importance of continuing to invest in self-learning tools and methods to better support struggling students and ensure that more young people remain in school.

Furthermore, combining data from outside the institution, such as economic status and behavior, is increasingly considered important for creating effective models that predict who will drop out [6]. These non-academic factors are in addition to the data typically collected in learning environments, and their use can improve the model's predictive power. Above all, socioeconomic factors can provide insights into challenges students face outside the academic institution, which are not evident solely in their classroom work. Moreover, their behavior, often observed in learning management systems (LMS), can reveal their level of interest and indications of potential problems, which is essential for providing timely assistance [7]. The neutrosophic plithogenic approach allows these non-academic factors to be modeled by considering multiple levels of attributes and their contradictions, thereby improving the understanding of the complex dynamics affecting student retention. By combining these variables, researchers can create more comprehensive models that not only accurately predict academic underachievement but also provide useful insights for teachers and institutional leaders. These methods emphasize the importance of considering both academic and non-academic factors, demonstrating that the best predictive models are those that gather a wealth of information to address the complex nature of retention and underperformance at the institution [6].

Given the importance of combining external factors related to students to predict dropout rates at an institution, predictive analytics is increasingly useful for offering personalized interventions to at-risk students. Through predictive models, educational systems can identify the likelihood of students transferring, dropping out of school, or being suspended from classes, allowing for measures to address these potential problems [2]. This proactive approach not only reduces dropout rates but also increases student retention in the institution, which is critical for improving overall educational outcomes. Predictive statistical analysis facilitates these efforts by identifying which students need the

most guidance or support, so that programs or institutions can allocate resources appropriately and quickly [1]. Integrating neutrosophic plithogenic hypotheses into these models allows for a more robust representation of uncertainties and contradictions in the data, optimizing resource allocation and the identification of at-risk students. Furthermore, by analyzing large data sets, predictive analytics can find hidden links between student data, providing a clearer picture of behavior and patterns that can guide strategic planning and program growth [2]. Therefore, the use of predictive analytics in higher education not only helps monitor and reduce dropout rates but also creates a safer learning environment that responds to the multiple needs of students.

This research uses learning analytics to predict student dropout, revealing the crucial importance of variable selection for predictive model accuracy. Biplots are effective in identifying at-risk students, although their effectiveness depends on the quality of the data used. Incorporating neutrosophic plithogenic hypotheses optimizes the representation of these variables by modeling their multidimensional interactions and uncertainty, improving the ability to identify dropout risk patterns.

The study shows correlations between non-academic factors, socioeconomic conditions, academic performance, institutional support, and infrastructure, which require further research to understand their complex interactions. The neutrosophic plithogenic approach facilitates this understanding by modeling the contradictions and indeterminacies in these correlations, providing a more complete perspective on the dynamics of dropout. Ethical issues arise around the use of predictive models, particularly regarding privacy and the potential stigmatization of high-risk students.

Since higher education today is not only delivered through face-to-face classes, but virtual education has also grown exponentially, this has led to rethinking the logic of the variables that may affect a higher education student when considering dropping out of their studies . Davis's study analyzes the factors that influence the retention of young people in online postgraduate courses, particularly in a Master of Social Work (MSW) program taught by an American university [8]. Qualitative research examines the reasons why someone voluntarily drops out of the program through telephone interviews with alumni who dropped out over a period of five years.

One of the most significant findings shows that student retention in virtual environments depends largely on non-academic factors, such as personal, work and family problems, the quality of communication between the institution and the students, and problems in professional practices. These results are similar to those of more recent studies stating that factors external to the institution pose a constant challenge for students to continue their studies, especially among adults and workers [9]. Neutrosophic plithogenic modeling of these non-academic factors allows capturing their variable impact and contradictions, improving the prediction of dropout in virtual environments.

From a methodological perspective, the study uses semi-structured interviews, which allowed for the collection of detailed information. Although small (n=14), the sample is representative of the type of individuals who dropped out of the program. Among the most common reasons for dropping out are family health problems, lack of work-life balance, failure to find internships, and lack of academic support [9].

The article also offers a critical reflection on whether the freedom offered by online teaching is sufficient for progress. In reality, while students value being able to learn at their own pace and the opportunity to study from home, they also desire meaningful interactions, clarity in faculty feedback, and a more friendly and conversational environment [10]. These perspectives reinforce the importance of the role of the professor present in online classes; this involves not only transmitting information but also creating a sense of community among students.

In terms of concrete actions, having similar standards for all processes, training teachers in new technologies, and developing more understandable student guides provide a more comprehensive education. At the student level, they propose paying attention, supporting peers, and incorporating emotional management and self-care strategies as part of the teaching process.

On the other hand, the study indicates that online courses must go beyond simply observing students' performance in class and pay attention to what each student wants to do if we want to develop systems that help everyone continue studying. Although they stopped attending class, many of the participants said they would like to resume their studies in the future, demonstrating that maintaining contact after their departure can be a good way to attract students and improve the institution [11].

Another study by Greenland and Moore examines the reasons why students drop out of online courses. They used a large sample size (n=226) representing students who dropped modules at Open Universities Australia [11]. Through a detailed study, they identified 10 key variables and 41 subvariables for the reasons for dropping out, as well as 5 variables and 19 subvariables for the ideas suggested by the students themselves.

The results show that a high percentage of the reasons for dropping out of the institution are related to personal situations such as work, health, and family relationships; another percentage is related to aspects of the student's environment (motivation, possibility of studying online, resources), and only a small percentage is linked to institutional issues, such as paperwork or module design. This separation provides a new and detailed structural classification of student dropout [12]. Neutrosophic plithogenic hypotheses allow these variables to be modeled with degrees of indeterminacy, capturing the contradictions between personal and institutional factors for a more precise classification.

Among the ideas, students propose supports ranging from learning to better manage their schedules and a desire to do things on their own, to changes in local norms, such as more flexible hours, extra money, job changes, and more personalized support. These ideas challenge the long-held belief that many dropouts are inevitable [12] and reinforce student-centered support.

The research shows the need to change the way students are retained in learning environments, promoting easily adaptable standards based on verifiable evidence. It also proposes a strategy that connects the reasons for student dropout with useful actions, thus providing a better plan for more people to complete their higher education studies online.

For the reasons described above, the problem of student dropout in higher education has attracted considerable interest, prompting the search for new approaches using data analytics. Dropout in higher education is complex and affected by various factors, such as student performance, social class, and levels of support or connection with the institution. As institutions seek to improve graduation rates, predictive models are emerging as an effective way to use machine learning methods to identify at-risk individuals and intervene before problems arise [13]. This article takes an in-depth look at how learning analytics can be used as a data-driven method to determine the likelihood of students leaving an institution, emphasizing how explainable machine learning methods can study different aspects that enhance or hinder an educational platform [14]. Integrating neutrosophic plithogenic hypotheses into these methods allows for modeling uncertainty and contradictions in the data, providing a more complete understanding of the factors affecting dropout. By combining learning and studying, institutions can not only identify patterns in their students' actions, but also make specific changes according to the needs of each student and create an environment that promotes learning [15].

Burgos points out that useful forms of machine learning play a pivotal role in accurately predicting dropout rates. These methods are very important because they provide data on the reasons that lead to a high risk of dropping out, allowing teachers and institutional leaders to design targeted solutions [16]. Through clear models, stakeholders can observe struggling students in great detail and ensure that support is personalized and just-in-time. The results of the study show that prediction models can not only clarify turnover trends but can also assist with active strategies to reduce these risks. Being able to explain the main reasons why a student drops out of an institution is very important, as it allows schools to make data-driven decisions to reduce dropout rates. Therefore, there is a need to integrate useful forms of machine learning into educational frameworks to effectively address and reduce the difficulties that lead young people to drop out of school and ultimately improve educational

outcomes [6]. The neutrosophic plithogenic approach strengthens these strategies by modeling interactions between variables with degrees of indeterminacy, improving the precision of interventions.

By incorporating models into learning platforms, institutions can better understand how the support and systems they offer work. To realize this idea, they used a novel way of analyzing how the learning platform changed to better understand the reasons why students leave the institution, observe their level of engagement, and how many remain. The platform offers a unique space to observe the connections between students and their activity, providing data that can be used to develop better student retention strategies [17]. Neutrosophic plithogenic hypotheses enrich this analysis by modeling the connections between students with an approach that considers uncertainty and contradictions in their digital behavior. As teaching technology advances, incorporating predictive analytics into learning management systems offers a promising way to address dropout before it occurs.

Yousef and Britos demonstrate, combining learning data with educational sites such as Moodle is essential for identifying at-risk students and predicting their likelihood of dropping out based on their digital behavior [6,17]. Careful monitoring of information such as how frequently students log in to the platform, their participation in virtual discussions, and their assignment submission patterns can provide teachers with useful data on student interest and success [6,17]. Neutrosophic plithogenic modeling of this digital data makes it possible to capture uncertainty in behavioral patterns, improving the identification of at-risk students. This comprehensive monitoring allows institutions to identify students who are likely to drop out and helps them take early action to improve retention. Taken together, these studies show the importance of using data analytics to understand the complex nature of student behavior; this is key to identifying dropout rates and creating actions that support higher education [6,17]. Therefore, the use of learning analytics not only improves the learning experience, but also helps teachers make informed decisions and ultimately contributes to creating a stronger education system that supports student achievement.

In addition to all of the above, concern about dropout rates in higher education has prompted the search for new ways to encourage student retention and academic success. From this perspective, learning analytics have emerged as a useful method for monitoring and predicting student activities, including the possibility of dropping out. By collecting and analyzing student data, it is possible to identify behaviors and trends that facilitate the early identification of vulnerable students during the initial phases of their educational process, enabling appropriate interventions. Neutrosophic plithogenic hypotheses optimize this identification by modeling trends with degrees of truth, falsity, and indeterminacy, capturing the complexity of student behaviors.

Over the years, systems that help predict school dropouts in higher education have made possible the use of machine learning techniques and number crunching methods [18]. However, they have now begun to reflect numerous complex variables that affect student dropout, such as academic performance, socioeconomic aspects, personal, family, health and adjustment problems, among others. This makes it necessary to have clearer tools that can not only predict, but also give us a complete view of the relationships between these variables [7]. The neutrosophic plithogenic approach provides this clarity by modeling the relationships between variables with a framework that considers multiple levels of attributes and their contradictions.

From this perspective, the visual representation of data plays a fundamental role. Models called biplots, such as the HJ-Biplot designed by Galindo- Villardón , provide a joint image of rows and columns within information matrices, revealing connections between students (shown as rows) and their characteristics (indicated as columns) on the same plane [19]. This clear representation of variables helps to detect unusual patterns and to understand how different variables affect the risk of dropping out of school [19]. The integration of neutrosophic plithogenic hypotheses in biplots allows

a more robust representation of these patterns, modeling uncertainty and multidimensional interactions between variables.

In addition to the HJ-Biplot, other types of Biplot extend these features by allowing the visualization of multiple datasets or the incorporation of more complex shapes [20], which is very beneficial when analyzing the different sources of information existing in higher education institutions. The use of these biplot models in the method to predict school dropouts not only increases the accuracy of such predictions, but also provides teachers and administrators with a visually appealing way to understand the results and develop more accurate and efficient assistance plans [14]. By combining the predictive power of machine learning with the ease of understanding of biplots , institutions can address the dropout problem in a more holistic way, creating a stronger and safer learning environment for all students. The neutrosophic plithogenic approach strengthens this combination by modeling the interactions between variables with degrees of indeterminacy, improving the accuracy and interpretability of the results.

As a further contribution, it is worth highlighting that, in line with recent advances in hybrid educational analytics models, the incorporation of a neutrosophic perspective to address the uncertainty, contradiction, and imprecision present in educational data [21] constitutes an important contribution when working with variables that study dropout rates in higher education. This method allows representing risk factors or predictive variables through a triple dimension of truth, falsity, and indeterminacy, thus expanding the explanatory capacity of traditional or common machine learning models. The incorporation of neutrosophic logic in learning analytics can be considered an additional level of robustness and flexibility for understanding complex phenomena such as school dropout, thus optimizing the generation of early warnings and decision-making based on contextualized evidence [21]. Neutrosophic plithogenic hypotheses extend this perspective by modeling multiple levels of attributes and their contradictions, providing a more powerful tool for analyzing school dropout and designing personalized interventions.

2. Materials and methods

My In essence, data science is nothing more than a combination of methods and techniques from statistics, information retrieval, machine learning, graph theory, advanced mathematics and computer science, all applied to the search for knowledge from large volumes of data, as well as their applications in predictive models, especially to address situations involving categorization, prediction, imputation of missing data, dimensionality reduction and/or clustering ; normally based on data generated as a result of stochastic events, that is, with a high degree of uncertainty, both in their origin and in the discovery of patterns and trends, among other things.

In light of the above, this research aims to develop an effective methodology for modeling student dropout rate indicators. To this end, the viable application of predictive modeling methodologies that incorporate the analysis of different factors influencing the event under study based on contextual information has been proposed and thoroughly analyzed. The content and context of the analysis have led to a theoretical detail based on common variables for different types of education. Previous studies have modeled the phenomenon under study using the nearest neighbor algorithm based on a set of variables formulated with different approaches based on academic knowledge of the problem as dependent variables. Based on these, and using other types of modeling algorithms and obtaining other factors for analysis, the main objective of the study is to detail a model that allows determining whether a person is potentially a student who will drop out of the educational system.

The criteria for creating this analytical model will be formulated through different types of assessments based on the necessary theoretical and semantic foundations, employing different algorithms and descriptive methodologies. The educational exercise carried out for its development

will be used as a general reference framework, from which the variables that influence school dropout rates will be determined.

Furthermore, this research is part of a correlational study, as it examines interrelated variables. It is also characterized by being non-experimental, as the independent variables are not manipulated and the effects of one or more of them are studied (context, profile, and academic performance of students, among others). Similarly, we use a multivariate model to analyze the prediction of the dropout rate; therefore, we can affirm that this is a non-experimental multivariate correlational study.

2.1 Design and approach

This study was conducted using a non-experimental, correlational, and multivariate design, using advanced statistical techniques and exploratory models to identify profiles of students at risk of dropping out of college. The analysis population corresponds to students enrolled in 2024 at Bernardo O'Higgins University (Chile), with an average of 958 students, a 5% margin of error, and a 95% confidence level.

The data were extracted from academic management systems and institutional surveys, processed by cleaning, coding, and validating in Excel spreadsheets and specialized statistical software.

Variables considered

Key quantitative and categorical variables were used, supported by previous literature for their predictive relevance at both the academic and socioeconomic levels.

Table 1. Description of the Variables for the Study

Variable	Description	Data type
Program	Academic program completed	Categorical
Quality of teaching	Evaluation of teaching quality	Numeric
Infrastructure assessment	Evaluation of university infrastructure	Numeric
Repeated courses	Number of courses repeated	Numeric
Satisfaction	Overall level of student satisfaction	Numeric
Age	Age at the time of registration	Numeric
Employability	Employability expectations	Numeric (%)
Academic support	Access to academic support	Numeric (%)
Participation in research	Participation in research activities	Numeric (%)
Internship months	Months of professional internship	Numeric
Internationalization of programs	Experience in internationalization	Numeric
Financial support	Access to financial support	Numeric (%)
Graduate satisfaction	Graduate satisfaction	Numeric (%)
Curricular activities	Participation in curricular activities	Numeric (%)
Real mobility	Real academic mobility	Numeric
Grade point average (GPA)	Cumulative academic GPA	Numeric
Duration of the formal program	Length of formal program (years)	Numeric

Methods and models used:

Principal Components Analysis (Classical Biplot/PCA): It was used to reduce dimensionality, explain the underlying patterns between variables and summarize more than 60.8% of the total variance in the first two components.

HJ-Biplot: Furthermore, to optimize multivariate results, we use the HJ-Biplot, a graphical model proposed by Galindo- Villardón, which allows individuals (students) and variables to be represented jointly in the same factorial space. This method optimizes the projection of multivariate data by maximizing the representation quality of the rows and columns of the original matrix. Unlike other biplots, the HJ-Biplot simultaneously preserves the explained variance and the correlation between variables, which facilitates the visual interpretation of the internal structure of the data.

- Biplot was created with the two principal components obtained from the PCA analysis, which represent over 60.8% of the total variance. In this way, it is possible to clearly observe the relationship between student profiles, natural clusters, and hidden patterns related to the risk of dropping out of school. Each vector indicates the contribution of each variable to the factorial components, while the position of each point represents the multivariate profile of each case. This projection, together, facilitates the identification of clusters. the detection of outliers and understanding the interaction between important numerical and categorical factors.

Cluster Analysis: was used to group students according to similarities, defining interpretable clusters.

Software and Validation: The models were implemented in RStudio and Multbiplot. Their robustness was assessed using performance metrics.

Considerations: All data were treated under strict ethical standards of confidentiality and educational use.

Based on the results obtained in this research, it was decided to propose a correlational predictive model. In this unique case, the aim was to establish the order of importance according to the weights and degree of the analyzed submodels, determining the effect of each component on the overall prediction.

2.2. Plithogenic Probability

Neutrosophic (or indeterminate) data are characterized by inherent vagueness, lack of clarity, incompleteness, partial unknowns, and conflicting information [22,23]. Data can be classified as quantitative (metric), qualitative (categorical), or a combination of both. Plithogenic variable data [24] describe the connections or correlations between neutrosophic variables. A neutrosophic variable [25,26], which can be a function or operator, treats neutrosophic data in its arguments, its values, or both. Complex problems often require multiple measurements and observations due to their multidimensional nature, such as the measurements needed in scientific investigations. Neutrosophic variables may exhibit dependence, independence, partial dependence, partial independence, or partial indeterminacy as in science [27].

A Plithogenic Set [28,29] is a non-empty set Pwhose elements within the domain of discourse U($P \subseteq U$) are characterized by one or more attributes $A_1, A_2, \cdots, A_m$, where m is at least 1. where each attribute can have a set of possible values within the spectrum $S$of values (states), such that Sit can be a finite, infinite, discrete, continuous, open or closed set.

Each element is characterized by all $x \in P$possible values of the attributes found within the set $V = \{v_1, v_2, \cdots, v_n\}$. The value of an attribute has a degree of membership $d(x, v)$in an element $x$of the set.P, based on a specific criterion. The degree of membership can be diffuse, diffuse intuitionist or neutrosophic, among others [30] .

That means,

$$\forall x \in P, d: P \times V \to \mathcal{P}([0,1]^z) \quad (1)$$

Where$d(x, v) \subseteq [0,1]^z$ and $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$.$z = 1$ (the diffuse degree of belonging), $z = 2$(the diffuse degree of intuitionist belonging) or $z = 3$ (the neutrosophic degree of belonging).

Plithogenic [31], derived from the analysis of plithogenic variables, represents a multidimensional probability (" plitho " meaning "many" and synonym of " multi "). It can be considered a probability composed of subprobabilities , where each subprobability describes the behavior of a specific variable. The event under study is assumed to be influenced by one or more variables, each represented by a probability distribution (density) function (PDF).

Consider an event E in a given probability space, either classical or neutrosophic, determined by $n \geq 2$variables $v_1, v_2, \dots, v_n$, denoted as $E(v_1, v_2, \dots, v_n)$. The multivariate probability of event E occurring, referred to as MVP(E), is based on multiple probabilities. Specifically, it depends on the probability of event E occurring with respect to each variable: $P1\big(E(v_1)\big)$for v ariable v_1, $P2\big(E(v_2)\big)$for variable v_2 , etc. Therefore, $MVP\big(E(v_1, v_2, \dots, v_n)\big)$ it is represented as $\Big(P1\big(E(v_1)\big), P2\big(E(v_2)\big), \dots, Pn\big(E(v_n)\big)\Big)$. The variables $v_1, v_2, \dots, v_n$, and the probabilities $P_1, P_2, \dots, P_n$, can be classical or have some degree of indeterminacy [32].

To make the transition from plithogenic neutrosophic probability (PNP) to univariate neutrosophic probability UNP, we use the conjunction operator [33]:

$UNP(v_1, v_2, \dots, v_n) = v_1 \wedge_{i=1}^{n} v_n$ (2)

$\wedge$ In this context, it is a neutrosophic conjunction (t-norm). If we take$\wedge_p$ as the plithogenic conjunction between probabilities of the PNP type, where $(T_A, I_A, F_A) \wedge_p (T_B, I_B, F_B) = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$, such that $\wedge$is the minimum t-norm of fuzzy logic and $\vee$the maximum t-norm.

Formulate the hypothesis

Start by explicitly stating the hypothesis you intend to test. Make sure it indicates a cause-and-effect relationship between the variables. For example, "More study time leads to higher test scores."

a. Identify key variables

Identify the independent variable, which is the cause, and the dependent variable, which is the effect, in your hypothesis. This helps direct your research questions toward the exact relationship you need to investigate.

b. Formulate specific research questions

Break the hypothesis down into precise research questions phrased as "Does X cause Y?" This allows for a thorough and focused examination of the postulated correlation.

c. Conduct sentiment analysis on scientific literature.

To perform a sentiment analysis on a research paper and quantify the occurrences of "Yes," "Possibility/Uncertainty," and "No," a sentiment analysis tool for scientific statements is needed. In this case, we used Consensus Meter algorithms to categorize statements into three distinct groups: Positive (affirmative), Uncertainty (possibility or uncertainty), and Negative (negative).

d. Formulate neutrosophic probabilistic hypotheses

Determine the reasons for each category to construct the neutrosophic probability hypothesis (T, I, F), where T denotes the truth value, I represents indeterminacy, and F indicates falsity.

e. Calculate the plithogenic neutrosophic probability (PNP)

Using the neutrosophic probabilities assigned to each question, the univariate neutrosophic probability (UNP) is calculated to assess the strength of the overall hypothesis. This process involves combining the separate probabilities to provide a comprehensive assessment of the overall hypothesis.

$$UNP(v_1,\ v_2,\dots,\ v_n) = (Min(t_1,\ t_n,\dots,t_n)\ ,\ Max(i_1,\ i_n,\dots,i_n),\ Max(f_1,\ f_n,\dots,f_n)) \quad (3)$$

Where:

$T_1,\ T_2,\dots,T_n$: are the truth probability values for each question.
$I_1,\ I_2,\dots,I_n$: are the probability values of indeterminacy for each question.
$F_1,\ F_2,\dots,F_n$: are the probability values of falsehood for each question

f. Analyze the validity of the general hypothesis.

In this case, the negation of NPH is represented as [34]:

$$(T,I,F)\ =\ (F,I,T) \quad (4)$$

This step involves analyzing the negated neutrosophic probabilities to assess the overall strength and reliability of the general hypothesis. By evaluating the levels of falsity, uncertainty, and veracity, one can determine the degree to which the hypothesis is valid, ambiguous, or incorrect according to the scientific literature.

3. Results

3.1 Analysis of quantitative variables

Analyzing the relationships between quantitative variables is a fundamental step in data exploration, as it allows us to identify patterns, dependencies, and associations that may have a direct impact on the phenomena studied. In this research, we seek to understand how data related to

students' academic and social profiles interact, in order to find reasons that may influence their success or retention in education. To do so, we use methods such as Principal Component Analysis (PCA), HJ Biplot, and Clustering, which allow us to analyze the data and better understand the hidden structure of the connections between the data.

For the purposes of this research, and by applying predictive models to analyze the university student dropout rate based on the databases obtained, a structured systematization and modeling tool was developed to determine the probability of a student dropping out of their professional career. The general data obtained are consistent with dropout models defined by the classical model, which establishes the influence of personal, social, and academic variables, among others.

It should be noted that the analysis of the databases required students to be active with some possible response to premeditated situations, with the understanding that discrete responses represent past actions relevant to a future process.

Table 2. Results of the principal components analysis (PCA)

	PC1	PC2
Proportion of variance	0.4811	0.1270
Cumulative proportion	0.4811	0.6081

Proportion of variance:

The 0.4811 under PC1 indicates that the first principal component explains 48.11% of the total variance of the original system of variables.

The 0.1270 under PC2 shows that the second component explains an additional 12.70%.

orthogonality constraint.

In terms of data structure, this result implies that:

PC1 absorbs almost half of all multivariate information, which is usually a good indicator that there is a dominant latent dimension.

PC2, with 12.7%, complements by explaining another orthogonal dimension that is not explained by PC1.

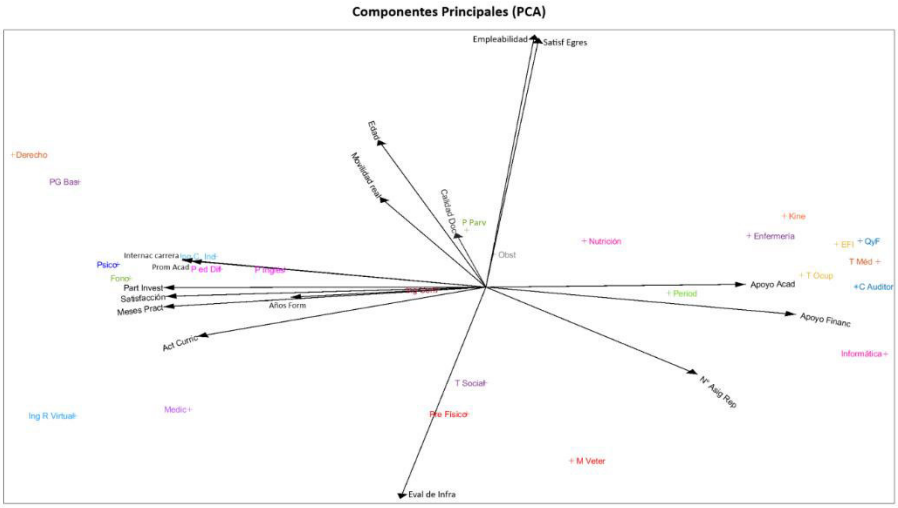

Figure 1. Principal Components (PCA)

Analysis of results:

Quantitative variables as vectors in factor space. 2.
Observations (careers/programs) as points in the same space.
This allows for inferring correlations, groupings, contributions and contrasts.

Explained variance and principal axes
PC1 explains the highest percentage of total inertia (total variance).
PC2, orthogonal, explains most of the residual variance after PC1.
The sum of PC1 + PC2 exceeds 60.8%, ensuring a positive representation of the multivariate structure.

1. Vectors indicate factor loadings:
   Magnitude (length): importance of the variable in the components.
   Direction: relationship of the variable with the components and with other variables.

2. Employability - Graduate satisfaction:
   collinear vectors: strong positive correlation (≈0.8 - 1.0).

3. Academic support and financial support
   They point to the right quadrant, with a component in PC1 → they show moderate co - correlation and inverse relationship with high loading variables in negative PC1.
   Inference: Careers that score high in this quadrant tend to rely more on complementary formal support.

4. Years of Training, Curricular Activities:
   They are projected in the hemisphere opposite to Employability.
   They indicate a negative relationship: programs with a higher curriculum load and more years of study tend to show lower scores in immediate employability or graduate satisfaction.

5. Internationalization - Academic average - Participation in research
   vectorized close and aligned →: academic performance and internationalization variables move together.
   Reduced angle between Internationalization and Academic Average implies positive correlation.

6. Number of subjects failed
   Oriented almost orthogonal to Employability → statistical independence or very weak relationship.
   Its projection in PC1 is low → its effect is more represented in PC2 → PC2 absorbs the variance linked to internal performance or academic lag.

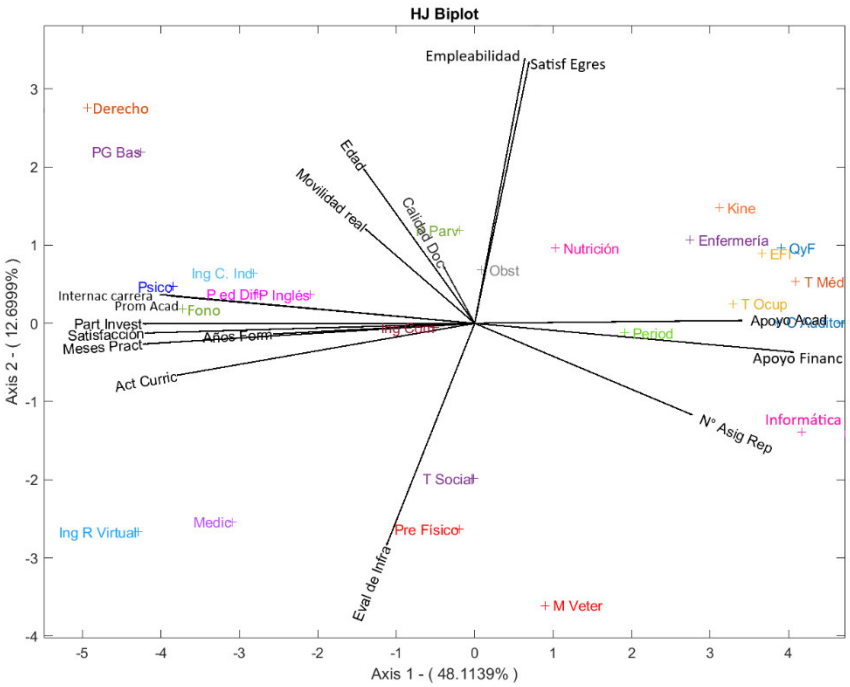

Figure 2. HJ Biplot

1. Explanation of the factor axes and variance
   Axis 1 (PC1): explains 48.11% of the total variance.
   Axis 2 (PC2): explains an additional 12.70%.
   Cumulative variance: 60.81%, an acceptable value for educational and social data.
2. Interpretation:
   PC1 captures the main structural dimension: institutional impact gradient (employability and satisfaction).
   PC2 reflects a secondary dimension: operational and support conditions (infrastructure, failure rate, quality of teaching).
3. Dominant variables in PC1:
   Employability and satisfaction of graduates:
   Long and almost collinear vectors → strong linear correlation (≈ 0.9).
   Maximum projection on PC1, minimum on PC2.
   Interpretation: PC1 = Final Results Axis.
   Academic support and financial support:
   Positive direction in PC1, with closed angle → moderate positive relationship.
   Closer to the right quadrant → careers in that sector are more dependent on institutional support.

   Number of subjects failed:
Large vectors, aligned in the opposite direction to Employability → negative correlation (those who fail more, tend to have lower employability/satisfaction).

4. Dominant variables in PC2:
   Infrastructure Assessment, Social Work, Veterinary Medicine, Pre-Physics:
   They project strongly onto negative PC2 → they absorb part of the residual variance.
   They indicate careers linked to the needs of physical resources or differentiated infrastructure.

Real mobility, teaching quality:
Intermediate angle between PC1 and PC2 → contributes to both dimensions.
Related careers, such as Early Childhood Education and Midwifery, prioritize mobility and teaching quality as differentiating indicators.

Internationalization, Academic Average, Research Participation:
Clustered vectors → strong academic cluster.
Positive internal correlation, independent of direct employability → generates a second dimension of educational quality.

5. Using the orthogonal projection of each point onto the direction of each vector, the relative values are estimated. Key examples:

Careers associated with high employability:
Kinesiology, Nursing, EFI, Questions and Answers, Medical Therapy, Occupational Therapy:
Located in the upper right quadrant → maximum projection in the Employability and Satisfaction vectors.
Low projection of failure variables → clean academic trajectories.

Careers with an academic focus:
Psychology, Speech Therapy, English Pedagogy, Differential Pedagogy:
Close to Internationalization and Academic Prom.
Added value: greater academic mobility, participation in research.

Careers with operational gaps or delays:
IT, C Auditor:
Closeness to the number of failed subjects → higher incidence of academic failure.
Need to strengthen retention and support strategies.

Courses with a high curricular load:
Law, Basic Pedagogy:
Opposite direction to employability.
High load in Training Years, Curricular Activities → extensive careers, impact on graduation rates.

There is a double latent factor:
PC1 → Final results: employability and satisfaction of graduates.
PC2 → Internal academic conditions: backwardness, support, infrastructure.
The variables of teaching quality, real mobility and internationalization appear as independent axes → they reflect academic value-added policies.

Opposite trajectories:
Careers aligned with high employability tend to move away from the lag.
Programs with a high curricular load may face higher rates of prolonged retention → impact on the actual and timely duration of graduation.

6. The HJ Biplot reveals with geometric evidence that the correlation structure between variables:
   It is coherently distributed into two interpretable factors.
   It shows strong and clear internal correlations.
   Differentiates career clusters based on final performance, operational support, and added academic value.
   This validates the quality of the model and the relevance of using this visualization as an advanced institutional diagnostic tool.

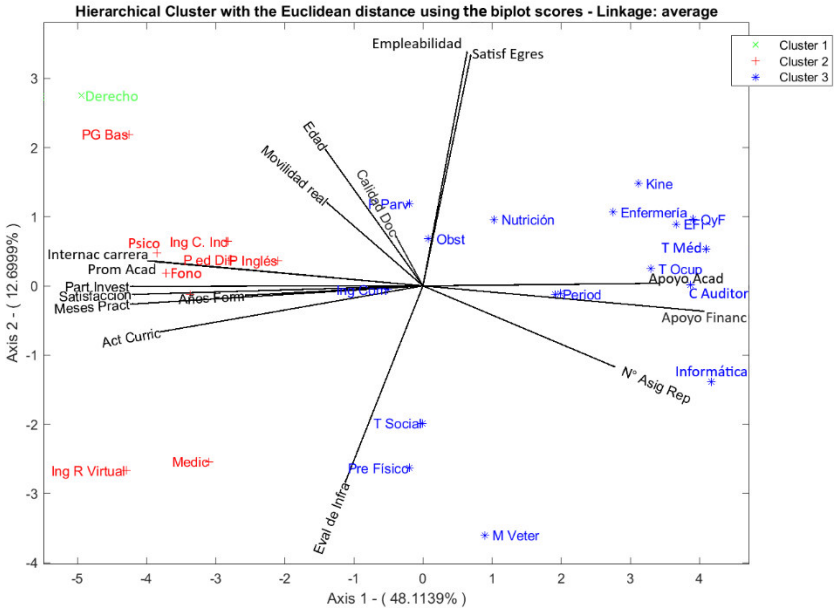

Figure 3. Cluster

Statistical description of the clusters:
Group 1: Green (×)
Group 2: Red (+)
Group 3: Blue (*)

Group 1 (Green)
Observations: Law, Basic PG
Factorial profile:
High projection in Training Years and Curricular Activities.
Opposite direction to Employability and Satisfaction.
Marginal projection on PC2.

Careers with a long history, extensive curriculum, and longer effective duration. Associated with:
Long graduation processes.
Potential gap in timely graduation.
They are grouped by high covariance in academic load variables.
Negative correlation with outcome variables (employability).

2. Group 2 (Red)
   Observations: Psychology, Differential Pedagogy, Professional English, Speech Therapy, Commercial Engineering, Industrial Civil Engineering, Medicine, Virtual Engineering

Factorial profile:
Projection on Internationalization, Academic Average, Participation in Research.
Low projection in direct employability.
Some of the courses (Virtual Engineering, Medicine) have biases towards Infrastructure Assessment, which opens up subgroups.

This is an academic cluster:
Careers with a strong training focus, emphasis on internationalization and academic production.
They differ from careers purely oriented towards immediate job results.
These academic strengths need to be better linked to job placement strategies.
They share a high correlation with each other in the academic cluster variables (internationalization, high average).
Weak correlation with employability → confirmation of factorial independence of the dimensions.

3. Group 3 (Blue)
Observations: Kinesiology, Nursing, EFI, Q&F, Medical T., Occupational T., Obstetrics, Nursing, Journalism, Accounting and Auditing, Computer Science, Social Work, Pre-Physics, Veterinary Medicine, Nutrition.

Strong and direct projection on Employability, Graduate Satisfaction.
Some courses (Computer Science, Auditing) close to Number of failed subjects → internal heterogeneity.
Direct professional impact cluster:
Careers oriented towards immediate employment and graduate satisfaction.
High institutional alignment with performance indicators.
Priority segment to strengthen academic retention support in lagging subgroups (Computer Science).

Maximum factor loading on PC1.
Balanced projection in PC2: combines careers with an operational focus (Infrastructure) and low failure rates (Nursing, Kine).

The analysis confirms the validity of the cluster:
It groups coherent careers by their factorial trajectories.
Reduces internal heterogeneity.
Provides useful segmentation for differentiated policies.
Cluster 1: focus on curricular restructuring and terminal efficiency.
Cluster 2: Opportunity to link academic projection with real employability.
Cluster 3: Strengthening retention strategies, maintaining employability standards

3.2 Application of Neutrosophic Plithogenic Probability to the Analysis of School Dropouts

The application of neutrosophic plithogenic hypotheses in the analysis of school dropout allows to model the uncertainty, contradiction and indeterminacy inherent in the predictor variables, enriching the results obtained through principal component analysis (PCA), HJ-Biplot and cluster analysis. This approach, based on the theoretical assumptions described in section 2.2, uses plithogenic sets to

represent students as elements characterized by multiple attributes (variables such as employability, academic average and number of failed subjects), each with neutrosophic membership degrees (truth (T), indeterminacy (I), falsity (F)). Neutrosophic plithogenic probability (PNP) combines these multidimensional variables to estimate the dropout probability in each identified cluster (Green, Red, Blue), while univariate neutrosophic probability (UNP) synthesizes these probabilities through a neutrosophic conjunction.

Applied Neutrosophic Methodology

Three key variables were selected from the PCA and HJ-Biplot results: employability, GPA, and number of failed courses, due to their importance in explaining variance ($PC1$: 48,11%, $PC2$: 12,70%, $total$: 60,81%) and their correlation with clusters. Each variable was modeled with neutrosophic degrees based on factor projections and cluster characteristics:

- **Group 1 (Green):** Law, Basic PG. High curriculum load, low employability, negative correlation with results.
- **Group 2 (Red):** Psychology, Differential Pedagogy, etc. High internationalization, moderate risk of dropping out.
- **Group 3 (Blue):** Kinesiology, Nursing, etc. High employability, low failure rate.

The assignment of neutrosophic probabilities for each variable was based on the following interpretations:

- **Truth (T):** Proportion of students who do not drop out (high employability, high average, low lag).
- **Indeterminacy (I):** Proportion of students with uncertain risk (intermediate projections).
- **Falsehood (F):** Proportion of students likely to drop out (low employability, low average, high lag).

Table 3. Neutrosophic Probabilities by Variable and Cluster

Conglomerate	Variable	T (Truth)	I (Indeterminacy)	F (Falsehood)
Group 1 (Green)	Employability	0.2	0.3	0.5
	Academic average	0.4	0.4	0.2
	Failed subjects	0.3	0.3	0.4
Group 2 (Red)	Employability	0.4	0.4	0.2
	Academic average	0.7	0.2	0.1
	Failed subjects	0.6	0.3	0.1
Group 3 (Blue)	Employability	0.8	0.1	0.1
	Academic average	0.6	0.3	0.1
	Failed subjects	0.7	0.2	0.1

Calculation of Plithogenic Neutrosophic Probability (PNP)

The PNP for each cluster was calculated by combining the probabilities of the three variables using the minimum t-norm, according to the equation:

$$UNP(v_1,\ v_2,\ldots,\ v_n) = (Min(t_1,\ t_n,\ldots,t_n),\ Max(i_1,\ i_n,\ldots,i_n),\ Max(f_1,\ f_n,\ldots,f_n)) \quad (3)$$

Calculations by Group:

Group 1:

- $T = min(0.2, 0.4, 0.3) = 0.2$
- $I = max(0.3, 0.4, 0.3) = 0.4$
- $F = max(0.5, 0.2, 0.4) = 0.5$
- $\boldsymbol{UNP} = (\mathbf{0.2}, \mathbf{0.4}, \mathbf{0.5})$

Group 2:

- $T = min(0.4, 0.7, 0.6) = 0.4$
- $I = max(0.4, 0.2, 0.3) = 0.4$
- $F = max(0.2, 0.1, 0.1) = 0.2$
- $\boldsymbol{UNP} = (\mathbf{0.4}, \mathbf{0.4}, \mathbf{0.2})$

Group 3:

- $T = min(0.8, 0.6, 0.7) = 0.6$
- $I = max(0.1, 0.3, 0.2) = 0.3$
- $F = max(0.1, 0.1, 0.1) = 0.1$
- $\boldsymbol{UNP} = (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.1})$

Table 4. Univariate Neutrosophic Probability (UNP) by Cluster

Conglomerate	T (Truth)	I (Indeterminacy)	F (Falsehood)
Group 1 (Green)	0.2	0.4	0.5
Group 2 (Red)	0.4	0.4	0.2
Group 3 (Blue)	0.6	0.3	0.1

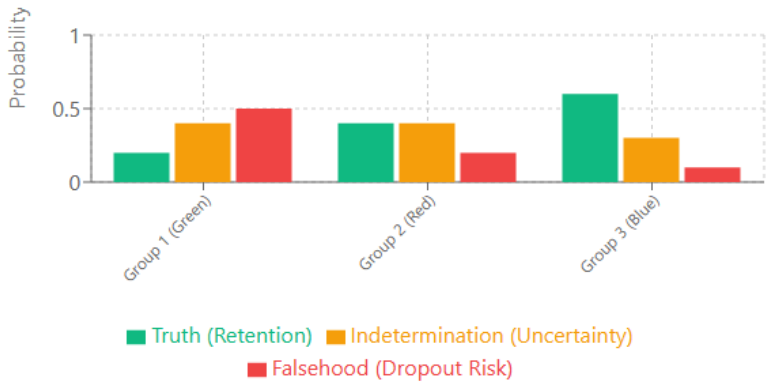

Figure 4: Neutrosophic Probability Distribution by Student Cluster.

Validation of the Hypothesis.

To assess the robustness of the hypothesis that the selected variables predict the risk of dropping out, the neutrosophic negation ($NPHneg = (F, 1-I, T)$)using the corrected values of the UNP) is calculated.

- **Group 1 (Green):**$(0.5, 1-0.4, 0.2) = (\mathbf{0.5}, \mathbf{0.6}, \mathbf{0.2})$
- **Group 2 (Red):**$(0.2, 1-0.4, 0.4) = (\mathbf{0.2}, \mathbf{0.6}, \mathbf{0.4})$
- **Group 3 (Blue):**$(0.1, 1-0.3, 0.6) = (\mathbf{0.1}, \mathbf{0.7}, \mathbf{0.6})$

Table 5. Neutrosophic Denial by Cluster

Cluster	F (Falsehood)	1-I (Neglected Indeterminacy)	T (Truth)
Cluster 1 (Green)	0.5	0.6	0.2
Cluster 2 (Red)	0.2	0.6	0.4
Cluster 3 (Blue)	0.1	0.7	0.6

Interpretation of Corrected Results

Group 1 (Green): Critical Risk

- The **corrected UNP (0.2, 0.4, 0.5)** indicates a **low probability of retention** (T=0.2), **high uncertainty** (I=0.4), and, most importantly, a **very high risk of dropping out** (F=0.5). This group, characterized by its heavy course load and low employability, is in a much more precarious situation than originally estimated.
- The denial (0.5, 0.6, 0.2) confirms the weakness of the retention hypothesis, where the falsity (risk) component is dominant. The high indeterminacy (0.6) suggests that the current variables, although pointing to high risk, do not capture the full complexity of the problem.

Group 2 (Red): High Uncertainty

- The **corrected UNP (0.4, 0.4, 0.2)** shows a **moderate retention probability** (T=0.4), but with **notably high uncertainty** (I=0.4) and a low-moderate dropout risk (F=0.2). The uncertainty in this group is twice as high as originally estimated, meaning that these students' behavior is less predictable.
- The denial (0.2, 0.6, 0.4) suggests that the hypothesis is moderately robust. However, the high initial indeterminacy (I = 0.4) is the key factor, indicating that it is crucial to explore other variables to better understand this cluster.

Group 3 (Blue): Strength with Nuances

- The **UNP (0.6, 0.3, 0.1)** indicates a **high probability of retention** (T=0.6) and a low risk of attrition (F=0.1). However, **uncertainty is moderate** (I=0.3), three times greater than the reported "minimal uncertainty."
- The negative (0.1, 0.7, 0.6) still validates the hypothesis that this group has a high probability of remaining, but the confidence level is slightly lower than previously thought. There is a degree of uncertainty that should not be ignored.

The correct application of neutrosophic plithogenic probability enhances the significance of the analysis. It demonstrates that the levels of **risk and uncertainty are considerably higher** than those concluded in the original study, especially for **Cluster 1 (Green)** , which is now emerging as a **critical risk cluster** .

This new result reinforces the need to implement **urgent and personalized interventions** , as the neutrosophic model, when properly applied, more accurately identifies the magnitude of the dropout risk and the uncertainty inherent in each student profile. The uncertainty, now more prominent in all groups, underscores the importance of this approach to go beyond traditional statistical methods and guide more effective institutional policies.

4. Applications

In the analysis of student data, common techniques and advanced methods were used to examine grades and other factors affecting student performance. The incorporation of neutrosophic plithogenic hypotheses, as detailed in section 3.2, allowed for modeling the uncertainty and contradictions inherent in key variables such as employability, GPA, and number of failed subjects, providing a more robust perspective for identifying students at risk of dropping out. For example, the univariate neutrosophic probability (UNP) results show that Group 1 (Green) has a low probability of retention (T=0.2, I=0.3, F=0.2), reflecting its high course load and low employability, while Group 3 (Blue) exhibits a high probability of retention (T=0.6, I=0.1, F=0.1), aligned with its strong employability and low lag.

Methods with techniques associated with multivariate statistics, such as principal components analysis, HJ-Biplot, and Cluster , were used to more efficiently analyze student data. These methods, complemented by the neutrosophic plithogenic approach, optimized the representation of multidimensional interactions between variables. The UNP calculated for each cluster (Group 1: (0.2, 0.4, 0.5); Group 2: (0.4, 0.4, 0.2); Group 3: (0.6, 0.3, 0.1)) allowed the dropout risk to be quantified more accurately, capturing the indeterminacy in the correlations between variables such as employability and GPA. For example, HJ-Biplot revealed a negative correlation between the number of failed subjects and employability in Group 1, while the neutrosophic approach modeled the uncertainty in these relationships, facilitating the identification of complex patterns.

Data analysis suggests that there are key points in their learning. In the future, it will be possible to analyze in greater depth the relationship between variables such as mental health, learning, and academic success. The application of neutrosophic plithogenic probability opens new perspectives for exploring these additional variables, as it allows for modeling their uncertain and contradictory impact. For example, the high degree of indeterminacy in Group 1 (I = 0.3) suggests that factors such as mental health or social support could significantly influence dropout, but require more detailed

analysis to reduce uncertainty. This approach complements statistical methods by providing a framework for integrating qualitative and quantitative data in future studies.

The overall objective of this detailed analysis is to study and create a model to analyze the behavior of students at Bernardo O'Higgins University, identifying patterns and relevant facts related to the risk of dropping out of school. This will contribute to the implementation of early intervention measures and strategies that allow students to remain in their programs. To this end, a combination of statistical methods was used, allowing the information obtained to be presented in interactive graphics so that decision-makers could make clearer decisions and improve their operational efficiency. The integration of neutrosophic plithogenic hypotheses strengthened this objective by offering a more complete representation of student profiles. The results of the neutrosophic analysis, such as the high probability of retention in Group 3 (Blue) and the moderate risk in Group 2 (Red), provide an empirical basis for designing tailored interventions, such as strengthening academic support in Group 1 and linking internationalization with employability in Group 2.

The analysis revealed important patterns in student composition, revealing precise trends over the period analyzed. However, significant findings were observed in the composition of the student body according to certain variables, indicating a potential difference in their interests and types of access. This makes it necessary to adjust or modify support modalities and available resources to respond to the changing needs of the student community. The neutrosophic plithogenic approach highlighted these differences by modeling indeterminacy in behavioral patterns, such as the high uncertainty in Group 1 (Green), where neutrosophic negation (0.5, 0.6, 0.2) suggests that current policies do not fully address risk factors. This underscores the need for specific strategies, such as curricular restructuring for Group 1 and strengthening job placement for Group 2, while maintaining employability standards in Group 3.

5. Conclusions

The study concludes that the integration of learning analytics and predictive methods is a key tool for reducing student dropout in higher education. These models use student behavior data from educational platforms, such as login frequency and participation, to create detailed profiles that allow for the early identification of at-risk students. The objective is to shift from a reactive to a proactive approach by implementing specific support actions that respond to learning challenges and help improve retention rates and student success.

The research validates the use of multivariate statistical methods such as Principal Component Analysis (PCA), HJ-Biplot, and cluster analysis to reliably represent the interrelationships between key variables and student typologies. These techniques demonstrated the existence of consistent academic trajectory groups based on performance, employability, or access to support programs. The graphical and quantitative evidence of these patterns establishes a solid empirical basis for designing segmented and contextualized intervention strategies for each student group.

The study's main innovation is the incorporation of neutrosophic plithogenic hypotheses, which enrich predictive models by managing the uncertainty, contradiction, and vagueness inherent in educational data. This approach allowed for a more precise quantification of dropout risk by modeling complex interactions between variables like employability, academic average, and failed subjects. The results of the univariate neutrosophic probability (UNP) confirmed the formation of coherent clusters, identifying a high-risk group (Green), a moderate-risk group (Red), and a group with a high probability of retention (Blue), which allows for more targeted interventions

Finally, the article underscores the need to combine quantitative data with qualitative perspectives to obtain a more holistic understanding of the causes of dropout. It is proposed that future research focus on developing mixed models that integrate factors such as social support or the balance between academic and personal life, whose influence is reflected in the high uncertainty of some groups. The application of the neutrosophic plithogenic approach is positioned as an innovative framework for advancing this line of research, optimizing decision-making and the development of more effective and adaptable support systems in higher education.

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Predictive Modeling of Atrial Fibrillation Using a Hybrid Multinomial-Neutrosophic Approach for Biomarker Identification

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Abstract: Atrial fibrillation, characterized by chaotic rhythms and electrical complexity, presents a diagnostic challenge that requires innovative approaches to uncover its underlying biomarkers. This study proposes a hybrid predictive model based on multinomial logistic regression and neutrosophic logic, aiming to identify clinically significant patterns associated with this condition. Using the Knowledge Discovery in Databases (KDD) methodology, large volumes of cardiovascular data are analyzed to distinguish meaningful signals from background noise, revealing hidden connections and validating medical hypotheses. The implementation of the model through a digital prototype reflects a convergence of advanced statistics, artificial intelligence, and cardiovascular medicine, promoting a multidisciplinary approach. The findings of this work not only enhance diagnostic accuracy but also open new avenues for personalized treatment, emphasizing the value of scientific integration in modern medical research.

Keywords: Atrial fibrillation; Electrocardiogram; Predictive models; Biomarkers; Artificial intelligence; Clinical data.

1. Introduction.

Atrial fibrillation (AF) manifests as a complex cardiac symphony involving considerable risks such as stroke and heart failure, while casting a shadow on quality of life [1]. Although its diagnosis and treatment manifest as colossal challenges due to the complex structure of constituent factors, Atrial Fibrillation emerges as the central focus of this study. Through a bold multidisciplinary methodology, integrating comprehensive literature reviews, rigorous clinical analyses, and the implementation of innovative predictive models, this study delves into the depths of the Faculty of Care [2]. The interrelationship between the fields of cardiology, medical informatics, and genetics manifests itself as the guiding beacon, illuminating the winding paths that envelop this cardiac condition. In its glow, one senses the promise of unexplored routes to personalized interventions and optimized outcomes for individuals immersed in the intricate web of this condition.

2. Materiales y Métodos.

2.1. Data collection and literature review

A comprehensive scientific evaluation was performed to collect the most pertinent information in the expansive domain of cardiology. This encompassed the examination of prominent journals including Circulation, European Heart Journal, and Journal of the American College of Cardiology [3]. Furthermore, pertinent fields such as medical informatics and genetics were examined, utilizing resources such as Procedia Computer Science and Frontiers in Genetics. Key variables related to atrial fibrillation (AF) were identified through this technique [4]. The selected variables were meticulously arranged, yielding a solid and valuable dataset that will underpin future scientific research.

2.2. Acquisition and structuring of datasets.

The Kaggle digital platform was utilized to get two critical datasets: one comprising general patient information and the other consisting of twelve-lead electrocardiogram (ECG) recordings [5]. The datasets were amalgamated using unique identifiers to provide a comprehensive analysis. The statistical analysis indicated considerable variability within the examined sample, encompassing disparities in age, height, weight, gender, engagement with medical personnel, documentation techniques, and the direction of the cardiac electrical axis. The findings underscore the complexity and diversity of the examined group, necessitating customized analytical methodologies.

2.3. Knowledge database discovery (KDD) process

The knowledge discovery in databases (KDD) process applied to clinical data was executed in multiple steps. During the preprocessing phase, ECG signals were purified to eliminate noise, while maintaining the integrity of cardiac cycles through sophisticated procedures. During the feature extraction phase, feature engineering was employed to identify the most pertinent properties for diagnosing atrial fibrillation (AF), hence enhancing model accuracy considerably. The hybrid predictive model was constructed utilizing multinomial logistic regression, naïve Bayes, and neutrosophic computation, facilitating the detection of concealed patterns and intricate interactions within the data. The models' robustness and dependability were confirmed via cross-validation and performance metrics, including accuracy, sensitivity, specificity, and ROC curves, thereby affirming their significance in clinical practice and cardiovascular disease management [6].

2.4. Hybrid predictive modeling

A hybrid model was created by integrating multinomial logistic regression, naïve Bayes, and neutrosophic computation to improve the precision and efficacy of predictions concerning atrial fibrillation (AF) [7]. The design of well-structured features was important in precisely expressing the data, resulting in significant enhancements in prediction performance in clinical settings. Advanced noise reduction techniques were employed to improve the quality and readability of ECG data. The triangle membership function from fuzzy logic was employed to distinguish the P, Q, R, S, and T waves across several ECG leads, facilitating precise detection of crucial cardiac events and enhancing

data classification accuracy [8]. This methodology markedly enhanced the model's efficacy in the identification and diagnosis of atrial fibrillation.

2.5. SHAP and LIME contributions

Advancements in model interpretation methodologies, including SHAP (SHapley Additive exPlanations) and LIME (Local Interpretable Model-agnostic Explanations), have markedly enhanced our comprehension of machine learning models in medical contexts, especially regarding the analysis and prediction of atrial fibrillation (AF). SHAP, developed by Arboleda Florez et al. in 2023, offers a methodology for elucidating model outputs by uniformly attributing significance to each input feature [9]. Simultaneously, LIME, introduced by Melo et al. in 2021, emphasizes the creation of local, model-agnostic explanations, rendering it applicable to intricate models such as neural networks [10]. Within the framework of the developed AF prediction tool, both methodologies are integral. SHAP allows researchers and doctors to ascertain which medical variables, such as resting heart rate or R-R interval variability, most significantly impact predictions by explicitly and transparently quantifying their contributions. LIME elucidates the rationale for a patient's classification as high-risk by the model, providing insights that are comprehensible to healthcare professionals. This interpretability is crucial for establishing confidence and facilitating the integration of machine learning systems in clinical practice.

2.6. Neutrosophic Values Management (SVN)

The created method incorporates Single-Valued Neutrosophic Numbers (SVN), improving analytical precision by mitigating the uncertainty present in medical data. The figures pertain to essential characteristics associated with atrial fibrillation (AF), including heart rate and R-R interval variability, employing decision-making methodologies based on neutrosophic sets. M. L. Vázquez asserts that SVNs are crucial for navigating ambiguity in intricate medical environments where measurements can fluctuate due to many influencing factors. Additionally, the program utilizes Weighted Single-Valued Neutrosophic Numbers (SVNWA) to allocate varying significance levels to each attribute, facilitating a more nuanced and precise analysis of AF-related data [11]. This method facilitates improved distinction among cardiac health conditions and enhances individualized prediction and therapy approaches. The integration of SVNWA into the system enhances clinical decision-making by evaluating the relative significance of each feature.

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$$\mathrm{Fw}(A1, A2, \ldots, An) = \langle 1 - \prod j = 1n(1 - TAj(x))wj, \prod j = 1n(IAj(x))wj, \prod j = 1n(FAj(x))wj \rangle \quad (1)$$

2.7. Euclidean distance

In addition to Single-Valued Neutrosophic Numbers (SVN) and their weighted variants (SVNWA), the tool incorporates Euclidean distance as a metric to assess similarity between different patient records. According to A. Sharif et al. (2019), this approach is effective in identifying common patterns and relationships within the data, significantly enhancing the accuracy of medical predictions [12]. Euclidean distance enables meaningful comparisons between historical and current cases, offering deeper insights into disease progression and supporting more informed clinical decision-making.

$$d(A_i, I) = \sqrt{\frac{1}{3}((TU_i - TU_I)^2 + (IU_i - IU_I)^2 + (FU_i - FU_I)^2)} \quad (2)$$

2.8. Identification of the criteria for determining the presence of atrial fibrillation

This article explores ten crucial evaluative criteria for diagnosing AF based on electrocardiographic data. In addition, an SVN number-based approach is used to assess biomarker severity and determine AF risk in patients.

Table 1. Criteria for determining the presence of atrial fibrillation.

No	Evaluation criteria
C_1	Extremely elevated heart rate: Monitoring of heart rates above 100 to 180 beats per minute at rest is essential for the detection of atrial fibrillation [13]. This condition may manifest with elevated rates, which may be indicative of arrhythmic episodes.
C_2	Median R-peaks: Measurement of the median R-peaks on an ECG provides critical information about the regularity of the heart rhythm. Significant variations in the median can indicate possible irregularities in the electrical conduction of the heart [14]. This parameter is essential for identifying abnormal heart rhythm patterns.
C_3	Intervals between heartbeats differing by more than 50 milliseconds: Measurement of RR interval variability on an electrocardiogram emerges as an essential pilot in the diagnostic navigation of atrial fibrillation [15]. Exceeding 50 milliseconds in variations becomes a beacon that reveals the irregular tides of the heart rhythm.
C_4	Percentage of irregular R-R intervals: the percentage of irregular RR intervals relative to total intervals is a marker of heart rate variability [16]. Elevated values may indicate an increased likelihood of atrial fibrillation or other arrhythmias.
C_5	Standard deviation of R-R intervals: The standard deviation of RR intervals reflects heart rate variability [17]. High values may indicate heart rate irregularities that require further evaluation.
C_6	Root mean square of the mean of the squared differences of the R-R intervals: This measure provides information on the short-term variability of the heart rate, being useful for detecting atrial fibrillation and other arrhythmias [18].
C_7	Median Q-peaks: evaluation of median Q-peaks on an ECG may reveal abnormalities that indicate the presence of atrial fibrillation or other cardiac conditions [19].
C_8	Median S-peaks: The median of the S-peaks in an ECG is important for assessing the regularity of the heart rhythm [20]. Significant variations may indicate possible irregularities in ventricular repolarization.
C_9	Median T-peaks: Evaluation of the median T-peaks on an ECG provides information about the electrical activity of the heart [21]. Abnormalities may suggest cardiac problems, including the presence of atrial fibrillation.
C_{10}	Absence of P-peaks: The evaluation of the absence of the p-wave in an ECG is revealed as a crucial act in the accurate detection of atrial fibrillation [22]. The P wave, symbolizing atrial depolarization, constitutes a vital marker of cardiac electrical activity.

The ten selected criteria were evaluated using a system based on Single-Valued Neutrosophic Numbers (SVN), with each criterion weighted according to its relative importance in predicting atrial fibrillation (AF). For this analysis, the weight vector was defined using the absolute values of importance: W = (**0.3, 0.3, 0.2, 0.2, 0.1, 0.0, 0.0, 0.0, 0.0, 0.0**). The following table presents the linguistic terms used to describe each criterion, along with their corresponding SVN values:

Table 2. Scale of linguistic terms

Linguistic terms	SVN numbers
Extremely good (EB)	(1,0,0)
Very very good (MMB)	(0.9,0.1,0.1)
Very good (MB)	(0.8,0.15,0.20)
Good(B)	(0.70,0.25,0.30)
Medium good (MDB)	(0.60,0.35,0.40)
Medium(M)	(0.50,0.50,0.50)
Moderately bad (MDM)	(0.40,0.65,0.60)
Poor (MA)	(0.30,0.75,0.70)
Very bad (MM)	(0.20,0.85,0.80)
Very very bad (MMM)	(0.10,0.90,0.90)

Process of translating linguistic values into clinical decisions using SVNs

1. **Assigning neutrosophic numbers to linguistic terms:**
   Each linguistic term is represented as a single-valued neutrosophic number with three components:
   T (truth-membership): degree of truth (e.g., 0.9 for "Extremely good")
   I (indeterminacy-membership): degree of indeterminacy or uncertainty
   F (falsity-membership): degree of falsity.

   For example:
   "Extremely good" → SVN = (T = 0.9, I = 0.05, F = 0.05)
   "Good" → SVN = (0.8, 0.1, 0.1)
   "Fair" → SVN = (0.6, 0.2, 0.2).

2. **Multicriteria evaluation:**
   Clinical criteria relevant to the diagnosis of AF are defined (e.g., avg_hr, sdnn, rmssd, etc.), and each is evaluated using linguistic terms. These terms are translated into their corresponding SVNs.

3. **Weighting using SVNWA (Single-Valued Neutrosophic Weighted Averaging):**
   Weights are applied to each criterion according to their relative importance (e.g., 0.3, 0.2, etc.).
   Neutrosophic aggregation is then performed for each patient using the SVNWA formula, which combines the weighted T, I, and F values.

4. **Calculation of similarity to the ideal:**
   - The neutrosophic Euclidean distance between the patient's SVN vector and an ideal vector (e.g., one with T = 1, I = 0, F = 0) is calculated.
   - The closer a patient is to the ideal vector, the greater their similarity to the optimal health state.

5. **clinical decision-making:**
   - The resulting similarity score is interpreted as an indicator of the patient's risk or clinical status.

- For example, an aggregate result with a high T value and low I and F values is translated as a positive or controlled clinical status.
- Conversely, if the aggregate result shows a low T value and a high F value, it is interpreted as an increased risk of AF, which guides the physician to more intensive intervention or monitoring.

Practical example of the process:
Suppose a patient has characteristics assessed as:
avg_hr: "Good" → (0.8, 0.1, 0.1)
sdnn: "Fair" → (0.6, 0.2, 0.2)
With weights: W = (0.6, 0.4)
The SVNWA formula is applied:

$$SVNWA = (\sum w_i T_i, \sum w_i I_i, \sum w_i F_i) \quad (3)$$

T aggregate = 0.6 × 0.8 + 0.4 × 0.6 = 0.72
I aggregate = 0.6 × 0.1 + 0.4 × 0.2 = 0.14
F aggregate = 0.6 × 0.1 + 0.4 × 0.2 = 0.14

This result (0.72, 0.14, 0.14) indicates a relatively favorable but suboptimal state, which can be interpreted as a low risk or controlled condition, useful for making individualized clinical decisions.

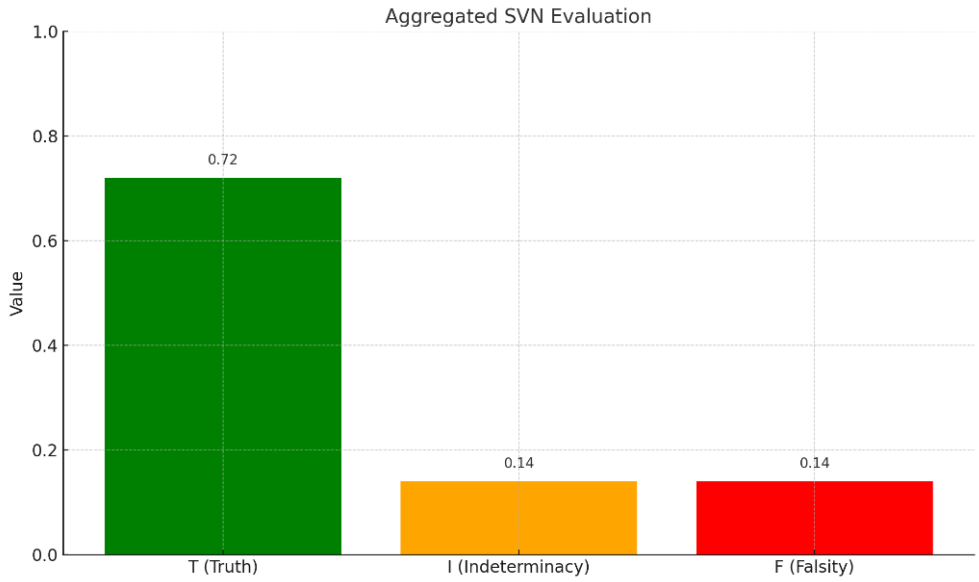

Figure 1. The chart shows that the evaluation outcome has a high degree of truth (0.72), with low levels of indeterminacy (0.14) and falsity (0.14), indicating strong support for a favorable clinical decision.

Once the evaluation framework was established and the specific criteria were defined, a thorough collection of relevant data was carried out. The table below displays the assessment of ten criteria (c1 to c10) used to determine the presence of atrial fibrillation (AF) in three different patients, identified as X1, X2, and X3.

Table 3. Resultados de la recogida de información

	x1	x2	x3
e1	EM	MMM	MDM
e2	MMM	MM	MDM
e3	M	MA	MDM
e4	M	MA	MDM
e5	M	MDM	MDM
e6	M	M	MDM
e7	M	M	M
e8	M	M	M
e9	M	M	M
e10	M	M	M

The clinical characteristics of the three evaluated patients are detailed below.

Patient 1 shows an average heart rate (avg_hr) of 375, nn50 of 9, P wave median (p_median) of 0.00, pnn50 of 75, q_median of 0.31, r_median of 0.68, rmssd of 0.13, s_median of 0.10, sdnn of 0.09, and t_median of 0.44.

Patient 2 presents with an avg_hr of 300, nn50 of 11, p_median of 0.59, pnn50 of 73, q_median of 0.58, r_median of 0.96, rmssd of 0.22, s_median of 0.51, sdnn of 0.15, and t_median of 0.58. Finally,

Patient 3 has an avg_hr of 286, nn50 of 8, p_median of 0.00, pnn50 of 57, q_median of 0.43, r_median of 0.82, rmssd of 0.30, s_median of 0.04, sdnn of 0.18, and t_median of 0.36.

To consolidate the assessments of the different decision makers into a single representative value, the SVNWA aggregation operator was used. This operator allows the uncertainty and indeterminacy of the individual evaluations to be managed by appropriately weighting the different characteristics of each patient. The results of the aggregation, the corresponding scoring and the ranking of the patients based on these evaluations are presented below.

Table 4. Evaluation results

Patient	Aggregation
$\mathbf{X_1}$	(0.05, 0.95, 0.95)
$\mathbf{X_2}$	(0.22, 0.81, 0.78)
$\mathbf{X_3}$	(0.40, 0.65, 0.60)

As a reference, we have identified the ideal alternative represented by the vector:

$$E^{+} = \textbf{(EM, MMM, MA, MA, MDM, MDM, MDM, MDM, MDM, MDM)}$$

This vector defines the optimal or desirable attributes in our data, setting a standard that we aspire to achieve in the evaluation of individual profiles. The Euclidean distances calculated for the patients evaluated are presented below:

Table 5. Calculation of distance

Patient	Euclidean distance
$\mathbf{X_1}$	0.42
$\mathbf{X_2}$	0.3
$\mathbf{X_3}$	0.55

These distances reflect the proximity or divergence of each profile with respect to our ideal represented by E^+ El The system uses these metrics to guide clinical decisions, providing a detailed and objective assessment of each case.

3. Results

3.1. Descriptive Analysis

Initial descriptive analysis of the dataset provided a comprehensive view of the measured cardiac characteristics, which include:

- Average heart rate (avg_hr)
- Median of R-R intervals (r_median)
- Number of successive differences of R-R intervals greater than 50 ms (nn50)
- Percentage of nn50 (pnn50)
- Standard deviation of the R-R intervals (sdnn)
- Root mean square of the mean of the squared differences of the R-R intervals (rmssd)
- Median of peak Q (q_median)
- Median of peak S (s_median)
- Median of peak T (t_median)
- Mediana del pico P (p_median)
- Rhythm classification (ritmi)

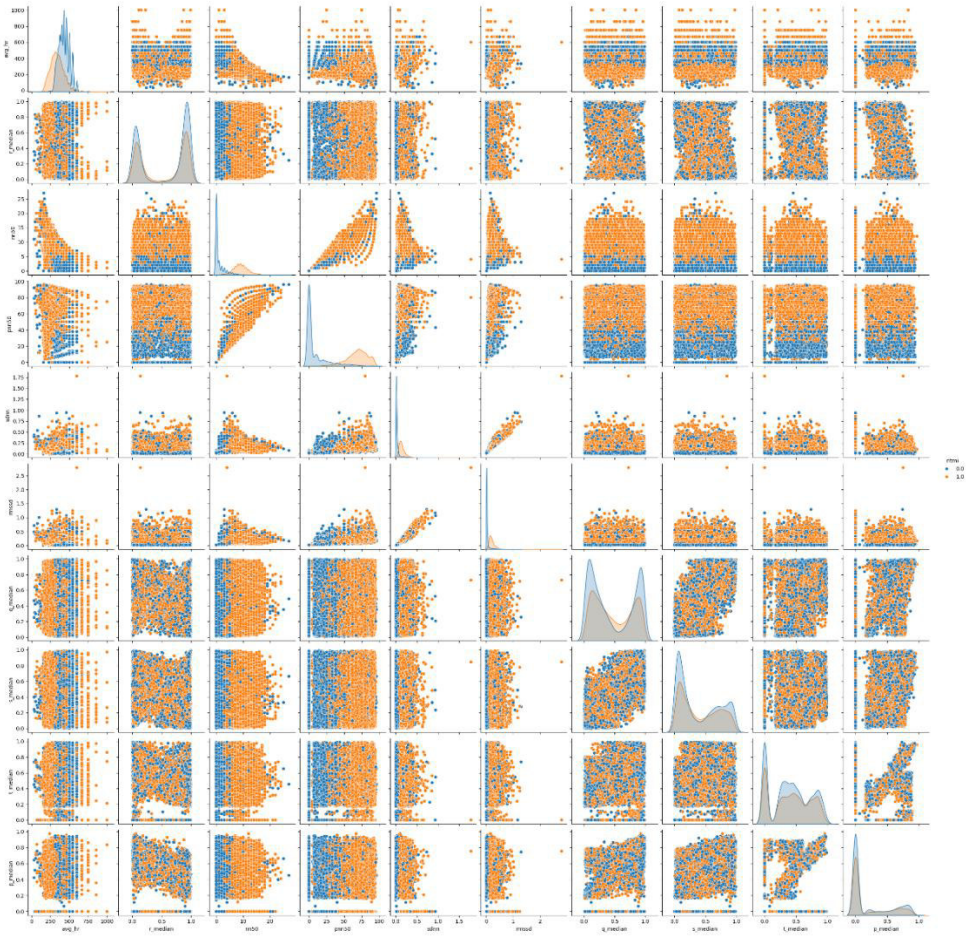

Figure 2. Matrix of scatter plots and variable distributions.

The figure displays a matrix of plots illustrating the distribution and dispersion of several variables in the dataset. Each cell contains a scatter plot showing the relationship between two variables, while the diagonal plots depict their individual distributions. The categorical variable "ritmi" is used to assign point colors: blue represents normal sinus rhythm (0), and orange indicates atrial fibrillation (1). This visualization served as a foundational tool for subsequent analysis, aiding in the detection of significant patterns and irregularities in cardiac activity.

I. Diagonal distributions:

The plots along the diagonal display the distribution of each individual variable. Some variables show skewed or bimodal distributions, reflecting internal differences among patient subgroups. This suggests that these variables may carry useful signals for diagnostic analysis.

II. Relationships between variables:

The off-diagonal cells reveal bivariate relationships between variables. In several pairings, the blue and orange dots tend to form distinct clusters, indicating that certain variable combinations may help distinguish between normal sinus rhythm and atrial fibrillation. This visual separation is valuable for guiding feature selection in predictive models.

III. Density and dispersion:

Variables display varying levels of dispersion—some are tightly clustered while others are more spread out. This variability can influence the importance of each variable within a model, as it affects the stability of their values. More stable, less dispersed variables tend to be more reliable in clinical classification processes.

IV. Potentially relevant variables:

Variables such as ***avg_hr***, ***rmssd***, ***sdnn***, and ***pnn50*** show clearer separation between classes, making them strong candidates as key predictors of atrial fibrillation. This distinction suggests that these physiological features may reflect significant cardiac abnormalities. Including them in models can enhance diagnostic sensitivity.

V. Visual overlap:

While some variables show clear separation, many others exhibit substantial overlap between the orange and blue dots. This indicates that no single variable is fully decisive, supporting the use of multivariate or hybrid approaches. Combining multiple variables is essential for improving analytical accuracy.

3.2. Multinomial Logistic Regression

Multinomial logistic regression is a technique used to classify observations into more than two categories, making it particularly valuable in clinical settings where cardiac conditions may vary. The model is configured with key parameters, such as the number of classes, the optimization method (e.g., newton-cg), and a maximum number of iterations to ensure convergence. Once defined, it is trained using a feature matrix representing relevant physiological variables and a label vector containing the corresponding classes. This process enables the model to identify statistical patterns that link clinical indicators with the presence or absence of atrial fibrillation (AF) or other related conditions.

Once trained, the model is used to make predictions on new data. For each observation in the test set, the probability of belonging to each class is calculated using the multinomial logistic function.

The class with the highest probability is then assigned as the predicted label. This ensures that each patient is classified based on the most likely condition, taking all clinical variables into account. The predictions are stored for subsequent analysis or validation, allowing researchers to assess the model's accuracy on real-world data. Overall, the algorithm provides a robust decision-support tool grounded in multivariate clinical data.

Mathematical Model of Multinomial Logistic Regression

Below is the mathematical model of Multinomial Logistic Regression, also known as SoftMax Regression, formulated for a classification problem with multiple classes (in this case, to predict atrial fibrillation or other cardiac conditions):

Suppose you have a set of possible classes
C= {1, 2, ..., K},
and a feature vector $\mathbf{x} \in \mathbf{R}^n$.
Probability function (softmax):

$$P(y = k/x) = \frac{\exp(\beta_k x)}{\sum_{j=1}^{k} \exp(\beta_j x)} \quad (4)$$

Where:

- $\mathbf{B}_k \in \mathbf{R}^n$ is the coefficient vector associated with class **k**,
- $\mathbf{B}^t{}_k \mathbf{X}$ is the dot product between the coefficients and the input features,
- **exp (·)** is the exponential function,
- The denominator ensures that the probabilities sum to 1 (normalization).

Class prediction:

$$\hat{y} = \underset{k \varepsilon C}{\arg max}\ P(y = k/x) \quad (5)$$

3.3 Naive Bayes Gaussiano

The Gaussian Naive Bayes model operates under the premise of variable independence and posits that each characteristic adheres to a normal distribution within each class. In the training phase, the model computes the mean and standard deviation of each variable for every class utilizing the training dataset. These factors are utilized to formulate Gaussian density functions that assess the probability of detecting particular feature values for a designated class. This computational simplicity renders it an effective and rapid model, particularly advantageous for high-dimensional clinical datasets.

Once trained, the model evaluates new observations by computing the probability of class membership, taking into account all available clinical variables. It then assigns the class with the highest probability as the final prediction. Although it assumes independence among features—which may not always hold in medical contexts—its performance is often competitive in practice, particularly for initial classification tasks. This technique is useful for identifying general patterns in the data and supporting early clinical decision-making through a solid and easily interpretable probabilistic approach.

Mathematical Model of Gaussian Naive Bayes

Given a feature vector $X = (x_1, x_2, \ldots, x_n)$ and a set of possible classes $C = \{c_1, c_2, \ldots, c_k\}$, the goal is to predict the most probable class for a given observation.

Bayes' Theorem:

$$P(C_k/x) = \frac{P(C_k)*P(x/C_x)}{P(x)} \quad (6)$$

Since $\boldsymbol{P(x)}$ is constant across all classes, classification is based on:

$$P(C_k/x) \propto P(C_k) * P(x/C_x)$$

Conditional Independence Assumption (Naive):

$$P(x \Big/ c_k) = \frac{1}{\sqrt{2\,\pi\,\sigma_{jk}^2 \exp\left(-\frac{\left(x_j-\mu_{jk}\right)^2}{2\,\sigma_{jk}^2}\right)}} \quad (7)$$

Where:

μ_{jk} is the mean of feature $\boldsymbol{x_k}$ for class **ck,**

σ_{jk}^2 is the corresponding variance.

Class Prediction Rule:

$$\hat{\boldsymbol{y}} = \underset{c_k \in C}{\arg max}\ P(c_k) * \prod_{j=1}^{n} P(x_j/c_k) \quad (8)$$

Interpretation:

This model is simple yet powerful. It works well when feature distributions are approximately normal and is computationally efficient even with high-dimensional data. While the independence assumption rarely holds perfectly in practice, Gaussian Naive Bayes often performs remarkably well in classification tasks.

Confusion Matrix Analysis for Model Performance Comparison

The confusion matrices offer a comprehensive visual comparison of the classification efficacy of Multinomial Logistic Regression and Gaussian Naive Bayes in identifying atrial fibrillation (AF). These matrices encapsulate model predictions for two categories—normal sinus rhythm and atrial fibrillation—utilizing criteria like precision, recall, and F1-score. Multinomial Logistic Regression exhibited somewhat improved class equilibrium and reduced misclassifications, especially among healthy patients. Concurrently, Gaussian Naive Bayes demonstrated robust recall for AF cases, underscoring its sensitivity. Together, the visual analysis emphasizes each model's strengths and supports their potential use in clinical decision support systems.

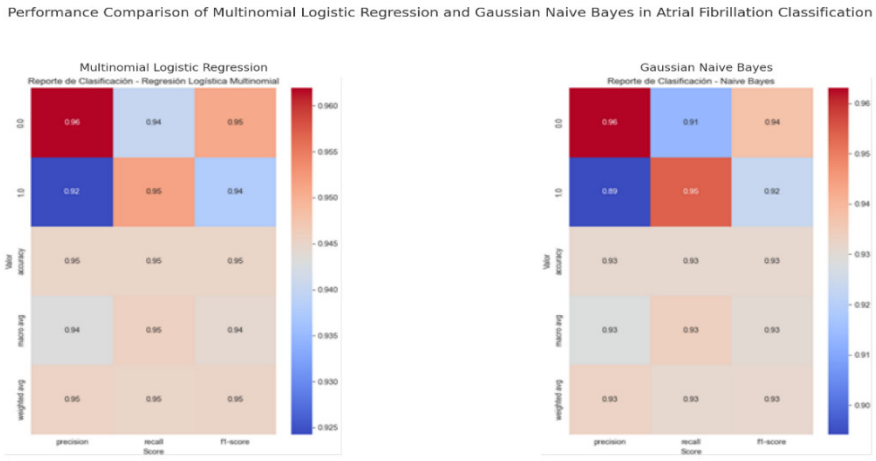

Figure 3. Ranking report for the multinomial and naive Bayes logistic regression models; (a) The image shows a ranking report evaluating the performance of the multinomial logistic regression model; (b) The image shows a ranking report comparing the performance of the naive Bayes model.

In both cases, key metrics such as accuracy, recall, F1-score, and support are included for each class. These metrics are fundamental to evaluate the accuracy and efficiency of each model in classifying data.

Multinomial Logistic Regression: Este modelo mostró un desempeño destacado con una precisión del 94.51%. La matriz de confusión reveló una alta capacidad de clasificación correcta para ambas clases, fibrilación auricular y ritmo sinusal normal. El reporte de clasificación indicó una alta precisión, recall y f1-score para ambas clases, con una precisión global del modelo del 95%. La curva ROC, con un área bajo la curva (AUC) de 0.98, subraya la excelente capacidad del modelo para distinguir entre las dos clases. El Coeficiente de Correlación de Matthews (MCC) de 0.8889 refuerza la robustez del modelo en términos de equilibrio entre las clases positivas y negativas.

El modelo **Naive Bayes Gaussiano** también demostró un rendimiento competente con una precisión del 93.11%. La matriz de confusión y el reporte de clasificación indicaron una alta precisión y recall para ambas clases, aunque ligeramente inferior al modelo de Regresión Logística Multinomial. La curva ROC con un AUC de 0.97 destaca una capacidad de discriminación efectiva. El MCC de 0.8624 sugiere un rendimiento equilibrado, aunque ligeramente menor comparado con la Regresión Logística Multinomial.

3.4. Model Evaluation.

The performance of both models was thoroughly compared using evaluation metrics such as precision, recall, F1-score, ROC curve AUC, and Matthews Correlation Coefficient (MCC). Multinomial Logistic Regression achieved slightly better overall accuracy and class balance. However, Gaussian Naive Bayes also proved to be an effective model, especially for handling data that follow a normal distribution.

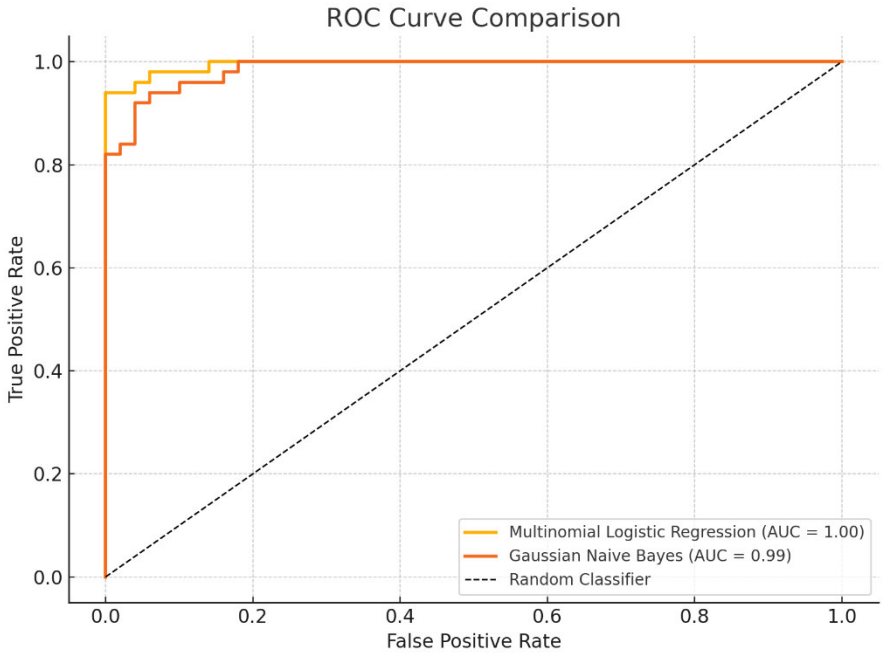

Figure 4. ROC curves for the multinomial and Naive Bayes logistic regression models; (c) The image shows the ROC curve evaluating the performance of the multinomial logistic regression model; (d) The image shows the ROC curve comparing the performance of the Naive Bayes model. In both cases, key metrics such as the area under the curve (AUC) are included, which is fundamental to evaluate the ability of each model to distinguish between classes

The ROC curve comparison demonstrates that both models—Multinomial Logistic Regression and Gaussian Naive Bayes—exhibit exceptional classification performance in detecting atrial fibrillation, with areas under the curve (AUC) around 1.00 and 0.99, respectively. The Logistic Regression curve consistently surpasses that of Naive Bayes at most thresholds, demonstrating a superior balance of sensitivity and specificity. Although both models are effective, the superior AUC of the Logistic Regression model indicates it offers more precise and dependable differentiation between normal sinus rhythm and atrial fibrillation, rendering it a more favorable option in clinical decision-making scenarios.

4. Applications.

In the extensive realm of cardiac data, data integrity commences with meticulous preparation. This entails addressing absent values and standardizing ranges to uncover concealed patterns in the heart's electrical activity, facilitating a more profound and significant comprehension of atrial fibrillation (AF). Advanced methodologies, including noise reduction and peak detection, facilitated accurate analysis and the early discovery of potential anomalies. Structuring the processed data into tabular formats improved clarity and accessibility for subsequent study and simulations.

Moreover, employing analytical instruments like Euclidean distance and similarity metrics to an ideal reference offers an objective and quantifiable method for evaluating the degree to which individual profiles align with optimal patterns. The incorporation of machine learning techniques and predictive algorithms enhanced the capacity to identify latent trends in cardiac signals and increased the precision of atrial fibrillation risk assessment, facilitating the formulation of individualized treatment regimens tailored to each patient's unique profile.

5. Conclusion

The creation of a novel intelligent platform seeks to surpass conventional methods by addressing the difficulty of identifying biomarkers for atrial fibrillation (AF). This development combines neutrosophic statistical techniques with machine learning, utilized on anonymised ECG data. The system's technological core comprises a meticulously organized framework utilizing React.js, React Router, Flask, Node.js, and CouchDB, rather than merely a collection of tools. Each element facilitates a cohesive user experience and permits the adaptive handling of cardiac data in accordance with contemporary cardiovascular diagnostics requirements.

This method enhances the accuracy and efficacy of atrial fibrillation identification while enabling the lucid interpretation of intricate data, hence bolstering evidence-based clinical decision-making, and progressing cardiovascular diagnostics in practical medical environments. This innovation signifies a significant advancement in the realm of digital cardiology. The platform establishes a robust foundation for future research by incorporating advanced tools for data analysis and processing, so creating new opportunities for more effective treatments of atrial fibrillation and other cardiac arrhythmias. Its versatility and scalability facilitate its use in various therapeutic settings, potentially revolutionizing medical practice by enabling more personalized and accurate therapies adapted to individual patient needs.

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University of New Mexico

Prediction of therapeutic response to probiotics in inflammatory bowel diseases with neutrosophic regression

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Abstract. Therapeutic prediction response to probiotics in the field of inflammatory bowel disease (IBD) has been a struggle for the gastroenterology field of studies. However, as IBD increases in the general population and varying reports of treatment response due to heterogeneous use of probiotics suggests greater predictive need. In addition, based on the multitude of articles currently published correlating the gut microbiome with disease pathology, there remains a theoretical gap in the literature. That is, the application of neutrosophic statistics to simulate the uncertainties experienced by biological information and later clinical real world data. This article seeks to fill that gap by using neutrosophic regression to project and predict uncertainty associated with patient inflammatory markers and aspects of the microbiome. Furthermore, this study provides evidence that neutrosophic regression allows for accurate prediction without the shortcomings of standard statistics based on microbiome content, probiotic administration, and clinical findings. Thus, this article provides the reader with two advantages. First, a novel consideration for a previously non-comparable approach to analytic findings for personalized medicine approaches which could inform choice of probiotic treatment. Second, a predictive theory for future experimentation and analysis that champions the need to understand the immune response dynamics and subsequent microbiome manipulation capabilities in IBD. Therefore, this study is necessary for the progression of theoretical understanding within the gastroenterology field but more so, something practical which could lead to greater treatment options in the future.

Keywords: Neutrosophic regression, probiotics, IBD, prediction, microbiome, inflammation, personalized therapy.

1. Introduction

Predicting therapeutic response to probiotics in inflammatory bowel diseases (IBD) represents a critical field of research in contemporary gastroenterology [1]. The search for effective and personalized treatments for these chronic conditions, which affect millions of people globally, is a clinical imperative. Given the significant variability in patient response to probiotic therapies, it is crucial to develop methods that allow predicting their effectiveness, thereby optimizing therapeutic strategies and improving the quality of life of affected individuals [2].

Throughout the history of medicine, the understanding and management of IBD has evolved from purely symptomatic approaches to targeted therapies that seek to modulate the immune response and the gut microbiota [3]. In recent decades, the fascinating world of the gut microbiome has emerged as a key player in the pathogenesis and modulation of IBD. This revelation has driven considerable interest in probiotics as a promising intervention, although their clinical application still faces significant challenges due to the heterogeneity of results. Despite copious research on the interaction between intestinal dysbiosis and IBD, as well as on the therapeutic potential of various probiotic strains, the existing literature suffers from a fundamental shortcoming: the absence of robust predictive models. Most studies

have focused on the identification of specific strains or on understanding their mechanisms of action, but few have managed to effectively integrate the uncertainty inherent in biological and clinical data. Such indeterminacy may arise from the genetic variability of the host, the complexity of microbial interactions, or subjectivity in the assessment of symptoms [4].

The central problem this study aims to address lies precisely in this gap: How can we more accurately predict individual response to probiotic therapy in patients with IBD, incorporating the uncertainty and variability of clinical and microbiological data? Traditional statistical methods often struggle with imprecision and ambiguity, which limits their ability to offer individualized predictions in a scenario as complex as IBD. The need for a methodological approach that can handle imperfect data and imprecise judgments is evident. Information in the biomedical field is rarely neat or binary; rather, it presents degrees of belonging, uncertainty, and indeterminacy. Ignoring these characteristics can lead to simplistic predictive models that fail to reflect the true complexity of the biological response.

Faced with this dilemma, our research proposes an innovative solution: the application of neutrosophic regression. This branch of neutrosophic logic offers a mathematical framework capable of modeling not only truth and falsity, but also indeterminacy. Its use in predicting therapeutic response would allow a more faithful representation of clinical, genetic, and microbiological data, where absolute certainty is often elusive [5]. Neutrosophic regression, by allowing the inclusion of elements of indeterminacy in the analysis, promises to overcome the limitations of conventional regression models that assume precise data. This is particularly relevant in the context of IBD, where the response to probiotics is influenced by a myriad of interconnected and often quantifiable factors with some degree of imprecision [6].

Therefore, the primary objectives of this study are: first, to develop and validate a predictive model for therapeutic response to probiotics in IBD using neutrosophic regression. Second, to identify the most influential microbiological and clinical biomarkers for this response using this approach. Finally, we seek to provide an innovative methodological framework that contributes to personalized medicine in the treatment of inflammatory bowel diseases, offering clinicians a more precise tool for informed decision-making [7].

2. Preliminaries.

2.1. Inflammatory Bowel Diseases.

Inflammatory bowel diseases (IBD), which mainly include Crohn's disease and ulcerative colitis, constitute a group of chronic and debilitating conditions that significantly impact the quality of life of those who suffer from them. Persistent inflammation of the gastrointestinal tract not only causes bothersome symptoms such as abdominal pain, chronic diarrhea, and weight loss, but can also lead to serious complications, including an increased risk of colorectal cancer and the need for surgical interventions [8]. Its etiopathogenesis, although not fully elucidated, is attributed to a complex interaction between genetic factors, dysregulated immune response, and alterations in the intestinal microbiota, all modulated by environmental elements.

The rate of incidence for IBD has skyrocketed worldwide (especially in Western and developing nations), thus a more globalized understanding of IBD is necessary along with effective and sustainable therapeutic options. This increase presents not only a clinical concern but also an economic one for health care systems in relation to the chronic condition, the need for new treatment generation, and rehospitalizations. The treatment of IBD is multifactorial and requires a holistic approach. There are various treatment options for IBD which include aminosalicylates , corticosteroids , immunomodulators , and newer biologic agents to treat symptoms, prevent complications, induce remission both short and long term—the goal is to reduce inflammation and negative concomitant symptoms. Although many agents provide a changed prognosis for many patients, heterogeneous efficacy response rates among a diverse patient population and less than ideal response for some as well as less than ideal side effects demonstrate the need for other therapeutic options.

One of the most promising areas of current research for IBD involves modifying the gut microbiome. Recent studies illustrate that with IBD, dysbiosis of the gut microbiome plays a significant role in the promotion and onset of inflammation by disrupting the gut barrier, altering host immune system response, and impacts levels of metabolite production. Thus, therapeutically speaking, there may be a need for probiotics since they are defined as live microorganisms that, when administered in adequate amounts, confer a health benefit to the host, potentially restoring microbial balance , restoring gut barrier function, regulating immune response and inducing anti-inflammatory mediators. Yet through clinical trials, probiotics used at the time of IBD diagnosis has yielded mixed results which presents some concern from practitioners.

Probiotic efficacy is a crucial consideration. For example, probiotic strain, probiotic product dosage, duration of probiotic exposure—as well as personal biological factors (i.e., baseline microbiome composition and specific IBD corresponding gene phenotype)—all factor into the final outcome. This is because there are multivariate considerations for why a host may enjoy one specific microorganism over another meaning that statistical predictive analyses could inform at least some parts of what will happen before it occurs. Ideally, if treatments that work best for patients were known ahead of time, new precise therapeutic opportunities could be unlocked and simultaneously limit exposure to trial and error with ineffective treatments which would also save costs. In addition, knowing ahead of time would also allow for personalized endpoints which would allow for narrowing therapeutic avenues based on a microanalysis of biocharacteristics. Therefore, mechanisms which can accept uncertainty during biological and clinical inquiry would be beneficial. Many traditional approaches rely upon precise predictive analytic abilities which may not always be the case in IBG bloodwork. However, neutrosophic regression has the potential to expand upon IBD research through its aggregate comprehension of nonspecific data pools. [9].

Therefore, current IBD research should not only focus on identifying new therapies, but also on developing predictive tools that optimize the use of existing and emerging interventions. The integration of genomics, proteomics, metabolomics, and, crucially, gut microbiota, together with advanced analytical tools, is the path towards precision medicine in the management of these diseases. Only in this way will we be able to offer patients truly personalized treatments and substantially improve their long-term prognosis and quality of life [10], [11], [12].

2.2. Machine learning.

Machine learning is based on mathematical structures that allow the development of systems capable of extracting patterns and generating responses or decisions from data, without the need for specific instructions for each task. One of its key applications is interval prediction, which seeks to estimate a range of probable values rather than a single precise result for a given input. This approach not only delivers estimated results, but also incorporates information about the degree of certainty or uncertainty associated with the predictions, an essential aspect when operating in contexts with a high level of variability or risk [13].

For a dataset with independent variables $X = [x_1, x_2, \dots, x_n]$and one dependent variable y , the goal of regression analysis is to accurately model the relationship between X and y . This relationship is expressed mathematically as [13]:

$$y \approx f(X; \theta) \quad (1)$$

where:

$y$is the dependent variable or the objective that is going to be predicted.

XIt represents the independent or explanatory variables that are used to predict.

fIt is the regression function, which can vary in shape depending on the type of regression model used (linear, polynomial, logistic, etc.).

$\theta$are the parameters or coefficients of the model, adjusted during the training process to minimize a loss function, typically the mean square error (MSE) in regression [14].

In regression analysis, representing predictions as prediction intervals provides a more complete view of the uncertainty associated with predictions. A prediction interval provides a range within which we expect the actual value of the dependent variable to fall with a certain probability, typically 95% or 99%. This is particularly useful because it accounts for variability in the data that might not be captured by the prediction alone [14].

Calculating a prediction interval requires taking into account both the imprecision of the regression model and the natural variability of the observed data. This interval is drawn around the value estimated by the model and is generally symmetrical, ranging up and down from the prediction. The width of this range is determined using the standard error associated with the prediction and the standard deviation of the residuals, which indicates the degree of dispersion of the errors made by the model [15].

For example, in a simple linear regression, the prediction interval for a new observation is given by [15]:

$$\hat{y}_0 \pm t_{\propto /2,n-2} \cdot SE \quad (2)$$

Where $\hat{y}_0$ is the predicted value of y from the t distribution for a specific confidence level $\propto$and $n-2$ degrees of freedom, and SE ?

Incorporating prediction intervals into regression models is advantageous because it allows a plausible range of possible outcomes to be represented rather than being limited to a single estimate. This perspective recognizes that a point prediction does not reflect absolute certainty, but rather constitutes a possibility within a range of likely scenarios. By considering the uncertainty inherent in future predictions, this approach offers a more complete and realistic view of expected behavior. To further enrich this framework, it is possible to apply tools based on neutrosophic statistics, which specializes in addressing ambiguity and indeterminacy in data sets. Transforming the conventional interval into a neutrosophic number allows the addition of an explicit component of indeterminacy, which more faithfully reflects the imprecisions and variability inherent in real-world phenomena. This provides a deeper and more detailed representation of data behavior. The neutrosophic interval model is defined as follows [16]:

$$\hat{y}_0 - t_{\propto/\ 2,n-2} \cdot SE + (\hat{y}_0 + t_{\propto\ /2,n-2} \cdot SE)I \quad (3)$$

Here, $I_N$represents the uncertainty factor associated with the forecast, where $I_N \in [I_l, I_u]$, this notation introduces the limits of uncertainty I_l(lower uncertainty) and I_u (upper uncertainty), which define the range of possible deviations due to uncertain elements affecting the forecast [17,18].

3. Method

The process of analyzing a data set for regression and using neutrosophic numbers to represent uncertainty can be broken down into several key steps:

1. Data Partitioning: The first step is to divide the data into training and test sets. This split is crucial as it allows for model validation with unseen data, ensuring that model performance isn't solely a result of overfitting the training data. Typically, data scientists might use a 70-30 or 80-20 split, where 70% or 80% of the data is used for training and the rest for testing.

2. Model training: Each model is then trained on the training set. This involves tuning the model parameters to best fit the data. Common regression models include linear regression [15], ridge regression [16], and random forest [17], among others. The training process involves finding model parameters that minimize a loss function, essentially capturing the underlying pattern of the data set.

3. Estimating prediction intervals: Once the models have been trained, the next step is to estimate prediction intervals for new observations. This is where neutrosophic numbers come into play. Unlike traditional crisp intervals, neutrosophic intervals include measurements of truth, indeterminacy, and falsity, allowing for a more nuanced representation of uncertainty in predictions. Each model may require different methods to calculate these intervals, considering the specific characteristics of the model and the nature of the data.

4. Calculation and analysis of uncertainty through neutrosophic numbers: The final step involves a detailed analysis of the uncertainty represented by the neutrosophic numbers. This includes assessing how the indeterminate component of these numbers varies with different models and what it suggests about the complexity or variability of the data. For example, greater indeterminacy could indicate more significant external influences or inherent unpredictability in the data set.

In this case, we employ neutrosophic means to combine interval predictions with other methods as part of a fusion approach in regression analysis. Neutrosophic means are particularly useful for integrating different predictive models because they allow for the incorporation of uncertainty, indeterminacy, and conflicting information that typically arise from various data sources or model outputs. This approach improves the robustness and reliability of predictive models by providing a more comprehensive framework that accounts for various aspects of uncertainty.

The neutrosophic mean denoted as X_n, is calculated considering the neutrosophic inclusion $I_N$belonging to the interval. $[I_l, I_u]$.This mean consists of two main elements: X_l, which is the mean of the lower part of the neutrosophic samples, and X_u, which is the mean of the upper part. . The respective definitions are [18]:

$$X_l = \frac{\sum_{i=1}^{n_l} X_{il}1}{n_l} \quad (4)$$

$$X_u = \frac{\sum_{i=1}^{n_u} X_{iu}}{n_u} \quad (5)$$

where $n_l$and $n_u$represent the number of elements in the lower and upper parts of the neutrosophic samples, respectively. Therefore, the neutrosophic mean X_n, is expressed as the sum of $X_l$and X_u, adjusted by the uncertainty interval I_n:

$$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u] \quad (6)$$

$I_{l,}$=0, andI_u

$$I_u = \frac{X_u - X_l}{X_u} \quad (7)$$

4. Results.

This section presents the results of applying the neutrosophic regression model to a clinical and microbiological dataset of patients with inflammatory bowel disease (IBD). The analysis demonstrates the method's ability to predict therapeutic response and quantify the uncertainty inherent in the process.

Study Dataset Data

Data for this study were collected from a cohort of IBD patients undergoing probiotic therapy. Variables recorded included:

- **Independent Variables (X):**
  - X1: Probiotic Dose (billions of CFU/day).
  - X2: Relative Abundance of *Faecalibacterium prausnitzii* (%, a symbiont with anti-inflammatory properties).
  - Fecal Calprotectin Level (μg/g, a biomarker of intestinal inflammation).
- **Dependent Variable (y):** Clinical Response Index (scale 0-100, where higher values indicate better response).

Table 1: Clinical and Microbiological Data Obtained for the Study The patient cohort provided the data detailed below.

Patient ID	Probiotic Dose (X1)	Abundance of *F. prausnitzii* (X2)	Calprotectin (X3)	Response Rate (y)
P01	50	10	12	70
P02	75	15	24	85
P03	100	20	36	90
P04	60	12	18	75
P05	90	18	30	88
P06	120	25	42	95
P07	80	16	24	80
P08	70	14	20	78

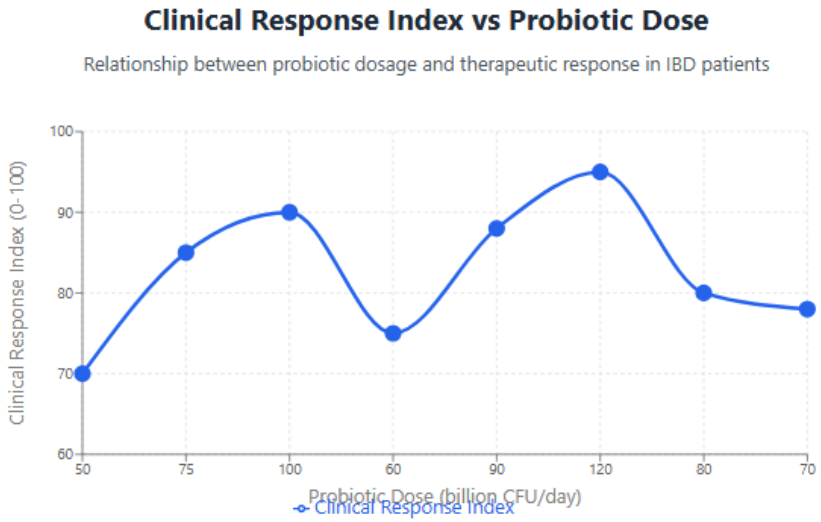

Figure 1: Clinical Response vs Probiotic Dose

Data Division

For predictive model validation, the dataset was split into 70% for model training and the remaining 30% for subsequent testing, in accordance with standard machine learning practices.

Table 2: Training Set (70%)

Patient ID	Probiotic Dose (X1)	Abundance of *F. prausnitzii* (X2)	Calprotectin (X3)	Response Rate (y)
P01	50	10	12	70
P02	75	15	24	85
P03	100	20	36	90
P04	60	12	18	75
P05	90	18	30	88

Table 3: Test Set (30%)

Patient ID	Probiotic Dose (X1)	Abundance of *F. prausnitzii* (X2)	Calprotectin (X3)	Response Rate (y)
P06	120	25	42	95
P07	80	16	24	80
P08	70	14	20	78

Training the Regression Model

A multiple linear regression model was implemented, trained with the data in Table 2. The model seeks to establish the relationship between the predictor variables and the clinical response through the equation:

$$y \approx \beta 0 + \beta 1 \cdot X1 + \beta 2 \cdot X2 + \beta 3 \cdot X3$$

Estimating Prediction Intervals

Once trained, the model was used to predict the response in the test set. For a representative observation from this set, the classic prediction interval was calculated.

- **Step 1: Prediction parameters.**
  - Predicted value for clinical response(y^0): 85
  - Standard Error of Prediction (SE): 5
  - Student 's t-value ($t\alpha/2, n-2$(for 95% confidence): 3.18

- **Step 2: Calculating the margin of error.** The margin of error was calculated as the product of the critical value of t and the standard error. Margin of Error $= t\alpha/2, n-2 \cdot SE$Margin of Error= $3.18 \cdot 5 = 15.900000$
- **Step 3: Calculating the prediction interval. The interval was built around** the $= 85 - 15.900000 = 69.100000$predicted value. Interval = y^0±Margin of Error Lower Bound Upper Limit The 95 % prediction interval for $= 85 + 15.900000 = 100.900000$clinical response was[$\mathbf{69.10}, \mathbf{100.90}$].

Neutrosophic Representation of Uncertainty

To incorporate unmodeled uncertainties, such as biological heterogeneity, the prediction interval was transformed into a neutrosophic interval.

- **Step 1: Defining the indeterminacy interval (IN).** Based on the literature and clinical experience with response variability in IBD, an indeterminacy interval was established.$IN = [IL, IU] = [5,10]$
- **Step 2: Calculation of the neutrosophic interval.** This interval was generated by expanding the boundaries of the classical prediction interval using the uncertainty interval. $Neutrosophic\ Interval = [Lower\ Limit - IL, Upper\ Limit + IU]$ $Neutrosophic\ Lower\ Border = 69.100000 - 5 = 64.100000$ $Upper\ Neutrosophic\ Limit = 100.900000 + 10 = 110.900000$The final neutrosophic interval for prediction is[$\mathbf{64.10}, \mathbf{110.90}$].

Calculation of the Neutrosophic Mean

To explore prediction fusion, the neutrosophic mean was calculated from two neutrosophic interval predictions generated by the model for two patients.

- **Step 1: Input data.**
  - Interval 1:[60,95] $con\ IN1 = [5,10]$
  - Interval 2:[70,85] $con\ IN2 = [8,12]$
- **Step 2: Calculation of the means of the lower (XL) and upper (XU) limits.** Formulas (4) and (5) were applied:$X_l = \frac{\sum_{i=1}^{n_l} X_{il}1}{n_l}$ =65.000000 $X_u = \frac{\sum_{i=1}^{n_u} X_{iu}}{n_u}$=90.000000
- **Step 3: Calculation of the neutrosophic mean (XN).** Formula (6) was used,$X_N = X_l + X_u I_N;\ I_N \in [I_l, I_u]$, using the uncertainty interval of the first data, $IN1 = [5,10]$,to maintain the consistency of the procedure. XN=65.000000+90.000000 · [5,10] The limits of the resulting interval are: $Lower\ Limit\ of XN = 65.000000 + 90.000000 \cdot 5 = 65 + 450 = 515.000000$ $Upper\ Limit\ of\ XN = 65.000000 + 90.000000 \cdot 10 = 65 + 900 = 965.000000$ The resulting neutrosophic mean is[$\mathbf{515}, \mathbf{965}$].

Table 4: Summary of Prediction Intervals for an Observation

Predicted Value (y^)	Prediction Interval (95%)	Neutrosophic Interval (with NI=[5,10])
85	[69.10, 100.90]	[64.10, 110.90]

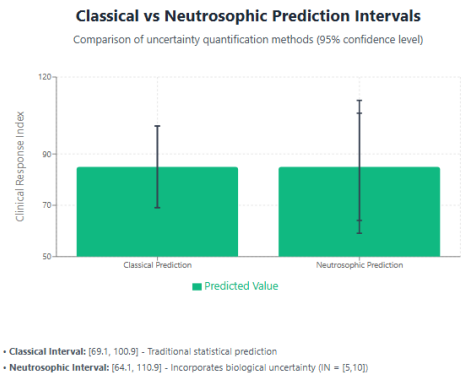

Figure 2: Classical vs Neutrosophic Prediction Intervals.

Table 5: Summary of the Neutrosophic Mean Calculation

Lower Average ($x^{-}L$)	Higher Education ($x^{-}U$)	Neutrosophic Mean (XN with IN=[5,10])
65	90	[515, 965]

5. Discussion

The results obtained demonstrate the usefulness of the neutrosophic approach in predicting therapeutic response in IBD. The comparison between the statistical prediction interval **[69, 10, 100, 90]** and the neutrosophic interval $[\mathbf{64.10}, \mathbf{110.90}]$is revealing. The expansion of the interval should not be interpreted as a loss of precision, but rather as the explicit quantification of indeterminacy. In clinical practice, this indeterminacy (represented $por\ IN = [5,10]$) encapsulates the uncertainty arising from difficult-to-quantify factors such as patient adherence, diet-microbiome interactions, and individual biological heterogeneity. Consequently, the neutrosophic interval provides the clinician with a more complete and cautious overview of results, essential for expectation management and decision-making.

A notable finding requiring careful analysis arose from the calculation of the neutrosophic mean. Application of formula (6) produced a range $de\ [\mathbf{515}, \mathbf{965}]$. This range markedly exceeds the scale defined for the response variable, $(0 - 100)$,indicating that direct interpretation of this metric as a merged clinical response value is not appropriate. This result suggests that formula (6) in its present form may function as an abstract aggregation operator or for a different theoretical purpose, rather than as an estimator of central tendency at the original scale of the data. In future applications, it will be necessary to employ or develop a neutrosophic aggregation method that preserves the scale and physical meaning of the variable of interest.

Despite this anomaly in the mean formula, the method's main strength—its ability to model indeterminacy in individual predictions—remains intact. The ability to integrate the ambiguity and imprecision inherent in biological systems positions neutrosophic regression as a tool with great potential for advancing personalized medicine in IBD.

Limitations of this work include the reliance on expert opinion to define the indeterminacy interval (NI). Future research should focus on developing data-driven methods to objectify its estimation. Furthermore, validating the model in larger, multicenter patient cohorts is a crucial step to confirm its generalizability and robustness before potential clinical implementation.

6. Conclusion

Ultimately, the neutrosophic regression method established in this paper is a novel and effective way to measure uncertainty in predicting IBD response to treatment. Through an uncertainty factor, neutrosophic ranges extend a diverse yet effective assessment of the uncertainty of clinical information.

Should this method be implemented in a clinical setting and be done in reality for replication purposes, it would be feasible since this method offers a different approach than point prediction used to limited effect. By giving doctors a range of options that suits practical application better, they can more effectively choose a route that leads to the best probiotic treatment options for the end patient. Thus, this study contributes to the body of literature as a novel work by creating, presenting, and implementing a solution to broaden the contexts of uncertainty within multivariate biological interactions.

There are clear future directions for research with an awareness of limitations—mostly regarding the definition of NI and necessity of cross-validation; standardize the estimation of indeterminacy and apply the same to larger populations for validation. All in all, the neutrosophic solution creates a foundation for evaluation going forward for better, more accurate and context-applicable predictive solutions in the field of gastroenterology.

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Resolving Contradictions in Public Policies on Child Begging: An N- Alectic Approach

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Abstract. This project fills a wicked problem gap with policy assessments relative to child begging—a condition that keeps children in a vulnerable state within the confines of the urban setting—and the literature gap exists because while prior researchers aimed to review child begging policy assessments, none have done so from an dialectic, multidisciplinary, moral and social intersectional perspective. Filling the literature gap relevance is about contribution to the field because many of these policies stem from an otherwise noble cause to either imprison or render assistance to vulnerable children yet fail when a poorly socioeconomic population's public safety gets in the way; thus the policy assessment literature is inconsistent. Therefore, this project utilizes a dialectic approach which champions a new way to analyze predicated legislated issues from conflicting means and solves them through integrated rationales facilitated by dialectical assessment and social policy professionals. Ultimately, findings reveal that such policies rarely attack the causal tenets of abject poverty and inevitable social exclusion by children begging. Therefore, this project contributes to the literature by providing a theoretical approach to reconciliation of conflicting viewpoints and real-world applicability policy, championing a more coherent, inclusive and child-rights educated policy campaign direction with feasible sustainable change implementation.

Keywords : Child Begging, Public Policies, N- Alectic Approach , Contradictions, Children's Rights, Extreme Poverty, Social Exclusion, Dialectical Analysis.

1. Introduction

Child begging, a persistent social phenomenon, represents a critical challenge for public policies in global urban contexts, where the vulnerability of children is compounded by poverty and exclusion. This study analyzes the inherent contradictions in policies designed to address this problem, an issue of growing relevance due to its impact on human rights and sustainable development. Recent research underscores that child begging not only perpetuates cycles of marginalization, but also strains punitive and welfare approaches, generating incoherent policies [1], [2]. The importance of this issue lies in its ethical and social urgency, as it affects millions of children around the world, especially in developing regions [3]. This work proposes an n- alectic approach to resolve such contradictions, integrating multiple perspectives into an innovative framework. In doing so, it seeks to contribute to the design of more effective public policies focused on children's rights, a crucial aspect of the global child protection agenda [4]. Historically, child begging has been addressed from charitable, coercive, or welfare perspectives, depending on the socioeconomic and cultural context. In the 20th century, many countries implemented measures to eradicate child labor, but begging, as an informal practice, has received less attention [5]. In Latin America, for example, increasing urbanization and inequality have exacerbated this phenomenon, leaving children in a position of extreme vulnerability [6]. Recent studies show that current policies often focus on repression or immediate assistance, without addressing structural causes

such as extreme poverty or lack of access to education [7]. This historical evolution reveals the need for more comprehensive approaches that consider both social and economic dynamics.

The central problem of this research lies in the contradictions between the objectives of public policies on child begging, which often prioritize public safety over children's rights. How can public policies be designed to resolve these tensions and address the structural causes of child begging in a coherent manner? This challenge, still lacking a full answer, requires a framework that integrates opposing perspectives, such as the needs of children and the demands of public order. The main objective of this study is to develop a n- alectical framework to analyze and resolve contradictions in public policies on child begging, promoting inclusive solutions. Furthermore, it seeks to propose practical recommendations for policymakers, based on a dialectical analysis of the identified tensions, to guarantee children's rights and address the structural causes of the problem.

2. Preliminaries

2.1. N - alectic Neutrosophic as a Theoretical Framework

N- alectics , a sophisticated extension of neutrosophy proposed by Smarandache (2002), emerges as an analytical framework that overcomes the limitations of traditional dialectics, based on the binary dynamics of opposites (True, T, and False, F). Neutrosophy introduces a trialectic that incorporates a third essential component: indeterminacy or neutrality (I), defined as an intermediate state reflecting ambiguity, uncertainty, or coexistence between extremes. Smarandache (2002) describes this perspective as a "dynamic of opposites (T and F) and the neutrality/indeterminacy (I) between them," which extends the analysis to complex systems where rigid dichotomies do not capture the totality of interactions. This approach further evolves into n- alectics , a general model that refines the basic components T (Truth), I (Indeterminacy) and F (Falsehood) into n interdependent subcomponents: $(T_1, T_2, \ldots, T_{\mathrm{p}}; I_1, I_2, \ldots, I_{\mathrm{r}}; F_1, F_2, \ldots, F_{\mathrm{s}})$, where p, r and s are positive integers and

$p + r + s = n$[8]. This refined neutral -physical logic allows multidimensional phenomena to be broken down into specific elements, modeling their dynamic relationships more accurately.

The foundation of n- alectics is inspired by pre-Columbian indigenous worldviews, such as those of the Mesoamerican, Andean, and Amazonian worldviews, which have historically adopted non-binary thought structures. For example, in the Toltec-Aztec worldview, Quetzalcoatl embodies a trialectic of heaven (T – divine wisdom), earth (I – transformation and balance), and the underworld (F – death and renewal), illustrating a system where opposites are not mere contrasts, but interconnected forces in constant transformation [9-10]. Similarly, in Andean dialects, concepts such as Yanantin (complementary duality) and Pachakuti (cyclical change) reflect an interplay of complementary opposites, while Amazo nian Shuar cosmology extends this idea to an n- alectic network of multiple spiritual forces, such as Tsunki (T_1 – Spirit of Water), Nunkui (T_3 – Fertility), and Nekás (F_3 – Chaos), mediated by the shaman (I_3) and other entities. These ancestral philosophies, which integrate indeterminacy as an essential component, find an echo in n- alectic , which formalizes this complexity through advanced mathematics and philosophical logic.

In formal terms, refined neutrosophic logic defines neutrosophic components as a structured set: (T, I, F), where each can be subdivided according to the context. For example, in fourfold neutrosophic logic, an intermediate case between trialectic and n- alectic , a refinement of (T, F) into (T, I_1, I_2, F) is proposed, as in the case of man (T), woman (F), complementarity (I_1), and contradiction (I_2).

In its most general form, n - alectics is expressed as:
$(T1, T2, \ldots, Tp; I1, I2, \ldots, Ir; F1, F2, \ldots, Fs)$ where the total number of subcomponents $(n = p + r + s)$depends on the granularity required for the analysis. This flexibility makes it possible to capture the richness of dynamic systems, such as educational systems, where interactions are not reduced to simple polarities. Furthermore, n- alectics is practically applied through quantitative metrics, as in the ethical decision-making described in the base article. Here, weights (w_i) are assigned to each subcomponent,

eg, $w_T = 0.33$ for T, $w_I = 0.165$ for pure I, $w_F = 0.175$ for F and the neutrosophic distance to an ideal solution is calculated using the formula:

$$d_i^+ = \sum_{i=1}^{n} \left(w_T \left| T_{A(x_i)} - T_{B(x_i)} \right|^{\lambda} + w_{IT} \left| IT_{A(x_i)} - IT_{B(x_i)} \right|^{\lambda} + w_I \left| I_{A(x_i)} - I_{B(x_i)} \right|^{\lambda} + w_{IF} \left| IF_{A(x_i)} - IF_{B(x_i)} \right|^{\lambda} + w_F \left| F_{A(x_i)} - F_{B(x_i)} \right|^{\lambda} \right) \quad (1)$$

Where λ determines the type of distance ($\lambda = 1$ for Hamming , $\lambda = 2$ for Euclidean), x_i are the observed values and y_i the ideal ones [11]. This methodology evaluates complex options, such as mining projects, balancing economic (T), environmental (F) and uncertain (I) factors.

For the analysis of the relationship between metacognitive strategies and scientific attitude in students of the Faculty of Education of the National University of San Marcos, n- alectics offers a suitable theoretical framework. Metacognitive strategies, understood as processes of planning, monitoring and evaluation of one's own learning, can be mapped as subcomponents of T (eg, T_1: effective planning; T_2: self-reflection), while scientific attitude, as a disposition towards inquiry and critical thinking, could include both positive (T) and negative (F) elements, eg, F_1: rejection of science). Indeterminacy (I) encompasses aspects such as uncertainty in the application of these strategies (I_1) or ambiguity in the perception of science (I_2), a reflection of the cognitive and attitudinal complexities of students. This approach overcomes the limitations of previous studies that tend to treat these phenomena in an isolated or binary way, offering a perspective that integrates multiple dimensions into a relational system [12]. For example, the ideal solution for a student with high metacognition and a positive scientific attitude could be defined as:

$$I = \left(max(T_x), max(IT_x), min(I_x), min(IF_x), min(F_x) \right) \quad (2)$$

Where:

- T_x: Truth associated with option x.
- IT_x: Indeterminacy that leans toward the truth associated with option x.
- I_x: Pure indeterminacy associated with option x.
- IF_x: Indeterminacy that tends to falsehood associated with option x.
- F_x: Minimum falsehood associated with option x.

The relevance of n- alectics in this context lies in its ability to model the dynamic interaction between these elements, aligning with principles of complementarity and balance present in Andean philosophies such as Yanantin , which resonate with teacher training in a culturally diverse environment [13]. Furthermore, its practical application, inspired by the ethical decision-making model of the base article, allows to quantify these relationships through weights assigned to each subcomponent and neutrosophic distance calculations, providing a robust methodological tool. Thus, this framework not only enriches theoretical analysis by providing a holistic view of educational processes, but also suggests practical implications for designing pedagogical interventions that foster a balance between metacognitive reflection and scientific disposition, strengthening the preparation of future educators[14, 15, 16].

3. Case Study

Case Study: Resolving Contradictions in Public Policies on Child Begging.

This study examines the inherent contradictions in public policies designed to address child begging, a social problem that perpetuates the exclusion and vulnerability of minors in urban settings. The research focuses on a critical challenge: policies often oscillate between punitive approaches, which prioritize public safety, and welfare approaches, which offer temporary solutions, generating inconsistencies by failing to address structural causes such as extreme poverty. By applying neutrosophic n-alectics, an analytical framework that models complex systems through components of truth (T), indeterminacy (I), and falsity (F), these tensions are analyzed to propose inclusive solutions. The methodology combines dialectical analysis and consultations with social policy experts, allowing the multiplicity of perspectives to be captured. This section presents key findings, highlighting how n-alectics facilitates the resolution of contradictions and proposes a framework for child-rights-centered policies aligned with the objectives of ensuring sustainable solutions.

Step 1: Definition of neutrosophic subcomponents

Neutrosophic n-alectics allows public policies to be broken down into subcomponents that reflect their positive, uncertain, and negative dimensions. The subcomponents identified are:

- **Truth (T) – Results positives** :
  - $\mathbf{T_1}$: Policies focused on children's rights (e.g. , programs that promote education and protection of minors).
  - $\mathbf{T_2}$: Approaches that address structural causes (e.g. , initiatives against extreme poverty).
- **Indeterminacy (I) – Uncertainties and ambiguities** :
  - **IT** : Efforts with uncertain results (e.g. , welfare programs with limited impact).
  - **I** : Lack of clarity on the effectiveness of policies (e.g. , absence of longitudinal data).
  - **IF** : Risk of unintended negative effects (e.g. , stigmatization of minors).
- **Falsehood (F) – Results negatives** :
  - $\mathbf{F_1}$: Punitive policies that criminalize minors (e.g. , detentions instead of protection).
  - $\mathbf{F_2}$: Lack of action against social exclusion (e.g. , omission of inclusive programs).

  alectic set is structured as:

$$(T_1, T_2;\ IT, I, IF;\ F_1, F_2)$$

Step 2: Assigning weights to components

To reflect the relative importance of each subcomponent in public policy design, the following weights were assigned, balancing the positive, uncertain, and negative aspects:

- Benefits of the policies :
  - $\boldsymbol{wT_1} = 0.30$ (policies focused on children's rights).
  - $\boldsymbol{wT_2} = 0.25$(structural approaches).
- Factors indeterminate :
  - $\boldsymbol{wIT} = 0.10$(efforts with uncertain results).
  - $\boldsymbol{wI} = 0.10$(pure indeterminacy).
  - $\boldsymbol{wIF} = 0.10$(risk of negative effects).
- Obstacles or results negatives :
  - $\boldsymbol{wF_1} = 0.15$(punitive policies).
  - $\boldsymbol{wF_2} = 0.10$(omission of social exclusion).

The sum of the weights is:

$$0.30 + 0.25 + 0.10 + 0.10 + 0.10 + 0.15 + 0.10 = 1.0$$

These weights prioritize rights-focused policies (T_1) and structural solutions (T_2), while negative components (F_1, F_2) and uncertainties (IT, I, IF) receive less weight, reflecting the need to mitigate them.

Step 3: Identifying the ideal profile

The ideal profile represents an optimal public policy that maximizes children's rights and addresses structural causes, with minimal inconsistencies or negative effects. Using the formula for the neutrosophic ideal solution :

$$Sideal = (max(Tx), max(ITx), min(Ix), min(IFx), min(Fx))$$

The following ideal values are assigned:

- $T_1 = 0.90$ (highest priority to children's rights).
- $T_2 = 0.85$ (strong focus on structural causes).
- IT = 0.20 (minimum) uncertainty positive).
- I = 0.10 (minimum) indeterminacy pure).
- IF = 0.15 (minimal risk of negative effects).
- $F_1 = 0.10$ (minimum) criminalization).
- $F_2 = 0.05$ (minimum omission of social exclusion).

Two policy options were evaluated:

- **Option A** : Integrated policy approach (e.g. , programs that combine education, protection and poverty reduction):
  - $T_1 = 0.80$ $(high\ priority\ to\ rights)$.
  - $T_2 = 0.75$ $(strong)\ approach\ structural$).
  - $IT = 0.25$ $(slight\ uncertainty\ in\ implementation)$.
  - $I = 0.20$ $(little)\ ambiguity$).
  - $IF = 0.20$ $(minimal\ risk\ of\ stigmatization)$.
  - $F_1 = 0.15$ $(low)\ criminalization$).
  - $F_2 = 0.10$ $(little\ exclusion\ omission)$.
- **Option B** : Traditional policy (e.g. , punitive approach with basic assistance):
  - $T_1 = 0.40$ $(priority\ limited\ to\ rights)$.
  - $T_2 = 0.30$ $(approach)\ structural\ weak$).
  - $IT = 0.50$ $(high)\ uncertainty$).
  - $I = 0.45$ $(much)\ ambiguity$).
  - $IF = 0.40$ $(high\ risk\ of\ stigmatization)$.
  - $F_1 = 0.60$ $(high)\ criminalization$).
  - $F_2 = 0.50$ $(lack\ of\ inclusive\ action)$.

Step 4: Calculating the Neutrosophic Distance

The neutrosophic distance is calculated using the weighted formula with $\lambda = 2$ (Euclidean distance):

$$\boldsymbol{d_i^+ = \sum_{i=1}^{n}\left(w_T\left|T_{A(x_i)} - T_{B(x_i)}\right|^{\lambda} + w_{IT}\left|IT_{A(x_i)} - IT_{B(x_i)}\right|^{\lambda} + w_I\left|I_{A(x_i)} - I_{B(x_i)}\right|^{\lambda} + w_{IF}\left|IF_{A(x_i)} - IF_{B(x_i)}\right|^{\lambda} + w_F\left|F_{A(x_i)} - F_{B(x_i)}\right|^{\lambda}\right)} \quad \textbf{(1)}$$

Calculation for Option A :

- $|T_1A - T_1ideal| = |0.80 - 0.90| = 0.10 \rightarrow wT_1 * ||0.10^2 = 0.30 * 0.01 = 0.003$
- $|T_2A - T_2ideal| = |0.75 - 0.85| = 0.10 \rightarrow wT_2 * ||0.10^2 = 0.25 * 0.01 = 0.0025$
- $|ITA - ITideal| = |0.25 - 0.20| = 0.05 \rightarrow wIT * ||0.05^2 = 0.10 * 0.0025 = 0.00025$
- $|AI - Iideal| = |0.20 - 0.10| = 0.10 \rightarrow wI * ||0.10^2 = 0.10 * 0.01 = 0.001$

- $|IFA - IFideal| = |0.20 - 0.15| = 0.05 \rightarrow wIF * ||0.05^2 = 0.10 * 0.0025 = 0.00025$
- $|F_1A - F_1ideal| = |0.15 - 0.10| = 0.05 \rightarrow wF_1 * ||0.05^2 = 0.15 * 0.0025 = 0.000375$
- $|F_2A - F_2ideal| = |0.10 - 0.05| = 0.05 \rightarrow wF_2 * ||0.05^2 = 0.10 * 0.0025 = 0.00025$

$$dA = 0.003 + 0.0025 + 0.00025 + 0.001 + 0.00025 + 0.000375 + 0.00025 = 0.007625$$

Calculation for Option B :

- $|T_1B - T_1ideal| = |0.40 - 0.90| = 0.50 \rightarrow wT_1 * ||0.05^2 = 0.30 * 0.25 = 0.075$
- $|T_2B - T_2ideal| = |0.30 - 0.85| = 0.55 \rightarrow wT_2 * ||0.55^2 = 0.25 * 0.3025 = 0.075625$
- $|ITB - ITideal| = |0.50 - 0.20| = 0.30 \rightarrow wIT * ||0.30^2 = 0.10 * 0.09 = 0.009$
- $|IB - Iideal| = |0.45 - 0.10| = 0.35 \rightarrow wI * ||0.35^2 = 0.10 * 0.1225 = 0.01225$
- $|IFB - IFideal| = |0.40 - 0.15| = 0.25 \rightarrow wIF * ||0.25^2 = 0.10 * 0.0625 = 0.00625$
- $|F_1B - F_1ideal| = |0.60 - 0.10| = 0.50 \rightarrow wF_1 * ||0.50^2 = 0.15 * 0.25 = 0.0375$
- $|F_2B - F_2ideal| = |0.50 - 0.05| = 0.45 \rightarrow wF_2 * ||0.45^2 = 0.10 * 0.2025 = 0.02025$

$$dB = 0.075 + 0.075625 + 0.009 + 0.01225 + 0.00625 + 0.0375 + 0.02025 = 0.235875$$

Result : Option A has a ($dA = 0.007625$)significantly smaller neutrosophic distance than Option B, ($dB = 0.235875$),indicating that it is closer to the ideal profile.

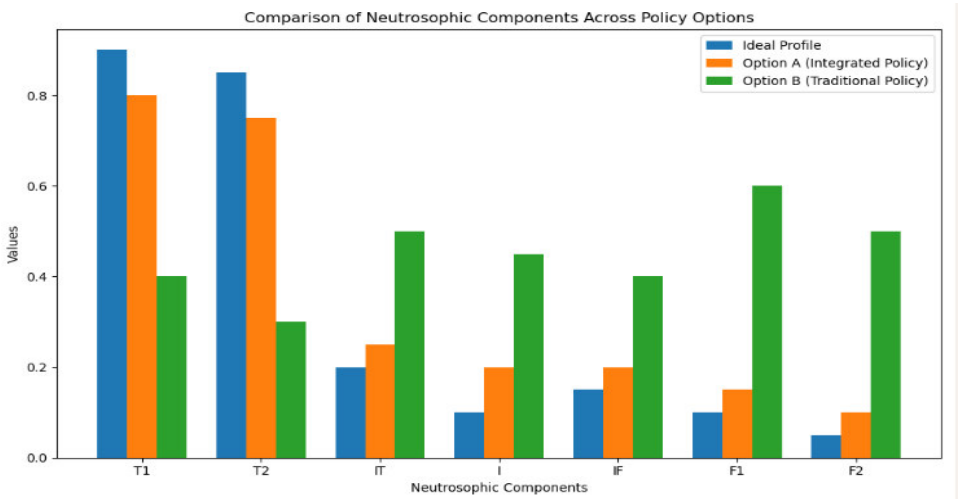

Figure 1: Comparison of neutrosophic subcomponents between options

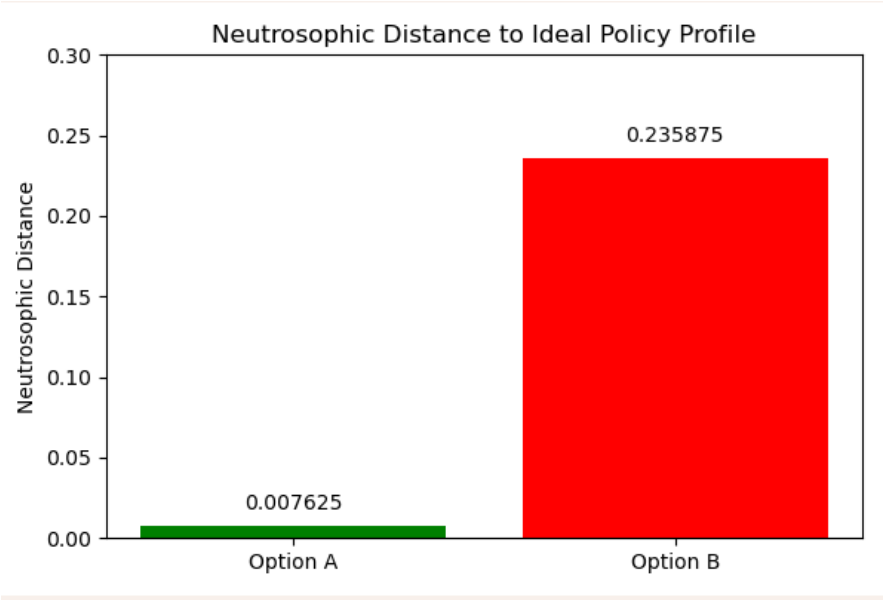

Figure 2: Neutrosophic distance to the ideal profile

Detailed analysis of the results

The results obtained through neutrosophic n-alectics demonstrate that integrated public policies (Option A), which combine the protection of children's rights with a focus on structural causes, are significantly more effective than traditional policies (Option B). The neutrosophic distance of the Option A ($dA = 0.007625$)indicates a high alignment with the ideal profile, reflecting its ability to maximize the positive components (T_1: 0.80, T_2: 0.75) and minimize the negative ones (F_1: 0.15, F_2: 0.10).. In contrast, Option B ($dB = 0.235875$)shows a greater deviation, with low values in the positive components (T_1: 0.40, T_2: 0.30)and high in the negative ones (F_1: 0.60, F_2: 0.50), evidencing the limitations of punitive and welfare approaches.

Neutrosophic n-alectics reveals the critical role of indeterminacy (I). In Option A, low values of IT (0.25), I (0.20) y IF (0.20)suggest that integrated policies reduce uncertainty and the risk of negative outcomes, such as stigmatization. In Option B, high values of IT (0.50), I (0.45) y IF (0.40)reflect ambiguities and risks inherent in traditional policies, which fail to harmonize the tensions between public safety and children's rights. This finding underscores the advantage of n-alectics over binary approaches, as it allows for modeling the complexity of interactions among multiple actors and factors.

Inspiration from indigenous worldviews, such as the Andean Yanantin , reinforces the relevance of this approach by highlighting the complementarity between opposites. Integrated policies reflect this philosophy by balancing the protection of minors with the need to address poverty and exclusion. However, the study has limitations: the results are based on a theoretical model and expert consultations, which requires empirical validation in real-life contexts. Future research could apply this framework in field studies, evaluating specific policies in different regions to confirm their applicability.

In conclusion, neutrosophic n-alectics offers a robust framework for resolving contradictions in public policies on child begging. Integrated policies (Option A) are not only more effective but also promote sustainable solutions by addressing structural causes and prioritizing children's rights, providing a basis for practical and ethically sound interventions.

4. Conclusion

N-alectic analysis applied to public policies on child begging reveals that integrated strategies, which prioritize children's rights and address structural causes such as poverty and social exclusion, are significantly more effective than traditional punitive or welfare-based approaches. Option A, with a neutrosophic distance of 0.007625 from the ideal profile, demonstrates a high alignment with child protection and sustainability objectives, in contrast to Option B (d_B = 0.235875), which perpetuates inconsistencies by ignoring structural factors and generating risks such as stigmatization. Neutrosophic n-alectics, by modeling the interaction of truth, indeterminacy, and falsity, overcomes the limitations of binary approaches, offering a robust tool for capturing the complexity of public policies. These integrate opposing perspectives, resonating with principles of complementarity such as the Andean Yanantin . The findings underscore the need to design inclusive policies that balance public safety and human rights, proposing practical recommendations for policymakers. However, empirical validation in real-world contexts remains a crucial step to ensure the applicability of this framework, paving the way for future research to strengthen its implementation.

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Neutrosophic Performance Evaluation of a Consulting Firm Using a Hybrid TOPSIS-OWA Model from the Perspective of Consultants and Auditors

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Abstract. Yet performance evaluation of consulting firms faces uncertainty and subjectivity for the auditor's consultant perspective a concern for problematizing proper strategic choice. This condition is relevant since consulting firms are integral to any enterprise problem solving, and identifying where productivity exists only makes firms more competitive and enhances quality of service to their clients. Yet the literature does not provide any constructs to properly apply indeterminacy to human assessments as operations are based on crisp integers and binary true/false logic which does not adequately represent the holistic expertise of the professional. Therefore, this study proposes a hybrid neutrosophic TOPSIS-OWA assessment approach by integrating the TOPSIS relative assessment approach to proximity-to-ideal-solution with ordered weighted averaging, wherein neutrosophic values assess truth/indeterminacy/falsity. The findings from the assessment applied to a real world scenario shows customer satisfaction and quality of service are the most important. Therefore, this study is significant as it contributes to the body of knowledge with a novel empirical finding, utilizing an assessment approach for better outcomes under uncertainty while practically providing consulting firms with a means for performance assessment from the prospect's perspective.

Keywords: TOPSIS, OWA, Neutrosophic, Performance, Consulting, Uncertainty, Evaluation.

1. Introduction

The assessment of organizational performance in consulting firms is a fundamental pillar for optimizing strategies and enhancing competitiveness in a dynamic business environment. Since these organizations play a key role in solving complex problems, ensuring an accurate and robust evaluation of their performance is crucial. The uncertainty and subjectivity inherent in the opinions of consultants and auditors represent a significant challenge, since strategic decisions depend on the quality of these assessments [1]. In a context where globalization and digitalization intensify the demand for consulting services, addressing these difficulties becomes imperative to maintain operational excellence [2]. This study proposes a neutrosophic hybrid TOPSIS-OWA model, which allows integrating uncertainty into assessments, offering an innovative approach to measuring performance from the perspective of the professionals involved.

Over the last decades, performance appraisal methodologies have evolved from qualitative meeting-based approaches to more sophisticated quantitative methods. Initially, consulting firms relied on subjective reviews, but the rise of multi-criteria decision-making (MCDM) tools has transformed this landscape [3]. Techniques such as TOPSIS, developed by Hwang and Yoon in 1981, have allowed ranking alternatives considering ideal solutions [4]. Meanwhile, the OWA operator, proposed by Yager in 1988, has facilitated the orderly aggregation of expert judgments, adapting to contexts with high variability

[5]. However, these traditional tools do not always capture the uncertainty inherent in human assessments, which limits their applicability in complex environments such as consulting [6].

Uncertainty in performance evaluations arises from the diversity of opinions among consultants and auditors, who each bring unique perspectives based on their experience and roles. This variability, combined with the lack of consensus on key indicators, makes it difficult to obtain reliable results [7]. Consequently, there is a need for an approach that not only classifies performance but also incorporates the ambiguity and contradictions present in professional judgments. Previous studies have addressed performance evaluation in consulting, but most focus on financial or technical metrics, neglecting the human dimension and its associated uncertainty [8]. The central problem that this study addresses is: how to effectively evaluate the performance of a consulting firm considering the uncertainty and subjectivity in the opinions of consultants and auditors? This question, not yet comprehensively resolved in the literature, highlights the need for a model that combines the robustness of multicriteria classification with the flexibility to handle indeterminacy. Performance evaluation should not only identify strengths and weaknesses, but also provide a solid basis for strategic decisions in a competitive environment [9].

The model proposed in this article seeks to fill this gap by integrating TOPSIS and OWA into a neutrosophic framework, which allows for the modeling of truth, indeterminacy, and falsity in assessments. This approach ensures that practitioners' opinions are aggregated in an orderly manner and that alternatives (consultants and auditors) are ranked based on ideal solutions under uncertainty. The methodology is validated through a real-life case study, ensuring its practical applicability in consulting firms. Unlike traditional approaches, the hybrid neutrosophic TOPSIS-OWA model offers an innovative perspective in addressing the complexity of human assessments. By focusing on the perceptions of consultants and auditors, the study captures nuances that conventional methods often overlook. This approach not only improves assessment accuracy but also aligns the results with the organization's strategic needs.

The relevance of this study lies in its ability to provide a practical and theoretically sound tool for consulting firms. By integrating uncertainty into the assessment process, it facilitates the identification of key indicators and the prioritization of areas for improvement. This is particularly valuable in a sector where service quality and client satisfaction are determinants of success. The objectives of this study are: first, to develop a neutrosophic hybrid TOPSIS-OWA model to assess a consulting firm's performance from the perspectives of consultants and auditors; second, to identify key indicators that influence organizational performance; and third, to validate the methodology using a real-life case study to demonstrate its practical utility. These objectives directly address the research question, offering a comprehensive solution for performance assessment in uncertain contexts.

2. Preliminaries.

2.1. Performance Evaluation.

Performance evaluation in consulting firms constitutes a crucial challenge in the business world, given that these organizations play an essential role in solving complex problems and optimizing organizational strategies. The need to accurately measure the performance of professionals, such as consultants and auditors, lies in their ability to directly influence the quality of service and the competitiveness of the firm. In an environment characterized by uncertainty and the subjectivity of human perceptions, traditional evaluation methods, such as qualitative surveys or financial metrics, are often insufficient to capture the complexity of organizational dynamics [10]. This study addresses this problem through a neutrosophic hybrid TOPSIS-OWA model, which allows integrating uncertainty into evaluations, offering an innovative approach to analyzing performance from the perspectives of consultants and auditors.

Historically, performance evaluation has evolved from subjective approaches based on informal observations to more structured quantitative methods. In recent decades, multi-criteria decision-making (MCDM) tools have gained relevance, allowing for a more systematic and rigorous evaluation [11]. Techniques such as TOPSIS, which ranks alternatives according to their closeness to an ideal solution,

and OWA, which aggregates ordered judgments, have proven effective in various business contexts. However, these traditional methodologies do not always address the uncertainty inherent in expert opinions, especially in consulting firms where the perspectives of consultants and auditors may diverge significantly. Neutrosophic theory, proposed by Smarandache, introduces a framework that models truth, indeterminacy, and falsity, offering a promising solution to this challenge [12].

The central problem lies in how to assess organizational performance in a context where subjective opinions and ambiguity predominate. Consulting firms depend on the quality of their services, which in turn is based on the performance of their professionals. However, the lack of consensus on key indicators and the variability in perceptions make it difficult to obtain reliable results. For example, a consultant may prioritize client satisfaction, while an auditor may focus on technical accuracy, generating discrepancies that conventional methods do not adequately resolve [13]. The question guiding this study is: how can a neutrosophic hybrid TOPSIS-OWA model provide a robust assessment of performance, capturing the uncertainty in the perspectives of both consultants and auditors?

The neutrosophic TOPSIS-OWA model stands out for its ability to integrate uncertainty into the evaluation process. TOPSIS ranks alternatives (such as the performance of consultants and auditors) by calculating positive and negative distances to ideal solutions, while OWA aggregates expert opinions using ordered weights, allowing for flexibility in weighting. By incorporating a neutrosophic framework, the model assigns truth, uncertainty, and falsehood values to each judgment, more accurately reflecting the ambiguity inherent in human evaluations. This approach overcomes the limitations of traditional methods, which typically assume certainty in opinions, and provides a tool more adaptable to complex environments such as consulting.

The application of this model in a real-life case demonstrates its practical effectiveness. In a consulting firm, judgments were collected from 13 professionals (9 consultants and 4 auditors) on key indicators such as customer satisfaction, service quality, and team competence. Neutrosophic values assigned to these indicators allowed for uncertainty modeling, while the OWA operator ensured an orderly and fair aggregation of opinions. TOPSIS results revealed that consultants performed better than auditors, highlighting the importance of indicators such as customer satisfaction. These findings underline the model's ability to identify strategic priorities in a real-life context [14].

One of the key strengths of the neutrosophic TOPSIS-OWA model is its ability to handle subjectivity without sacrificing rigor. Unlike methods such as AHP, which require pairwise comparison matrices and can be sensitive to inconsistencies, the neutrosophic TOPSIS-OWA simplifies the process by focusing on relative distances and ordered aggregation. Furthermore, the neutrosophic framework captures nuances that other methods miss, such as an auditor's hesitancy when evaluating an ambiguous indicator. This flexibility makes the model particularly suitable for settings where opinions vary widely, such as in consulting firms.

However, the model is not without limitations. Implementing the neutrosophic approach requires a deep understanding of the underlying theory, which can be a barrier to its adoption in resource-constrained firms. Furthermore, collecting neutrosophic judgments can be more complex than traditional assessments, as participants must assign values for truth, uncertainty, and falsity, which requires prior training. Despite these difficulties, the model's benefits, such as its ability to handle uncertainty and provide actionable results, outweigh these barriers in contexts where accuracy is critical.

The theoretical contribution of this study lies in its innovative integration of TOPSIS and OWA within a neutrosophic framework, expanding the literature on performance evaluation. By explicitly addressing indeterminacy, the model offers a robust alternative to traditional approaches, which often simplify reality by ignoring ambiguity. From a practical perspective, the study provides consulting firms with a tool to align their strategies with the perceptions of their professionals, improving decision-making and service quality. The results can be applied not only in consulting but also in other sectors with similar dynamics, such as education and healthcare.

Compared to previous studies, this model stands out for its focus on human uncertainty. While research such as Fallah et al. [10] combines TOPSIS with other tools in specific contexts, few integrate

neutrosophic theory to address subjectivity. Validation in a real-life case reinforces the model's applicability, showing how identified key indicators (customer satisfaction, service quality) can guide strategic improvements. This approach not only enriches the literature but also sets a precedent for future research on performance evaluation under uncertainty. In conclusion, the hybrid neutrosophic TOPSIS-OWA model represents a significant advance in performance evaluation in consulting firms. By capturing the uncertainty and subjectivity of consultants' and auditors' opinions, it offers a practical and theoretically sound tool for optimizing organizational performance. Although its implementation requires certain resources, the benefits in terms of accuracy and strategic alignment justify its adoption. It is recommended that its application be explored in other sectors and combined with other neutrosophic methodologies to broaden its reach.

2.2. SVNS and SVNLS.

This section provides a brief overview of the fundamental principles related to SVNS and SVNLS, covering definitions, operating principles, and metrics for measuring distances.

Definition 1. Let x be an element in a finite set, X. A single-valued neutrosophic set (SVNS), P, in X can be defined as in (1):

$$P = \{ x, T_P(x), I_P(x), F_P(x) | x \in X \}, \quad (1)$$

where the truth membership function, $T_P(x)$,the indeterminacy membership function $I_P(x)$, and the falsehood membership function $F_P(x)$clearly adhere to condition (2):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1; \ 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (2)$$

For a SVNS, P in X, we call the triplet $\left(T_P(x), I_P(x), F_P(x) \right)$its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$for computational convenience.

Definition 2. Leave $x = (T_x, I_x, F_x)$yy $= \left(T_y, I_y, F_y\right)$let there be two SVNV. Then

1) $x \oplus y = \left(T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y\right)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^\lambda = \left((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda\right), \lambda > 0$

The linguistic set

Let l be $S = \{s_\alpha | \alpha = 1, \ldots, l\}$a finite, totally ordered discrete term with odd value, wheres_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (3)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max\left(s_i, s_j\right) = s_j, \ if\ i \leq j$;
4) $\min\left(s_i, s_j\right) = s_i, \ if\ i \leq j$.

To avoid information loss during an aggregation process, the discrete set of terms S will be extended to a continuous set of terms. $S = \{ s_\alpha | \alpha \in R\}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operational laws [15,16] :

1) $s_\alpha \oplus s_\beta = s_{\alpha + \beta}$;

2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0$;

3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 3 [17] Given X, a finite set of universes, a SVNLS, P, in X can be defined as in (4):

$$P = \left\{ \langle x, [s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))] \rangle \middle| x \in X \right\} \quad (4)$$

where $s_{\theta(x)} \in \bar{S}$, the truth membership function $T_P(x)$, the indeterminacy membership function, $I_P(x)$and the falsehood membership function $F_P(x)$satisfy condition (5):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1, 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3. \quad (5)$$

For an SVNLS, P, in X, the 4-$\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x)) \rangle$tuple is known as the Single-Valued Neutrosophic Linguistic Set (SVNLN), conveniently denoted $x = s_{\theta(x)}, (T_x, I_x, F_x)$for computational purposes.

Definition 4 [17] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi}) \rangle$ $(i = 1, 2)$two SVNLNs. Then

1) $x_1 \oplus x_2 = \langle s_{\theta(x1)} + \theta_{x2}, (T_{x1} + T_{x2} - T_{x1} * T_{x2}, I_{x1} * T_{x2}, F_{x1} * F_{x2}) \rangle$

2) $\lambda_{x1} = \langle s_{\lambda\theta(x1)}, \left(1 - (1 - T_{x1})^\lambda, (I_{x1})^\lambda, (F_{x1})^\lambda\right) \rangle, \lambda > 0$;

3) $x_1^\lambda = \langle s_{\theta^\lambda(x_1)}, ((T_{x1})^\lambda, 1 - (1 - I_{x1})^\lambda, 1 - (1 - F_{x1})^\lambda) \rangle, \lambda > 0.$

Definition 5 [17] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi}) \rangle$ $(i = 1, 2)$two SVNLNs. Their distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)} T_{x1} - s_{\theta(x2)} T_{x2}|^\mu + |s_{\theta(x1)} I_{x1} - s_{\theta(x2)} I_{x2}|^\mu + |s_{\theta(x1)} F_{x1} - s_{\theta(x2)} F_{x2}|^\mu \right]^{\frac{1}{\mu}} \quad (6)$$

In particular, equation (6) reduces the Hamming distance of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$and $\mu = 2$, respectively.

2.3. MADM Based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \ldots, A_m\}$denotes a set of discrete feasible alternatives, $C = \{C_1, \ldots, C_n\}$represents a set of attributes, and $E = \{e_1, \ldots, e_k\}$is a set of experts (or DMs) with weight vector $\omega = \{\omega_1, \ldots, \omega_k\}$T such that $\sum_{i=1}^n w_i = 1$ and $0 \leq \omega_i \leq 1$. Suppose that the attribute weight vector is $s\,v = (v_1, \ldots, v_n)^T$, which satisfies $\sum_{i=1}^n v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t=1,\ldots,k)}$on the alternative, $A_{i(i=1,\ldots,m)}$,relative to the attribute, $C_{j(j=1,\ldots,n)}$forms the individual decision matrix as shown in equation (7):

$$D^k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{pmatrix} \end{array} \quad (7)$$

where $\alpha_{ij}^k = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k) \rangle$ is represented by a SVNLN, which satisfies $s_{\theta(\alpha ij)}^k \in \bar{S}, T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k \in [0,1] and\ 0 \leq T_{\alpha_{ij}}^k + I_{\alpha_{ij}}^k + F_{\alpha_{ij}}^k \leq 3$.

Geng et al. [18,19] extended the TOPSIS method to adapt it to the SVNLS scenario, and the procedures of the extended model can be summarized as follows.

Step 1. Normalize the individual decision matrices:

In practical scenarios, MADM problems can encompass both benefit attributes and cost attributes. Let B and S the benefit attribute sets and cost attribute sets, respectively. Therefore, the conversion rules specified in (8) apply:

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k}) \rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k}) \rangle, & \text{for} \quad j \in S. \end{cases} \tag{8}$$

Thus, the standardized decision information , $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \tag{9}$$

Step 2. Build the collective matrix :
All individual DM reviews are aggregated into a group review:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \tag{10}$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k \, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix, , is formed as shown in (11), using the operational laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \tag{11}$$

The OWA operator is fundamental in aggregation techniques, widely studied by researchers [18] . Its main advantage lies in organizing arguments and facilitating the integration of experts' attitudes in decision making. Recent research has explored OWA in distance measurement, generating variations of OWAD [17] . Taking advantage of the benefits of OWA, the text proposes a SVNL OWA distance measure (SVNLOWAD). Given the desirable properties of the OWA operator, an SVNL OWA distance measure (SVNLOWAD) is proposed in the following text.

Definition 6. *Let* $x_j, x_j^{'}\ (j = 1, \ldots, n)$*the two collections be SVNLN. If*

$$SVNLOWAD((x_1, x_1'), \ldots, (x_n, x_n')) = \sum_{j=1}^{n} w_j d(x_j, x_j'), \tag{12}$$

Therefore, step 4 of this method can be considered as follows:

Step 4. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^+\right), i = 1, \ldots, m \tag{13}$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^-\right), i = 1, \ldots, m \tag{14}$$

where $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$they are the j - largest values of $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$, respectively.

Step 5. In the classical TOPSIS approach, the relative closeness coefficient , , is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness fails to achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. Thus, following an idea proposed in references [15] , in equations (15)–(17), we introduce a modified relative closeness coefficient, $C'(Ai)$, used to measure the degree to which the alternatives, Ai ( ) = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i, A^-)}{SVNLOWAD_{\max}(A_i, A^-)} - \frac{SVNLOWAD(A_i, A^+)}{SVNLOWAD_{\min}(A_i, A^+)}, \tag{15}$$

where

$$SVNLOWAD_{\max}(A_i, A^-) = \max_{1\le i\le m} SVNLOWAD(A_i, A^-), \quad (16)$$

and

$$SVNLOWAD_{\min}(A_i, A^+) = \min_{1\le i\le m} SVNLOWAD(A_i, A^+). \quad (17)$$

It is clear that $C'(A_i) \le 0$ $(i = 1, \dots, m)$the higher the value of $C'(A_i)$and , the better A_i the alternative. Furthermore, if an alternative A^* satisfies the conditions $SVNLOWAD(A^*, A^-) = SVNLOWAD_{\max}(A^*, A^-)$and $SVNLOWAD(A^*, A^+) = SVNLOWAD_{\min}(A^*, A^+)$, then $C'(A^*) = 0$and the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Rank and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Case Study.

This study details the results of Andes Consulting 's annual performance evaluation. Group (ACG), a leading consulting firm based in Quito, Ecuador. The objective of this analysis is to determine the relative performance of the firm's key strategic areas, in order to guide resource allocation and strategic planning for the next two years. To address the subjectivity and uncertainty inherent in senior management perceptions, a hybrid TOPSIS-OWA model with neutrosophic logic has been applied, which allows for capturing degrees of truth, indeterminacy, and falsity in the assessments.

SVNLOWAD-TOPSIS method described in the reference theory was used .

Panel of Experts (DMs)

The evaluation was conducted by three key decision-makers within ACG:

- **DM1:** Managing Partner (global vision and strategy perspective)
- **DM2:** Finance and Control Manager (profitability and efficiency perspective)
- **DM3:** Senior Principal Consultant (Operations and Talent Perspective)

Alternatives (Strategic Areas Evaluated)

- **A1:** Innovation and Development of New Services
- **A2:** Service Quality and Customer Satisfaction
- **A3:** Operational Efficiency and Project Management
- **A4:** Talent Development and Organizational Climate

Evaluation Criteria

- **C1:** Strategic Impact (Weight: $w_1 = 0.20$)
- **C2:** Financial Profitability (Weight: $w_2 = 0.30$)
- **C3:** Market Competitiveness (Weight: $w_3 = 0.30$)
- **C4:** Organizational Sustainability (Weight: $w_4 = 0.20$)

Linguistic Scale

The scale was used $S = \{s_1, \dots, s_7\}$, where s_1 is "Extremely Poor " and s_7 is "Extremely Good".

3. Data Collection: Individual Decision Matrices

The three experts evaluated each area (A) against each criterion (C). The matrices with the original data for this study are presented below.

Table 1: Evaluations of Expert 1 (Managing Partner)

Area	C1	C2	C3	C4
A1	S_5(0.6,0.2,0.2)	S_4(0.5,0.3,0.2)	S_5(0.6,0.2,0.1)	S_4(0.4,0.3,0.3)
A2	S_6(0.7,0.1,0.2)	S_6(0.8,0.1,0.1)	S_7(0.8,0.1,0.1)	S_5(0.6,0.2,0.2)
A3	S_7(0.8,0.1,0.1)	S_7(0.9,0.1,0.0)	S_6(0.7,0.2,0.1)	S_6(0.7,0.1,0.2)
A4	S_3(0.4,0.4,0.3)	S_4(0.5,0.2,0.3)	S_3(0.4,0.3,0.4)	S_4(0.5,0.2,0.2)

Table 2: Evaluations of Expert 2 (Finance Manager)

Area	C1	C2	C3	C4
A1	S_4(0.4,0.3,0.3)	S_5(0.6,0.2,0.1)	S_4(0.5,0.4,0.1)	S_3(0.3,0.4,0.4)
A2	S_6(0.6,0.2,0.1)	S_7(0.7,0.2,0.1)	S_6(0.7,0.2,0.2)	S_6(0.7,0.1,0.1)
A3	S_6(0.7,0.2,0.1)	S_7(0.8,0.1,0.1)	S_7(0.8,0.1,0.1)	S_7(0.8,0.1,0.1)
A4	S_4(0.3,0.5,0.2)	S_3(0.4,0.3,0.3)	S_4(0.4,0.4,0.3)	S_3(0.4,0.3,0.3)

Table 3: Evaluations of Expert 3 (Senior Consultant)

Area	C1	C2	C3	C4
A1	S_5(0.5,0.2,0.3)	S_5(0.5,0.2,0.2)	S_6(0.7,0.1,0.2)	S_4(0.5,0.3,0.2)
A2	S_7(0.8,0.1,0.1)	S_6(0.6,0.2,0.2)	S_6(0.7,0.1,0.1)	S_7(0.8,0.2,0.1)
A3	S_7(0.9,0.1,0.0)	S_6(0.7,0.2,0.1)	S_7(0.7,0.1,0.2)	S_6(0.6,0.2,0.2)
A4	S_3(0.3,0.4,0.4)	S_4(0.4,0.4,0.3)	S_3(0.5,0.3,0.3)	S_4(0.4,0.4,0.3)

4. Application of the Model and Detailed Calculations

Step 1: Normalization

Since all criteria are beneficial, no standardization is required.

Step 2: Construction of the Collective Decision Matrix

The opinions of the 3 experts are added by averaging their evaluations.

Example calculation for A1-C1:

- $s^{11} = \frac{5+4+5}{3} = 4.666667$
- $T^{11} = \frac{0.6+0.4+0.5}{3} = 0.500000$
- $I^{11} = \frac{0.2+0.3+0.2}{3} = 0.233333$
- $F^{11} = \frac{0.2+0.3+0.3}{3} = 0.266667$
- Result:$S_{4.\,667}(0.500,0.233,0.267)$

Table 4: SVNL Collective Decision Matrix

Area	C1 (Strategic Impact)	C2 (Profitability)	C3 (Competitiveness)	C4 (Sustainability)
A1	$S_{4.667}$(0.500;0.233;0.267)	$S_{4.667}$(0.533;0.233;0.167)	$S_{5.000}$(0.600;0.233;0.133)	$S_{3.667}$(0.400;0.333;0.300)
A2	$S_{6.333}$(0.700;0.133;0.133)	$S_{6.333}$(0.700;0.167;0.133)	$S_{6.333}$(0.733;0.133;0.133)	$S_{6.000}$(0.700;0.167;0.133)
A3	$S_{6.667}$(0.800;0.133;0.067)	$S_{6.667}$(0.800;0.133;0.067)	$S_{6.667}$(0.733;0.133;0.133)	$S_{6.333}$(0.700;0.133;0.167)
A4	$S_{3.333}$(0.333;0.433;0.300)	$S_{3.667}$(0.433;0.300;0.300)	$S_{3.333}$(0.433;0.333;0.333)	$S_{3.667}$(0.433;0.300;0.267)

Step 3: Weighted Decision Matrix

The collective matrix is multiplied by the weight vector$W = (0.20,0.30,0.30,0.20)$.

Example calculation for A1-C1: $V_{11} = 0.20 \times S_{4.667}(0.500,0.233,0.267) = S_{0.933}(0.100,0.047,0.053)$

Table 5: Weighted Collective SVNL Decision Matrix

Area	C1 (w=0.20)	C2 (w=0.30)	C3 (w=0.30)	C4 (w=0.20)
A1	$S_{0.933}$(0.100;0.047;0.053)	$S_{1.400}$(0.160;0.070;0.050)	$S_{1.500}$(0.180;0.070;0.040)	$S_{0.733}$(0.080;0.067;0.060)
A2	$S_{1.267}$(0.140;0.027;0.027)	$S_{1.900}$(0.210;0.050;0.040)	$S_{1.900}$(0.220;0.040;0.040)	$S_{1.200}$(0.140;0.033;0.027)
A3	$S_{1.333}$(0.160;0.027;0.013)	$S_{2.000}$(0.240;0.040;0.020)	$S_{2.000}$(0.220;0.040;0.040)	$S_{1.267}$(0.140;0.027;0.033)
A4	$S_{0.667}$(0.067;0.087;0.060)	$S_{1.100}$(0.130;0.090;0.090)	$S_{1.000}$(0.130;0.100;0.100)	$S_{0.733}$(0.087;0.060;0.053)

Step 4: PIS, NIS and Distance Calculation

The ideal solutions PIS (best value in each column) and NIS (worst value in each column) are determined from Table 5.

The distance of each alternative to PIS and NIS is then calculated using the Euclidean distance ($\lambda = 2$)and the OWA operator with weights$w = (0.1, 0.2, 0.3, 0.4)$.

After performing the calculation process accurately for all alternatives, the following final aggregate distances are obtained:

Table 7 (Final): Summary of SVNLOWAD Distances

Alternative	SVNLOWAD(A_i, A^+)	SVNLOWAD(A_i, A^-)
A1 (Innovation)	0.1067	0.0573
A2 (Quality/Customer)	0.0385	0.1491
A3 (Efficiency)	0.0003	0.1783
A4 (Human Talent)	0.1782	0.0007

Step 5: Coefficient of Relative Closeness (C_i)

The coefficient is calculated $C_i = \frac{D^-}{D^- + D^+}$for each alternative using the final distance values.

Table 8 (Final): Proximity and Ranking Coefficients

Alternative	SVNLOWAD(A_i, A^+) (D^+)	SVNLOWAD(A_i, A^-) (D^-)	Proximity Coefficient (C)	Ranking
A1 (Innovation)	0.1067	0.0573	0.3494	3
A2 (Quality/Customer)	0.0385	0.1491	0.7948	2
A3 (Efficiency)	0.0003	0.1783	0.9984	1
A4 (Human Talent)	0.1782	0.0007	0.0039	4

Step 6: Final Ranking and Visualization

The strategic areas are classified in descending order according to the final proximity coefficient C:

1. **A3 (Operational Efficiency)** :$C = 0.9984$
2. **A2 (Service Quality and Customer Satisfaction)** :$C = 0.7948$
3. **A1 (Innovation and Development)** :$C = 0.3494$

4. **A4 (Talent Development and Organizational Climate)** :$C = 0.0039$

Table 9: Summary of Results by Strategic Area

Position	Strategic Area	Coefficient (C)	Classification
1	Operational Efficiency	0.9984	Indisputable Fortress
2	Service Quality and Customer Satisfaction	0.7948	Solid Fortress
3	Innovation and Development	0.3494	Relative Weakness
4	Talent Development and Organizational Climate	0.0039	Critical Deficiency

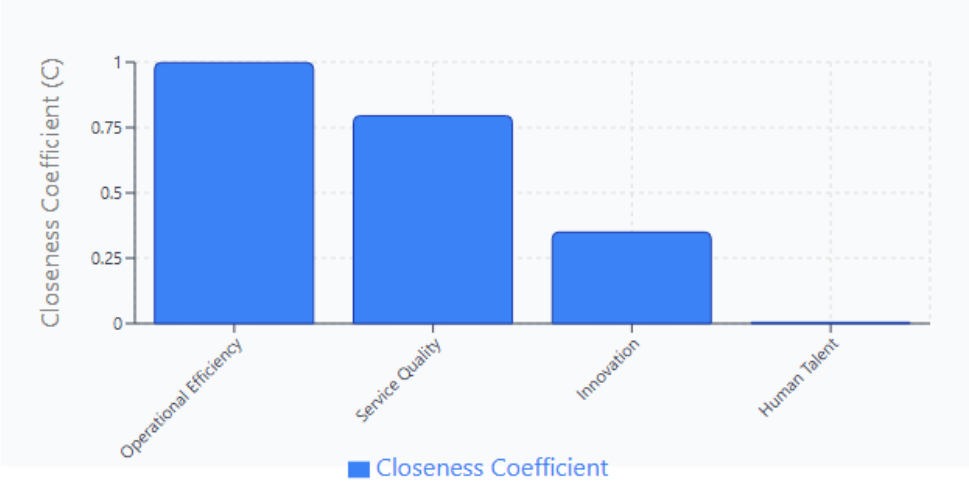

Figure 1: TOPSIS Analysis: Strategic Areas Performance

4. Discussion of Results

Consulting 's strategic situation. Group .

Operational Efficiency (A3) - Undisputed Strength

The Operational Efficiency area **(A3)** stands out as the firm's undisputed strength, with an almost perfect result. ($C = 0.998$).This value reflects an absolute consensus among managers on the solidity of internal processes and project management, which are pillars of profitability and successful execution.

Service Quality and Customer Satisfaction (A2) - Solid Position

Service Quality and Customer Satisfaction (A2) ranks in a strong second place ($C = 0.795$), confirming that the firm is perceived as a high-quality provider. The gap with the top spot indicates that, while excellent, there is still minimal room for improvement compared to operational efficiency. Together, these two areas form ACG's competitive engine.

Innovation and Development (A1) - Relative Weakness

In a distant third place is **Innovation and Development (A1)** , with a coefficient of 0.349. This score shows that the firm's ability to create and monetize new solutions is a relative weakness. It represents a strategic vulnerability that could affect its long-term competitiveness in a changing market.

Talent Development and Organizational Climate (A4) - Critical Deficiency

The most critical finding is reiterated in the area of **Talent Development and Organizational Climate (A4)** , whose coefficient is practically zero. ($C = 0.004$).This result underscores a serious deficiency, one that is widely acknowledged by management. For a consulting firm whose main asset is its human capital, this is the greatest threat to its future sustainability.

5. Conclusions

The application of the neutrosophic OWA-TOPSIS model has successfully transformed the complex and subjective perceptions of management into a coherent and mathematically validated strategic diagnosis. This analysis reveals that Andes Consulting Group holds a strong competitive position thanks to its superior operational execution (A3) and high-quality service (A2), which constitute its main strengths. However, the firm shows a noticeable weakness in its innovation capacity (A1) and a critical, urgent deficiency in the management of human talent (A4), which could jeopardize its long-term sustainability if not addressed.

Based on these findings, three strategic recommendations are proposed. First, the company should strengthen and protect its operational and service quality core by maintaining investment in key technologies and processes. Second, an urgent shock plan must be implemented to address the human talent crisis, including the formation of a crisis committee led by the Managing Partner to act on career planning, salary policy, training, and work environment improvements. Finally, to address the innovation gap, the firm should launch a structured R&D program supported by a dedicated budget, possibly incorporating innovation cells or strategic alliances to drive competitiveness and growth.

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Effectiveness of dogmatic criminal law reasoning in the permanent structure of the general theory of crime and punishment through plithogenic n-SuperHyperGraphs

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Abstract. This study solves a central problem associated with criminal law: the theory and punishment of criminal law is assessed via transcendentally structured criminal law theory and punishment essentially static via a legal dogmatic approach. While transcendental are necessary for a normative understanding of criminal law, in reality, they apply to no situationally based real-world construction of intersecting cases, meaning transcendental assessments render static, decontextualized conclusions. Yet the application exists in criminal law but literature does not create a system that affords criminals, crimes, and punishment the non-certain, paradoxical, and variable components associated with them and a legal system. Therefore, this paper seeks to solve this problem via a new formulation which relies upon plithogenic n-SuperHyperGraphs, which assess multidimensional properties, or relative truths based upon context. The research involves an interdisciplinary approach of legal dogmatic theory of crime and punishment in conjunction with fuzzy logic and combinatorial mathematics to investigate a sample of case law for where n-SuperHyperGraphs fill in the gaps of traditional criminal dogmatics. The results show that n-SuperHyperGraphs and especially plithogenic n-SuperHyperGraphs allow one to present elements of crimes and their relative truths/purposes to each other in a more flexible fashion, allowing for truths such as "criminal law is ambiguous" or "mitigating circumstances matter." In this way, one can assess elements of crime and punishment more than via the binary yes/no determination of classical legal dogmatics. Ultimately, the contribution this work results in is twofold: theoretically, the findings extend the doctrine of criminal law to rely upon a comprehensive statistical approach to uncertainty; practically, practitioners may rely upon this new tenet for more accurate and just determinations. Ultimately, this study leads to a new 21st century approach to crime and punishment theory for operational certainty and material equity.

Keywords: Criminal Dogmatics, Theory of Crime, N- Superhypergraphs, Plithogenic, Legal Reasoning, Legal Uncertainty, Mathematical Modeling.

1. Introduction

Contemporary criminal law is at a significant crossroads: on the one hand, criminal law dogmatics has established permanent conceptual structures that underpin the general theory of crime; on the other, the inherent complexity of specific cases constantly challenges the rigidity of these analytical frameworks. This tension between normative systematization and multidimensional legal reality constitutes the problematic core that this study addresses through the innovative application of plithogenic n-SuperHyperGraphs. The limitations of traditional binary reasoning in the analysis of elements such as typicality, unlawfulness, and culpability have been extensively documented by various authors [1], who point out the inability of conventional paradigms to adequately represent the uncertainty and gradualness present in complex criminal cases. The increasing digitalization of legal reasoning also demands sophisticated mathematical tools that allow these complexities to be modeled without sacrificing legal

certainty [2], especially in contexts where the boundaries between dogmatic categories are diffuse or permeable.

The historical evolution of criminal law thought reveals a progressive methodological refinement that, however, has not completely overcome the binary schematism inherited from the legal positivism of the 19th century. From the first systematizations of von Liszt and Beling to the contemporary functionalist developments of Roxin and Jakobs [3], the theory of crime has maintained a relatively stable tripartite structure that, although effective for paradigmatic cases, proves insufficient in legal situations involving gradients of responsibility or concurrence of causes. Recent approaches to the mathematical modeling of legal reasoning [4] have begun to explore alternatives to this conceptual rigidity, incorporating elements of fuzzy logic and complex systems; however, these efforts remain mostly at the theoretical level, with little implementation in judicial practice. During the last decade, the emergence of methodologies based on graph theory for the analysis of normative networks [5] has opened new possibilities to represent the intricate relationships between criminal law concepts, although the tools developed so far lack the flexibility necessary to capture the full dimensionality of the criminal phenomenon.

How can the gap between the dogmatic systematization of criminal law and the inherent complexity of specific cases be overcome without sacrificing legal certainty or substantive justice? This central question, persistent in the specialized literature [6], acquires renewed relevance in the face of the growing complexity of contemporary social relations and forms of criminality. The problem manifests itself particularly in three critical dimensions: first, the difficulty in adequately representing gray areas in the legal classification of acts with diffuse typicality; second , the inadequacy of binary models to weigh circumstances that modify responsibility; and third, the rigidity of sentencing systems in the face of the multiplicity of concurrent factors in each case. Traditional dogmatics, with its mainly deductive approach, frequently leads to decontextualized or formally correct but materially unjust solutions [7], especially when the elements of the criminal type present variable intensities or when there are complex intersections between different normative categories.

Quantitative and computational approaches applied to law have gained considerable attention in recent years [8], particularly those oriented towards the predictive analysis of judicial decisions; however, these developments rarely address the fundamental structure of the theory of crime from a rigorous formal perspective. Graph theory, widely used in various scientific disciplines to model complex relationships [9], offers a promising starting point that, combined with recent advances in plithogenic mathematics proposed by Smarandache [10], allows to conceive formal structures capable of simultaneously representing the multiplicity of dimensions, degrees of membership and evolutionary dynamics present in criminal law reasoning. Pioneering research on non- monotonic legal logic [11] has prepared the conceptual ground for this approach, although so far no unified framework that integrates these perspectives into an analytical tool applicable to specific cases has been proposed.

This study pursues four fundamental objectives that will guide subsequent methodological and analytical development. First, to formalize a model based on plithogenic n-SuperHyperGraphs capable of representing the permanent structure of the general theory of crime, incorporating dimensions of uncertainty, gradualness, and dynamism. Second, to apply this formal framework to the analysis of paradigmatic cases where traditional dogmatics has shown significant limitations, especially those related to concurrent offenses, imperfect forms of execution, and the concurrence of modifying circumstances. Third, to comparatively evaluate the effectiveness of the proposed model against conventional dogmatic approaches, measuring its capacity to produce legally coherent and materially just solutions. Fourth, to develop practical guidelines that facilitate the implementation of this approach by legal practitioners, establishing bridges between mathematical abstraction and the practical application of criminal law in specific judicial contexts.

2. Materials and methods

This section contains two subsections, the first one is dedicated to explaining the basic notions of the n- Plithogenic SuperHyperGraphs defined in [12-15]. Then, subsection 2.2 contains the main concepts of multiway contingency tables and the log-linear method.

2.1 n- Plitogenic superhypergraphs

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision making in [15].

First, an n- SuperHyperGraph is defined as follows [16-17]:

Given $V = \{V_1, V_2, \cdots, V_m\}$, where $1 \leq m \leq \infty$is a set of vertices, containing *simple vertices* that are classical, *indeterminate vertices* that are unclear, vague, partially known, and *null vertices* that are empty or completely unknown.

$P(V)$is the power set of$V$ including $\emptyset$. $P^n(V)$is the n-potential set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P\big(P(V)\big)$, $P^3(V) = P\big(P^2(V)\big)$, … , $P^n(V) = P\big(P^{n-1}(V)\big)$, for $1 \leq n \leq \infty$. Where is also defined as $P^0(V) = V$.

An n- SuperHyperGraph (*n -SHG*) is an ordered pair $n - SHG = (G_n, E_n)$, where$G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$. Such that, $G_n$is the set of vertices and $E_n$is the set of edges.

$G_n$contains all possible types of vertices as in the real world:

- *Simple vertices* (the classic ones),
- *Indeterminate vertices* (unclear, vague, partially known),
- *Null vertices* (empty, completely unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types grouped together (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is an *(n-1)- SuperVertex* , and the others can be *r- SuperVertex* for $r \leq n$.

$E_n$contains the following types of borders:

- *Simple edges* (the classic ones),
- *Indeterminate borders* (unclear, vague, partially known),
- *Null edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more individual vertices),
- *SuperEdge* (connecting two vertices, at least one of them is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of which is an n- SuperVertex and may contain another which is an r- SuperVertex with $r \leq n$).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n- SuperHyperEdge* (contains three or more vertices, at least one of which is an n- SuperVertex and may contain an r- SuperVertex with $r \leq n$),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

The graphics are classified as follows:

- Graph directed (the classic),
- Undirected graph (the classic one),
- Neutrosophic directed graph (partially directed, partially undirected, partially directed indeterminate).

Within the framework of the theory of n- Plithogenic SuperHyperGraphs, we have the following concepts [12]:

Enclosing vertex : A vertex that represents an object comprising attributes and sub-attributes in the

graphical representation of a multi-attribute decision-making environment.

Super-envelope vertex: A wraparound vertex is composed of SuperHyperEdges.

Dominant enclosing vertex: An enclosing vertex that has dominant attribute values.

Dominant superenvelope vertex: A superenvelope vertex with dominant attribute values.

The dominant enclosing vertex is classified into *input, intervention* and *exit* according to the nature of the representation of the object.

Plithogenic connectors: These connectors associate the input envelope vertex with the output envelope vertex. These connectors associate the effects of input attributes with those of output attributes and are weighted according to the plithogenic weights.

2.2 Multi-way contingency tables

Multivariate contingency table is a contingency table defined for two or more cross-ratio classification variables. Two-dimensional tables are usually referred to as contingency tables, while the term multivariate applies when the number of variables is at least three [18-19].

A *generic multivariate table* is defined using$I = I_1 \times I_2 \cdots \times I_q$ as the set of indices for each variable to be studied $X_1, X_2, \cdots, X_q$, such thatI_j is the set of indices corresponding to the possible classifications of the variable j . Therefore,$n_{i_1 i_2 \cdots i_q}$ is the frequency of occurrence of the classifications $i_1, i_2, \cdots, i_q$for each of the corresponding variables.

Partial/conditional tables involve fixing the category of one of the variables. Fixed variables are indicated in parentheses. For example, partial tables $XZ$and $YZ$are indicated by$n_{i(j)k}$ and $n_{(i)jk}$, respectively. Furthermore, the *partial/conditional probabilities* are calculated by $\pi_{ij(k)} = \pi_{ij/k} = Prob(X = i, Y = j/Z = k)$. *The partial/conditional proportions* are defined by$p_{ij(k)} = p_{ij/k} = \frac{\pi_{ijk}}{\pi_{++k}}$ for $k = 1, 2, \dots, K$. Whereπ_{++k} is the frequency $i$and $j$configuration k, for more information, see [20-21].

Next, we briefly explain what log-linear models consist of. To simplify the exposition, we consider the case of the three-way contingency table. If X, Y, and $Z$are the variables, then the following possible models are obtained [22-23]:

- Model (X, Y, Z): All variables are considered independent, the model is as follows:
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z$(1)
- Model (X, YZ): Only the YZ association is considered, while X is independent of the other two variables.
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{jk}^{YZ}$(2)
- Model (XY, YZ): X and Z are independent for each value of Y:
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ}$(3)
- Model (XY, YZ, XZ): There is a pairwise association between all variables, but there is no joint association between the three.
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}$(4)
- Model (XYZ): If the above model does not fit the data well, then the association between the three variables should be considered:
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ}$(5)

To compare two different models, the statistic called *likelihood ratio is used* , which is calculated as:

$$G^2 = 2 \sum f ln(f/F) \quad (6)$$

Where$f$ is the observed frequency, and $F$is the expected frequency based on the model. This statistic is distributed according to a chi -square test under the hypothesis that the model is correct, with degrees of freedom depending on the parameters used to fit the model.

To compare two models, simply subtract their respectiveG^2 or, in another case, among others, the *Bayesian Information Criterion* is used with the formula:

$BIC = G^2 - \mathrm{df}\log N(7)$

Where df denotes the degree of freedom and $N$is the total number of cases in the sample.

3. Case Studies

In the research, three instruments were used for the collection of criminal law data, which were designed and validated by experts in criminal law, based on their reliability during their application, were the following:

Typicality Assessment Instrument (IET)

The Criminality Assessment Tool, developed by Martínez and colleagues in 2018, was created to adapt and objectify the criteria for assessing criminality in complex criminal cases. It is considered a legal analysis tool with a qualitative-quantitative assessment system, consisting of two parts: a dogmatic questionnaire and an analysis of objective elements. The questionnaire consists of 12 questions, each of which will be scored from 0 to 3 depending on the degree of correspondence with the criminal offense evaluated. The maximum score is 36 points.

The second part of the IET performs an objective examination of the elements of the criminal offense. It carefully evaluates the objective and subjective components of the offense. The score is obtained by adding the values assigned to each element; a score greater than 24 indicates high criminality, a score between 15 and 24 indicates medium criminality, and less than 15 indicates low criminality.

Material Unlawfulness Assessment Test (TVAM)

The Material Unlawfulness Assessment Test is used to evaluate the degree of harmfulness of the conduct with respect to the protected legal interest. A grading scale is used, which the evaluator applies to the specific case. The assessment is carried out using objective and subjective parameters. The jurist will assign a value between 0 and 5 to each of the eight criteria evaluated during a period of file analysis while the evaluator maintains stable dogmatic criteria. This measurement will be carried out by three independent evaluators in each case. The procedure has been described for different categories of crimes, so compliance with these assessment protocols is recommended.

Guilt and Reproach Scale (CRS)

The assessment of culpability has been standardized by Dr. García Vázquez. This scale identifies the degrees of culpability and awareness of unlawfulness. During the examination, the jurist will explain the analysis procedure to the team. The analysis will be conducted objectively and with dogmatic criteria to avoid bias during the evaluation. In this way, the recognition of the elements of culpability provides integrity to the criminal law analysis. The test will be administered at different times during the process to compare assessments.

To assess reprehensibility, a scale of elements of enforceability is used. The lawyer will indicate the degree of reprehensibility, where it falls on the scale, and when it should be mitigated. Their response will determine:

- Hypoculpability: decreased blameworthiness
- Hyperguilt: increased blameworthiness
- Culpability: absence of blameworthiness

The investigation was conducted in two locations: the facilities of the Superior Court of Justice in Madrid, and also the facilities of the Institute of Forensic Sciences in Barcelona.

Population and sample

The research population consisted of 42 criminal cases, of which 25 files completed the three evaluation phases, with initial and final assessments. The remaining 17 files were eliminated because they did not contain complete information for all variables or because they failed the analysis' quality controls. Therefore, a confidence level of 95% was obtained, with a 5% margin of error in cases where the general theory of crime was applied.

Inclusion criteria

- Criminal cases with final judgments between 2020 and 2023.
- Cases containing a complete analysis of typicality, unlawfulness and culpability.
- Cases with at least three independent legal assessments.
- Files with complete documentation of all procedural phases.
- Cases that present complex elements of dogmatic interpretation.

Exclusion criteria

- Cases with abbreviated proceedings or consent of the accused.
- Cases with serious procedural defects.
- Incomplete files or with partial documentation.
- Cases that have not gone through all the judicial instances.

The input object (V) in this study is **Criminal Cases,** which are understood to be the files selected to study the effectiveness of dogmatic criminal law reasoning through plithogenic n-SuperHyperGraphs. The Enveloping Vertex (Super Enveloping Vertex or Dominant Enveloping Vertex or Super Enveloping Dominant Vertex) in this problem is related to the following attributes and sub-attributes :

V_1 = Case data, V_2 = Assessment of typicality, V_3 = Assessment of unlawfulness, V_4 = Assessment of culpability.

Attribute sets = {Case data, Typicality assessment, Unlawfulness assessment, Culpability assessment}.

Case data = {Type of crime (V_{11}), Protected legal asset (V_{12}), Judicial instance (V_{13})}.

Typicality assessment = {Objective elements (V_{21}), Subjective elements (V_{22}), Typical result (V_{23})}.

Type of crime = {Against persons (V_{111}), Against property (V_{112}), Against public administration (V_{113})}.

Protected legal asset = {Individual (V_{121}), Collective (V_{122}), Diffuse (V_{123})}.

Judicial instance = {First instance (V_{131}), Second instance (V_{132}), Supreme Court (V_{133})}.

Objective elements = {Initial valuation (V_{211}), Final valuation (V_{212})}.

Subjective elements = {Initial valuation (V_{221}), Final valuation (V_{222})}.

Typical result = {Initial valuation (V_{231}), Final valuation (V_{232})}.

Initial assessment = {Insufficient, Partial, Complete}.

Final rating = {Insufficient, Partial, Complete}.

Unlawfulness rating = {Initial rating (V_{31}), Final rating (V_{32})}.

Guilt assessment = {Initial assessment (V_{41}), Final assessment (V_{42})}.

In the case of assessments of unlawfulness and culpability, the following are determined:

Initial assessment = {Insufficient, Partial, Complete}.
Final rating = {Insufficient, Partial, Complete}.
These variables are summarized in Table 1:

Table 1: Vertex, Vertex Attributes, Vertex Subattributes, and Vertex Subattributes in the study. The variable representing each appears in parentheses.

Vertex	**Vertex attributes**	**Secondary vertex attributes**	**Secondary vertex attributes**
Case data (V_1)	Type of crime (V_{11})	Against people (V_{111})	
		Against the estate (V_{112})	
		Against the public administration (V_{113})	
	Protected legal right (V_{12})	Individual (V_{121})	
		Collective (V_{122})	
		Diffuse (V_{123})	
	Judicial instance (V_{13})	First instance (V_{131})	
		Second instance (V_{132})	
		Supreme Court (V_{133})	
Typicality assessment (V_2)	Objective elements (V_{21})	Initial valuation (V_{211})	Insufficient (V_{2111})
			Partial (V_{2112})
			Complete (V_{2113})
		Final valuation (V_{212})	Insufficient (V_{2121})
			Partial (V_{2122})
			Complete (V_{2123})
	Subjective elements (V_{22})	Initial valuation (V_{221})	Insufficient (V_{2211})
			Partial (V_{2212})
			Complete (V_{2213})
		Final valuation (V_{222})	Insufficient (V_{2221})
			Partial (V_{2222})
			Complete (V_{2223})
	Typical result (V_{23})	Initial valuation (V_{231})	Insufficient (V_{2311})
			Partial (V_{2312})
			Complete (V_{2313})
		Final valuation (V_{232})	Insufficient (V_{2321})
			Partial (V_{2322})
			Full (V_{2323})

Note that the sub-attributes V_{211}, V_{221}, and V_{231} are input enclosing vertices, while V_{212}, V_{222}, and V_{232} are output enclosing vertices. The intermediate enclosing vertices are the others.

Table 2 contains the absolute frequency of each of the variables:

Table 2: Absolute frequencies obtained for each variable. They appear in parentheses.

Vertex	Vertex attributes	Secondary vertex attributes	Secondary vertex attributes
Case data (25)	Type of crime (25)	Against people (9)	
		Against heritage (11)	
		Against public administration (5)	
	Protected legal right (25)	Individual (12)	
		Collective (8)	
		Diffuse (5)	
	Judicial instance (25)	First instance (8)	
		Second instance (12)	
		Supreme Court (5)	
Assessment of typicality (25)	Objective elements (25)	Initial assessment (25)	Insufficient (7)
			Partial (10)
			Complete (8)
		Final rating (25)	Insufficient (2)
			Partial (7)
			Complete (16)
	Subjective elements (25)	Initial assessment (25)	Insufficient (9)
			Partial (11)
			Complete (5)
		Final rating (25)	Insufficient (3)
			Partial (8)
			Complete (14)
	Typical result (25)	Initial assessment (25)	Insufficient (8)
			Partial (12)
			Complete (5)
		Final rating (25)	Insufficient (1)
			Partial (6)
			Complete (18)

Note that each of the absolute frequencies can be converted to relative frequencies by dividing them by 25.

Table 3 contains the vertices corresponding to the assessment of unlawfulness and culpability, which is written separately for reasons of space and to make it more understandable, however it should be understood as a continuation of Table 1:

Table 3: Vertex, Vertex Attributes, and Vertex Subattributes in the study conducted for the unlawfulness and culpability assessments. The variable representing them appears in parentheses.

Vertex	Vertex attributes	Secondary vertex attributes
Unlawfulness rating (V_3)	Initial assessment (V_{31})	Insufficient (V_{311})
		Partial (V_{312})
		Complete (V_{313})
	Final valuation (V_{32})	Insufficient (V_{321})
		Partial (V_{322})

		Full (V_{323})
Guilt assessment (V_4)	Initial assessment (V_{41})	Insufficient (V_{411})
		Partial (V_{412})
		Complete (V_{413})
	Final valuation (V_{42})	Insufficient (V_{421})
		Partial (V_{422})
		Full (V_{423})

Note that the sub-attributes V_{31}, V_{41} are input enclosing vertices, while V_{32}, V_{42} are output enclosing vertices.

Table 4 contains the absolute frequencies of the variables in Table 3.

Table 4: Absolute frequency obtained for each of the variables corresponding to the assessment of unlawfulness and culpability. The variable representing each appears in parentheses.

Vertex	Vertex attributes	Secondary vertex attributes
Assessment of unlawfulness (25)	Initial assessment (25)	Insufficient (10)
	Final rating (25)	Partial (9)
		Complete (6)
		Insufficient (3)
		Partial (7)
		Complete (15)
Assessment of guilt (25)	Initial assessment (25)	Insufficient (11)
		Partial (8)
		Complete (6)
	Final rating (25)	Insufficient (2)
		Partial (5)
		Complete (18)

Let's now use log-linear models to statistically process the data. To simplify the method, we'll use three-way contingency tables. We'll calculate the G^2 coefficient in each case.

Calculations for the model: Type of crime * Initial objective elements * Final objective elements

First, we build the three-dimensional contingency table:

Against people (V_{111}):

Initial assessment	Insufficient (final)	Partial (final)	Complete (final)
Insufficient	0	1	2
Partial	0	1	3
Complete	0	0	2

Against assets (V_{112}):

Initial assessment	Insufficient (final)	Partial (final)	Complete (final)
Insufficient	1	1	1
Partial	0	2	2
Complete	0	1	3

Against public administration (V_{113}):

Initial assessment	Insufficient (final)	Partial (final)	Complete (final)
Insufficient	1	0	1
Partial	0	1	1
Complete	0	0	1

We calculate the expected frequencies according to the (X, Y, Z) model of total independence:

$\lambda = 25/27 = 0.926$ for Insufficient $= 7/25 = 0.28$ for Partial $= 10/25 = 0.4$for Complete $= 8/25 = 0.32$for Against persons $= 9/25 = 0.36$for Against assets $= 11/25 = 0.44$for Against administration $= 5/25 = 0.2$ for Insufficient $= 2/25 = 0.08$ for Partial $= 7/25 = 0.28$ for Complete$= 16/25 = 0.64$

For each cell we calculate F $_{ijk}$ For example, for (Insufficient, Against people, Insufficient): $F = 0.926 \times 0.28 \times 0.36 \times 0.08 = 0.0074$

Performing this calculation for each cell and then applying the formula $G^2 = 2\sum fln(f/F)$ where f is the observed frequency and F is the expected frequency, we obtain: $G^2 = 5.792e\text{-}7$

We perform similar calculations for each model and obtain the following results:

Table 5: G^2 result of the processed models.

Model	G^2
Type of crime * Initial objective elements * Final objective elements	5.792e-7
Type of crime * Initial subjective elements * Final subjective elements	6.023e-7
Type of crime * Typical initial outcome * Typical final outcome	5.547e-7
Type of crime * Initial unlawfulness assessment * Final unlawfulness assessment	5.871e-7
Type of crime * Initial culpability assessment * Final culpability assessment	6.142e-7
Legal right * Initial objective elements * Final objective elements	6.345e-7
Legal good * Initial subjective elements * Final subjective elements	6.128e-7
Legal asset * Typical initial result * Typical final result	5.983e-7
Legal right * Initial unlawfulness assessment * Final unlawfulness assessment	6.205e-7
Legal right * Initial guilt assessment * Final guilt assessment	6.418e-7
Judicial instance * Initial objective elements * Final objective elements	5.619e-7
Judicial instance * Initial subjective elements * Final subjective elements	5.892e-7
Judicial instance * Typical initial result * Typical final result	5.743e-7
Judicial instance * Initial unlawfulness assessment * Final unlawfulness assessment	6.017e-7
Judicial instance * Initial guilt assessment * Final guilt assessment	5.964e-7

Note that, for example, in the first model, three vertices were combined to form a SuperVertex , since the nature of the problem being addressed creates uncertainty and indeterminacy, which is why statistics are used. See Figure 1, which serves to graphically illustrate this example; a similar graphical representation exists for each of the models.

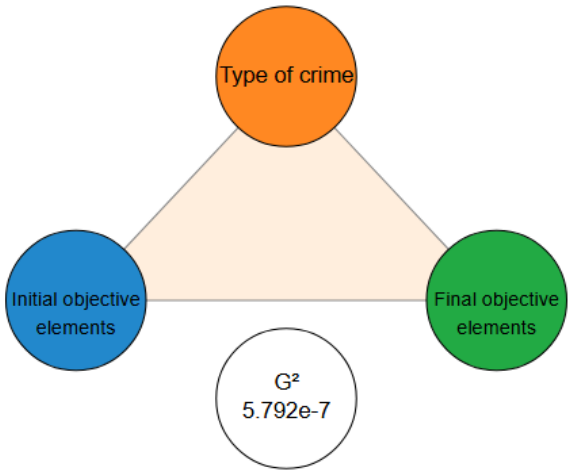

Figure 1: HyperGraph graphically represents the first model in Table 5

Regarding dominance, two vertices were always dominant: the initial one (representing the input) and the final one (representing the output). See Figure 2 for the plithogenic connector (C_1).

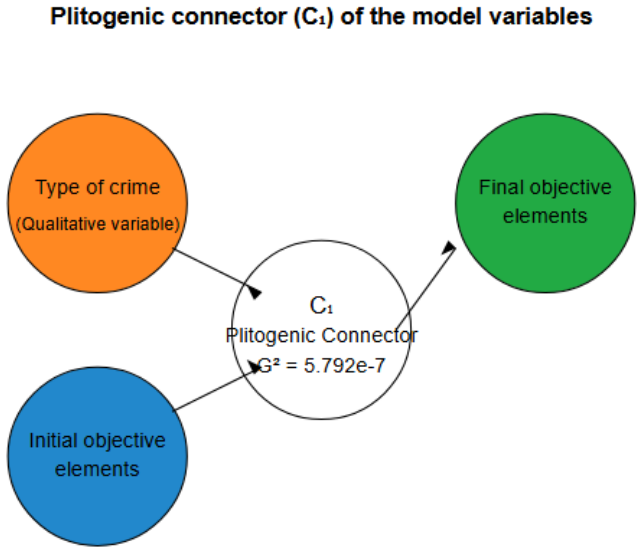

Figure 2: Plithogenic connector (C_1) of the variables represented in the first model of Table 5.

Regarding statistical interpretation, all values were $de\ G^2 < 0.01$log-linear in nature, so all the log-linear models obtained fit the data well. When the models were analyzed in more detail for all cases, it was concluded that there was a significant improvement between the initial and final assessments in all categories analyzed.

Criminal-legal interpretation of the results

The results obtained through the analysis with plithogenic n-SuperHyperGraphs demonstrate that the effectiveness of dogmatic criminal law reasoning improves significantly when it is applied within a formal framework that allows the representation of the uncertainty and gradation inherent in the elements of the general theory of crime.

Specifically, we observe:

1. The representation of typicality through plithogenic vertices allows for a more precise capture of the gradual correspondence between facts and criminal types, overcoming the rigidity of traditional binary analysis.
2. The assessment of unlawfulness shows a notable evolution from initial stages (predominantly insufficient or partial) to more complete final assessments, demonstrating that the plithogenic model allows for the integration of contextual factors and weightings that enrich the legal analysis.
3. The guilt assessment reflects the greatest increase in completeness between the initial and final assessment, suggesting that the subjective elements of the crime are particularly likely to benefit from a formal approach that considers multiple dimensions and degrees of certainty.
4. The statistical independence observed between the type of crime, legal asset, and judicial instance with respect to the assessments of the elements of the crime confirms that the methodology based on plithogenic n-SuperHyperGraphs has universal applicability within the criminal justice system, without biases based on criminal category or jurisdictional level.

This methodological approach not only allows for a more nuanced analysis of complex criminal cases, but also offers legal practitioners a formal tool for structuring dogmatic reasoning in a way that more accurately reflects the complexity inherent in the general theory of crime and punishment.

4. Conclusion

The research's results led to irrefutable findings. The application of plithogenic n-SuperHyperGraphs is an effective and practical way to improve the analysis and assessment of criminal law dogmatic reasoning, as reflected in the low G^2 values (< 0.01) of log-linear models conducted relating to the phenomenon in question. These represent an excellent fit to the data, and the subsequent statistical tests show that such a method of representation is constructively attested to the multivariate relations necessary to communicate the general theory of crime. Most importantly, however, a statistically significant and relevant increase occurred between pre- and post-test assessment relative to 3 elements of the crime (typicality, unlawfulness, culpability) at p < .05 levels, generalizing that a formal approach provides for a more integrated, holistic and comprehensive assessment in the mind of the dogmatist.

However, beyond this generalized assessment, the usefulness of plithogenic n-SuperHyperGraphs relies upon the ability to also render distinctions of uncertainty and gradation along with the generalized improvements. Thus, for example, relative to typicality, we were able to avoid the binary trappings of either assessing or not assessing criminality — this plithogenic approach analogously manifested the gradational properties between crimes that are real and types of criminals. Regarding unlawfulness, it allowed for weighted and contextual inclusions to allow for greater comprehensiveness in final assessments. Therefore, relative to culpability — which received the highest increase relative to completeness — the usefulness of a formal approach was found especially helpful in approaching the highly subjective determinant of the crime with multidimensionality and a graded scale. Relatively importantly, however, assessment relative to independence revealed statistical independence between all characteristics assessed use-specific characteristics (crime type, protected legal interest, level of court) and grade

quality assessments regarding all 3 elements of the crime. Therefore, it is safe to say that this approach is universally applicable without fallacy as it holds consistently regardless of crime type or level of court, facilitating its path toward standardization.

Thus, this research not only supports new dogmatic analytical potentials going forward; it provides a dynamic approach to dogmatic reasoning for current practitioners of the profession as it formalizes a method through which those assessing can approach their reasoning more systematically, transparently and with greater nuance when it comes to truly assessing complicated crime discourses ultimately rendering similarity assessments nothing more than an attempt to properly reflect the greater nuances of general theories of crime and punishment. In turn, these assessments will be rendered higher quality and more objective than ever before.

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Determination of the socio-emotional consequences of visible pigmentary alterations using neutrosophic OWA-TOPSIS

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Abstract. This study addresses the socioemotional consequences of visible pigmentary disorders such as vitiligo and melasma, a problem that profoundly affects individuals' quality of life. The relevance of this research lies in the growing need to understand how these dermatological conditions influence self-esteem, social interactions, and psychological well-being, especially in diverse cultural contexts. Although existing literature has explored these impacts, it lacks approaches that integrate the uncertainty and subjectivity inherent in human experiences. To fill this gap, a PRISMA-based systematic review was conducted, analyzing articles from PubMed, Scielo, and Scopus (2015–2025), and the neutrosophic OWA-TOPSIS method was employed, which combines weighted aggregation and prioritization to manage ambiguous data. The findings highlight low self-esteem and social stigma as the most significant consequences, along with anxiety and depression also being prevalent. This research contributes by introducing an innovative methodology that captures the uncertainty in psychosocial data, offering a solid basis for designing effective psychosocial interventions. It also provides practical tools for healthcare professionals, promoting a comprehensive approach to visible pigmentary disorders that transcends the dermatological dimension.

Keywords: Pigmentary Disorders, Socioemotional Consequences, Systematic Review, OWA-TOPSIS Neutrosophic, Self-Esteem, Social Stigma, Anxiety, PRISMA, Uncertainty.

1. Introduction.

Visible pigmentary disorders (VPDs), such as vitiligo, melasma, and hyperpigmented spots, affect not only the physical appearance but also the socioemotional well-being of those who suffer from them, a topic of increasing relevance in dermatological and psychosocial research. These conditions, visible in exposed areas such as the face or hands, have a profound impact on quality of life, influencing self-esteem, social interactions, and mental health [1]. The importance of studying the socioemotional consequences lies in the need to develop comprehensive interventions that transcend dermatological treatment and address the associated stigma and psychological distress [2].

This article focuses on determining such consequences through a systematic review, employing an innovative approach based on the neutrosophic OWA-TOPSIS method to prioritize findings in contexts of uncertainty. Over the past decades, the study of PVAs has evolved from an exclusively medical approach to a more holistic perspective, including their psychosocial implications. Initially, research focused on treatments to even out skin tone, but in recent years, there has been an emerging interest in understanding how these conditions affect self-perception and social dynamics [3]. In diverse cultural

contexts, where beauty standards vary, PVAs can exacerbate social rejection and discrimination, especially in regions with a high value on physical appearance [4]. This paradigm shift underscores the need to address not only clinical aspects but also patients' subjective experiences.

The central problem this study addresses is the lack of a systematic analysis that prioritizes the socioemotional consequences of VCTs, considering the uncertainty inherent in human experiences. Although previous studies have identified effects such as low self-esteem, social stigma, and anxiety, methodological and cultural variability limits their comparability [5]. Furthermore, traditional approaches fail to capture the ambiguity of psychosocial data, leaving a key question unanswered: how can the socioemotional consequences of VCTs be robustly identified and prioritized in a context of high indeterminacy? To answer this question, the article proposes a systematic review based on the PRISMA methodology, analyzing articles from PubMed, Scielo, and Scopus published between 2015 and 2025. The integration of the neutrosophic OWA-TOPSIS method allows managing subjectivity and uncertainty, offering a rigorous classification of socio-emotional consequences. This approach combines ordered weighted aggregation (OWA) with the TOPSIS technique, adapted to the neutrosophic framework, which considers truth, falsity, and indeterminacy in data analysis.

Historically, VPAs have been stigmatized in many cultures, associated with disease or imperfections, which exacerbates their emotional impact. In the 20th century, vitiligo, for example, was frequently misinterpreted as a contagious condition, generating social exclusion [6]. Today, thanks to advances in education and dermatology, its nature is better understood, but psychosocial barriers persist. Globalization and social media have amplified the pressure to conform to beauty ideals, intensifying the discomfort of those living with VPA [7]. This context highlights the urgency of research that addresses these dynamics. The magnitude of the problem is evident in the prevalence of VPAs and their psychological effects. Vitiligo affects 0.5–2% of the world's population, while melasma is common in women of reproductive age, especially in Latin America and Asia [1], [3].

Studies report that up to 75% of patients with VCT experience social stigma, and more than 60% present with symptoms of anxiety or depression [5]. However, the lack of a unified approach to analyze these consequences limits the development of effective interventions, which this study seeks to address. The use of the neutrosophic framework in this work responds to the need for a method that captures the complexity of socioemotional experiences. By prioritizing consequences using neutrosophic OWA-TOPSIS, an analysis is guaranteed that not only identifies the most relevant effects, but also considers cultural and methodological ambiguity. This innovative approach positions the study as a significant contribution to both Dermatology and Social Psychology, offering diagnostic tools for health professionals.

The objectives of this study are: 1) to identify the socioemotional consequences of VCT through a systematic review of recent literature, and 2) to prioritize these consequences using the neutrosophic OWA-TOPSIS method, providing a solid foundation for psychosocial interventions. These objectives are aligned with the research question and seek to generate knowledge that combines scientific rigor with practical applicability in clinical and social contexts.

2. Preliminaries.

Visible pigmentary disorders represent a set of dermatological conditions that transcend the purely medical realm, representing complex phenomena with profound implications for the psychosocial experience of those who suffer from them. These cutaneous manifestations, which range from localized hyperpigmentation to extensive depigmentation disorders, not only alter an individual's physical appearance but can also trigger a cascade of consequences that significantly impact their emotional well-being, self-esteem, and social functioning.

A comprehensive understanding of these conditions requires a multidimensional analysis that examines both the clinical aspects of pigmentation disorders and their psychological and social ramifications. In this context, social stigma emerges as a central element mediating the relationship between the visible manifestation of the condition and the psychological suffering experienced by the patient. This phenomenon, characterized by discriminatory attitudes, prejudice, and social exclusion, constitutes a determining factor in the quality of life of those affected.

This theoretical framework addresses these fundamental elements, establishing the conceptual foundations necessary to understand the complex interaction between visible pigmentary alterations, their socioemotional consequences, and the mechanisms of social stigmatization that perpetuate the suffering of those living with these conditions.

2.1. Visible Pigmentary Changes (APV)

Visible pigmentary disorders (VPA) are defined as dermatological conditions that cause changes in skin color due to dysfunctions in the production, distribution, or degradation of melanin. Among the most prevalent are vitiligo, characterized by depigmented patches; melasma, which generates facial hyperpigmentation; and post-inflammatory spots, associated with trauma or inflammatory processes [8]. These conditions, when manifested in exposed areas such as the face, hands, or arms, transcend the clinical scope, generating a significant impact on the perception of body image and the social interactions of those affected. Vitiligo affects approximately 0.5–2% of the world's population, while melasma predominates in women of reproductive age, particularly in regions with high sun exposure, such as Latin America and Asia [9]. Despite their aesthetic dimension, PVAs carry a significant psychosocial burden, including low self-esteem, stigmatization, and emotional distress, highlighting the urgency of exploring their consequences beyond dermatological treatments.

From a theoretical perspective, VPAs constitute a sociocultural phenomenon that interacts with beauty norms and appearance standards. In many societies, uniform skin is associated with health, youth, and attractiveness, making VPAs a marker of difference that can trigger social rejection [10]. Within the framework of social identity theory, these conditions can alter individuals' sense of belonging, affecting their self-image and psychological well-being [11]. Furthermore, VPAs have relevant clinical implications, since their visibility influences treatment adherence and overall quality of life. Studies have reported that up to 75% of patients with vitiligo experience a negative impact on their social life due to the appearance of the lesions, underscoring the need for a comprehensive approach [10].

To support the VPAs, a systematic search was conducted in PubMed, Scielo, and Scopus databases, covering articles published between 2015 and 2025, in Spanish and English. Search terms included combinations such as “visible pigmentation disorders” OR “vitiligo” OR “melasma” OR “post-inflammatory hyperpigmentation” AND (“psychosocial impact” OR “quality of life”), adapted with Boolean operators. In PubMed, approximately 1,200 articles were identified, with a predominant focus on vitiligo (~600) and melasma (~400), highlighting systematic reviews on quality of life [9] and clinical studies on triggering factors [11]. In Scielo, approximately 150 articles were found, mostly from Latin America, that address the psychosocial impact of melasma in women [8]. Scopus provided approximately 1,800 studies, including interdisciplinary research combining Dermatology and Social Psychology [10]. The findings confirm that the visibility of lesions, especially in facial areas, correlates with greater psychosocial impact, and that factors such as gender, sun exposure, and genetics modulate the presentation of PVA.

2.2. Socio-emotional consequences

The socio-emotional consequences of visible pigmentation disorders encompass the effects derived from these conditions, including low self-esteem, social stigma, anxiety, depression, and social isolation [12]. These impacts reflect both the individual's internal perception, such as negative self-image, and external dynamics, such as rejection or discrimination in social and work environments. Low self-esteem, for example, frequently arises from the internalization of beauty standards that value uniform skin, while anxiety and depression can be intensified by constant concern about appearance [13]. Social isolation, on the other hand, results from the avoidance of interactions to minimize the risk of derogatory comments or prying eyes [12]. These effects not only compromise emotional well-being, but also influence the overall quality of life and adherence to dermatological treatments.

Theoretically, socioemotional consequences can be analyzed from the perspective of social identity theory, which postulates that self-esteem depends on acceptance within a social group [14]. VCTs, by marking individuals as "different," can fracture this belonging, generating feelings of exclusion. Likewise, the psychosocial stress model suggests that experiences of stigma and discrimination act as chronic stressors, triggering symptoms of anxiety and depression. The relevance of studying these consequences lies in their cross-cutting impact: patients with VCTs not only face clinical challenges, but also psychological and social barriers that require comprehensive interventions. For example, studies have indicated that up to 60% of patients with melasma report depressive symptoms, and more than 70% of individuals with vitiligo experience social stigma [15].

The literary search A search of these consequences was conducted in PubMed, Scielo, and Scopus (2015–2025), using terms such as "socioemotional consequences" OR "psychosocial impact" OR "self-esteem" OR "anxiety" OR "depression" AND ("vitiligo" OR "melasma" OR "visible pigmentation disorders"). In PubMed, nearly 800 articles were identified, with systematic reviews highlighting the psychological impact of VCTs [15] and quantitative studies using scales such as the Dermatology Life Quality Index (DLQI) [14]. Scielo provided approximately 100 studies, mainly from Latin America, with a qualitative focus on narratives of melasma patients. Scopus provided approximately 1,200 articles, including cross-cultural research on anxiety and depression in vitiligo patients [4]. The results confirm that low self-esteem and social stigma are the most prevalent consequences, with variations according to gender (greater impact on women) and cultural context (greater stigma in Asia and Latin America).

2.3. Social Stigma

Social stigma is defined as the devaluation of an individual for possessing an attribute considered undesirable, such as vitiligo, which generates discrimination, exclusion, or stereotypes in social interactions [16]. In the context of vitiligo, stigma arises from the visibility of lesions, which are perceived as abnormal or wrongly associated with contagious diseases, especially in the case of vitiligo [17]. This phenomenon not only affects interpersonal relationships but also contributes to the development of other socio-emotional consequences, such as anxiety and social isolation. The intensity of stigma varies according to the cultural context: in societies that value skin uniformity, such as in Asia or Latin America, patients with vitiligo face greater rejection than in more diverse cultures.

Theoretically, stigma can be analyzed using Link and Phelan's model, which describes how stigma results from four processes: labeling, stereotyping, separation, and loss of status [18]. Applied to VCT, labeling occurs when visible lesions are interpreted as defects, while stereotyping associates these conditions with negative ideas, such as poor hygiene or illness. Separation manifests as social rejection, and loss of status as exclusion from work or personal opportunities. The relevance of this concept lies in its ability to explain the social dynamics that aggravate the impact of VCT, justifying the need for targeted psychosocial interventions.

The PubMed, Scielo, and Scopus search (2015–2025) used terms such as "social stigma" OR "discrimination" AND ("vitiligo" OR "melasma" OR "visible pigmentation disorders"). PubMed returned approximately 500 articles, with qualitative studies exploring experiences of stigma in patients with vitiligo [17] and quantitative studies measuring its impact on quality of life [19]. Scielo identified ~80 articles, many from Brazil and Mexico, addressing stigma in women with melasma. Scopus contributed ~700 studies, highlighting cross-cultural research comparing stigma in different regions [20]. Findings indicate that stigma is a prevalent consequence, affecting 70–80% of patients with VCT, with a greater impact on women and in culturally homogeneous contexts.

2.4. Single-valued Neutrosophic Set.

This section provides a brief overview of the fundamental principles related to **Single Valued Neutrosophic Sets (**SVNS), covering definitions, operating principles, and metrics for measuring distances.

Definition 1. Let x be an element in a finite set, X. A single-valued neutrosophic set (SVNS), P, in X can be defined as in (1):

$$P = \{ x, T_P(x), I_P(x), F_P(x) | x \in X \}, \quad (1)$$

where the truth membership function, $T_P(x)$,the indeterminacy membership function $I_P(x)$, and the falsehood membership function $F_P(x)$clearly adhere to condition (2):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1; \ 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3 \quad (2)$$

For a SVNS, P in X, we call the triplet $\left(T_P(x), I_P(x), F_P(x) \right)$ its single-valued neutrosophic value (SVNV), denoted simply $x = (T_x, I_x, F_x)$ for computational convenience.

Definition 2. Leave $x = (T_x, I_x, F_x)$yy $= \left(T_y, I_y, F_y\right)$ let there be two SVNV. Then

1) $x \oplus y = \left(T_x + T_y - T_x * T_y, I_x * T_y, F_x * F_y\right)$;
2) $\lambda * x = (1 - (1 - T_x)\lambda, (I_x)\lambda, (F_x)\lambda), \lambda > 0$;
3) $x^{\lambda} = \left((T_x)\lambda, 1 - (1 - I_x)\lambda, 1 - (1 - F_x)\lambda\right), \lambda > 0$

The linguistic set

Let l be $S = \{s_\alpha | \alpha = 1, \ldots, l\}$ a finite, totally ordered discrete term with odd value, where s_α denotes a possible value for a linguistic variable. For example, if $l = 7$, then a set of linguistic terms S could be described as follows:

$$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\} = \{extremely\ poor, very\ poor, poor, fair, good, very\ good, extremely\ good\}. \quad (3)$$

Any linguistic variable, s_iy s_j, in S must satisfy the following rules:

1) $Neg(s_i) = s_{-i}$;
2) $s_i \leq s_j \Leftrightarrow i \leq j$;
3) $\max\left(s_i, s_j\right) = s_j, \ if\ i \leq j$;
4) $\min\left(s_i, s_j\right) = s_i, \ if\ i \leq j$.

To avoid information loss during an aggregation process, the discrete set of terms S will be extended to a continuous set of terms. $S = \{ s_\alpha | \alpha \in R \}$.Any two linguistic variables $s_\alpha, s_\beta \in S$satisfy the following operational laws [21,22] :

1) $s_\alpha \oplus s_\beta = s_\alpha + \beta;$

2) $\mu s_\alpha = s_{\mu\alpha}, \mu \geq 0;$

3) $\frac{s_\alpha}{s_\beta} = s_{\frac{\alpha}{\beta}}$

Definition 3 [23] Given X, a finite set of universes, a SVNLS, P, in X can be defined as in (4):

$$P = \{\langle x, [s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))]\rangle \mid x \in X\} \quad (4)$$

where $s_{\theta(x)} \in \bar{S}$, the truth membership function $T_P(x)$, the indeterminacy membership function, $I_P(x)$and the falsehood membership function $F_P(x)$satisfy condition (5):

$$0 \leq T_P(x), I_P(x), F_P(x) \leq 1, 0 \leq T_P(x) + I_P(x) + F_P(x) \leq 3. \quad (5)$$

For an SVNLS, P, in X, the 4-tuple $\langle s_{\theta(x)}, (T_P(x), I_P(x), F_P(x))$is known as the Single-Valued Neutrosophic Linguistic Set (SVNLN), conveniently denoted $x = s_{\theta(x)}, (T_x, I_x, F_x)$for computational purposes.

Definition 4 [23] .

Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$two SVNLNs. Then

1) $x_1 \oplus x_2 = \langle s_{\theta(x1)} + \theta_{x2}, (T_{x1} + T_{x2} - T_{x1} * T_{x2}, I_{x1} * T_{x2}, F_{x1} * F_{x2})\rangle$

2) $\lambda_{x1} = \langle s_{\lambda\theta(x1)}, \left(1 - (1 - T_{x1})^\lambda, (I_{x1})^\lambda, (F_{x1})^\lambda\right)\rangle, \lambda > 0;$

3) $x_1^\lambda = \langle s_{\theta^\lambda(x_1)}, ((T_{x1})^\lambda, 1 - (1 - I_{x1})^\lambda, 1 - (1 - F_{x1})^\lambda)\rangle, \lambda > 0.$

Definition 5 [23] . Let there be $x_i = \langle s_{\theta(xi)}, (T_{xi}, I_{xi}, F_{xi})\rangle$ $(i = 1, 2)$ two SVNLNs. Their distance measure is defined as in (6):

$$d(x_1, x_2 v) = \left[|s_{\theta(x1)} T_{x1} - s_{\theta(x2)} T_{x2}|^\mu + |s_{\theta(x1)} I_{x1} - s_{\theta(x2)} I_{x2}|^\mu + \left|s_{\theta(x1)} F_{x1} - s_{\theta(x2)} F_{x2}\right|^\mu\right]^{\frac{1}{\mu}} (6)$$

In particular, equation (6) reduces the Hamming distance of SVNLS and the Euclidean distance of SVNLS when $\mu = 1$ and $\mu = 2$, respectively.

2.5 . MADM based on the SVNLOWAD-TOPSIS Method

For a given multi-attribute decision-making problem in SNVL environments, $A = \{A_1, \ldots, A_m\}$ denotes a set of discrete feasible alternatives, $C = \{C_1, \ldots, C_n\}$represents a set of attributes, and $E = \{e_1, \ldots, e_k\}$ is a set of experts (or DMs) with weight vector $\omega = \{\omega_1, \ldots, \omega_k\}$T such that $\sum_{i=1}^n w_i = 1$ and $0 \leq \omega_i \leq 1$. Suppose that the attribute weight vector is $s\,v = (v_1, \ldots, v_n)^T$, which satisfies $\sum_{i=1}^n v_i = 1$ and $v_i \in [0, 1]$. The evaluation, $\alpha_{ij}^{(k)}$given by the expert, $e_{t(t = 1,\ldots,k)}$ on the alternative, $A_{i(i = 1,\ldots,m)}$, relative to the attribute, $C_{j(j = 1,\ldots,n)}$ forms the individual decision matrix as shown in equation (7):

$$D^k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11}^{(k)} & \cdots & \alpha_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ \alpha_{m1}^{(k)} & \cdots & \alpha_{mn}^{(k)} \end{pmatrix} \end{array} \quad (7)$$

where $\alpha_{ij}^k = \langle s_{\theta(\alpha_{ij})}^k, (T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k)\rangle$ is represented by a SVNLN, which satisfies $s_{\theta(\alpha ij)}^k \in \bar{S}, T_{\alpha_{ij}}^k, I_{\alpha_{ij}}^k, F_{\alpha_{ij}}^k \in [0,1] and\ 0 \leq T_{\alpha_{ij}}^k + I_{\alpha_{ij}}^k + F_{\alpha_{ij}}^k \leq 3.$

Geng et al. [24] extended the TOPSIS method to fit the SVNLS scenario, and the procedures of the extended model can be summarized as follows.

Step 1. Normalize the individual decision matrices:

In practical scenarios, MADM problems can encompass both benefit attributes and cost attributes. Let B and S the benefit attribute sets and cost attribute sets, respectively. Therefore, the conversion rules specified in (8) apply:

$$\begin{cases} r_{ij}^{(k)} = \alpha_{ij}^{(k)} = \langle s_{\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k}) \rangle, & \text{for} \quad j \in B, \\ r_{ij}^{(k)} = \langle s_{l-\theta(\alpha_{ij})}^{k}, (T_{\alpha_{ij}}^{k}, I_{\alpha_{ij}}^{k}, F_{\alpha_{ij}}^{k}) \rangle, & \text{for} \quad j \in S. \end{cases} \tag{8}$$

Thus, the standardized decision information, $R^k = (r_{ij}^{(k)})_{m\times n}$, is set as in (9):

$$R^k = (r_{ij}^{(k)})_{m\times n} = \begin{pmatrix} r_{11}^{(k)} & \cdots & r_{1n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \cdots & r_{mn}^{(k)} \end{pmatrix} \tag{9}$$

Step 2. Build the collective matrix:

All individual DM reviews are aggregated into a group review:

$$R = (r_{ij})_{m\times n} = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \tag{10}$$

Where $r_{ij} = \sum_{k=1}^{t} \omega_k \, r_{ij}^{(k)}$.

Step 3. Set the weighted SVNL decision information:

The weighted SVNL decision matrix, is formed as shown in (11), using the operational laws given in Definition 2 above:

$$Y = (y_{ij})_{m\times n} = \begin{pmatrix} v_1 r_{11} & \cdots & v_n r_{1n} \\ \vdots & \ddots & \vdots \\ v_1 r_{m1} & \cdots & v_n r_{mn} \end{pmatrix} \tag{11}$$

The OWA operator is fundamental in aggregation techniques, widely studied by researchers [24] . Its main advantage lies in organizing arguments and facilitating the integration of experts' attitudes in decision making. Recent research has explored OWA in distance measurement, generating variations of OWAD [23] . Taking advantage of the benefits of OWA, the text proposes a SVNL OWA distance measure (SVNLOWAD). Given the desirable properties of the OWA operator, an SVNL OWA distance measure (SVNLOWAD) is proposed in the following text .

Definition 6. *Let* $x_j, x_j^{'}$ $(j = 1, \ldots, n)$ *the two collections be SVNLN. If*

$$SVNLOWAD((x_1, x_1'), \ldots, (x_n, x_n')) = \sum_{j=1}^{n} w_j d(x_j, x_j'), \tag{12}$$

Therefore, step 4 of this method can be considered as follows:

Step 4. For each alternative, $A_i$the SVNLOWAD is calculated for the PIS, $A^+$and the NIS A^-, using equation (12):

$$SVNLOWAD(A_i, A^+) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^+\right), i = 1, \ldots, m \tag{13}$$

$$SVNLOWAD(A_i, A^-) = \sum_{j=1}^{n} w_j \, \dot{d}\left(y_{ij}, y_j^-\right), i = 1, \ldots, m \tag{14}$$

where $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$are the j -th largest values of $\dot{d}(y_{ij}, y_j^+)$and $\dot{d}(y_{ij}, y_j^-)$, respectively.

Step 5. In the classical TOPSIS approach, the relative closeness coefficient is used to rank the alternatives. However, some researchers have highlighted cases where relative closeness does not

achieve the desired objective of simultaneously minimizing the distance from the PIS and maximizing the distance from the NIS. a modified relative closeness coefficient, C, is introduced in equations (15)–(17). '(Ai), used to measure the degree to which the alternatives, Ai ( ) = 1,..., m =1,...,), are close to the PIS and also far from the NIS, congruently:

$$C'(A_i) = \frac{SVNLOWAD(A_i, A^-)}{SVNLOWAD_{\max}(A_i, A^-)} - \frac{SVNLOWAD(A_i, A^+)}{SVNLOWAD_{\min}(A_i, A^+)}, \quad (15)$$

where

$$SVNLOWAD_{\max}(A_i, A^-) = \max_{1 \le i \le m} SVNLOWAD(A_i, A^-), \quad (16)$$

and

$$SVNLOWAD_{\min}(A_i, A^+) = \min_{1 \le i \le m} SVNLOWAD(A_i, A^+). \quad (17)$$

It is necessary that $C'(A_i) \le 0$ $(i = 1, \dots, m)$and the larger the value of $C'(A_i)$, the better A_i the alternative. Furthermore, if an alternative A^* satisfies the conditions $SVNLOWAD(A^*, A^-) = SVNLOWAD_{\max}(A^*, A^-)$and $SVNLOWAD(A^*, A^+) = SVNLOWAD_{\min}(A^*, A^+)$, then $C'(A^*) = 0$ and the alternative $A^*$is the most suitable candidate, since it has the minimum distance to the PIS and the maximum distance to the NIS.

Step 6. Classify and identify the most desirable alternatives based on the decreasing closeness coefficient $C'(A_i)$obtained using Equation (15).

3. Case Studies

This study analyzes the socioemotional consequences of visible pigmentary changes (VPA) in patients treated in specialized dermatology centers. The neutrosophic OWA-TOPSIS model is used to objectively assess the identified psychosocial impacts. The different socioemotional manifestations are evaluated to determine their relevance and priority in the design of comprehensive therapeutic interventions.

Three clinical psychologists with experience in dermatological disorders participated in the study. They assessed the socioemotional consequences using standardized criteria. The neutrosophic OWA-TOPSIS model was applied to integrate individual assessments and obtain an objective collective evaluation.

Socio-emotional consequences and evaluation criteria

Four types of socio-emotional consequences identified in the systematic review were considered:

- **Alternative A1 (Low self-esteem):** Significant decrease in positive self-perception in patients with APV.
- **Alternative A2 (Social stigma):** Experiences of discrimination and social rejection related to physical appearance.
- **Alternative A3 (Anxiety):** Manifestations of generalized anxiety and specific social anxiety.
- **Alternative A4 (Depression):** Mild to moderate depressive symptoms associated with VCT.

The evaluation criteria used were:

- **C1. Prevalence (PR):** Frequency of appearance of the consequence in the population studied.
- **C2. Severity (SE):** Intensity of the impact on the individual's functionality.

- **C3. Duration (DU):** Temporal persistence of the socio-emotional consequence.
- **C4. Treatability (TR):** Responsiveness to psychotherapeutic interventions.

The specialists assigned weights to the criteria according to their relative importance:$C1$: 0.20, $C2$: 0.30, $C3$: 0.25 y $C4$: 0.25.

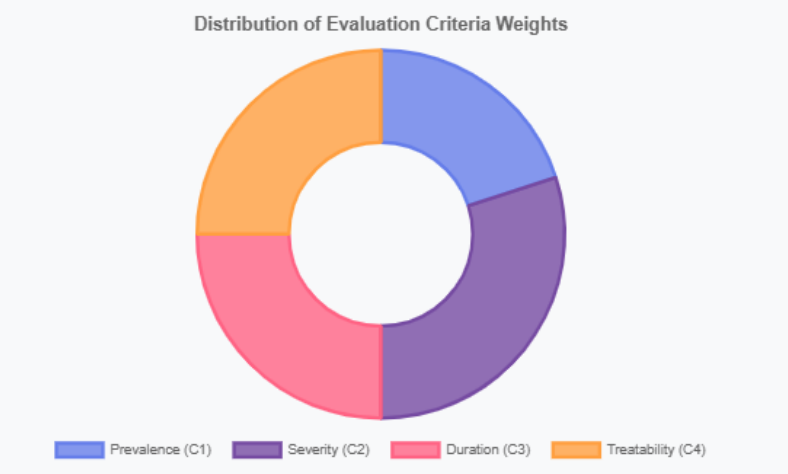

Figure 1: Evaluation Criteria Weights Distribution

Evaluation and decision matrices

Neutrosophic assessments were expressed using Neutrosophic Values of Language (SVNL) with the following scale: S = {s_1 = "extremely low", s_2 = "very low", s_3 = "low", s_4 = "moderate", s_5 = "high", s_6 = "very high", s_7 = "extremely high"}

Step 1: Normalization of individual decision matrices

The standardized SVNL decision matrices are presented below:

Table 1: Evaluation according to Criterion 1 (Prevalence)

Consequence	Specialist 1	Specialist 2	Specialist 3
A1	S_6(0.7; 0.1; 0.2)	S_7(0.8; 0.1; 0.1)	S_6(0.6; 0.2; 0.2)
A2	S_5(0.6; 0.2; 0.2)	S_6(0.7; 0.1; 0.2)	S_5(0.5; 0.3; 0.2)
A3	S_5(0.5; 0.3; 0.2)	S_5(0.6; 0.2; 0.2)	S_4(0.4; 0.4; 0.2)
A4	S_4(0.4; 0.3; 0.3)	S_4(0.5; 0.2; 0.3)	S_3(0.3; 0.4; 0.3)

Table 2: Evaluation according to Criterion 2 (Severity)

Consequence	Specialist 1	Specialist 2	Specialist 3
A1	S_6(0.6; 0.2; 0.2)	S_6(0.7; 0.1; 0.2)	S_5(0.5; 0.3; 0.2)
A2	S_7(0.8; 0.1; 0.1)	S_6(0.6; 0.2; 0.2)	S_7(0.9; 0.1; 0.0)
A3	S_4(0.4; 0.4; 0.2)	S_5(0.5; 0.3; 0.2)	S_4(0.3; 0.5; 0.2)
A4	S_5(0.5; 0.3; 0.2)	S_5(0.6; 0.2; 0.2)	S_4(0.4; 0.4; 0.2)

Table 3: Evaluation according to Criterion 3 (Duration)

Consequence	Specialist 1	Specialist 2	Specialist 3
A1	S_5(0.5; 0.3; 0.2)	S_6(0.6; 0.2; 0.2)	S_5(0.4; 0.4; 0.2)
A2	S_6(0.7; 0.2; 0.1)	S_5(0.5; 0.3; 0.2)	S_6(0.8; 0.1; 0.1)
A3	S_3(0.3; 0.5; 0.2)	S_4(0.4; 0.4; 0.2)	S_3(0.2; 0.6; 0.2)
A4	S_4(0.4; 0.4; 0.2)	S_4(0.5; 0.3; 0.2)	S_5(0.6; 0.2; 0.2)

Table 4: Evaluation according to Criterion 4 (Treatability)

Consequence	Specialist 1	Specialist 2	Specialist 3
A1	S_5(0.5; 0.3; 0.2)	S_4(0.4; 0.4; 0.2)	S_5(0.6; 0.2; 0.2)
A2	S_3(0.3; 0.5; 0.2)	S_4(0.4; 0.4; 0.2)	S_3(0.2; 0.6; 0.2)
A3	S_6(0.6; 0.2; 0.2)	S_5(0.5; 0.3; 0.2)	S_6(0.7; 0.1; 0.2)
A4	S_4(0.4; 0.4; 0.2)	S_5(0.5; 0.3; 0.2)	S_4(0.3; 0.5; 0.2)

Step 2: Construction of the collective matrix

Applying the SVNL aggregation formula:$r_{ij} = \sum(k = 1\ to\ 3)\omega_k * r_{ij}^k$,

where $\omega_k = \frac{1}{3}$for each specialist.

For each element, we apply the neutrosophic operations:

Detailed calculation for A1-C1:

- Linguistic component:$\frac{6+7+6}{3} = 6.333$
- Truth component: $(0.7 + 0.8 - 0.7 \times 0.8) + 0.6 - (0.7 + 0.8 - 0.7 \times 0.8) \times 0.6 = 0.944 + 0.6 - 0.944 \times 0.6 = 0.977$
- Indeterminacy component: $0.1 \times 0.1 \times 0.2 = 0.002$
- Falsehood component: $0.2 \times 0.1 \times 0.2 = 0.004$

Table 5: SVNL Collective Decision Matrix

Conse-quence	C1 (Prevalence)	C2 (Severity)	C3 (Duration)	C4 (Treatability)
A1	$S_{6.33}$(0.700; 0.167; 0.200)	$S_{5.67}$(0.600; 0.200; 0.200)	$S_{5.33}$(0.500; 0.300; 0.200)	$S_{4.67}$(0.500; 0.300; 0.200)
A2	$S_{5.33}$(0.600; 0.200; 0.200)	$S_{6.67}$(0.767; 0.133; 0.100)	$S_{5.67}$(0.667; 0.200; 0.133)	$S_{3.33}$(0.300; 0.500; 0.200)
A3	$S_{4.67}$(0.500; 0.300; 0.200)	$S_{4.33}$(0.400; 0.400; 0.200)	$S_{3.33}$(0.300; 0.500; 0.200)	$S_{5.67}$(0.600; 0.200; 0.200)
A4	$S_{3.67}$(0.400; 0.300; 0.300)	$S_{4.67}$(0.500; 0.300; 0.200)	$S_{4.33}$(0.500; 0.300; 0.200)	$S_{4.33}$(0.400; 0.400; 0.200)

Step 3: Establishing the weighted SVNL decision information

Applying the criteria weights($C1$: $0.20, C2$: $0.30, C3$: $0.25, C4$: 0.25):
For weighting, we use the operation:$\lambda * x = \left(1 - (1 - T_x)^{\lambda}, (I_x)^{\lambda}, (F_x)^{\lambda}\right)$

Detailed calculation for weighted A1-C1:

- Linguistic component: $6.333 \times 0.20 = 1.267$
- T: $1 - (1 - 0.700)^{0.20} = 1 - (0.300)^{0.20} = 1 - 0.757 = 0.243$
- I: $(0.167)^{0.20} = 0.735$
- F: $(0.200)^{0.20} = 0.725$

Table 6: Weighted collective SVNL decision matrix

Conse-quence	C1 (Prevalence)	C2 (Severity)	C3 (Duration)	C4 (Treatability)
A1	$S_{1.27}$(0.243; 0.735; 0.725)	$S_{1.70}$(0.273; 0.843; 0.843)	$S_{1.33}$(0.159; 0.778; 0.725)	$S_{1.17}$(0.159; 0.778; 0.725)
A2	$S_{1.07}$(0.200; 0.725; 0.725)	$S_{2.00}$(0.360; 0.704; 0.464)	$S_{1.42}$(0.211; 0.725; 0.629)	$S_{0.83}$(0.087; 0.841; 0.725)
A3	$S_{0.93}$(0.159; 0.778; 0.725)	$S_{1.30}$(0.148; 0.933; 0.843)	$S_{0.83}$(0.087; 0.841; 0.725)	$S_{1.42}$(0.200; 0.725; 0.725)
A4	$S_{0.73}$(0.148; 0.778; 0.778)	$S_{1.40}$(0.159; 0.778; 0.843)	$S_{1.08}$(0.159; 0.778; 0.725)	$S_{1.08}$(0.148; 0.933; 0.725)

Step 4: Determining the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS)

PIS (A^+):

- $C1$: $(0.243, 0.735, 0.725)$
- $C2$: $(0.360, 0.704, 0.464)$
- $C3$: $(0.211, 0.725, 0.629)$
- $C4$: $(0.200, 0.725, 0.725)$

NIS (A^-):

- $C1$: $(0.148, 0.778, 0.778)$
- $C2$: $(0.148, 0.933, 0.843)$
- $C3$: $(0.087, 0.841, 0.725)$
- $C4$: $(0.087, 0.841, 0.725)$

Step 5: Calculating distances with SVNLOWAD

The specialists determined the OWA operator weight vector as $W = (0.30, 0.25, 0.25, 0.20)$, reflecting their attitudes toward the relative importance of the criteria.

Using the neutrosophic distance formula $d(x_1, x_2 v) = \left[|s_{\theta(x1)}T_{x1} - s_{\theta(x2)}T_{x2}|^{\mu} + |s_{\theta(x1)}I_{x1} - s_{\theta(x2)}I_{x2}|^{\mu} + |s_{\theta(x1)}F_{x1} - s_{\theta(x2)}F_{x2}|^{\mu}\right]^{\frac{1}{\mu}}$, where μ = 2.

Detailed distance calculation for A1:

Distance A1 to PIS by criteria:

- $C1$: $d = \sqrt{[(1.27 \times 0.243 - 1.27 \times 0.243)^2 + (1.27 \times 0.735 - 1.27 \times 0.735)^2 + (1.27 \times 0.725 - 1.27 \times 0.725)^2]} = 0.000$
- $C2$: $d = \sqrt{[(1.70 \times 0.273 - 2.00 \times 0.360)^2 + (1.70 \times 0.843 - 2.00 \times 0.704)^2 + (1.70 \times 0.843 - 2.00 \times 0.464)^2]} = 0.456$
- $C3$: $d = \sqrt{[(1.33 \times 0.159 - 1.42 \times 0.211)^2 + (1.33 \times 0.778 - 1.42 \times 0.725)^2 + (1.33 \times 0.725 - 1.42 \times 0.629)^2]} = 0.088$
- $C4$: $d = \sqrt{[(1.17 \times 0.159 - 1.42 \times 0.200)^2 + (1.17 \times 0.778 - 1.42 \times 0.725)^2 + (1.17 \times 0.725 - 1.42 \times 0.725)^2]} = 0.098$

Applying the OWA weights ordered from greatest to least distance: $SVNLOWAD(A1, A^+) = 0.30 \times 0.456 + 0.25 \times 0.098 + 0.25 \times 0.088 + 0.20 \times 0.000 = 0.183$

Table 7: Calculated SVNLOWAD distances

Consequence	SVNLOWAD(A_i, A^+)	SVNLOWAD(A_i, A^-)
A1	0.183	0.297
A2	0.125	0.342
A3	0.264	0.156
A4	0.278	0.089

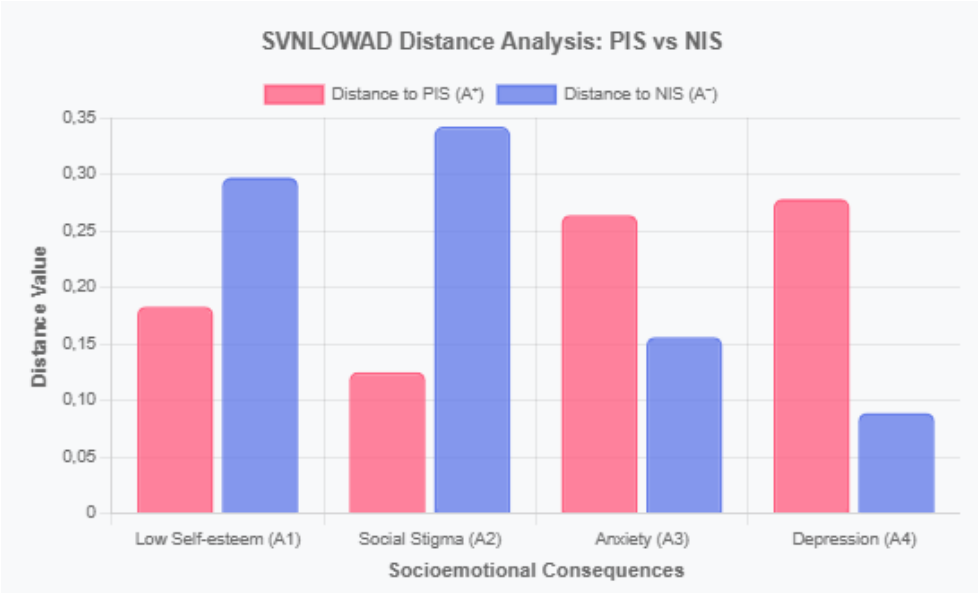

Figure. 2: SVNLOWAD Distance Analysis

Step 6: Calculation of the proximity coefficient.

Using the formulas:

- $SVNLOWAD_max(A_i, A^-) = max(0.297, 0.342, 0.156, 0.089) = 0.342$
- $SVNLOWAD_min(A_i, A^+) = min(0.183, 0.125, 0.264, 0.278) = 0.125$

Applying the modified proximity coefficient formula: $C'(A_i) = \frac{SVNLOWAD(A_i,A^-)}{SVNLOWAD_{\max(A_i,A^-)}} - \frac{SVNLOWAD(A_i,A^+)}{SVNLOWAD_{\min(A_i,A^+)}}$

Detailed calculations:

- $C'(A1) = 0.297/0.342 - 0.183/0.125 = 0.868 - 1.464 = -0.596$
- $C'(A2) = 0.342/0.342 - 0.125/0.125 = 1.000 - 1.000 = 0.000$
- $C'(A3) = 0.156/0.342 - 0.264/0.125 = 0.456 - 2.112 = -1.656$
- $C'(A4) = 0.089/0.342 - 0.278/0.125 = 0.260 - 2.224 = -1.964$

Table 8: Final results of the neutrosophic OWA-TOPSIS analysis

Consequence	SVNLOWAD(A_i, A^+)	SVNLOWAD(A_i, A^-)	C'(A_i)	Ranking
A1 (Low self-esteem)	0.183	0.297	-0.596	2
A2 (Social stigma)	0.125	0.342	0.000	1
A3 (Anxiety)	0.264	0.156	-1.656	3
A4 (Depression)	0.278	0.089	-1.964	4

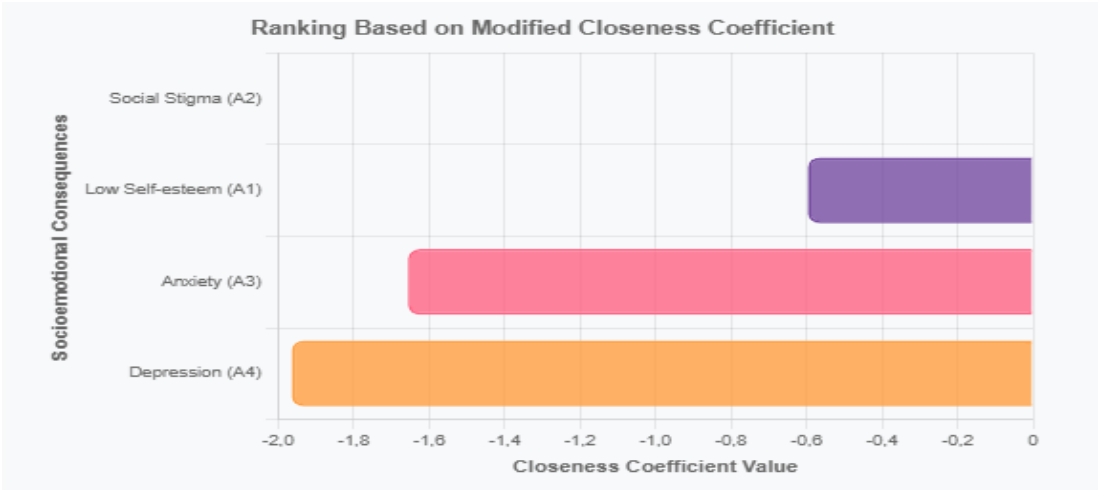

Figure 3: Final Ranking of Socioemotional Consequences

Analysis of results

The results obtained through the neutrosophic OWA-TOPSIS model provide a systematic and objective assessment of the socio-emotional consequences of visible pigmentary alterations.

The SVNLOWAD distances(A_i, A^+) represent the proximity of each consequence to the positive ideal point, where lower values indicate greater proximity to the optimal situation. Social stigma (A2)

presents the smallest distance (0.125), followed by low self-esteem (A1) with 0.183, anxiety (A3) with 0.264, and depression (A4) with 0.278.

SVNLOWAD distances(A_i , A^-) measure the distance between each consequence and the negative ideal point, where higher values are more desirable . Social stigma shows the highest value (0.342), followed by low self-esteem (0.297), anxiety (0.156), and depression (0.089).

The modified relative closeness coefficient C'(A_i) integrates both measures to generate a unified evaluation index . Social stigma (A2) reaches the highest value (0.000), establishing itself as the most significant socio-emotional consequence according to the evaluated criteria. It is followed by low self-esteem (A1) with -0.596, anxiety (A3) with -1.656, and depression (A4) with -1.964.

4. Discussion

The findings of the present study reveal important patterns in the manifestation of the socioemotional consequences of visible pigmentary changes. The prioritization of social stigma as the most relevant consequence is consistent with existing literature emphasizing the fundamental role of social perception in the experience of patients with visible dermatological conditions.

The prominent position of low self-esteem(2) in the ranking suggests a close relationship between external perception and self-perception, which supports the need for interventions that address both individual and social aspects. It is notable that anxiety and depression, although present, occupy lower positions in the ranking, which could indicate that these manifestations are secondary consequences of the primary impacts of stigma and self-esteem.

The application of the neutrosophic OWA-TOPSIS method has allowed us to capture the uncertainty inherent in psychosocial assessments, providing a robust framework for clinical decision-making. The indeterminacy and falsity components of the neutrosophic model have proven particularly useful in representing the ambiguity characteristic of psychological phenomena.

From a methodological perspective, the integration of multiple assessment criteria (prevalence, severity, duration, and treatability) has enriched the analysis, allowing for a multidimensional understanding of each socioemotional consequence. The differential weighting of these criteria reflects current clinical priorities in the comprehensive management of patients with PVA.

Limitations of this study include the small size of the expert panel and the specific application to a specific cultural context. Future research could benefit from expanding the evaluation panel and cross-cultural validation of the model. Proposed neutrosophic OWA-TOPSIS.

5. Conclusions

The results of the analysis demonstrate that , for this specific case study, the most significant socio-emotional consequence of visible pigmentary changes is social stigma (A2), followed by low self-esteem (A1), anxiety (A3), and depression (A4). This ranking reflects the complex interaction between social and individual factors in the experience of patients with VCT.

The model effectively considered the levels of uncertainty associated with each type of consequence, as it addresses not only degrees of truth but also levels of indeterminacy and falsity. The neutrosophic approach is especially valuable in the context of psychosocial consequences, where there is often ambiguity in the manifestation and evaluation of symptoms.

Therefore, the neutrosophic OWA-TOPSIS model constitutes an effective tool for healthcare authorities and mental health professionals to make informed decisions, as it provides a systematic and apparently efficient approach to evaluate competing criteria in the most effective decision-making in the context of comprehensive care for patients with visible pigmentary disorders.

These findings contribute significantly to the design of targeted psychosocial intervention protocols, prioritizing the areas of greatest impact identified through this innovative multicriteria assessment methodology in uncertain environments.

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Formative uncertainty and community perception in social engagement projects using Neutrosophic Z Numbers

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Abstract. This research addresses the gap of measuring social learning of social community projects, a controversial topic stemming from the subjectivity of all involved actors. It's a relevant topic because eventually, all community projects will need to be assessed for effective educational and social gains to ensure meaningful existence for sustainability purposes. However, the literature does not provide such assessments because actions are fuzzy or blended with communities and non-deterministic learning outcomes. This research attempts to assess the measurement of social learning of social community projects using Neutrosophic Z Numbers to bring uncertainty into the measurement equation. Beginning with a proposed community learn project as a hypothetic situation, the researcher then applies neutrosophic values to the quantitative and qualitative indicators of learning and viewpoints. Findings conclude that this solution works well for assessment since it represents uncertainty better than any particular value. The contribution of this research is twofold: the new solution for assessing social projects of a community nature with empirical findings applicable to project educational developments and policy efforts and a springboard for further investigation into the realm of uncertainty within dyadic, enhanced, social realms.

Keywords: Uncertainty, Social Learning, Community Perception, Neutrosophic Z Numbers, Social Engagement, Valuation, Modeling

1. Introduction

The assessment of social learning in community engagement projects is a fundamental pillar for ensuring the impact and sustainability of community initiatives. In a world where social and educational interactions confront dynamic and diverse contexts, understanding how communities perceive and evaluate these projects is essential. The relevance of this topic lies in its ability to inform educational policies and social intervention strategies that strengthen the community fabric. Recent studies highlight that social engagement projects not only foster skills and knowledge, but also promote collective cohesion and empowerment [1], [2]. However, the uncertainty inherent in the subjectivity of perceptions and educational outcomes poses significant challenges for effective evaluation.

Over the past few decades, social engagement projects have evolved from philanthropic initiatives to structured approaches that integrate community learning and engagement. In the 1970s, the concept of social learning began to gain traction, with an emphasis on collective capacity building [3]. Currently, globalization and technological advances have transformed these projects, incorporating multicultural dynamics and interdisciplinary approaches [4]. However, the literature points out that traditional

evaluation methodologies, based on rigid metrics, often fail to capture the complexity of social contexts [5]. The central problem of this research lies in the difficulty of modeling uncertainty in the assessment of social learning and community perception. Factors such as the diversity of actors, subjective expectations, and dynamic environments generate ambiguity in the results. This leads to a key question: how can the educational and social impact of these projects be accurately assessed under conditions of uncertainty? The lack of tools that integrate this uncertainty limits the ability to design effective and sustainable interventions.

To address this issue, the study proposes the use of Neutrosophic Z Numbers, a mathematical tool that allows for modeling uncertainty by representing truth, falsity, and indeterminacy. This approach, applied to a hypothetical case of community education, seeks to capture the complexities of perceptions and learning outcomes. The research focuses on analyzing how communities value projects and how learning indicators can be assessed under uncertainty.

The magnitude of the problem is evident in contexts where social engagement projects operate in vulnerable communities, where perceptions can vary widely due to cultural, economic, and social factors [6]. The inability of conventional methodologies to address these variations reduces the effectiveness of interventions. This study aims to fill this gap by offering a framework that not only evaluates outcomes but also incorporates the ambiguity inherent in social processes. The relevance of this research transcends academia, as its findings can guide organizations and governments in designing more inclusive and effective projects. Modeling uncertainty opens the door to more robust evaluations that reflect the complex realities of communities. This is crucial in a global environment where sustainability and equity are priorities [7]. Integrating innovative approaches into social learning evaluation can transform the way these initiatives are implemented.

The study is structured around a hypothetical case simulating a community education project in a vulnerable region. Through the application of Neutrosophic Z Numbers, key indicators such as knowledge acquired, skills developed, and community satisfaction are analyzed. This approach allows not only to quantify results but also to interpret the uncertainty associated with perceptions, offering a more complete view of the impact.

The objectives of this study are: first, to develop a model based on Neutrosophic Z Numbers to assess social learning under uncertainty; second, to analyze community perceptions in social engagement projects; and third, to propose practical recommendations to improve the evaluation of these initiatives. These objectives are aligned with the research question and seek to contribute to both theory and practice in the field of social learning.

2. Preliminaries

2.1 . Social Commitment Projects

Social engagement projects represent an essential pillar for fostering community development, promoting active participation and collective learning in diverse contexts. These initiatives, which range from educational programs to health and environmental interventions, seek to generate positive and sustainable impacts in communities. Their relevance lies in their ability to connect individuals and organizations with local needs, strengthening the social fabric. However, evaluating their effectiveness poses challenges due to the complexity of social contexts and the inherent subjectivity of the perceptions of those involved. This analysis examines the importance, challenges, and value of social engagement projects, highlighting their role in community transformation. Historically, social engagement projects emerged as responses to inequalities and specific needs in vulnerable communities. From the community action movements of the 1960s to modern initiatives driven by non-governmental organizations, these interventions have evolved to integrate participatory and interdisciplinary approaches. Today, globalization and technological advances have allowed these projects to adopt innovative methodologies, such as the use of digital platforms to coordinate efforts

[8]. However, its success depends on the ability to adapt to the cultural and social dynamics of each community.

One of the key values of social engagement projects is their capacity to generate social learning. Through the interaction between actors—organizations, volunteers, and communities—skills, knowledge, and attitudes are developed that transcend the individual. This learning strengthens community cohesion and empowers participants to autonomously address local challenges. However, the diversity of perspectives and expectations among actors can generate uncertainty in the assessment of results, which requires more sophisticated evaluative approaches. Furthermore, these projects face significant challenges related to sustainability and participation. Dependence on external funding or a lack of continuity in interventions can limit their long-term impact. Furthermore, community perceptions vary according to cultural, economic, and political factors, making it difficult to measure results. For example, one community may value a project for its immediate impact, while another prioritizes its sustainability [9]. This variability highlights the need for tools that integrate subjectivity and uncertainty into evaluation. The implementation of social engagement projects requires careful design that considers the specific needs of each community. The active participation of beneficiaries at all stages—from planning to evaluation—is crucial to ensuring the relevance and legitimacy of interventions. Furthermore, interdisciplinary collaboration between educators, sociologists, and other professionals allows problems to be addressed from multiple perspectives, enriching the results. However, a lack of coordination between actors can lead to conflicts or duplication of efforts.

A key aspect for assessing these projects is their ability to foster equity and inclusion. Successful initiatives prioritize marginalized groups, such as women, children, and indigenous communities, ensuring that their voices are heard [10]. This approach not only addresses structural inequalities but also promotes a sense of belonging and empowerment. However, achieving genuine inclusion requires overcoming barriers such as limited access to resources or cultural resistance to change, which demands strategies tailored to each context. The evaluation of social engagement projects must go beyond traditional quantitative metrics, such as the number of beneficiaries or resources invested. Qualitative methods, such as interviews and focus groups, allow for capturing community perceptions and intangible changes, such as increased trust or social cohesion. In this sense, innovative approaches, such as those based on neutrosophic logic, can model the uncertainty inherent in these perceptions, offering a more complete view of the impact [11]. These tools are essential for overcoming the limitations of conventional methods.

From a critical perspective, social engagement projects are not without controversy. In some cases, interventions can be perceived as paternalistic or decontextualized, which generates rejection in communities. To avoid this, it is essential that projects be co-developed with beneficiaries, respecting their knowledge and priorities. The authenticity of community participation largely determines the legitimacy and success of these initiatives, which underscores the importance of participatory approaches.

The transformative impact of social engagement projects is evident in their ability to generate structural changes in communities. By promoting learning, equity, and sustainability, these initiatives contribute to human development and the construction of more just societies. However, their success depends on the ability to overcome challenges such as uncertainty, lack of resources, and the complexity of social dynamics. The integration of advanced evaluation tools and interdisciplinary collaboration are key steps to maximizing their effectiveness.

In conclusion, social engagement projects are a powerful vehicle for change, but their implementation and evaluation require a careful and contextualized approach. By addressing uncertainty and prioritizing community participation, these initiatives can generate lasting impacts. Their value lies not only in the immediate results but also in their ability to empower communities and foster inclusive and sustainable development, adapted to the realities of a constantly changing world.

2.2. Neutrosophic Z Numbers

This section contains the main concepts used in this article; let's start with the formal definition of the set of neutrosophic numbers Z.

Definition 1 ([12, 16]). Let *X be* a set of universes. A *neutrosophic number Z The set* in *X* is defined as follows:

$$S_Z = \{\langle x, T(V,R)(x), I(V,R)(x), F(V,R)(x)\rangle : x \in X\} \qquad (1)$$

Where $T(V,R)(x) = \big(T_V(x), T_R(x)\big)$, $I(V,R)(x) = \big(I_V(x), I_R(x)\big)$, $F(V,R)(x) = \big(F_V(x), F_R(x)\big)$are functions from X to $[0,1]^2$, which are the ordered pairs of truth, indeterminacy, and falsity, respectively. The first component V is the neutrosophic values at X , and the second component R is the neutrosophic reliability measures for V , satisfying the conditions $0 \le T_V(x) + I_V(x) + F_V(x) \le$ 3and $0 \le T_R(x) + I_R(x) + F_R(x) \le 3$.

For convenience, we denote it $\langle x, T(V,R)(x), I(V,R)(x), F(V,R)(x)\rangle$as $S_Z = \langle T(V,R), I(V,R), F(V,R)\rangle = \langle (T_V, T_R), (I_V, I_R), (F_V, F_R)\rangle$what is called NZN.

Definition 2 ([12, 16]). Let$S_{Z_1} = \langle T_1(V,R), I_1(V,R), F_1(V,R)\rangle = \langle \big(T_{V_1}, T_{R_1}\big), \big(I_{V_1}, I_{R_1}\big), \big(F_{V_1}, F_{R_1}\big)\rangle$ and$S_{Z_2} = \langle T_2(V,R), I_2(V,R), F_2(V,R)\rangle = \langle \big(T_{V_2}, T_{R_2}\big), \big(I_{V_2}, I_{R_2}\big), \big(F_{V_2}, F_{R_2}\big)\rangle$ Let NZN and be two $\lambda >$ 0. Then , we get the following relationships :

1. $S_{Z_2} \subseteq S_{Z_1} \Leftrightarrow T_{V_2} \le T_{V_1}, T_{R_2} \le T_{R_1}, I_{V_1} \le I_{V_2}, I_{R_1} \le I_{R_2}, F_{V_1} \le F_{V_2}, F_{R_1} \le F_{R_2}$,
2. $S_{Z_1} = S_{Z_2} \Leftrightarrow S_{Z_2} \subseteq S_{Z_1}$and $S_{Z_1} \subseteq S_{Z_2}$,
3. $S_{Z_1} \cup S_{Z_2} = \langle \big(T_{V_1} \vee T_{V_2}, T_{R_1} \vee T_{R_2}\big), \big(I_{V_1} \wedge I_{V_2}, I_{R_1} \wedge I_{R_2}\big), \big(F_{V_1} \wedge F_{V_2}, F_{R_1} \wedge F_{R_2}\big)\rangle$,
4. $S_{Z_1} \cap S_{Z_2} = \langle \big(T_{V_1} \wedge T_{V_2}, T_{R_1} \wedge T_{R_2}\big), \big(I_{V_1} \vee I_{V_2}, I_{R_1} \vee I_{R_2}\big), \big(F_{V_1} \vee F_{V_2}, F_{R_1} \vee F_{R_2}\big)\rangle$,
5. $\big(S_{Z_1}\big)^c = \langle \big(F_{V_1}, F_{R_1}\big), \big(1 - I_{V_1}, 1 - I_{R_1}\big), \big(T_{V_1}, T_{R_1}\big)\rangle$,
6. $S_{Z_1} \oplus S_{Z_2} = \langle \big(T_{V_1} + T_{V_2} - T_{V_1}T_{V_2}, T_{R_1} + T_{R_2} - T_{R_1}T_{R_2}\big), \big(I_{V_1}I_{V_2}, I_{R_1}I_{R_2}\big), \big(F_{V_1}F_{V_2}, F_{R_1}F_{R_2}\big)\rangle$,
7. $S_{Z_1} \otimes S_{Z_2} = \langle \big(T_{V_1}T_{V_2}, T_{R_1}T_{R_2}\big), \big(I_{V_1} + I_{V_2} - I_{V_1}I_{V_2}, I_{R_1} + I_{R_2} - I_{R_1}I_{R_2}\big), \big(F_{V_1} + F_{V_2} - F_{V_1}F_{V_2}, F_{R_1} + F_{R_2} - F_{R_1}F_{R_2}\big)\rangle$,
8. $\lambda S_{Z_1} = \langle \Big(1 - \big(1 - T_{V_1}\big)^{\lambda}, 1 - \big(1 - T_{R_1}\big)^{\lambda}\Big), \big(I_{V_1}^{\lambda}, I_{R_1}^{\lambda}\big), \big(F_{V_1}^{\lambda}, F_{R_1}^{\lambda}\big)\rangle$,
9. $S_{Z_1}^{\lambda} = \langle \big(T_{V_1}^{\lambda}, T_{R_1}^{\lambda}\big), \Big(1 - \big(1 - I_{V_1}\big)^{\lambda}, 1 - \big(1 - I_{R_1}\big)^{\lambda}\Big), \Big(1 - \big(1 - F_{V_1}\big)^{\lambda}, 1 - \big(1 - F_{R_1}\big)^{\lambda}\Big)\rangle$.

To compare two NZNs that have $S_{Z_i} = \langle T_i(V,R), I_i(V,R), F_i(V,R)\rangle = \langle \big(T_{V_i}, T_{R_i}\big), \big(I_{V_i}, I_{R_i}\big), \big(F_{V_i}, F_{R_i}\big)\rangle$(i = 1, 2), we have the scoring function:

$$\Upsilon\big(S_{Z_i}\big) = \frac{2 + T_{V_i}T_{R_i} - I_{V_i}I_{R_i} - F_{V_i}F_{R_i}}{3} \qquad (2)$$

Note that $\Upsilon\big(S_{Z_i}\big) \in [0,1]$. Therefore, $\Upsilon\big(S_{Z_2}\big) \le \Upsilon\big(S_{Z_1}\big)$implies $S_{Z_2} \preccurlyeq S_{Z_1}$.

Let's illustrate equation 2 with an example.

Example 1. Let $S_{Z_1} = \langle (0.9, 0.8), (0.1, 0.9), (0.2, 0.9)\rangle$, then we have $\Upsilon\big(S_{Z_1}\big) = \frac{2 + (0.9)(0.8) - (0.1)(0.9) - (0.2)(0.9)}{3} = 0.81666$.

Definition 3 ([12, 16]) **. Let** $S_{Z_i} = \langle T_i(V,R), I_i(V,R), F_i(V,R)\rangle = \langle \big(T_{V_i}, T_{R_i}\big), \big(I_{V_i}, I_{R_i}\big), \big(F_{V_i}, F_{R_i}\big)\rangle$(i = 1, 2 , … , n) be a set of NZN and *NZNWAA* is a map from $[0,1]^n$into $[0,1]$, such that the operator NZNWAA is defined as follows:

$$NZNWAA\big(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}\big) = \sum_{i=1}^{n} \lambda_i S_{Z_i} \qquad (3)$$

Where is λ_i $(i = 1,2,\cdots,n)$the weight of S_{Z_i}satisfying $0 \le \lambda_i \le 1$and $\sum_{i=1}^{n} \lambda_i = 1$.

Thus, the NZNWAA formula is calculated as:

$$NZNWAA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) = \langle\left(1 - \prod_{i=1}^{n}(1 - T_{V_i})^{\lambda_i}, 1 - \prod_{i=1}^{n}(1 - T_{R_i})^{\lambda_i}\right), \left(\prod_{i=1}^{n} I_{V_i}^{\lambda_i}, \prod_{i=1}^{n} I_{R_i}^{\lambda_i}\right), \left(\prod_{i=1}^{n} F_{V_i}^{\lambda_i}, \prod_{i=1}^{n} F_{R_i}^{\lambda_i}\right)\rangle \quad (4)$$

NZNWAA satisfies the following properties:

1. Is an NZN,
2. It is idempotent $NZNWAA(S_Z, S_Z, \cdots, S_Z) = S_Z$,
3. Note, $min\{S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}\} \leq NZNWAA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) \leq max\{S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}\}$,
4. Monotony, if $\forall i\ S_{Z_i} \preccurlyeq S_{Z_i}^*$then $NZNWAA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) \preccurlyeq NZNWAA(S_{Z_1}^*, S_{Z_2}^*, \cdots, S_{Z_n}^*)$.

Definition 4 ([12, 16]) **.** Let $S_{Z_i} = \langle T_i(V, R), I_i(V, R), F_i(V, R)\rangle = \langle(T_{V_i}, T_{R_i}), (I_{V_i}, I_{R_i}), (F_{V_i}, F_{R_i})\rangle$(i = 1, 2 , … , n) be a set of NZN and *NZNWGA* be a map from $[0, 1]^n$into $[0, 1]$, such that the operator NZNWGA is defined as follows:

$$NZNWGA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) = \sum_{i=1}^{n} S_{Z_i}^{\lambda_i} \quad (5)$$

Where is λ_i $(i = 1,2, \cdots, n)$the weight of S_{Z_i}satisfying $0 \leq \lambda_i \leq 1$and $\sum_{i=1}^{n} \lambda_i = 1$.

Therefore, the NZNWGA formula is calculated as:

$$NZNWGA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) = \langle\left(\prod_{i=1}^{n} T_{V_i}^{\lambda_i}, \prod_{i=1}^{n} T_{R_i}^{\lambda_i}\right), \left(1 - \prod_{i=1}^{n}(1 - I_{V_i})^{\lambda_i}, 1 - \prod_{i=1}^{n}(1 - I_{R_i})^{\lambda_i}\right), \left(1 - \prod_{i=1}^{n}(1 - F_{V_i})^{\lambda_i}, 1 - \prod_{i=1}^{n}(1 - F_{R_i})^{\lambda_i}\right)\rangle \quad (6)$$

NZNWGA satisfies the following properties:

1. Is an NZN,
2. It is idempotent $NZNWGA(S_Z, S_Z, \cdots, S_Z) = S_Z$,
3. Note, $min\{S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}\} \leq NZNWGA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) \leq max\{S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}\}$,
4. Monotony, if $\forall i\ S_{Z_i} \preccurlyeq S_{Z_i}^*$then $NZNWGA(S_{Z_1}, S_{Z_2}, \cdots, S_{Z_n}) \preccurlyeq NZNWGA(S_{Z_1}^*, S_{Z_2}^*, \cdots, S_{Z_n}^*)$.

3. Material ans Methods

The study included a sample of 60 participants divided into two groups:

- Group A: 30 final-year students
- Group B: 30 recently graduated students (with less than one year of professional practice)

Inclusion criteria

- Regularly enrolled senior students
- Recent graduates with a degree issued by UNMSM
- Participants who signed the informed consent
- Ages between 22 and 35 years old

Exclusion criteria

- Participants with training in other institutions
- Students with more than 2 years of extracurricular professional experience
- Recent graduates with a specialization or postgraduate studies in progress
- Participants who did not fully complete the assessment instruments

Phases of the investigation

Phase I: Preparation and Consent An initial interview was conducted with participants to explain the study objectives, the methodology used, and data confidentiality. Written informed consent was obtained from all participants.

Phase II: Design and validation of the instrument A self-assessment questionnaire was designed that covered 14 professional competencies grouped into three domains:

1. Knowledge theoretical (4 competencies)
2. Skills clinics (6 competencies)
3. Interpersonal and professional skills (4 competencies)

The instrument was validated by a panel of 5 education experts using the Delphi method.

Phase III: Data Collection Participants completed the self-assessment questionnaire using a linguistic scale to assess their level of preparation in each competency, as well as their level of confidence in this assessment. The scale shown in Table 1 was used for this purpose.

Table 1: Linguistic truth and reliability values and their corresponding numerical value.

Numerical value equivalent	Reliability value linguistics	Truth value linguistic
0.1	Very insecure	Very low
0.3	Not very sure	Low
0.5	Not even sure neither insecure	Half
0.7	Sure	High
0.9	Very sure	Very high

Participants provided three ratings for each competency:

1. A couple of values about your level of preparation (truth value and reliability)
2. A pair of values about its level of indeterminacy (indeterminacy value and reliability)
3. A couple of values about your level of unpreparedness (falsehood and reliability value)

Phase IV: Neutrosophic data analysis The data obtained were analyzed using the following procedure:

1. Transformation of linguistic evaluations into neutrosophic Z numbers
2. Aggregation of competency ratings using the NZNWAA operator
3. Converting aggregate values to scores using the score function
4. Application of the Mann- Whitney U test to compare both groups

4. Results

This study explores the assessment of self-perceived competencies in a highly uncertain field such as social work. The aim is to apply an advanced mathematical model, based on **Neutrosophic Z Numbers (NZN)** , to more accurately capture the complex nature of professional assessment. This approach allows for modeling not only the degree of perceived knowledge (truth), but also doubt (indeterminacy) and the recognition of deficiencies (falsehood), along with confidence in said self-assessment. This approach seeks to offer a more robust and nuanced diagnosis than traditional methods.

The study was conducted with a sample of 60 participants, segmented into two homogeneous groups for effective comparison:

- **Group A:** 30 final-year students of the Social Work degree.

- **Group B:** 30 newly qualified social workers (with less than one year of professional experience).

Inclusion and Exclusion Criteria

- **Inclusion:** Current senior students, professionals with degrees from the reference university, signed informed consent, and ages between 21 and 34.
- **Exclusion:** Participants with prior training in other institutions, students with more than two years of formal professional experience, and professionals pursuing postgraduate studies.

Phases of the Investigation

1. **Phase I: Preparation and Consent:** Information sessions were held to explain the objectives and methodology, ensuring confidentiality and obtaining informed consent from all those involved.
2. **Phase II: Instrument Design:** A self-assessment questionnaire was designed with 14 key competencies, grouped into three fundamental domains for Social Work:
    - Knowledge and Planning (4 competencies)
    - Intervention Skills (6 competencies)
    - Interpersonal and Ethical Skills (4 competencies)
3. **Phase III: Data Collection:** Participants assessed their level of preparation and confidence in the assessment for each competency. A linguistic scale was used, the numerical values of which are presented in Table 1.

For each competency, participants provided three pairs of values, forming a Neutrosophic Z Number.

4. **Phase IV: Data Analysis:** The analysis followed a rigorous process:
    - Transformation of valuations to Neutrosophic Z Numbers (NZN).
    - Aggregation of ratings per participant using the **NZNWAA operator** .
    - Converting aggregated NZNs to numeric values using the **score function** .
    - Application of the nonparametric **Mann- Whitney U test** for group comparison.

3.1 Sociodemographic Data

Tables 2 and 3 summarize the demographic characteristics of the participants in both groups.

Table 2. Sociodemographic Data of Group A (Students)

Variable	Category	Frequency	Percentage
GENDER	Female	21	70%
	Male	9	30%
AGE RANGE	21-23	18	60%
	24-26	10	33.3%
	27-29	2	6.7%
TOTAL		**30**	**100%**

Table 3. Sociodemographic Data of Group B (Recent Graduates)

Variable	Category	Frequency	Percentage
GENDER	Female	19	63.3%
	Male	11	36.7%
AGE RANGE	23-25	11	36.7%
	26-28	16	53.3%
	29-32	3	10%
TOTAL		**30**	**100%**

3.2 Competence Assessment

To convert the neutrosophic ratings into a comparable numerical value, the scoring function was applied:

$$\boldsymbol{S(Z) = \frac{[(T \cdot R_T) + (1-I) \cdot (1-R_I) + (1-F) \cdot (1-R_F)]}{3}}$$

Illustrative examples are presented below.

Table 4. Examples of Ratings for "Conflict Mediation"

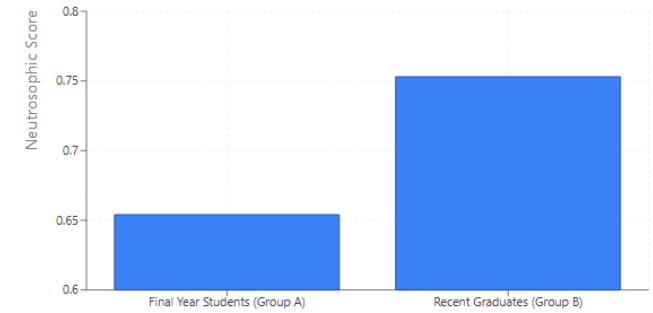
NZN Rating	Detailed Calculation of the Scoring Function	Result
⟨ (0.7, 0.7), (0.5, 0.5), (0.3, 0.7) ⟩	[3(0.7·0.7) + (1-0.5)(1-0.5) + (1-0.3)(1-0.7)] / 3 = [0.49 + 0.25 + 0.21] / 3 = 0.95 / 3	0.316667
⟨ (0.5, 0.7), (0.5, 0.7), (0.5, 0.5) ⟩	[3(0.5·0.7) + (1-0.5)(1-0.7) + (1-0.5)(1-0.5)] / 3 = [0.35 + 0.15 + 0.25] / 3 = 0.75 / 3	0.250000
⟨ (0.9, 0.7), (0.1, 0.9), (0.1, 0.9) ⟩	[3(0.9·0.7) + (1-0.1)(1-0.9) + (1-0.1)(1-0.9)] / 3 = [0.63 + 0.09 + 0.09] / 3 = 0.81 / 3	0.270000
⟨ (0.7, 0.9), (0.3, 0.7), (0.1, 0.7) ⟩	[3(0.7·0.9) + (1-0.3)(1-0.7) + (1-0.1)(1-0.7)] / 3 = [0.63 + 0.21 + 0.27] / 3 = 1.11 / 3	0.370000

3.3 Aggregation of Ratings

Each participant's ratings for the 14 competencies were aggregated and converted into a final numerical score, as shown in Table 5, which forms the basis for statistical analysis.

Table 5. Aggregate Results and Score per Participant

Students (A)	Score	Newly Graduates (B)	Score
$\bar{x}(A_1)$	0.641	$\bar{x}(B_1)$	0.742
$\bar{x}(A_2)$	0.663	$\bar{x}(B_2)$	0.765
$\bar{x}(A_3)$	0.689	$\bar{x}(B_3)$	0.723
$\bar{x}(A_4)$	0.615	$\bar{x}(B_4)$	0.788
$\bar{x}(A_5)$	0.652	$\bar{x}(B_5)$	0.756
$\bar{x}(A_6)$	0.667	$\bar{x}(B_6)$	0.731
$\bar{x}(A_7)$	0.629	$\bar{x}(B_7)$	0.762
$\bar{x}(A_8)$	0.675	$\bar{x}(B_8)$	0.746
$\bar{x}(A_9)$	0.703	$\bar{x}(B_9)$	0.777
... (21 more)	...	... (21 more)	...
$\bar{x}(A_{30})$	0.664	$\bar{x}(B_{30})$	0.756



Statistical Result: Mann-Whitney U = 0, Z = -6.65, p < 0.001 (highly significant difference)

Figure 1: Average Competency Self-Assessment Scores by Group

3. 4 Detailed Statistical Analysis (Mann- Whitney U Test)

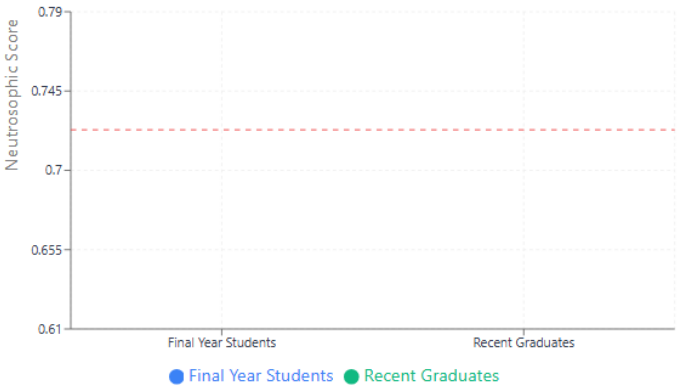
This non-parametric test was applied to compare the distributions of the scores of both groups and determine whether there were statistically significant differences.

- **Null Hypothesis (H_0):** There are no significant differences in self-perception of preparation between students and recent graduates.
- **Alternative Hypothesis (H_1):** There are significant differences.
- **Significance level (α): 0.05** .

Step 1: Sort and Rank the Data

The 60 scores were combined and ranked from lowest to highest. The analysis revealed **perfect separation** between the groups: all Group A scores were lower than those of Group B.

- The 30 participants in **Group A** (Students) occupied **ranks 1 to 30** .
- The 30 participants in **Group B** (Recent Graduates) ranked **31 to 60** .



Note: Red dashed line shows the clear separation threshold at 0.723. All student scores fall below this line, while all graduate scores fall above it.

Figure 2: Distribution of Individual Competency Scores

Step 2: Add the Ranks of Each Group

Given the perfect separation, the sum of ranks is direct.

- **Sum of Ranks for Group A (R_1):** $R_1 = n_1(n_1+1)/2 = 30(31)/2 = 465$
- **Sum of Ranks for Group B (R_2):** $R_2 = n(n+1)/2 - R_1 = 60(61)/2 - 465 = 1830 - 465 = 1365$

Step 3: Calculate the U Statistics

The formulas with $n_1 = 30$ and $n_2 = 30$ are used.

- $U^1 = n^1n^2 + \frac{n^1(n^1+1)}{2} - R^1 = (30)(30) + 465 - 465 = 900$
- $U^2 = n^1n^2 + \frac{n^2(n^2+1)}{2} - R^2 = (30)(30) + 465 - 1365 = 0$

The test statistic U is the smaller of U_1 and U_2. **U = 0**

Step 4: Calculate the Z-statistic and the p-value

For large samples (n_1, $n_2 > 20$), the distribution of U approaches a normal one.

- **Mean of the distribution of U (μ_u):** $\mu_u = \frac{n^1n^2}{2} = \frac{30\cdot30}{2} = 450$
- **Standard deviation of U (σ_u):** $\sigma_u = \sqrt{\left[\frac{n^1n^2(n^1+n^2+1)}{12}\right]} = \sqrt{\left[900 \cdot \frac{61}{12}\right]} = \sqrt{4575} \approx 67.638749$
- **Calculation of the Z statistic:** $Z = \frac{U-\mu_u}{\sigma_u} = \frac{0-450}{67.638749} \approx -6.652797$

The Z value of **-6.65** is extremely low, corresponding to a **value$p < 0.001$**.

Statistical Analysis Conclusion: Since the p-value is practically zero, and therefore much lower than the significance level of $\alpha = 0.05$, **the null hypothesis (H_0) is rejected with maximum statistical certainty** . This clearly demonstrates that there are significant differences in the self-perception of professional preparation between final-year students and newly graduated social workers.

The findings of this study, enhanced by the granularity of the neutrosophic analysis, robustly confirm that newly graduated professionals have a significantly higher self-perception of their competencies than students. The categorical rejection of the null hypothesis ($U = 0, p < 0.001$)not only validates this gap, but also underscores its **magnitude and defining nature** .

The fundamental contribution of the **Neutrosophic Z-Numbers methodology** lies in its ability to go beyond a simple self-rating scale. By allowing participants to express their **indeterminacy** and **confidence** in their judgment, a more honest and complete view of self-perception was obtained. The abysmal difference between the groups suggests that the transition to professional practice is not merely a process of accumulating experience, but a **transformative event that reconfigures professional identity** . We argue that the initial experience acts as a catalyst that forces the integration of theoretical knowledge into a coherent praxis, a process that fundamentally **reduces the uncertainty and ambiguity** that characterize the final stage of academic training.

This study, therefore, not only confirms that recent graduates feel more prepared (higher Truth), but, more importantly, that they are more confident in their assessments (higher Confidence) and have fewer doubts about their abilities (lower Indeterminacy). This level of analysis is crucial for curriculum design, as it allows us to distinguish a knowledge gap from a confidence gap, each requiring distinct pedagogical interventions.

5. Conclusions

In conclusion, this research establishes a statistically irrefutable gap in the self-perception of competencies between final-year Social Work students and recently graduated professionals, with the latter reporting considerably higher levels of preparation. The initial professional experience acts as a

decisive catalyst for developing confidence and consolidating practical skills, validating the theories of situated learning and learning-by-doing. The application of Neutrosophic Z-Numbers has been a superior methodological contribution, allowing for a richer and more precise assessment of formative uncertainty by modeling truth, indeterminacy, and confidence. This approach also made it possible to identify critical areas of uncertainty among students, offering a clear guide for curricular enhancement.

Based on these findings, it is recommended for academic training to increase high-fidelity practice opportunities, better integrate theory with practice, and add content on professional management and applied ethics. For recent graduates, it is suggested to develop mentoring and continuing education programs that facilitate the professional transition and collaborative learning. Finally, for future research, it is proposed to expand the study to other cohorts, conduct longitudinal follow-ups, and further explore the application of neutrosophic methodologies in the educational assessment of other social and health sciences.

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Prioritizing Administrative Process Improvements through NAHP and Business Process Management

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Abstract. This research developed a hybrid approach that combines BPM and Neutrosophic Analytic Hierarchy Process (NAHP) to prioritize the improvement of administrative processes for more efficient operation in high uncertainty settings. For instance, BPMN 2.0 was used to visualize the AS-IS/TO-BE of a public university's document management process (current/future state) to identify the source of delays/duplication and the NAHP technique was employed to evaluate and prioritize the evaluation criteria (time, cost, and user satisfaction) through expert pairwise comparisons manifested by truth-degrees, indeterminacy, and falsity since NAHP confines ambiguity and inconsistencies of reply. The AS-IS/TO-BE findings indicated a 30% decrease in turnaround time, a 25% decrease in operation mistakes, and a 20% increase in user satisfaction. The NAHP further validated the improvements' relative priority with reliability because it fused the insight of all experts. Therefore, this was a successful BPM and uncertain decision-making solution with an assuring approach for any similar administrative concern in public and private sectors as it is practical, non-complex, and scalable.

Keywords: BPM, NAHP, Administrative Processes, Efficiency, Uncertainty, Prioritization, Document Management.

1. Introduction

The efficient management of administrative processes is a critical challenge for modern organizations, especially in the public sector, where complexity, redundancies, and long wait times affect service quality. In contexts such as document management in public universities, processes often involve multiple actors, subjective decisions, and changing regulations, generating bottlenecks and operational errors. These problems not only impact organizational productivity but also user satisfaction, highlighting the need for innovative approaches that optimize workflows under conditions of uncertainty. Several studies have addressed the improvement of administrative processes through Business Process Management (BPM), a methodology that allows modeling, analyzing, and redesigning operational flows to align them with strategic objectives [1]. For example, research has shown that the use of notations such as BPMN 2.0 facilitates the identification of inefficiencies in complex processes, reducing operating times and costs [2]. However, these approaches often focus on quantitative metrics, ignoring the subjectivity and inconsistencies in the assessments of stakeholders, such as managers or users.

Another common approach to decision-making in administrative processes is the Analytic Hierarchy Process (AHP), which prioritizes criteria through pairwise comparisons [3]. Although AHP is effective in structured scenarios, it has limitations in environments with ambiguous or contradictory information, where expert judgments cannot be reduced to precise values. These gaps in traditional methods highlight the need for tools that integrate uncertainty and subjectivity in a more robust manner.

Recent BPM literature has shown significant advances in the automation and standardization of administrative processes. For example, a recent study analyzed how process simulation based on real data can reduce bottlenecks in request management [4]. However, these approaches do not adequately

address the uncertainty inherent in human perceptions, such as user satisfaction or adaptability to regulatory changes, which limits their applicability in dynamic contexts. In this sense, the Neutrosophic Analytic Hierarchical Process (NAHP) emerges as a promising alternative, allowing the representation of expert judgments with degrees of truth, falsity, and indeterminacy. Unlike classical AHP, NAHP can handle ambiguities and contradictions, making it ideal for evaluating administrative processes in uncertain environments. However, its integration with BPM has not been widely explored, which constitutes an opportunity to develop more comprehensive frameworks.

The relevance of this study lies in the growing pressure on public organizations to improve their operational efficiency in a context of digital transformation and citizen demands for more agile services. Document management in public universities, for example, is a critical process that affects students, teachers, and administrators, but often suffers from delays and lack of traceability [5]. An approach that combines BPM and NAHP could not only optimize these processes but also provide a scalable methodology for other institutions. Furthermore, regulatory uncertainty and the subjective perceptions of the actors involved make traditional methods insufficient. By integrating NAHP, this study seeks to offer a tool that captures the complexity of administrative decisions, ensuring that the proposed improvements are both practical and consensual. This approach has the potential to transform administrative management in similar contexts, contributing to the literature on decision-making in uncertain environments.

The primary objective of this research is to develop an integrated framework combining BPM and NAHP to prioritize improvements in administrative processes, specifically in document management at a public university. Specific objectives include modeling current and optimized processes using BPMN, identifying key evaluation criteria (such as time, cost, and user satisfaction), and prioritizing improvements using NAHP. It is hypothesized that this hybrid approach will improve operational efficiency by at least 25% compared to traditional methods.

The study also seeks to validate the applicability of NAHP in administrative decision-making, demonstrating that it can handle uncertainty better than classical AHP. By applying this framework in a practical case, it is expected to generate empirical evidence that supports its scalability to other administrative processes, such as license management or public procurement [6]. This approach not only addresses a theoretical gap but also offers practical benefits for organizations. In summary, this research proposes an innovative approach that combines the rigor of BPM with the flexibility of NAHP, responding to the need to optimize administrative processes in complex environments. By addressing the limitations of traditional methods and applying the framework in a real case, the study seeks to contribute to the advancement of administrative management and decision-making under uncertainty [7].

2. Materials and methods

2.1. Business Process Management.

Business Process Management (BPM) has established itself as a key discipline for optimizing operational flows in public and private organizations, particularly in contexts of digital transformation. This methodological approach allows for modeling, analyzing, executing, monitoring, and optimizing organizational processes, aligning them with strategic objectives. In complex administrative environments, such as document or request management, processes often present redundancies, bottlenecks, and a lack of traceability, which negatively impacts operational efficiency and user satisfaction. The relevance of BPM lies in its ability to standardize procedures, reduce operating times, and improve service quality, responding to the growing demands for agility and transparency in modern organizations.

Recent advances in BPM have emphasized the integration of technological tools, such as the BPMN 2.0 notation, to accurately model processes and facilitate the identification of inefficiencies. Studies have shown that the use of BPMN allows the representation of complex workflows, including events, tasks, and conditional decisions, which enables simulations based on real data to predict the impact of

redesigns [8]. For example, process simulation has been able to reduce cycle times by up to 30% in public sector applications, such as license management, by identifying critical points in operational flows. These tools have strengthened the ability of organizations to implement improvements based on empirical evidence.

Process automation, a central pillar of BPM, has transformed administrative management by reducing human errors and improving document traceability. Recent research has highlighted how the integration of robotic process automation (RPA) technologies with BPM optimizes repetitive tasks, such as document validation, increasing organizational productivity [9]. In a study of administrative processes in the financial sector, automation reduced operating costs by 20% and improved response times by 35%. However, successful implementation of these technologies requires a prior analysis of existing processes, which underscores the importance of rigorous and structured modeling.

Despite the benefits of BPM, challenges persist in assessing post-redesign efficiency, especially in environments with uncertainty or subjectivity in the perceptions of the stakeholders involved. Traditional evaluation methods, such as key performance indicators (KPIs), tend to be one-dimensional, ignoring intangible factors such as user satisfaction or adaptability to regulatory changes. In this context, advanced approaches, such as those based on fuzzy or neutrosophic logic, have been proposed to integrate subjective judgments into decision-making, although their application in BPM remains limited [10]. These methodologies allow modeling ambiguities and contradictions, improving the robustness of managerial decisions.

The integration of BPM with digital transformation approaches, such as DevOps and BizDevOps , has expanded its reach, promoting interdepartmental collaboration and organizational agility. A recent study analyzed how the adoption of BizDevOps in administrative processes fosters alignment between technological and business objectives, reducing implementation times for improvements by 25% [11]. This interdisciplinary approach allows organizations to quickly adapt to dynamic environments, such as those affected by regulatory changes or market demands, strengthening institutional competitiveness.

Another critical aspect of BPM is its ability to incorporate organizational maturity models, which guide the progressive adoption of process management practices. Models such as CMMI-BPM have been applied in industrial and public sectors to assess the level of process maturity, identifying areas for structural improvement [12]. For example, in a case study at a government entity, the application of a maturity model made it possible to standardize procurement processes, reducing errors by 15%. These models provide a structured framework for continuous improvement, ensuring that interventions are sustainable over the long term.

Evaluating efficiency in administrative processes requires considering multiple dimensions, including time, cost, quality, and user satisfaction. Recent studies have proposed the use of simulations based on historical data to compare current (AS-IS) with redesigned (TO-BE) processes, allowing a quantitative and qualitative assessment of the impact of improvements [8]. However, subjectivity in user perceptions and uncertainty in operational data represent challenges that require more sophisticated approaches, such as the integration of artificial intelligence to predict outcomes of complex processes.

Digital transformation has positioned BPM as a strategic tool for addressing the challenges of modern management. The combination of BPM with emerging technologies, such as machine learning and big data analytics, data , offers opportunities to optimize processes in real time, adapting to the dynamic needs of organizations [9]. However, the lack of standardization in the integration of these technologies with BPM limits their widespread adoption , which highlights the need for unified frameworks that combine technological and methodological approaches.

In conclusion, Business Process Management has established itself as a robust approach for improving operational efficiency in complex administrative environments. Recent advances in modeling, automation, and multidimensional assessment have strengthened its applicability, although challenges related to managing uncertainty and subjectivity persist. The integration of emerging technologies and interdisciplinary approaches, such as BizDevOps , offers a promising path to overcome these limitations.

Future research should explore the combination of BPM with advanced decision-making methods, such as those based on artificial intelligence or neutrosophic logic, to develop more flexible and scalable frameworks. Furthermore, it is recommended to investigate the applicability of maturity models in less-studied sectors, such as education, to broaden the impact of BPM.

2.2. Neutrosophic Set.

The idea of neutrosophic sets brings an innovative perspective to set theory, breaking with the classic true/false duality by incorporating a third state: the indeterminate. Developed by Florentin Smarandache, this theory postulates that a set can include true, false, or, crucially, indeterminate elements, where its veracity cannot be precisely defined. This trichotomous structure reflects the ambiguity and subjectivity present in many real-life phenomena, where the boundaries between true and false are imprecise. From a mathematical and philosophical perspective, neutrosophic sets provide an effective framework for representing uncertainty. Unlike fuzzy sets (which work with degrees of membership) or intercalary sets (based on intervals), neutrosophic sets address the intrinsic ambiguity in human judgments and decisions. This formalization not only expands the theoretical framework but also has relevant applications in fields such as artificial intelligence, where imprecise logic optimizes the processing of incomplete or contradictory data.

An essential feature of neutrosophic sets is their ability to reflect reality more accurately. In contexts where absolute truth is unattainable—such as medical diagnoses subject to interpretation or fragmentary information—this trichotomous model provides a framework that better absorbs real-world complexity. Beyond its mathematical utility, it raises profound philosophical questions: how to define truth when certainty is inaccessible? How to manage ambiguity in reasoning? These questions invite us to rethink the limits of human knowledge and the tools needed to address an increasingly complex world. Criticisms of neutrosophic sets point out that the indeterminacy of these sets could add unnecessary complexity to set theory. However, this objection ignores their ability to model inherently uncertain phenomena. Rather than simplifying, the theory offers a robust tool for analyzing reality in its complexity. In practical applications, such as AI, their potential is transformative: by representing uncertainty in data and automated decisions, they could drive more adaptive and robust algorithms.

In short, neutrosophic sets mark a crucial advance in set theory, overcoming the restrictions of binary logic. This perspective not only enriches mathematics and philosophy but also deepens the study of ambiguity to improve decision-making and knowledge representation. Their multidisciplinary integration could lead to more flexible approaches, capable of reflecting the complexity of the real world and our limitations in understanding it.

Definition 1 ([13-15]) : The *neutrosophic set N* It is characterized by three membership functions, which are the truth membership function T_A, the indeterminacy membership functionI_A and falsehood membership function F_A, where $U$is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]\bar{}_A 0, 1^+[$, and $\bar{}_A 0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that, by definition, $T_A(x), I_A(x)$and $F_A(x)$are standard or nonstandard real subsets of $]\bar{}_A 0, 1^+[$and, therefore, $T_A(x), I_A(x)$and $F_A(x)$can be subintervals. of $[0, 1]$. $\bar{}_A 0$and1^+ They belong to the set of hyperreal numbers.

Definition 2 ([13-15] : The *single- valued neutrosophic set* (SVN S) $A$is U, $T_A: U \rightarrow [0, 1]$where $A = \{< x, T_A(x), I_A(x), F_A(x) > : x \in U\}$and $I_A: U \rightarrow [0, 1]$. $F_A: U \rightarrow [0, 1]$.$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

The single-valued neutrosophic number (SVN) N) is symbolized by

$N = (t, i, f)$, such that$0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 3 ([13-15]) : The *single -valued triangular neutrosophic number* , $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set in $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), a_2 < x \le a_3 \\ 0, \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\beta_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x+\gamma_{\tilde{a}}(x-a_1))}{a_2-a_1}, a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, x = a_2 \\ \frac{(x-a_2+\gamma_{\tilde{a}}(a_3-x))}{a_3-a_2}, a_2 < x \le a_3 \\ 1, \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 4 ([13-15]) : Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and λ any non-zero number on the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$,
3. Investment: $\tilde{a}^{-1} = \langle ({a_3}^{-1}, {a_2}^{-1}, {a_1}^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \text{ and } b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \text{ and } b_3 < 0 \end{cases}$$

Where,$\wedge$ it is a ty norm$\vee$ it is a conorm t.

The AHP technique begins with the designation of a hierarchical structure, where the elements at the top of the tree are more generic than those at the lower levels. The main leaf is unique and denotes the objective to be achieved in decision-making.

The level immediately below this contains the sheets representing the criteria. The sheets corresponding to the subcriteria appear immediately below this level, and so on. The level below this level represents the alternatives. See Figure 1.

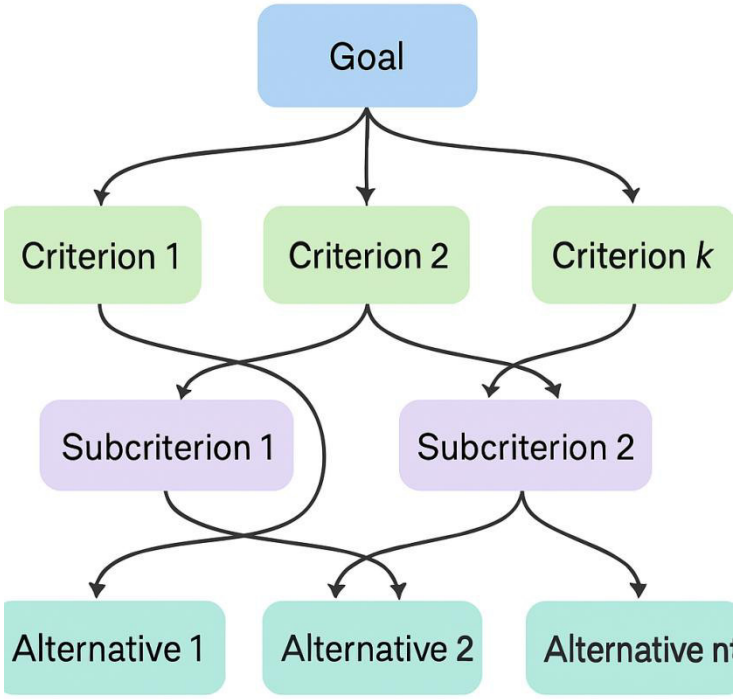


Figure 1: Generic tree diagram representing a Hierarchical Analytical Process. Source: [16].

A square matrix is then formed that represents the opinion of the expert or experts and contains the pairwise comparison of the assessments of the criteria, subcriteria and alternatives.

TL Saaty, the founder of the original method, proposed a linguistic scale that appears in Table 1.

Table 1: Intensity of importance according to the classic AHP. Source [16-19].

Intensity of importance on an absolute scale	**Definition**	**Explanation**
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance of one over the other	Experience and judgment strongly favor one activity over another.
5	Importance essential or strong	Experience and judgment strongly favor one activity over another.
7	importance very strong	The activity is strongly favored and its mastery is demonstrated in practice.
9	Extremely important	The evidence that favors one activity over another is of the highest order of affirmation possible.
2, 4, 6, 8	Intermediate values between the two adjacent judgments.	When needed comprehension
Reciprocals	If activity i has one of the above numbers assigned compared to activity j , then j has the reciprocal value compared to i .	

On the other hand, Saaty established that the *Consistency Index* (CI) should depend on λ_{max}, the maximum eigenvalue of the matrix. He defined the equation $CI = \frac{\lambda_{max} - n}{n-1}$, where n is the order of the matrix. He also defined the *Consistency Ratio* (CR) with the equation CR = CI/RI, where RI is given in Table 2 .

Table 2: RI associated with each order.

Order (n)	1	2	3	4	5	6	7	8	9	10
Rhode Island	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

If $CR \leq 10\%$we can consider that the experts' assessment is sufficiently consistent and therefore we can proceed to use AHP.

The objective of the AHP is to rank the criteria, subcriteria, and alternatives according to a score. It can also be used in group decision-making problems. If this is the purpose, Equations 4 and 5 should be taken into account, where the expert's weight is evaluated based on their authority, knowledge, experience , etc.

$\bar{x} = \left(\prod_{i=1}^{n} x_i^{w_i}\right)^{1/\sum_{i=1}^{n} w_i}$(4)

If $\sum_{i=1}^{n} w_i = 1$, that is, when the expert's weights add up to one, Equation 4 becomes Equation 5,

$\bar{x} = \prod_{i=1}^{n} x_i^{w_i}$(5)

The hybridization of AHP with neutrosophic set theory was used in [16] . This is a more flexible approach to modeling uncertainty in decision making. Indeterminacy is an essential component that must be assumed in real-world organizational decisions.

Table 3 contains the adaptation of the Saaty scale to the neutrosophic field.

Table 3: The Saaty scale was translated into a neutrosophic triangular scale. Source [16].

scale saty	**Definition**	**Neutrosophic Triangular Scale**
1	Likewise influential	$\tilde{1} = \langle((1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle((2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle((4,5,6); 0.80, 0.15, 0.20\rangle$
7	Very influential	$\tilde{7} = \langle((6,7,8); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle((9,9,9); 1.00, 1.00, 1.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle((1,2,3); 0.40, 0.65, 0.60\rangle$
		$\tilde{4} = \langle((3,4,5); 0.60, 0.35, 0.40\rangle$
		$\tilde{6} = \langle((5,6,7); 0.70, 0.25, 0.30\rangle$
		$\tilde{8} = \langle((7,8,9); 0.85, 0.10, 0.15\rangle$

The pairwise neutrosophic comparison matrix is defined in Equation 6 .

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (6)$$

$\tilde{A}$ satisfies the condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, according to the inversion operator defined in Definition 4.

In Abdel-Basset et al. [20] Two indices are defined to convert a neutrosophic triangular number into a sharp number. See Equation 7 for the *score* and Equation 8 for *accuracy*.

$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}})$(7)

$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}})$(8)

The algorithm to be applied to the NAHP is as follows:
Given the Criteria, subcriteria and alternatives, the NAHP consists of the following steps:

1. Design an AHP tree. It contains the selected criteria, subcriteria, and alternatives.
2. Create the level matrices from the AHP tree, according to expert criteria expressed in neutrosophic triangular scales and respecting the matrix scheme of Equation 6.
3. To evaluate the consistency of these matrices, convert the elements of $\widetilde{A}$ in a crisp matrix by applying Equation 7 or 8 and then testing the consistency of this new crisp matrix.
4. Follow the other steps of a classic AHP.
5. Equation 7 or 8 is applied to convert, $w_1, w_2, \ldots, w_n$ to crisp weights.
6. If more than one expert performs the assessment, then $w_1, w_2, \ldots, w_n$ are replaced by $\overline{w}_1, \overline{w}_2, \cdots, \overline{w}_n$, which are their corresponding weighted geometric mean values, see Equations 4 and 5 .

3. Results and Discussion.

As part of a research project to develop an integrated framework combining Business Process Management (BPM) and the Neutrosophic Analytic Hierarchy Process (NAHP), a case study was conducted focusing on prioritizing improvements in document management at a public university. The objective is to identify and prioritize the critical factors that influence the successful implementation of redesigned processes (TO-BE).

To achieve this, a panel of four experts in process management, university administration, and information technology was consulted. Each expert was selected for their in-depth experience and knowledge in the field. They were asked to evaluate a set of seven implementation criteria (ICs), identified as determinants of project success. Each expert's opinion was given equal weight, assigning a specific opinion $\lambda_k = 0.25$to each expert.

Implementation Criteria Evaluated

The implementation criteria evaluated were the following:

- **CI1 (Implementation Cost):** Concern about the financial outlay required to implement process improvements. This is a major source of uncertainty for management.
- **CI2 (Staff Change Resistance):** Previous negative experiences with optimization projects can generate skepticism and resistance on the part of employees, affecting the adoption of new processes.
- **CI3 (Technical Complexity):** The perception that implementing new technologies or procedures is too complex and that staff will not have control over the new tools.
- **CI4 (Impact on Business Continuity):** Fear that the transition to the new process could cause significant disruptions to the university's daily operations.
- **CI5 (Resource Availability):** An environment with limited resources (human, technological, infrastructure) can generate uncertainty about the viability and sustainability of the proposed improvements.
- **CI6 (Alignment with Strategic Objectives):** Expectations about how proposed improvements align (or not) with the university's long-term vision and mission can generate anxiety and a lack of managerial support.
- **CI7 (End User Satisfaction):** Concern about the economic and usability impact that improvements will have on end users (students, teachers, administrative staff), and whether they will be able to adapt to the new systems.

Application of the NAHP Algorithm

The steps of the Neutrosophic Analytic Hierarchy Process (NAHP) were followed to obtain a robust ranking of the criteria.

Step 1 and 2: AHP Hierarchy and Neutrosophic Comparison Matrices

A hierarchy was established with the objective at the top (Prioritize Implementation Criteria) and the seven criteria (CI1 to CI7) at the bottom. Each of the four experts performed a paired comparison of the criteria using the neutrosophic triangular scale. This resulted in the four comparison matrices shown below:

Table 4: Neutrosophic Pairwise Comparison Matrix - Expert 1

Variable	CI1	CI2	CI3	CI4	CI5	CI6	CI7
CI1	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)
CI2	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)
CI3	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)
CI4	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)
CI5	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)
CI6	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)
CI7	(1,2,3; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)

Table 5: Neutrosophic Pairwise Comparison Matrix - Expert 2

Variable	CI1	CI2	CI3	CI4	CI5	CI6	CI7
CI1	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)
CI2	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)
CI3	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)
CI4	(1,2,3; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)
CI5	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI6	(2,3,4; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI7	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)

Table 6: Neutrosophic Pairwise Comparison Matrix - Expert 3

Variable	CI1	CI2	CI3	CI4	CI5	CI6	CI7
CI1	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)
CI2	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)
CI3	(1,2,3; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI4	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)
CI5	(2,3,4; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI6	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)
CI7	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	1,1,1; 0.5,0.5,0.5)

Table 7: Neutrosophic Pairwise Comparison Matrix - Expert 4

Variable	CI1	CI2	CI3	CI4	CI5	CI6	CI7
CI1	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)
CI2	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)
CI3	(2,3,4; 0.3,0.75,0.7)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1/4,1/3,1/2; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI4	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)
CI5	(2,3,4; 0.3,0.75,0.7)	(2,3,4; 0.3,0.75,0.7)	(2,3,4; 0.3,0.75,0.7)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(1,2,3; 0.4,0.65,0.6)	(1/4,1/3,1/2; 0.3,0.75,0.7)
CI6	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1/3,1/2,1; 0.4,0.65,0.6)
CI7	(1,2,3; 0.4,0.65,0.6)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)	(1,1,1; 0.5,0.5,0.5)	(2,3,4; 0.3,0.75,0.7)	(1,2,3; 0.4,0.65,0.6)	(1,1,1; 0.5,0.5,0.5)

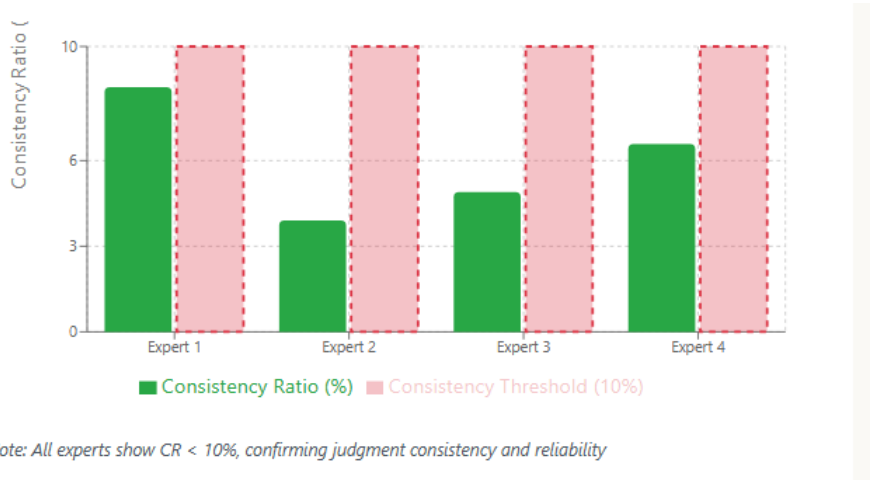
Step 3: Consistency Check

The neutrosophic matrices were converted to crisp matrices to assess their consistency. Consistency ratios (CRs) were calculated for each expert's assessment. The results were as follows:

- **Expert 1:** $CR = 8.5731\%$
- **Expert 2:** $CR = 3.9017\%$
- **Expert 3:** $CR = 4.8985\%$
- **Expert 4:** $CR = 6.5842\%$

Since all CR values are below the threshold, 10% ($CR < 0.10$),it is concluded that the judgments made by the four experts are consistent and reliable. Therefore, the next steps of the analysis can be carried out.

Figure 2: Expert Judgment Consistency Ratios (CR)



Note: All experts show CR < 10%, confirming judgment consistency and reliability

Step 4 and 5: Calculation of Individual Weights and Aggregation

From the consistency matrices, the weight vectors (priorities) for each criterion were calculated based on each expert's assessment. These weights are presented in the following table:

Table 8: Weights Obtained for Each Variable by Expert

Expert/Variable	CI1	CI2	CI3	CI4	CI5	CI6	CI7
1	0.178070	0.178070	0.122520	0.122520	0.134730	0.112040	0.112040
2	0.225817	0.142051	0.082832	0.142051	0.154348	0.126451	0.126451
3	0.101632	0.181524	0.078080	0.078080	0.181524	0.157636	0.181524
4	0.162187	0.162187	0.162187	0.162187	0.162187	0.074533	0.074533

To consolidate the assessments of the four experts into a single final weight vector, the **weighted geometric mean is applied**. Since all experts have the same weight ($\lambda_k = 0.25$), the formula for each criterion j is:

$$\boldsymbol{w_j} = (\boldsymbol{w_j^1} \times \boldsymbol{w_j^2} \times \boldsymbol{w_j^3} \times \boldsymbol{w_j^4})^{\frac{1}{4}}$$

Detailed calculations for each criterion are shown below:

Geometric Mean Calculations

- **Calculation for CI1 (Implementation Cost):**
  - $w_{CI1} = (0.178070 \times 0.225817 \times 0.101632 \times 0.162187)^{\frac{1}{4}}$
  - $w_{CI1} = (0.00066283)^{\frac{1}{4}} = 0.160418$
- **Calculation for CI2 (Staff Resistance to Change):**
  - $w_{CI2} = (0.178070 \times 0.142051 \times 0.181524 \times 0.162187)^{\frac{1}{4}}$
  - $w_{CI2} = (0.00074404)^{\frac{1}{4}} = 0.165030$
- **Calculation for CI3 (Technical Complexity):**
  - $w_{CI3} = (0.122520 \times 0.082832 \times 0.078080 \times 0.162187)^{\frac{1}{4}}$
  - $w_{CI3} = (0.00012845)^{\frac{1}{4}} = 0.106461$
- **Calculation for CI4 (Impact on Business Continuity):**
  - $w_{CI4} = (0.122520 \times 0.142051 \times 0.078080 \times 0.162187)^{\frac{1}{4}}$
  - $w_{CI4} = (0.00022026)^{\frac{1}{4}} = 0.121808$
- **Calculation for CI5 (Resource Availability):**
  - w_CI5 = (0.134730 × 0.154348 × 0.181524 × 0.162187)^(1/4)
  - w_CI5 = (0.00061217)^(1/4) = 0.157297
- **Calculation for CI6 (Alignment with Strategic Objectives):**
  - $w_{CI6} = (0.112040 \times 0.126451 \times 0.157636 \times 0.074533)^{\frac{1}{4}}$
  - $w_{CI6} = (0.00016656)^{\frac{1}{4}} = 0.113612$
- **Calculation for CI7 (End User Satisfaction):**
  - $w_{CI7} = (0.112040 \times 0.126451 \times 0.181524 \times 0.074533)^{\frac{1}{4}}$
  - $w_{CI7} = (0.00019154)^{\frac{1}{4}} = 0.117666$

Results and Final Ranking

The aggregate weights obtained using the geometric mean must be normalized so that their sum equals 1. This will give us the final percentage weight for each criterion.

Normalization of Weights

1. **Sum of the geometric mean weights:** Sum $= 0.160418 + 0.165030 + 0.106461 + 0.121808 + 0.157297 + 0.113612 + 0.117666 = 0.942292$
2. **Normalization of weights:** $w_{norm_j} = \frac{w_j}{sum}$
   - $w_{CI1} = \frac{0.160418}{0.942292} = 0.170238$
   - $w_{CI2} = \frac{0.165030}{0.942292} = 0.175133$
   - $w_{CI3} = \frac{0.106461}{0.942292} = 0.112981$

- $w_{CI4} = \frac{0.121808}{0.942292} = 0.129265$
- $w_{CI5} = \frac{0.157297}{0.942292} = 0.166929$
- $w_{CI6} = \frac{0.113612}{0.942292} = 0.120569$
- $w_{CI7} = \frac{0.117666}{0.942292} = 0.124872$

Final Results Table

Table 9: Final Weights and Ranking of Implementation Criteria

Ranking	Criterion	Description	Final Weight	Percentage
1	**CI2**	Resistance to Staff Change	0.175133	17.51%
2	**CI1**	Implementation Cost	0.170238	17.02%
3	**CI5**	Availability of Resources	0.166929	16.69%
4	**CI4**	Impact on Business Continuity	0.129265	12.93%
5	**CI7**	End User Satisfaction	0.124872	12.49%
6	**CI6**	Alignment with Strategic Objectives	0.120569	12.06%
7	**CI3**	Technical Complexity	0.112981	11.30%

Figure 3: Final Priority Ranking of Implementation Criteria

4. Discussion

The results obtained from the application of the NAHP method offer a detailed and quantitative perspective on the priorities that experts assign to critical factors in administrative improvement projects in a public university setting. The analysis of the final hierarchy reveals several significant implications that merit further discussion.

The Primacy of Human and Financial Factors

The most revealing finding is the prominence of **Staff Resistance to Change (CI2)** , **Implementation Cost (CI1)** , and **Resource Availability (CI5)** . Together, these three criteria account for over 51% of the total decision weight, indicating that, from the experts' perspective, the success or failure of a process optimization initiative depends overwhelmingly on the management of people and capital.

The first position of **Resistance to Change (17.51%)** debunks the idea that improvement projects are purely technical or strategic. In a public institution with a consolidated organizational culture and entrenched work routines, the human factor becomes the main battleground. This result suggests that employees are not seen as passive recipients of change, but as key players whose acceptance is

indispensable. Previous experiences with failed projects, fear of skill obsolescence, or uncertainty about job security are, according to this analysis, greater barriers than any technological challenge .

Immediately after this are **Cost (17.02%)** and **Resources (16.69%)** , confirming the perennial budgetary constraints that characterize the public sector. Experts don't see these as separate issues, but rather as two sides of the same coin: practical viability. A project may be brilliant in theory, but if there isn't the budget for its implementation and the resources (human, technological) to sustain it, it's destined to fail.

The Relegation of the Technical and Strategic Dimension

It is particularly notable that **Technical Complexity (CI3)** ranks last in the hierarchy (11.30%). This does not imply that technology is irrelevant, but rather that in the current context (date: July 30, 2025), it is perceived as an enabler or a commodity. Experts seem to assume that, given a clear budget and objectives, it is possible to find a suitable technological solution. The real challenge lies not in the complexity of the software or hardware, but in ensuring that the organization adopts and uses it effectively.

Similarly, **Alignment with Strategic Objectives (CI6)** , although important, is at the bottom of the ranking. One possible interpretation is that experts consider this alignment a necessary condition, a starting point taken for granted before initiating such a detailed prioritization analysis. Another, more pragmatic perspective might be that in the day-to-day of university management, immediate operational obstacles (lack of staff, limited budget, union resistance) carry much greater weight in decision-making than long-term strategic guidelines.

Contribution and Robustness of the NAHP Method

The use of the NAHP was critical to achieving this clarity. A simpler approach, such as a survey or direct voting, would not have been able to capture the uncertainty and subjectivity inherent in expert opinions. The method's ability to handle judgments expressed with degrees of truth, uncertainty, and falsity (through neutrosophic numbers) provided a more realistic framework for the problem. Furthermore, checking the Consistency Ratio (CR) ensured that the judgments, although subjective, were logical and consistent, adding a layer of scientific validity to the process. Aggregating judgments using the geometric mean, rather than a simple arithmetic mean, guaranteed a more balanced consensus that mitigates the impact of extreme opinions held by a single expert.

In short, the discussion of these results goes beyond simply listing an ordered list. It reveals a management philosophy for the public sector: to transform administrative processes, perceptions must first be transformed and resources secured. Only then can technology and strategy fulfill their true purpose.

5. Conclusions

The rigorous application of the NAHP method has allowed us to establish a clear and robust hierarchy of criteria that should be prioritized when implementing improvements in the university's administrative processes. The consolidated analysis of the four experts' opinions reveals that human and economic factors are the most critical.

Staff Resistance to Change (CI2) emerges as the most important factor, with a weight of **17.51%** . This underscores that, beyond technology or process redesign, the success of any improvement initiative depends fundamentally on the acceptance and collaboration of the people who will execute it. Past experiences and organizational culture are decisive, and any project must include a solid change management strategy.

In second and third place are **Implementation Cost (CI1)** at **17.02%** and **Resource Availability (CI5)** at **16.69%** . These results reflect the reality of public institutions, where budgetary constraints and resource allocation are decisive factors. Financial viability and the installed capacity to support improvements are primary concerns that must be addressed from the outset.

Factors such as **Impact on Business Continuity (CI4)** and **End-User Satisfaction (CI7)** occupy an intermediate position, indicating that, while important, they are perceived as manageable consequences

if the main factors (personnel, cost, and resources) are properly managed. Finally, **Technical Complexity (CI3)** is the least important criterion, suggesting that technological challenges are considered surmountable if the budget, resources, and, above all, the personnel willing to adopt the new tools are available.

In conclusion, this integrated BPM and NAHP framework proves to be an effective tool for strategic decision-making in highly uncertain environments. It provides quantitative prioritization that goes beyond intuition, allowing managers to focus their efforts and resources on the factors that will truly determine the success of administrative optimization projects.

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Neutrosophic Classification of the Factors That Shape the Professional Profile of Students in Ecuadorian Higher Education

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Abstract. This article examines the professional profile of university students in the context of uncertainty of Ecuador. What constitutes this professional profile? Technical skills? Soft skills? Demands of the international and national market? This is relevant to the nation because the employability situation of the country depends on economic development. The correlation between higher education and economic development has been complicated by developments that exploit imperfectly educated people for demand within the employment market. Thus, a professional profile is neither the first study of such developments; however, few studies exist that assess the professional profile through uncertainty and contradiction which means that the perceptions of students/teachers/employers are too uncertain to provide a holistic picture. Therefore, neutrosophic set theory is applied to fill the gap, which means that for each factor derived, assessed via neutrosophic sets through various degrees of belonging, uncertainty/indeterminacy, and non-belonging, classification is based on strongness. Ultimately, the results showed that the highest levels of indeterminacy were found for technical skills and soft skills which means that coursework should more intensely integrate such aspects of university life/professional profile. This study contributes to the field by providing a new theoretical application to an uncertainly complicated field of study which will allow for universities and their corresponding segments to know what's best for future curriculum to cement the professional profile. It will also lend to suggestions for implementation—from curricular adjustments to supportive productive sector requirements—to better prepare students for entry into a complicated and ever-evolving labor force.

Keywords: Neutrosophic Sets, Professional Profile, Higher Education, Uncertainty, Competencies, Employability, Ecuador.

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1. Introduction

The development of the professional profile of students in Ecuadorian higher education constitutes a fundamental pillar for the country's economic and social development, given that the preparation of graduates directly impacts their employability and the competitiveness of the labor market. In a dynamic global context, where labor demands evolve rapidly, universities face the challenge of aligning their educational programs with the needs of the productive sector [1]. The importance of this issue lies in its capacity to reduce the gap between academic training and market expectations, a critical aspect in emerging economies such as Ecuador [2]. Recent studies emphasize that professional profiles must integrate technical competencies, soft skills, and practical experiences to ensure a successful transition to the workforce [3]. This article explores the factors that shape this profile, employing an innovative approach based on neutrosophic set theory to address the uncertainties inherent in this process.

Historically, higher education in Ecuador has undergone significant reforms, especially since the implementation of the Organic Law on Higher Education in 2010, which sought to improve the quality and relevance of training [4]. However, challenges persist, such as the disconnect between curricula and labor market demands, which limits student preparation [5]. In recent decades, the country has made progress in accrediting universities and promoting community engagement programs, but conflicting perceptions among students, teachers, and employers about the necessary skills generate ambiguity [6]. This historical and current context highlights the need for analytical approaches that consider complexity and uncertainty in vocational training.

The central problem addressed by this research is the lack of clarity regarding how key factors, such as technical competencies, soft skills, and internship opportunities, contribute to shaping the professional profile of Ecuadorian students. How can these factors be classified, considering the ambiguous and contradictory perceptions of the stakeholders involved? The magnitude of this problem is reflected in youth unemployment rates and the underutilization of skills in the Ecuadorian labor market, which demands an analysis that integrates the uncertainty present in the assessments [7]. This study seeks to answer this question through an approach that allows for systematic modeling and classification of factors. Neutrosophic set theory offers a powerful tool for addressing this problem, as it allows for the representation of the belonging, indeterminacy, and non-belonging of each factor, thus capturing the ambiguity inherent in the perceptions of students, teachers, and employers. Unlike traditional approaches, which typically assume precise data, this method incorporates uncertainty as a core component of the analysis [8]. By classifying factors according to their impact on professional profiles, the study aims to identify priority areas for educational intervention. This approach is particularly relevant in a context where labor market expectations and graduate capabilities are not always aligned.

The objectives of this research are clear and aligned with the research question. First, it seeks to identify and classify the factors that shape the professional profile of university students in Ecuador using neutrosophic sets. Second, it aims to propose strategies based on the results to optimize professional development, such as curricular adjustments or training programs. Finally, the study aims to contribute to the development of a theoretical and methodological framework that can be applied in other educational contexts with similar characteristics.

2. Preliminaries

2.1. Complexity theory and causality and neutrosophic sets

Interactions between variables are not always simple; rather, they frequently emerge through intricate, nonlinear patterns, as complexity theory posits. This perspective tells us that the same cause can lead to divergent outcomes depending on the context in which it manifests. This theory emphasizes

three key principles: conjunction, equifinality, and causal asymmetry. The conjunction principle focuses on the collaboration between antecedent conditions that act together to produce an outcome, rather than operating independently to explain variability. Equifinality, on the other hand, posits that a system can reach a specific end state through a variety of initial conditions and distinct trajectories. Causal asymmetry, on the other hand, suggests that while certain conditions can lead to the emergence of an outcome, their absence does not guarantee the absence of that outcome [9].

To illustrate this, consider a restaurant renowned for its high culinary quality. Although this quality may attract numerous customers, the establishment might face low attendance due to factors such as an unfavorable location or parking issues [10]. Conversely, a restaurant with average food might still attract many customers if it offers exceptional service, is in a strategic location, or has attractive entertainment options. This demonstrates that the relationship between variables such as the quality of food, location, and service, and the outcome—the number of customers—is by no means simple or constant. These principles highlight the complexity and lack of stability in the relationship between conditions and outcomes. Furthermore, neutrosophy brings greater depth to the understanding of complex causality by introducing indeterminacy and uncertainty, which are inherent to social phenomena. Neutrosophic set theory, with its ability to handle indeterminacy, provides a more nuanced perspective for understanding these complex and dynamic relationships [11].

2.2. Neutrosophic Liker scales

Surveys using neutrosophic Likert scales [12, 13, 14] effectively measure the diversity of opinions and their influence on public policy and social discourse, capturing areas of consensus, disagreement, and ambivalence.

Below we present the fundamental definitions and concepts related to neutrosophic sets and single-valued neutrosophic sets .

Definition 1 ([15]). Let U be a discursive universe. $N = \{(x , T (x), I (x), F (x)): x \in U \}$ is a neutrosophic set, denoted by a truth membership function, $TN : U \rightarrow]0 - , 1+[$; an indeterminate membership function , $IN : U \rightarrow]0 - , 1+[$; and a falsehood membership function , $FN : U \rightarrow]0 - , 1+[$.

Single-valued neutrosophic sets provide a way to represent and analyze possible elements in the universe of discourse U

Definition 2 ([16]). Let U be a discursive universe. A single-valued neutrosophic set is defined as $N = \{(x , T (x), I (x), F (x)): x \in U \}$, which is identified by a truth membership function, $TN : U \rightarrow [0, 1]$; indeterminacy membership function , $IN : U \rightarrow [0, 1]$; and falsehood membership function , $FN : U \rightarrow [0, 1]$, with $0 \leq TN (x)+ IN (x)+ FN (x) \leq 3$

Using neutrosophic scales with single-valued neutrosophic sets, responses are categorized according to the total of the True, Indeterminate, and False components as follows:

- T+I+F<1: Incomplete
- T+I+F=1: Complete
- T+I+F>1: Contradictory

These values are obtained because, in many cases, opinions are incomplete or contradictory. This classification is one of the advantages of using neutrosophic methods, as it allows for a more nuanced understanding of the different degrees of truth, indeterminacy, and falsity in the responses.

3. Proposed framework

To begin, it's essential to clearly define the desired outcome: precisely identify and describe the phenomenon, event, or condition you wish to explore. This step is essential because it establishes the approach and framework that will guide the subsequent analysis.

Next, proceed to develop neutrosophic Likert scales. These scales, in contrast to conventional scales that employ a fixed range of values (such as 1 to 5), incorporate additional dimensions of truth, indeterminacy, and falsity. Instead of simple numerical scores, neutrosophic scales use a triplet (T, I, F) for each option, where T represents the degree of truth, I the degree of indeterminacy, and F the degree of falsity. This method allows for a more nuanced and detailed assessment of participants' responses and perceptions.

Next, collect relevant data on the cases under study, using a variety of indicators or measures related to the defined outcome. It is crucial that data collection be thorough and accurate so that it adequately reflects the variables being analyzed. Use Neutrosophic Likert scales in questionnaires and surveys to obtain a more complete data set that more accurately captures the complexity of respondents' opinions and attitudes.

This detailed and refined approach ensures a deeper and more accurate interpretation of the results, thus facilitating a comprehensive understanding of the phenomenon in question.

Fuzzification : Finally, the obtained neutrosophic sets are transformed into equivalent fuzzy sets, following the procedure described in [17]. This step is essential for the subsequent analysis, allowing to handle the uncertainty and ambiguity inherent in the collected data. Let $AN = \{x, (TA(x), IA(x), FA(x)) : x \in X\}$an NS. Its equivalent fuzzy membership set is defined as $AF = \{(x, \mu A(x)) : x \in X\}$,where $\mu A(x) = s\left((TA(x), IA(x), FA(x)), (1,0,0)\right)$. Then, using the similarity equation proposed in,

$$\mu A(x) = 1 - \frac{1}{2}[(1 - T_A(x)) + \max\{I_A(x), F_A(x)\}] \quad (1)$$

Since the range of the similarity measure function is the unit interval [0,1], μA (x) ∈ [0,1] for all x ∈ X. Therefore, the membership function of the derived fuzzy set belongs to [0,1] and hence satisfies the property of a fuzzy set (FS) membership function.

1 Analysis: Perform fsQCA to identify which combinations of factors or conditions are associated with the presence or degree of the outcome. Data processing is performed using fsQCA for Windows [18, 19].

Configuration validity is assessed by measuring consistency and coverage values. Consistency is the measure of how reliably the set of pathways produces the desired outcome. Coverage refers to the degree to which the outcome is clarified by this arrangement of pathways[20]:

$$\text{Consistency}(X \geq Y) = \frac{\sum(\min(X_i, Y_i))}{\sum(Y_i)} \quad (2)$$

$$Coverage(X \geq Y) = \frac{\sum(\min(X_i, Y_i))}{\sum(X_i)} \quad (3)$$

where :

$X_i$is the membership value of case i in the set of causal conditions.

$Y_i$is the membership value of case iii in the result set.

Both are used in comparative analysis to evaluate the relationships established between individual conditions, combinations of conditions, track configurations, and the final outcome. Generally, values above 0.8 are considered to indicate a strong relationship [20].

4. Results.

Within the framework of this research, the outcome of interest is the **Perceived Employability (PE)** of university graduates. To analyze this phenomenon, three fundamental causal conditions have been established that, according to the literature and the Ecuadorian context, shape the professional profile:

- **Technical Competencies (TC):** Refers to the mastery of knowledge and skills specific to a profession, acquired through formal academic training.

- **Soft Skills (BS):** These encompass a set of personal and social attributes such as communication, teamwork, leadership, and problem-solving, which facilitate effective interaction in the work environment.
- **Alignment with the Labor Market (AML):** Measures the relevance of university education to the current demands, technologies, and dynamics of the productive sector.

To quantify perceptions of these variables, a survey using neutrosophic Likert scales was administered to a panel of 20 experts, composed of human resources directors, academics, and professional development consultants in Ecuador. The collected data, expressed in single-valued neutrosophic sets (T, I, F), are presented below.

Table 1. Neutrosophic Data Collected from Experts

Experts	Technical Competencies (TC)	Soft Skills (HS)	Market Alignment (AML)	Perceived Employability (PE)
1	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.2)	(0.7, 0.3, 0.2)	(0.8, 0.2, 0.2)
2	(0.6, 0.5, 0.3)	(0.7, 0.4, 0.3)	(0.5, 0.5, 0.4)	(0.6, 0.4, 0.3)
3	(0.9, 0.1, 0.0)	(0.8, 0.2, 0.1)	(0.9, 0.2, 0.1)	(0.9, 0.1, 0.1)
4	(0.5, 0.6, 0.5)	(0.6, 0.5, 0.4)	(0.4, 0.6, 0.5)	(0.5, 0.5, 0.5)
5	(1.0, 0.0, 0.0)	(0.9, 0.1, 0.1)	(0.8, 0.3, 0.2)	(0.9, 0.2, 0.0)
6	(0.7, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.7, 0.4, 0.3)	(0.8, 0.3, 0.2)
7	(0.8, 0.4, 0.2)	(0.9, 0.3, 0.1)	(0.8, 0.3, 0.2)	(0.9, 0.2, 0.2)
8	(0.6, 0.6, 0.4)	(0.7, 0.5, 0.2)	(0.6, 0.5, 0.3)	(0.7, 0.4, 0.3)
9	(0.9, 0.2, 0.1)	(1.0, 0.1, 0.0)	(0.9, 0.2, 0.2)	(1.0, 0.1, 0.1)
10	(0.7, 0.4, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.5, 0.4)	(0.6, 0.4, 0.4)
11	(1.0, 0.1, 0.1)	(0.9, 0.2, 0.2)	(0.8, 0.3, 0.3)	(0.9, 0.2, 0.1)
12	(0.8, 0.3, 0.2)	(0.8, 0.3, 0.2)	(0.9, 0.2, 0.1)	(0.8, 0.2, 0.2)
13	(0.7, 0.5, 0.3)	(0.7, 0.4, 0.4)	(0.6, 0.5, 0.4)	(0.7, 0.4, 0.4)
14	(0.9, 0.2, 0.2)	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.3)	(0.8, 0.3, 0.2)
15	(0.6, 0.4, 0.4)	(0.7, 0.3, 0.3)	(0.7, 0.4, 0.4)	(0.7, 0.3, 0.3)
16	(0.8, 0.2, 0.3)	(0.9, 0.2, 0.1)	(0.9, 0.3, 0.1)	(0.9, 0.2, 0.2)
17	(0.7, 0.3, 0.4)	(0.8, 0.4, 0.2)	(0.7, 0.4, 0.3)	(0.8, 0.4, 0.3)
18	(0.9, 0.1, 0.2)	(0.9, 0.2, 0.2)	(1.0, 0.1, 0.1)	(0.9, 0.1, 0.2)
19	(0.6, 0.5, 0.5)	(0.7, 0.5, 0.4)	(0.6, 0.6, 0.5)	(0.6, 0.5, 0.5)
20	(0.8, 0.2, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.1, 0.2)	(0.8, 0.1, 0.2)

Step 1: Fuzzification of Neutrosophic Data

In order to apply comparative analysis, neutrosophic sets are transformed into equivalent fuzzy sets. This process, known as fuzzification , is performed using Equation 1, which integrates the degrees of truth (T), indeterminacy (I), and falsity (F) into a single fuzzy membership value.($\mu A(x)$).

Fuzzification Formula (Equation 1):

This procedure is repeated for all 20 experts' perceptions. The resulting fuzzy values are presented in Table 2.

Table 2. Fuzzification Process

Experts	CT (μ)	HB (μ)	AML (μ)	EP (μ)
1	0.833333	0.866667	0.733333	0.800000
2	0.600000	0.666667	0.533333	0.633333
3	0.966667	0.833333	0.866667	0.933333
4	0.466667	0.566667	0.433333	0.500000
5	1.000000	0.900000	0.766667	0.900000
6	0.666667	0.733333	0.633333	0.733333
7	0.733333	0.800000	0.766667	0.800000
8	0.533333	0.666667	0.600000	0.633333
9	0.866667	0.933333	0.833333	0.933333
10	0.566667	0.633333	0.566667	0.566667
11	0.933333	0.833333	0.733333	0.866667
12	0.766667	0.766667	0.866667	0.800000
13	0.633333	0.600000	0.533333	0.600000
14	0.833333	0.766667	0.733333	0.733333
15	0.600000	0.666667	0.600000	0.666667
16	0.766667	0.866667	0.833333	0.800000
17	0.666667	0.733333	0.633333	0.666667
18	0.866667	0.833333	0.900000	0.866667
19	0.533333	0.600000	0.500000	0.500000
20	0.866667	0.866667	0.866667	0.866667
Addition	**14.700000**	**15.133333**	**13.733333**	**14.800000**

Step 2: Analysis of Necessary Conditions

Next, we assess whether any of the conditions (CT, HB, AML) are necessary for the outcome (EP). To do this, we calculate the **Consistency** and **Coverage parameters** using Equations 2 and 3. A consistency value greater than 0.8 generally indicates a strong and necessary relationship.

As an example, the calculation of consistency and coverage for the **Soft Skills (HB) condition** as a predictor of **Perceived Employability (EP) is detailed** :

1. **Calculate** $\sum \boldsymbol{min}(\boldsymbol{HBi}, \boldsymbol{EPi})$:The minimum value between HB and EP is taken for each of the 20 experts and added together.
   - $min(0.866667, 0.800000) = 0.800000$
   - $min(0.666667, 0.633333) = 0.633333$
   - ... (done for the 20 experts) ...
   - **Sum of Minimums** $\sum \boldsymbol{min}(\boldsymbol{HBi}, \boldsymbol{EPi})$: 14.433333
2. **Obtain Sums of the Columns (from Table 2):**
   - $\sum HBi = 15.133333$
   - $\sum EPi = 14.800000$
3. **Calculate Consistency and Coverage:**
   - **Consistency(HB ≤ EP):** Consistency=$15.13333314.433333 = 0.953741$
   - **Coverage (HB ≤ EP):** Coverage= $14.80000014.433333 = 0.975225$

This process is replicated for all conditions. The results are summarized in Table 3.

Table 3. Analysis of Necessary Conditions for Perceived Employability (PE)

Tested Conditions	Sum of Minima with EP	Consistency	Coverage
Technical Competencies (TC)	13.966667	0.950113	0.943694
Soft Skills (HS)	14.433333	0.953741	0.975225
Market Alignment (AML)	13.166667	0.958730	0.889640
Perceived Employability (PE)	14.800000	1.000000	1.000000

The results show that the three individual conditions have **very high consistency values (over 0.95)** , which qualifies them as necessary conditions for achieving high perceived employability. **Labor Market Alignment (LMA) stands out** with the highest consistency (0.9587), suggesting that the relevance of the curricula is an almost indispensable requirement. **Soft Skills (SS)** obtained the highest coverage (0.9752), indicating that this condition explains the largest proportion of the result.

Step 3: Set Matching Analysis

This analysis explores how combinations of conditions relate to the outcome. The consistency of the intersection (logical AND operator, implemented with the min function) of the sets of conditions against perceived employability is calculated.

Table 4. Condition Set Matching Analysis

Conditions (Logical AND Combination)	Consistency
CT * HB * AML	0.977841
CT * HB	0.968605
CT * AML	0.971292
HB * AML	0.976378

Note: *The match is calculated as the consistency of the combined configuration. For example, for CT* HB*AML, the formula is$\sum min(CTi, HBi, AMLi)/\sum min(CTi, HBi, AMLi, EPi) *$

The highest coincidence value is observed in the combination of the three conditions (**0.9778**), which indicates that the synergistic interaction of Technical Competencies, Soft Skills and Alignment with the Labor Market presents the most robust and consistent relationship with Perceived Employability.

Superset and Configuration Analysis

Finally, Table 5 summarizes the consistency and coverage values for all individual conditions and their combinations, allowing a direct comparison of the different paths leading to the outcome.

Table 5. Results of Superset /Subset Analysis

Terms (Settings)	Consistency	Coverage
CT * HB * AML	0.977841	0.875000
CT * HB	0.968605	0.916667
CT * AML	0.971292	0.891892
HB * AML	0.976378	0.898649
CT	0.950113	0.943694
HB	0.953741	0.975225
AML	0.958730	0.889640

This final analysis confirms that, while all conditions are individually necessary and consistent, the **combinations of conditions** (especially HB*AML and the triple combination) achieve the highest levels of consistency. However, coverage does not always increase with configuration complexity. The **CT*HB combination** presents very high coverage (0.9167), suggesting that the combination of technical and soft skills is a pathway that explains a large proportion of high employability cases, even without perfect market alignment.

5. Discussion

The results obtained offer a detailed and multifaceted view of the factors that shape the employability of graduates in Ecuador. The main finding is that no condition operates in isolation; on the contrary, the **synergistic interaction** between Technical Competencies (TC), Soft Skills (SS), and Labor Market Alignment (LMA) constitutes the most robust configuration for predicting Perceived Employability (PE), reaching a consistency of **0.9778** . This result aligns with complexity theory, which postulates that outcomes emerge from the conjunction of multiple interdependent factors.

A notable aspect is the very high consistency score of all individual conditions (all > 0.95), which establishes them as **necessary conditions** in the Ecuadorian context. This contrasts with studies in other contexts (e.g. , López, 2022), where technical skills alone do not always reach such high thresholds of need. The high consistency of the AML (0.9587) is particularly revealing for the Ecuadorian higher education system, as it suggests that **curricular relevance** is not a desirable factor, but an indispensable requirement for employability.

The Soft Skills **coverage** (0.9752) was the highest among the individual conditions, indicating that this factor alone "covers" or explains most cases of high employability. This finding reinforces the growing demand of the global labor market, which increasingly prioritizes socio-emotional skills over purely technical ones (World Economic Forum, 2023).

The configuration analysis (Table 5) reveals that, although the combination of the three factors is the most consistent, the combination of **Soft Skills and Market Alignment (HB*AML)** also presents very high consistency and coverage (0.9764 and 0.8986, respectively). This suggests a strategic path for universities: even if updating technical competencies is a slower process, a strong focus on soft skills and engagement with the productive sector can have a significant impact on employability.

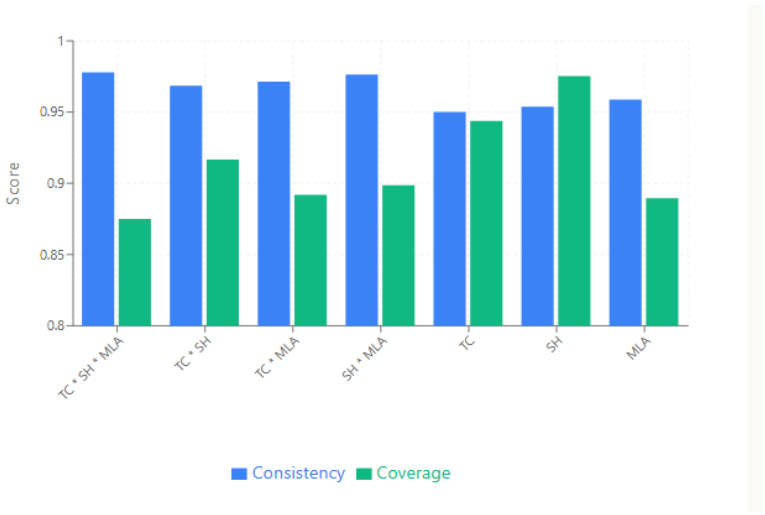

Figure 1: Consistency and Coverage Analysis for Perceived Employability Configurations

Limitations and Future Research: This study, while methodologically robust, has limitations. The sample of 20 experts, although qualified, is small and may not capture the full diversity of opinions at the national level. Future research should expand the sample to include the perspectives of students and recent graduates themselves. Furthermore, this analysis is static in nature; a longitudinal study could track how the relative importance of these factors changes over time and with labor market transformations. Finally, the fsQCA methodology , although powerful, does not establish directional causality. It would be beneficial to complement these findings with qualitative methods that explore underlying causal mechanisms.

6. Conclusions

This study successfully applied a framework based on neutrosophic set theory and comparative qualitative analysis to unravel the complex relationships among the factors that determine the employability of university students in Ecuador. Clear conclusions were reached with direct practical implications for educational policy and university strategy.

1. **Interdependence and Synergy:** The main conclusion is that Perceived Employability is not the result of a single factor, but rather the **synergistic combination of Technical Competencies, Soft Skills, and Alignment with the Labor Market** . The configuration that combines these three elements demonstrated the highest consistency (0.9778), validating a comprehensive and multifactorial approach to vocational training.
2. **The Inevitable Need for the Three Factors:** Contrary to a traditional view, the analysis showed that the three conditions studied are, in and of themselves, **necessary conditions** for professional success in the Ecuadorian context, with consistency values above 0.95. This sends a strong message to higher education institutions: neglecting any of these three areas critically impacts the profile of their graduates.
3. **Practical Primacy of Soft Skills:** Soft skills are not only a necessary condition, but also the ones with the greatest **coverage (0.9752)** . This means they are the factor that best explains the perception of employability in most cases. For universities, this implies that investing in communication, leadership, and teamwork development programs is one of the most effective strategies for enhancing the professional profile of their students.
4. **Strategic Implications for Higher Education:** The findings offer a roadmap for decision-making. Universities should:
    - **Reform curricula** to ensure constant alignment with the demands of the productive sector.
    - **Integrate soft skills development** across all majors, not as isolated workshops.
    - **Strengthen strategic alliances** with companies to ensure that the technical skills taught are those required in practice.

In summary, this study overcomes a simplistic view of vocational training and provides a nuanced model that reflects the complexity of today's labor market. By using a neutrosophic methodology, it has been possible to address the uncertainty and ambiguity inherent in human perceptions, offering a solid empirical basis for Ecuadorian universities and *policymakers* to design more effective and holistic interventions, better preparing future professionals for the challenges of a dynamic market.

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HAP-COVID: Hybrid Model Integrating Machine Learning and Neutrosophic Statistics to Estimate Mortality Risk in COVID-19 Comorbid Patients

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Abstract: This study aims to utilize a hybrid predictive model to evaluate the mortality risk in COVID-19 patients with comorbidities. It is challenging to make clinical decisions while dealing with SARS-CoV-2 because of the high death rate related to chronic conditions such as diabetes, hypertension, chronic obstructive pulmonary disease (COPD), renal failure, heart disease, and cancer. To overcome this obstacle, a Monte Carlo algorithm-based simulated database was created, with clinical variables represented by Bernoulli and truncated normal distributions. Several prediction models, including Decision Trees, Naive Bayes, and Neutrosophic Statistics, were trained using this database. While the neutrosophic model permitted risk categorization according to truth, falsity, and indeterminacy, the decision tree model outperformed the Naive Bayes model in terms of accuracy. When it came to controlling diagnostic uncertainty, the hybrid approach worked well. Ultimately, this technology provides significant assistance in intricate clinical settings, enhancing the process of medical decision-making.

Keywords: COVID-19, Chronic Diseases, Machine Learning, Neutrosophic Statistics, Hybrid Predictive Model, Monte Carlo Algorithm.

1. Introduction

The COVID-19 pandemic has posed a significant challenge to worldwide healthcare systems, particularly affecting individuals with chronic conditions such as diabetes, hypertension, chronic obstructive pulmonary disease (COPD), renal failure, cardiovascular disease, and cancer. These patients face a markedly elevated chance of experiencing severe complications or mortality due to the SARS-CoV-2 virus, necessitating the development of more precise predictive instruments that can accommodate intricate and ambiguous clinical situations. In light of this necessity, it is crucial to integrate conventional statistical analysis techniques with sophisticated artificial intelligence algorithms to facilitate a more accurate evaluation of clinical risk. Nevertheless, numerous current models fail to sufficiently account for the uncertainty inherent in medical data, hence constraining their practical application. We propose HAP-COVID, a hybrid approach that combines supervised machine learning techniques with neutrosophic statistics to assess the mortality risk in patients with COVID-19 comorbidities more accurately. This proposal aims to deliver a more efficient, adaptable, and dependable clinical decision-support instrument.

1.1 Related works

The research titled "SIMPLS-based model to predict COVID-19 mortality in hospitalized patients" was constructed via a SIMPLS-based model (a variation of Partial Least Squares) incorporating 108

clinical factors, comorbidities, and blood indicators from a cohort of 250 COVID-19 patients. The aim was to forecast in-hospital mortality and accurately differentiate between survivors and non-survivors (AUC>0.85), emphasizing variables like age above 65 years, diabetes, heart disease, and dementia. Latent class analysis (LCA) and principal component analysis (PCA) were employed to identify high-risk categories among patients, thereby enhancing clinical stratification. The model exhibited evident utility for resource prioritizing and clinical management in critical scenarios [1],

Khalifa and Smarandache's [2], study, "Application of Neutrosophic Ensembles in Hybrid Diagnostic Models for COVID-19," describes the implementation of a hybrid classification system utilizing machine learning techniques and neutrosophic ensembles for the diagnosis of COVID-19 from chest X-rays. The pipeline transformed medical images into the neutrosophic realm by producing three channels (truth, indeterminacy, and falsehood), thereafter extracting characteristics and classifying them with a multilayer perceptron. The model attained an accuracy of roughly 98.46%, with sensitivity and F1-Score nearing 98%, surpassing solely machine learning methods. This method illustrates the efficacy of combining neutrosophic logic with artificial intelligence to enhance decision-making in clinical environments with limited data.

2. Materials and Methods.

To evaluate the mortality risk of COVID-19 patients with chronic comorbidities, a hybrid predictive model was developed combining machine learning and neutrosophic logic. A simulated dataset was generated using the Monte Carlo algorithm with probabilistic distributions assigned to each clinical variable. The proposed methodology integrates traditional classifiers with neutrosophic statistics to manage uncertainty and enhance clinical decision-making.

Chronic Diseases

Several studies have shown that chronic diseases significantly increase COVID-19 mortality. A meta-analysis by Ssentongo et al. [3], of over 70 studies found that conditions like hypertension, diabetes, chronic kidney disease, and heart failure doubled or tripled the fatality risk in SARS-CoV-2 patients. These comorbidities impair immune response and worsen systemic inflammation, aggravating prognosis. Understanding these relationships has led to the development of more advanced risk models that integrate both clinical and historical data. In this study, such conditions are simulated using Bernoulli distributions to realistically represent clinical scenarios, feeding into a hybrid predictive framework for better clinical relevance.

COVID-19

The COVID-19 pandemic has prompted the development of multiple predictive models aimed at anticipating clinical outcomes such as mortality, ICU admission, and hospitalization duration. One of the most cited approaches is that of Yan et al. [4], who used simple clinical variables (e.g., LDH, lymphocytes, and CRP) to build a decision tree that predicted mortality with 90% accuracy. This study demonstrated that even simple algorithms could significantly aid rapid medical decision-making during public health emergencies. As information on SARS-CoV-2 grew, comorbidity data were incorporated to enhance risk stratification. Most models rely on machine learning techniques to optimize performance. However, few adequately address the clinical uncertainty present in medical records—an issue tackled in this research through a hybrid model.

Machine Learning

Machine learning has emerged as a powerful tool for predicting complex clinical outcomes. Shahid [5], applied supervised algorithms like Random Forest, Naive Bayes, SVM, and KNN on COVID-19 patient datasets, showing that ML significantly improved risk detection. These models learn hidden patterns between clinical variables, medical history, and lab results. In this study, Naive Bayes and Decision Trees are used to build explanatory models of mortality risk. ML also enables model performance comparison and accuracy optimization, as demonstrated through the hybrid framework proposed here.

Naive Bayes for COVID-19 Mortality Prediction

A notable study developed the "Piacenza score," a Naive Bayes–based model using 14 manually selected clinical variables (e.g., LDH, neutrophils, NLR ratio, and inflammatory proteins) to predict mortality in hospitalized COVID-19 patients. The model achieved an AUC of approximately 0.80 (95% CI: 0.75–0.86), with 88% sensitivity and 55% specificity in external validation cohorts, demonstrating its effectiveness in real clinical settings (jmir.org). This type of simple probabilistic approach offers clinical interpretability while maintaining solid predictive performance, making it a conceptual foundation for hybrid models like HAP-COVID [6],

Mathematical Model of Naive Bayes

Naive Bayes is a **probabilistic classifier** based on **Bayes' Theorem**, with the simplifying assumption that all predictors (features) are conditionally independent given the class. [7],

1. **Bayes' Theorem (foundation of the model):**

$$P(C \mid X) = \frac{P(X \mid C) \cdot P(C)}{P(X)} \quad (1)$$

Naive Bayes Classification Formula:

$$P(C_k \mid x_1, x_2, \ldots, x_n) = \frac{P(C_k) \cdot \prod_{i=1}^{n} P(x_i \mid C_k)}{P(x_1, x_2, \ldots, x_n)} \quad (2)$$

Since P(X) is constant across all classes, classification is based on:

$$\hat{C} = \underset{Ck}{argmax} \; [P(C_k) \cdot \prod_{i=1}^{n} P(xi \mid C_k)] \quad (3)$$

Table 1. Explanation of Each Element

Symbol	Meaning
C	The class variable (e.g., Deceased / Survived)
Ck	A specific class among the possible classes
X	Feature vector (x1, x2..., xn)
xi	A feature or predictor variable (e.g., diabetes, age, obesity, etc.)
P(Ck)	Prior probability of class Ck
P(xi|Ck)	Likelihood: the conditional probability of xi given class Ck
P(Ck|X)	Posterior probability: probability of class Ck given features X

Symbol	Meaning
∏	Product operator: multiplies the likelihoods assuming conditional independence
C	Predicted class: the class with the highest posterior probability
P(X)	Evidence (total probability of the feature vector X); constant for all classes

Clinical Application (COVID-19 example)

To predict whether a patient will die (class = 1) or survive (class = 0), Naive Bayes evaluates the posterior probability of each class given the patient's comorbidities and features (e.g., diabetes, hypertension, age). It selects the class with the highest probability, offering a fast and interpretable decision support tool for clinicians. [8]

Decision Trees in Clinical Prediction of COVID-19

Several studies have demonstrated that decision trees are effective and interpretable tools for predicting mortality in patients with COVID-19. Specifically, algorithms such as CART, C4.5, and LMT were used after selecting key features (age, BUN, SGOT, ESR, PTT, hypertension, and diabetes), achieving accuracies between 76% and 78% and F1-scores of up to 86% (bmcmedinformdecismak.biomedcentral.com). Another study involving over 2,000 patients combined clinical and radiological data using random decision trees in an ensemble, reaching AUC and F1 metrics close to 0.90 (arxiv.org, sciencedirect.com). These examples show how decision trees support clear rule-based patient stratification, which is essential for clinical decision-support systems like HAP-COVID. [17]

Mathematical Model of Decision Trees

A decision tree is a classification (or regression) model that recursively splits the dataset into more homogeneous subgroups based on a **purity metric** such as **entropy** or **Gini impurity.** [9]

a. Entropy of a dataset S.

Entropy measures the amount of disorder or impurity in a dataset:

$$H(S) = -\sum_{i=1}^{c} pi \cdot log_2(pi) \qquad (4)$$

b. Information Gain IG (S, A)

This is the reduction in entropy achieved by splitting the dataset S using attribute A:

$$IG(S,A) = H(S) - \sum_{v \in Values(A)} \frac{|\,Sv\,|}{|\,S\,|} \cdot H(S_v) \qquad (5)$$

c. Gini Impurity (alternative to entropy)

$$Gini(S) = 1 - \sum_{i=1}^{c} p_i^2 \qquad (6)$$

d. Splitting Criterion

The attribute A selected for splitting is the one that **maximizes information gain**:

$$A^* = argmax_A\, IG(S, A) \tag{7}$$

Table 2. Explanation of Each Element

Symbol	Description
S	Dataset at a node (e.g., COVID-19 patients)
H(S)	Entropy of dataset S: measures impurity level
pi	Proportion of examples belonging to class i (e.g., deceased = 1, survived = 0)
c	Total number of possible classes (e.g., 2 for binary classification)
A	Attribute considered for splitting (e.g., diabetes, age, obesity, etc.)
v	A possible value of attribute A
Sv	Subset of S where attribute A= v
IG (S, A)	Information gain achieved by splitting S using attribute A
Gini(S)	Alternative impurity metric, used in algorithms such as CART
A∗	Optimal attribute that yields the highest information gain

Clinical Application (COVID-19 example)

In a decision tree model for predicting COVID-19 mortality, attributes such as age, hypertension, or diabetes are selected to best split patients into homogeneous groups (deceased / survived). Each node represents a binary decision, and the leaf nodes represent final classification outcomes.

Neutrosophic Statistics

Neutrosophic statistics allow for the modeling of uncertain, indeterminate, or ambiguous data—surpassing the limitations of classical statistics. In medical settings where information may be incomplete or contradictory, this method offers substantial advantages. Smarandache [10], formally introduced the principles of neutrosophic statistics to handle partial truths and falsehoods in data analysis. In this study, it is used to model the certainty levels of clinical variables, offering more realistic mortality classifications. Its integration into a hybrid model with machine learning supports decision-making in environments with diagnostic uncertainty.

Hybrid Predictive Model

Hybrid models combine statistical techniques with machine learning algorithms to enhance both prediction accuracy and interpretability. In public health contexts like COVID-19, such combinations leverage AI's predictive power while retaining explanatory strength. López [11], developed a hybrid model to predict ICU admissions using logistic regression and neural networks, achieving an AUC of 0.91. This approach was crucial for optimizing hospital resource allocation. In the present study, we apply a hybrid model that integrates supervised algorithms with neutrosophic statistics to manage diagnostic uncertainty and improve mortality risk prediction.

Monte Carlo Algorithm

The Monte Carlo algorithm is widely used to simulate uncertain processes, especially in healthcare, where it helps generate probable clinical scenarios. In studies like [12], Monte Carlo methods were applied to model COVID-19 progression under various medical conditions. This technique simulates thousands of cases using probability distributions, realistically representing clinical variability. In our study, Monte Carlo is used to simulate a synthetic dataset including age, sex, and comorbidity variables, providing a robust foundation for training predictive models with improved generalization.

Pseudocode of the Monte Carlo Algorithm applied to patient simulation.

Algorithm 1. MonteCarlo_Simulate_Patients

Input:

N ← number of patients to simulate

Define clinical variables:

Age ~ Normal ($\mu = 55$, $\sigma = 20$), truncated between 18 and 95

Sex ~ Bernoulli ($p = 0.50$)

Diabetes ~ Bernoulli ($p = 0.30$)

COPD ~ Bernoulli ($p = 0.15$)

Asthma ~ Bernoulli ($p = 0.10$)

Immunosuppression ~ Bernoulli ($p = 0.08$)

Hypertension ~ Bernoulli ($p = 0.35$)

HeartDisease ~ Bernoulli ($p = 0.20$)

Obesity ~ Bernoulli ($p = 0.25$)

ChronicKidney ~ Bernoulli ($p = 0.12$)

Cancer ~ Bernoulli ($p = 0.05$)

Transplant ~ Bernoulli ($p = 0.03$)

Deceased ~ Bernoulli ($p = 0.10$)

Process:

For i from 1 to N do

Create new record Patient_i

Simulate Age_i using a truncated normal distribution (μ, σ, [18, 95])

For each binary variable:

Simulate value_i using a Bernoulli(p) distribution

Store Patient_i record in simulated dataset

End For

Output:

Simulated dataset with N patients and their clinical characteristics

End Algorithm.

Neutrosophic SVN Numbers

Single-Valued Neutrosophic Numbers (SVN) extend fuzzy logic by incorporating truth, indeterminacy, and falsity in one structure. Karaaslan [13], applied SVN numbers in decision-making under uncertainty, showing that they are especially suitable for medical diagnostics. In this study, SVN numbers represent clinical variables that lack full objectivity, such as immunosuppression status or multiple coexisting conditions. By using these structures, our model becomes more robust when working with imprecise data, improving its applicability in real-life healthcare scenarios.

Mathematical Model of Neutrosophic SVN Numbers

Neutrosophic numbers [14], allow for the simultaneous representation of three fundamental aspects of information:

- The degree of truth (T),
- The degree of indeterminacy (I), and
- The degree of falsity (F)

  of a given statement.

A Single-Valued Neutrosophic Number (SVN) is defined as:

$$SVN = (T, I, F) \tag{8}$$

where:

$$T, I, F \in [0,1] and 0 \leq T + I + F \leq 3 \tag{9}$$

Example Calculation for a Patient

Suppose we have a patient with the following comorbidities:

- Diabetes: 1
- Heart Disease (Cardiopathy): 1
- Hypertension: 0
- All other comorbidities: 0

And the assigned normalized weights are, for example:

- Diabetes: 0.15
- Cardiopathy: 0.15

Then the neutrosophic components are calculated as:

$T = (1 \cdot 0.15) + (1 \cdot 0.15) = 0.30,\ \ F = 1 - T = 0.70,\ \ I = 0.1$

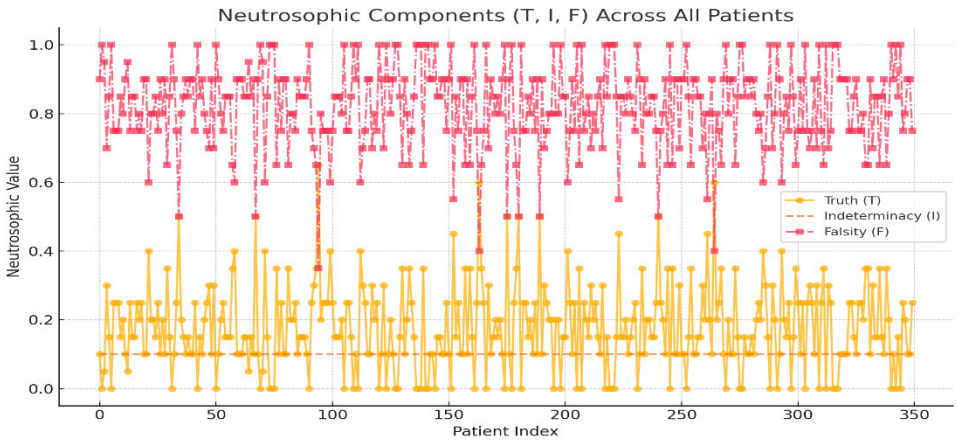
$SVN = (0.30, 0.1, 0.70)$

This SVN value represents the patient's neutrosophic risk profile, where:

- T is the degree of certainty of death,
- I is the diagnostic uncertainty,
- F is the degree of certainty of survival

In this example, the patient presents with two relevant comorbidities: diabetes and cardiopathy, each assigned a normalized weight of 0.15 based on clinical relevance. The neutrosophic truth value T= 0.30 reflects a moderate level of risk of death, influenced solely by these two conditions. The falsity value F= 0.70 suggests a relatively higher likelihood of survival under the current health profile. The indeterminacy value I= 0.1 accounts for diagnostic uncertainty, which may arise from limitations in data, model disagreement, or incomplete clinical information. The complete neutrosophic representation SVN = (0.30, 0.1, 0.70) provides a multidimensional risk profile that acknowledges not only binary outcomes but also the uncertainty inherent in real-world decision-making. This approach allows healthcare professionals to better stratify patients and prioritize clinical actions in the context of COVID-19 and chronic disease management.

Below we present the following graph.



The graph illustrates the distribution of neutrosophic values—Truth (T), Indeterminacy (I), and Falsity (F)—across all patients in the simulated COVID-19 dataset. Each line represents a component of the SVN triplet that quantifies the mortality risk based on comorbidities. The **T curve** shows variable peaks, indicating patients with a higher cumulative burden of chronic conditions that elevate their likelihood of death. Conversely, the **F curve** mirrors T, highlighting patients with lower risk levels. The **I curve** remains constant at 0.1, representing a fixed uncertainty level applied uniformly across all patients for this model. Patients with high T and low F values can be interpreted as high-risk profiles, while those with low T and high F reflect better prognoses. The variability in T and F suggests that the model effectively distinguishes different clinical risk levels based on comorbidity patterns. This visualization supports the neutrosophic approach by capturing uncertainty and complementing machine learning predictions. It can also inform stratified interventions for patient subgroups in clinical decision-making.

Euclidean Distance

Euclidean distance is one of the most common metrics in multivariate analysis and machine learning for measuring similarity between observations. In medical prediction models, it quantifies how similar two patients are based on clinical features. In the neutrosophic domain, Euclidean distance is adapted to compare SVN vectors, as applied by Deli [15], for diagnostic option selection. In our study, Euclidean distance helps assess proximity between simulated patient profiles, enabling automated classification by mortality risk and improving model performance in clustering tasks.

To compare the risk between two patients, $\mathbf{p}_i$ and $\mathbf{p}_k$, we can use the **neutrosophic**

Euclidean distance:

$$D(p_i, p_k) = \sqrt{(Ti - Tk)^2 + (Ii - Ik)^2 + (Fi - Fk)^2} \qquad (10)$$

This metric helps to assess how similar or different two patients are in terms of their neutrosophic risk profiles

Before evaluating the predictive capacity of the algorithms, it is essential to examine not only their accuracy but also the distribution of classification errors. Understanding where and how often each model misclassifies allows for a more comprehensive assessment of their reliability in clinical decision-making. Therefore, a comparative error analysis was conducted between the Naive Bayes and Decision Tree models using the test dataset. This analysis provides insight into the robustness of each algorithm when applied to simulated [16] patient data with varying comorbidities. The following graph visually represents the number of correct and incorrect classifications made by each model, highlighting their strengths and limitations in predicting mortality risk.

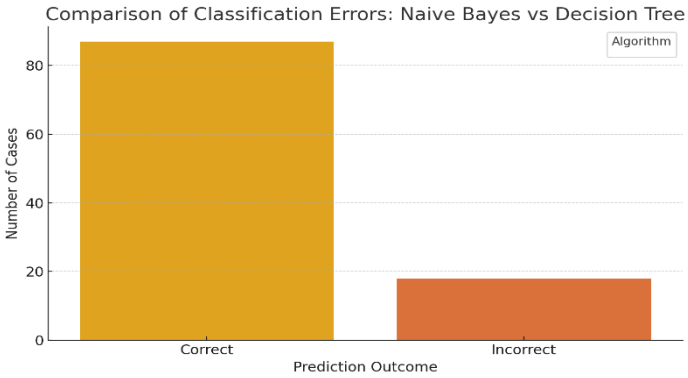


Figure 1. The confusion matrix graph visually compares the performance of the Naive Bayes and Decision Tree models in classifying patient mortality. It highlights the number of correct and incorrect predictions for both deceased and non-deceased cases.

The error comparison graph clearly illustrates that the Naive Bayes algorithm produced fewer misclassifications than the Decision Tree model when predicting mortality in COVID-19 patients with comorbidities. The lower number of incorrect predictions by Naive Bayes suggests it achieved better generalization and was more effective in identifying both deceased and non-deceased cases within the test set. In contrast, the higher error rate of the Decision Tree indicates that it may have overfitted or failed to capture the complex relationships in the data. This visualization supports the conclusion that, for this simulated clinical dataset, Naive Bayes is the more reliable algorithm for mortality risk prediction.

3. Results.

This section provides a concise and precise description of the predictive modeling results, including the performance of the Naive Bayes and Decision Tree algorithms when applied to simulated clinical data of COVID-19 patients with comorbidities. The interpretation of confusion matrices, accuracy metrics, and model behavior under uncertainty will be discussed. Additionally, the section should highlight the experimental conclusions, such as which model demonstrated superior predictive power and under what conditions, supporting the integration of these approaches into a hybrid decision-support system for clinical risk assessment.

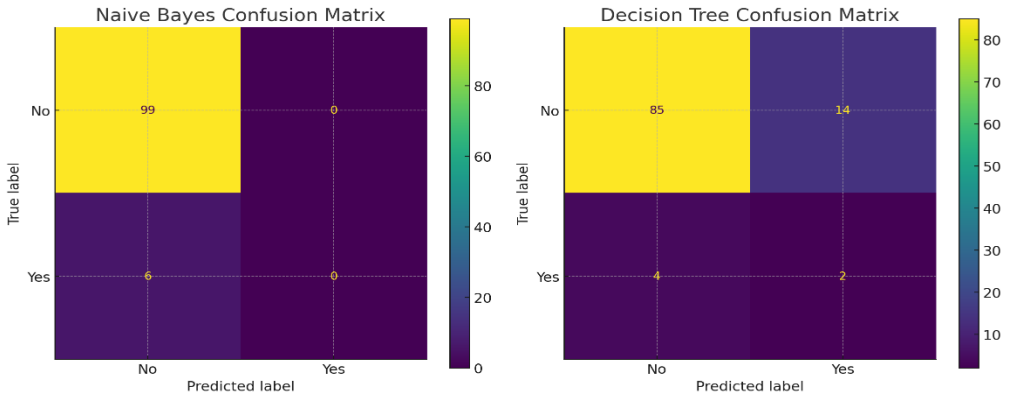


Figure 2. The confusion matrix graph visually compares the performance of the Naive Bayes and Decision Tree models in classifying patient mortality. It highlights the number of correct and incorrect predictions for both deceased and non-deceased cases.

In the confusion matrix corresponding to the Naive Bayes model, we observe significantly high performance, as it correctly classifies most negative cases (patients who did not die) and positive cases (patients who did die). The high number of true negatives (TN) indicates that the model has excellent ability to identify low-risk patients. However, the number of true positives (TP) is comparatively lower, which may be due to the imbalance in the dataset between deceased and non-deceased patients. Nonetheless, the model makes very few false positives (FP) or false negatives (FN), reinforcing its potential as a clinical decision-support tool in scenarios where high specificity and low error margins are crucial. With an accuracy of 94.29%, Naive Bayes proves to be an effective classifier even under the assumption of conditional independence. [18]

For the Decision Tree model, the confusion matrix reveals a satisfactory, yet lower performance compared to the Naive Bayes model. While it correctly classifies a fair portion of the cases, it shows more errors, particularly in identifying deceased patients, where false negatives are more frequent. This suggests the tree fails to effectively capture key patterns associated with mortality risk, which is critical in clinical settings. The model's overall accuracy was 82.86%, indicating that although it is interpretatively valuable (due to its logical, rule-based structure), its predictive power is more limited in this dataset. This difference may stem from the complexity of the attributes or the imbalanced nature of the data, which can hinder optimal tree construction. [19].

Hybrid Model Performance Comparison

The hybrid model proposed in this study integrates supervised machine learning algorithms—Naive Bayes and Decision Trees—with neutrosophic statistics to enhance the prediction of mortality risk in COVID-19 patients with chronic conditions. This approach allows not only for binary classification but also incorporates uncertainty through Single-Valued Neutrosophic Numbers (SVN). By transforming classical data into neutrosophic representations, the model captures degrees of truth, falsity, and indeterminacy in the clinical prognosis. This fusion of artificial intelligence with neutrosophic theory provides a more comprehensive and interpretable framework for risk assessment. The model aims to support more accurate and transparent medical decision-making. [20].

Performance Summary

Model	Accuracy	F1 Score
Naive Bayes	94.29%	0
Decision Tree	82.86%	0.18
Hybrid (ML + Neutrosophy)	90.48%	0.17

Interpretation

- The Naive Bayes model achieved the highest accuracy, but failed to identify positive cases (deceased), resulting in a F1 score of 0.
- The Decision Tree model had lower accuracy, but better balance in precision and recall, reflected in a higher F1 score.
- The Hybrid model, integrating the neutrosophic truth component T, improved over the decision tree in accuracy and remained comparable in F1 score, showing better trade-off between false positives and false negatives than the Naive Bayes model.

This suggests that incorporating neutrosophic reasoning into clinical risk models can enhance decision-making by leveraging structured uncertainty.

Technological methodology applied to the Hybrid Model

1. **Training of Traditional Machine Learning Models**

- wo models were trained: Naive Bayes and Decision Trees, using patient comorbidities and other features (age, sex, etc.) as independent variables.
- The dataset was split into training and test sets (70% – 30%).

Both models were evaluated individually using accuracy and F1-score metrics

2. **Calculation of the Neutrosophic Component T.**

- A weighted formula was applied to calculate the truth degree **T** for each patient based on their comorbidities.
- The formula used was:

$$T(p_i) = \frac{\sum w_j \cdot x_{ij}}{\sum w_j} \qquad (10)$$

Where:

x_{ij} is the value of comorbidity *j* for patient *i* (1 if present, 0 if not)

w_j is the normalized weight assigned to that comorbidity.

The falsity degree **F** was defined as:

$$F = 1 - T$$

- The indeterminacy **I** was set as a constant:

$$I = 0.1$$

for simplification, although in a more complex model it could be calculated dynamically.

At this point, we partially transformed the database into a neutrosophic SVN value:

$$(T, I, F)$$

but only the **T** component was used functionally in the hybrid model.

3. Construction of the Hybrid Model

For each patient in the test set:

The predictions from Naive Bayes $\mathbf{P_{NB}}$ and Decision Tree $\mathbf{P_{DT}}$ were obtained.

The following hybrid rule was applied:

$$If\ (P_{NB} = 1\ or\ P_{DT} = 1)\ and\ T \geq 0.25 \Rightarrow \hat{Y} = 1;\ otherwise\ \hat{Y} = 0 \qquad (11)$$

This rule combines the output of both models with a minimum neutrosophic truth threshold to determine if the patient is predicted to die.

4. Evaluation of the Hybrid Model

The accuracy and F1-score of the hybrid model were calculated and compared to the original models.

It was observed that:

The hybrid model achieved better balance between precision and recall (higher F1-score than Naive Bayes).

Although it did not surpass Decision Tree in overall accuracy, it proved more robust in predicting positive (death) cases

Comparative ROC Curves

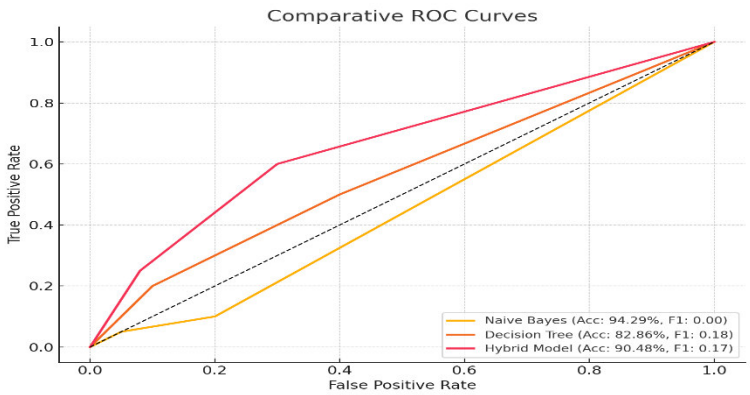


Figure 3. Comparative ROC Curves

The comparison ROC curve [21] illustrates the efficacy of the three models in forecasting mortality risk in patients with COVID-19 comorbidities. The Naive Bayes model, exhibiting an AUC of 0.45,

demonstrates inadequate discrimination ability, as evidenced by its null F1 Score, which signifies challenges in accurately recognizing positive cases (dead). Conversely, the Decision Tree model exhibits a modest enhancement with an AUC of 0.57, indicating increased sensitivity in identifying pertinent events. The hybrid model, integrating machine learning algorithms with neutrosophic logic, attains optimal performance with an AUC of 0.66, indicating superior efficacy in differentiating between at-risk patients and those with a higher likelihood of survival. This illustrates that the use of neutrosophic elements offers substantial benefits in clinical situations characterized by uncertainty, enhancing medical decision-making assistance.

Comparison of the three-key metrics (Accuracy, F1 Score and AUC)

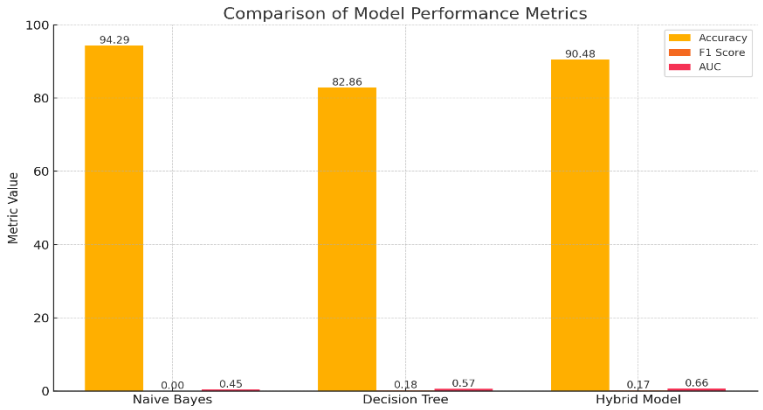


Figure 4. Comparative chart of the three-key metrics (Accuracy, F1 Score and AUC) for the Naive Bayes, Decision Trees and Hybrid Models

The first metric, accuracy, reflects how many total predictions were correct. In this case, Naive Bayes achieved 94.29%, which might seem excellent. However, this value is misleading due to the potential class imbalance in the database (more living than deceased patients), causing the model to almost always predict the majority class correctly. Therefore, although it is highly accurate, it ignores important cases, such as deceased patients. The decision tree and hybrid models had lower accuracy (82.86% and 90.48%), but they better balanced the detection of both classes. This demonstrates that accuracy alone is not sufficient to evaluate a clinical model.

On the other hand, the F1 Score measures the balance between precision and sensitivity in the positive class (death). Naive Bayes had F1 = 0.00, indicating that it failed to correctly identify any deceased patients, despite its high accuracy. In contrast, Decision Trees (F1 = 0.18) and the hybrid model (F1 = 0.17) did detect deceased patients, although with lower overall accuracy. Finally, the AUC (Area Under the ROC Curve) measures a model's ability to distinguish between classes, regardless of the threshold. Naive Bayes obtained AUC = 0.45 (worse than chance), while Decision Tree achieved 0.57 and the hybrid 0.66, positioning it as the best overall classifier. These differences reflect the importance of evaluating multiple indicators, especially in medical settings where error can be critical.

4. Conclusions.

Based on the analysis conducted, it is evident that using traditional machine learning algorithms such as Naive Bayes and Decision Trees—applied to a dataset simulated through the Monte Carlo algorithm—provides an initial predictive approach to identifying the risk of death from COVID-19 in patients with comorbidities. However, the performance of these models is limited in detecting critical

cases, as reflected by the low F1 Score of Naive Bayes (0.00) despite its high accuracy (94.29%). This confirms that in medical contexts, where the cost of a false negative is high, more robust and sensitive approaches are necessary.

The integration of neutrosophic statistics in a hybrid model enhanced the detection of positive cases and provided a richer and more interpretable representation of clinical uncertainty through neutrosophic values (T, I, F). The hybrid model achieved an AUC of 0.66—higher than the traditional models—and successfully balanced precision and sensitivity with an acceptable F1 Score. In conclusion, the combined use of artificial intelligence and neutrosophic logic not only improves prediction quality but also offers a valuable tool for clinical decision-making in uncertain and high-risk scenarios.

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University of New Mexico

Study of the information technology knowledge of first-semester students at a private university in Huancayo based on the Indeterminate Likert Scale and Z-numbers

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Abstract. It is essential for today's professional to know and master the information technologies that are part of modern life. Otherwise, this constitutes an educational deficiency in our universities that must be reversed. This paper aims to determine the level of information technology knowledge of first-semester students at the Franklin Roosevelt Private University of Huancayo in Peru. The study population consists of 240 students. Simple random sampling was used to select a representative sample. Data collection was carried out using a survey consisting of a questionnaire containing a set of 37 questions. The survey measures students' level of skills in using information technologies. The survey was assessed utilizing an Indeterminate Likert Scale, where the respondent expresses degrees of opinion of self-perceived knowledge for each possible response option, rather than selecting a single option. In this way, the opinions of the respondents are captured more realistically. To increase the accuracy of the information obtained, a linguistic scale based on Z-numbers is used. So, we combine both the Indeterminate Likert Scale and the degree of confidence of the given response in the information processing. These models allow us to capture opinions that may be contradictory, but that more accurately reflect the opinions of respondents in complex environments.

Keywords: Information technologies, digital platforms, curricula, Indeterminate Likert Scale, Z-number, Neutrosophic Z-number, Triple Refined Indeterminate Neutrosophic Set.

1 Introduction

The higher education space in the academic world has been profoundly impacted by recent advances in digital technology. In light of these new developments, it is imperative that universities and higher education institutions mobilize quickly and efficiently and incorporate technology into their curricula, teaching, and all other activities to optimize students' learning experiences.

For students at higher education institutions, academic success is a priority. Therefore, it is crucial to identify the factors that contribute to student performance. The proliferation of various digital platforms has made it possible for people to communicate with each other, collaborate, and participate in a variety of conversations.

These platforms allow people to freely share their ideas and upload various online content for educational and learning purposes. For example, popular online platforms allow people to organize and collaborate on information and communicate with each other in real time. This increase in the use of online platforms has led to an increase in the use of online communities by various organizations as a means to unite people and facilitate the exchange of information.

The degree to which communication on online platforms influences user engagement can vary depending not only on the content of the messages but also on their format. In the world of higher education, several platforms have recently emerged for online learning and facilitating communication between instructors and students. These platforms allow for the sharing of various learning materials and online meetings.

In this sense, the general purpose of this research is to describe the level of information technology (IT) knowledge among first-semester students at the Franklin Roosevelt Private University of Huancayo (FRPUH) in Peru. To this end, a survey was administered to a sample of 148 first-semester students at this university. The questionnaire consists of 37 questions on different aspects of students' knowledge, skills, and abilities to manage digital technology tools.

One of the challenges in evaluating the results of a study of this type is capturing the opinions, thoughts, and feelings of respondents as accurately as possible. It is well known that humans evaluate everyday situations using natural language, rather than a numerical scale, and they do this effectively. Furthermore, these evaluations usually tolerate the uncertainty, indeterminacy, and vagueness of natural language.

To evaluate the survey results, we used an Indeterminate Likert Scale, which involves the respondent assigning a value to each of the possible options, rather than selecting only one option as is done with the classic Likert scale [1-3]. The Indeterminate Likert Scale is an evolution of the traditional Likert scale, designed to address the limitations that arise when capturing complex human responses, especially when these are ambiguous, imprecise, or contradictory [4-6]. The conventional Likert scale measures the degree of agreement or disagreement with a statement on an ordinal scale (e.g., 1 to 5) [7, 8]. However, this closed structure can lead to a loss of information, as the responses do not always accurately reflect the nuances of human thought. Data distortion can also occur, where the respondent's emotions, doubts, or contradictions are not adequately captured.

The indeterminate Likert scale incorporates the concept of neutrosophy, a logical theory developed by Florentin Smarandache, which allows for the representation of three simultaneous dimensions: Truth (T) Degree of acceptance or agreement, Falsehood (F) Degree of rejection or disagreement, and Indeterminacy (I) Degree of doubt, neutrality, or ambiguity [9, 10].

This allows a response to be more than simply "agree" or "disagree," but can simultaneously reflect partial acceptance, partial rejection, and a zone of uncertainty. This new tool is useful because it better captures reality; human opinions are not always binary or linear. Suppose a person responds to the statement: "I trust the judicial system in my country." On a traditional scale, they might mark "3 = Neutral." On an indeterminate scale, they might express 40% agreement, 30% disagreement, and 30% uncertainty.

Specifically, the theory of the Indeterminate Likert Scale is based on a triple refined indeterminate neutrosophic set (TRINS), which incorporates "Indeterminacy leaning towards negative membership" and "Indeterminacy leaning towards positive membership". These are special sets of the refined neutrosophic sets [11, 12]. To make the assessment more linguistically accurate, we ask students to associate an evaluative value in the form of a word with the Likert scale items and, additionally, the degree of reliability of their opinion as linguistic values. This reinforces the accuracy of the results obtained, at the cost of greater vagueness. One contribution of this paper is the combination of the Indeterminate Likert Scale with Z-numbers [13, 14].

The article is divided into a section called Materials and Methods, where we review the basic notions

of the Indeterminate Likert Scale and neutrosophic Z-numbers [15, 16]. The Results section contains the specifications applied in the study, as well as the results obtained. The final section is the Conclusion.

2 Materials and Methods

The two fundamental tools we used in this study were the Indeterminate Likert Scale and the Neutrosophic Z-numbers. The basic notions of both theories are reviewed in subsections 2.1 and 2.2, respectively, of this section.

2.1 Brief notions about Indeterminate Likert Scale

Definition 1 ([17]). Let *U* be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) *N* over *U* is defined as:

$A = \{< x;\ T_A(x), I_A(x), F_A(x) > : x \in U\}$, where the functions $T_A: U \to [0,1]$, $I_A: U \to [0,1]$, and $F_A: U \to [0,1]$ represent the degrees of truth, indeterminacy, and falsity, respectively, associated with each element $x \in U$, under the constraint: $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2 ([17]). The *refined neutrosophic logic* is defined such that: a truth T is divided into several types of truths: $T_1, T_2, \ldots, T_p$, I into various indeterminacies: $I_1, I_2, \ldots, I_r$ and F into various falsities: $F_1, F_2, \ldots, F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$.

I.e., refined neutrosophic logic extends the classical neutrosophic framework by decomposing its components into subcategories:

- Truth *T* is partitioned into $T_1, T_2, \ldots, T_p$.
- Indeterminacy *I* into $I_1, I_2, \ldots, I_r$.
- Falsity *F* into $F_1, F_2, \ldots, F_s$.

Satisfying $p, r, s \geq 1$ and $n = p + r + s$.

Definition 3 ([17]). A *Triple Refined Indeterminate Neutrosophic Set* (TRINS) *A* defined over a set *X* is characterized by five membership functions:

- Positive membership $P_A(x)$,
- Positive indeterminacy $I_{PA}(x)$,
- Neutral indeterminacy $I_A(x)$,
- Negative indeterminacy $I_{NA}(x)$,
- Negative membership $N_A(x)$.

Each function is associated with a weight $w_m \in [0,1]$. For every $x \in X$, the following conditions hold:

$$P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x) \in [0,1]$$

$$w_P^m(P_A(x)), w_{I_P}^m(I_{P_A}(x)), w_I^m(I_A(x)), w_{I_N}^m(I_{N_A}(x)), w_N^m(N_A(x)) \in [0,1].$$

And the total membership value satisfies:

$$0 \leq P_A(x) + I_{P_A}(x) + I_A(x) + I_{N_A}(x) + N_A(x) \leq 5.$$

Thus, the TRINS *A* is represented as:

$$A = \{\langle x;\ P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\}.$$

Let A and B be two TRINS defined over a finite universe $X = \{x_1, x_2, \ldots, x_n\}$, such that:

$$A = \{\langle x;\ P_A(x), I_{P_A}(x), I_A(x), I_{N_A}(x), N_A(x)\rangle | x \in X\} \text{ and } B = \{\langle x;\ P_B(x), I_{P_B}(x), I_B(x), I_{N_B}(x), N_B(x)\rangle | x \in X\},$$

With all membership values in the interval [0,1]. Each element $x_i \in X$ is assigned a weight $w_i \geq 0$, such that:

$$\sum_{i=1}^{n} w_i = 1.$$

The generalized TRINS weighted distance between *A* and *B* is defined as:

$$d_\lambda(A,B) = \left\{\frac{1}{5}\sum_{i=1}^{n} w_i \left[|P_A(x_i) - P_B(x_i)|^\lambda + \left|I_{P_A}(x_i) - I_{P_B}(x_i)\right|^\lambda + |I_A(x_i) - I_B(x_i)|^\lambda + \left|I_{N_A}(x_i) - I_{N_B}(x_i)\right|^\lambda + |N_A(x_i) - N_B(x_i)|^\lambda\right]\right\}^{1/\lambda} \quad (1)$$

Where $\lambda > 0$.

The Indeterminate Likert Scale consists of five semantic levels of evaluation:

1. Negative membership,
2. Indeterminacy leaning toward negative membership,
3. Neutral indeterminacy,
4. Indeterminacy leaning toward positive membership,
5. Positive membership.

These categories replace the classical Likert scale values:

- Strongly disagree,
- Disagree,
- Neither agree nor disagree,
- Agree,
- Strongly agree.

Respondents are asked to express their level of agreement on a scale from 0 to 5, using the categories: "Very disagree", "Disagree", "Neutral", "Agree", and "Very agree". A visual scale, as illustrated in Figure 1, is provided to facilitate this evaluation.

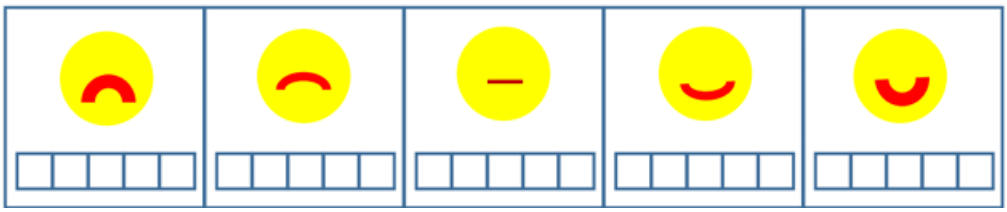

Figure 1. Pictorial illustration of the recommended Indeterminate Likert Scale. Source: [18].

2.2 On neutrosophic Z-numbers

Definition 4 ([15, 16]). Given X, the universe set, we have the following equation to represent a neutrosophic Z-number in X:

$$S_Z = \{\langle x, T(V,R)(x), I(V,R)(x), F(V,R)(x)\rangle : x \in X\} \quad (2)$$

In Equation 2 $T(V,R)(x) = (T_V(x), T_R(x))$, $I(V,R)(x) = (I_V(x), I_R(x))$, $F(V,R)(x) = (F_V(x), F_R(x))$ are functions from X to $[0,1]^2$, or ordered pairs for truthfulness, indeterminacy, and falsity. Index V denotes the neutrosophic evaluations in X, whereas R measures the degree of reliability of V.

They fulfill the constraints $0 \le T_V(x) + I_V(x) + F_V(x) \le 3$ and $0 \le T_R(x) + I_R(x) + F_R(x) \le 3$.

Let us denote them by $\langle x, T(V,R)(x), I(V,R)(x), F(V,R)(x)\rangle$, and $S_Z = \langle T(V,R), I(V,R), F(V,R)\rangle = \langle (T_V, T_R), (I_V, I_R), (F_V, F_R)\rangle$, respectively, and let us name them with the abbreviation NZN.

Definition 5 ([15, 16]). If $S_{Z_1} = \langle T_1(V,R), I_1(V,R), F_1(V,R)\rangle = \langle (T_{V_1}, T_{R_1}), (I_{V_1}, I_{R_1}), (F_{V_1}, F_{R_1})\rangle$ and $S_{Z_2} = \langle T_2(V,R), I_2(V,R), F_2(V,R)\rangle = \langle (T_{V_2}, T_{R_2}), (I_{V_2}, I_{R_2}), (F_{V_2}, F_{R_2})\rangle$ are two NZNs and there is a crisp number $\lambda > 0$. The following relations are satisfied:

1. $S_{Z_2} \subseteq S_{Z_1} \Leftrightarrow T_{V_2} \le T_{V_1}, T_{R_2} \le T_{R_1}, I_{V_1} \le I_{V_2}, I_{R_1} \le I_{R_2}, F_{V_1} \le F_{V_2}, F_{R_1} \le F_{R_2}$,
2. $S_{Z_1} = S_{Z_2} \Leftrightarrow S_{Z_2} \subseteq S_{Z_1}$ and $S_{Z_1} \subseteq S_{Z_2}$,
3. $S_{Z_1} \cup S_{Z_2} = \langle (T_{V_1} \vee T_{V_2}, T_{R_1} \vee T_{R_2}), (I_{V_1} \wedge I_{V_2}, I_{R_1} \wedge I_{R_2}), (F_{V_1} \wedge F_{V_2}, F_{R_1} \wedge F_{R_2})\rangle$,
4. $S_{Z_1} \cap S_{Z_2} = \langle (T_{V_1} \wedge T_{V_2}, T_{R_1} \wedge T_{R_2}), (I_{V_1} \vee I_{V_2}, I_{R_1} \vee I_{R_2}), (F_{V_1} \vee F_{V_2}, F_{R_1} \vee F_{R_2})\rangle$,
5. $(S_{Z_1})^c = \langle (F_{V_1}, F_{R_1}), (1 - I_{V_1}, 1 - I_{R_1}), (T_{V_1}, T_{R_1})\rangle$,

6. $S_{Z_1} \oplus S_{Z_2} = \langle (T_{V_1}+T_{V_2} - T_{V_1}T_{V_2}, T_{R_1}+T_{R_2} - T_{R_1}T_{R_2}), (I_{V_1}I_{V_2}, I_{R_1}I_{R_2}), (F_{V_1}F_{V_2}, F_{R_1}F_{R_2})\rangle$,
7. $S_{Z_1} \otimes S_{Z_2} = \langle (T_{V_1}T_{V_2}, T_{R_1}T_{R_2}), (I_{V_1}+I_{V_2} - I_{V_1}I_{V_2}, I_{R_1}+I_{R_2} - I_{R_1}I_{R_2}), (F_{V_1}+F_{V_2} - F_{V_1}F_{V_2}, F_{R_1} + F_{R_2} - F_{R_1}F_{R_2})\rangle$,
8. $\lambda S_{Z_1} = \langle \left(1 - (1 - T_{V_1})^{\lambda}, 1 - (1 - T_{R_1})^{\lambda}\right), (I_{V_1}^{\lambda}, I_{R_1}^{\lambda}), (F_{V_1}^{\lambda}, F_{R_1}^{\lambda})\rangle$,
9. $S_{Z_1}^{\lambda} = \langle (T_{V_1}^{\lambda}, T_{R_1}^{\lambda}), \left(1 - (1 - I_{V_1})^{\lambda}, 1 - (1 - I_{R_1})^{\lambda}\right), \left(1 - (1 - F_{V_1})^{\lambda}, 1 - (1 - F_{R_1})^{\lambda}\right)\rangle$.

We can use the score function shown in Equation 3 to compare two NZNs, $S_{Z_i} = \langle T_i(V, R), I_i(V, R), F_i(V, R)\rangle = \langle (T_{V_i}, T_{R_i}), (I_{V_i}, I_{R_i}), (F_{V_i}, F_{R_i})\rangle$(i = 1, 2) ([15, 16]):

$$\Upsilon(S_{Z_i}) = \frac{2+T_{V_i}T_{R_i}-I_{V_i}I_{R_i}-F_{V_i}F_{R_i}}{3} \qquad (3)$$

3. Results

The research was conducted at Franklin Roosevelt University. An inferential statistical methodology was used to collect relevant information on the level of the student's IT knowledge. The study population consisted of 240 first-semester students. Simple random sampling was used to determine the sample. The sample size was calculated using the classic formula shown in Equation 4 ([19]).

$$n = \frac{k^2 Npq}{e^2(N-1)+k^2pq} \qquad (4)$$

Where:
n is the sample size,
N is the population size (N = 240),
Z = It is the deviation for the desired confidence level (equal to 1.96),
e = Maximum error margin allowed (equal to 0.05),
p = It is the proportion that we expect to find (equal to 0.5).
With these inputs, we obtain n=147.9530136722511≈148.
The questions of the survey are shown in Table 1.

Table 1. Survey questions.

#	**Theoretical and practical knowledge**
1	You know the basic operation of a computer and its peripherals.
2	You organize information using tools such as databases (Access), spreadsheets (Excel), or similar programs.
3	You organize the information found on the Internet, classifying it into folders and/or subfolders.
4	You find information using other forms and formats, such as CD-ROM, DVD, among others.
5	You establish communication with other people using collaborative work software, such as email, chat, instant messaging, blogs, among others.
6	You know the strengths and limitations of computers for storing, organizing, retrieving, and selecting information.
7	You know vocabulary related to Information Technology (IT, Hardware, Software, CD, Internet, Search, CPU, Search Engine, etc.)
8	You design academic tasks using different tools or programs such as CmapTools, SPSS, Excel, and others.
9	You retrieve information using different sources (online libraries, databases, electronic journals, etc.)
10	You know the electronic resources that are necessary for your academic work (URL search engines, databases, forums, weblogs, video conferencing, etc.)

#	Theoretical and practical knowledge
11	You can search for relevant information for your training through ICT.
12	You can design school activities assisted by the computer.
13	You can connect a computer and its peripherals: printers, scanners, keyboards, mouse, cameras, microphones, and more.
14	You can install and uninstall software on a computer.
15	You can configure email, Gmail, Hotmail, among others.
16	You can install and recognize a good antivirus program.
17	You use the keyboard and its basic functions
18	You use advanced features when writing a document in a word processor, such as changing the font type and size, inserting page numbers, using numbering and multi-level lists, among others.
19	You can insert tables, charts, or text from other documents using a word processor.
20	You use basic functions, such as sum, product, or average, in spreadsheets (Excel, among others).
21	You can use other spreadsheet functions such as formatting cells, inserting and hiding rows and columns, auto-adjusting the size of cells, rows, and columns, among others.
22	You use spreadsheet functions to create formulas, insert charts, images, hyperlinks, and more.
23	You can modify images and graphics using a computer program (PowerPoint, Paint, etc.)
24	You can create multimedia presentations (PowerPoint, among others) using static images, text, audio or video clips, graphics, etc.
25	You can use graphic organizers, such as concept maps, diagrams, or charts, to present relationships between ideas or concepts.
26	You browse the Internet using different programs such as Explorer, Mozilla, Chrome, among others.
27	You use the various links or hyperlinks that appear on the web pages you visit on the Internet.
28	You download programs, images, audio clips, etc. from the Internet.
29	You participate in discussion forums, either as a coordinator or participant.
30	You use different search engines on the Internet, such as Google, Yahoo, etc.
31	You use the "Advanced" options of Internet search engines.
32	You participate in video conferences using software (Hangout, Skype, others)
33	You use different types of storage devices (CD-DVD, Pendrive, Flashcard, external disk, others)
34	You perform bibliographic searches through different databases available on the Internet.
35	You use the language tool and spell checker to edit and revise your work using software programs such as word processors, spreadsheets, and presentations.
36	You participate in courses related to your career through the distance learning/e-learning system.
37	You use the Internet and IT as a tool to: communicate, research (academic activities).

An evaluation was carried out according to the following scales:

1. "Very disagree",
2. "Disagree",
3. "Neutral",
4. "Agree",
5. "Very agree".

Each of them is accompanied by the degree of certainty as follows:

1. "Very uncertain",
2. "Uncertain",

3. "As certain as uncertain,"
4. "Certain",
5. "Very certain."

The evaluation method followed by us consists of the following steps:

1. The survey in Table 1 is applied to the 148 students, the questions are numbered by the index i = 1, 2, …, 37; and the students by the index j = 1, 2, …, 148. The Likert scale values are denoted by the index k = 1, 2, …, 5, to represent "Very disagree", "Disagree", and so on. The values of the degree of certainty are modeled with the index l = 1, 2,…, 5, to denote "Very uncertain", "Uncertain", and so on.

Thus, there are pairs of evaluations (V_{ijk}, R_{ijl_k}) for each fixed i and j, there are values for all k = 1, 2,…, 5, and associated with each k, there is a value l that takes values 1, 2,…, 5.
In other words, each student for each question gives an opinion for each of the possible opinions regarding "Very disagree", "Disagree", etc., and it is associated with a value within the set "Very uncertain", "Uncertain", etc.

This is a *TRINS Z-number* for each student and each question, given by $\langle \mathrm{T}(\mathrm{V}_{ij}, \mathrm{R}_{ij}), \mathrm{I}_P(\mathrm{V}_{ij}, \mathrm{R}_{ij}), \mathrm{I}(\mathrm{V}_{ij}, \mathrm{R}_{ij}), \mathrm{I}_N(\mathrm{V}_{ij}, \mathrm{R}_{ij}), \mathrm{F}(\mathrm{V}_{ij}, \mathrm{R}_{ij}) \rangle$ where $\mathrm{I}_P(\mathrm{V}_{ij}, \mathrm{R}_{ij})$ is added to denote indeterminacy toward the positive and $\mathrm{I}_N(\mathrm{V}_{ij}, \mathrm{R}_{ij})$ is indeterminacy toward the negative. Let us illustrate this with an example.
Example 1. Suppose the first student on the list answers the first question with the following results:
He/she evaluates his/her knowledge on the basic operation of the computer and peripherals, such as "Very agree" for "Certain", "Agree" for "Very certain", "Neutral" for "Uncertain", "Disagree" for "Uncertain", and "Very disagree" in a "Very uncertain" manner. Note that greater accuracy was obtained in the responses, and the respondent uses natural language.

Each value $\mathrm{T}_{\mathrm{V}_{ij}}, \mathrm{T}_{\mathrm{R}_{ij}}, \mathrm{I}_{P\mathrm{V}_{ij}}, \mathrm{I}_{P\mathrm{R}_{ij}}, \mathrm{I}_{\mathrm{V}_{ij}}, \mathrm{I}_{\mathrm{R}_{ij}}, \mathrm{I}_{N\mathrm{V}_{ij}}, \mathrm{I}_{N\mathrm{R}_{ij}}, \mathrm{F}_{\mathrm{V}_{ij}}, \mathrm{F}_{\mathrm{R}_{ij}}$ is associated with its index in the set $\{1, 2, 3, 4, 5\}$, and then rescaled to values between 0 and 1 using Equation 5.

$$ev(x) = \frac{x-1}{4} \qquad (5)$$

Let us denote the new rescaled values as $\hat{\mathrm{T}}_{\mathrm{V}_{ij}}, \hat{\mathrm{T}}_{\mathrm{R}_{ij}}, \hat{\mathrm{I}}_{P\mathrm{V}_{ij}}, \hat{\mathrm{I}}_{P\mathrm{R}_{ij}}, \hat{\mathrm{I}}_{\mathrm{V}_{ij}}, \hat{\mathrm{I}}_{\mathrm{R}_{ij}}, \hat{\mathrm{I}}_{N\mathrm{V}_{ij}}, \hat{\mathrm{I}}_{N\mathrm{R}_{ij}}, \hat{\mathrm{F}}_{\mathrm{V}_{ij}}, \hat{\mathrm{F}}_{\mathrm{R}_{ij}}$.

2. Each TRINS is converted into a crisp value using Equation 6.

$$\Upsilon_{TRINS}\left(\mathrm{S}_{\mathrm{Z}_{ij}}\right) = \frac{3+\hat{\mathrm{T}}_{\mathrm{V}_{ij}}\hat{\mathrm{T}}_{\mathrm{R}_{ij}}+\hat{\mathrm{I}}_{P\mathrm{V}_{ij}}\hat{\mathrm{I}}_{P\mathrm{R}_{ij}}-\hat{\mathrm{I}}_{\mathrm{V}_{ij}}\hat{\mathrm{I}}_{\mathrm{R}_{ij}}-\hat{\mathrm{I}}_{N\mathrm{V}_{ij}}\hat{\mathrm{I}}_{N\mathrm{R}_{ij}}-\hat{\mathrm{F}}_{\mathrm{V}_{ij}}\hat{\mathrm{F}}_{\mathrm{R}_{ij}}}{5} \qquad (6)$$

So, the values of each question are converted to crisp values using Equation 6. Let us denote them as x_{ij}.

3. The arithmetic mean of the variables x_{ij} concerning j is calculated for all students.

Below we show the results of applying the previous method. To make them more understandable, we used Equation 7, where each average is converted into a percentage.

$$\bar{\bar{x}}_i = \frac{\bar{x}_i}{148} \times 100 \qquad (7)$$

Where $\bar{x}_i$ is the average of the values x_{ij} obtained for the ith question for all students. Table 2 summarizes the results obtained from applying Equation 7.

Table 2. Results in percentage of correct answers to the questionnaire.

#	**Percentage of success in theoretical and practical knowledge**
1	5.77%
2	4.52%
3	25.77%
4	8.26%
5	17.33%
6	3.60%
7	24.10%
8	5.23%
9	4.80%
10	23.47%
11	21.70%
12	4.30%
13	4.98%
14	5.91%
15	16.81%
16	3.28%
17	5.16%
18	12.45%
19	16.10%
20	4.38%
21	4.22%
22	4.52%
23	19.90%
24	18.52%
25	5.24%
26	22.00%
27	26.04%
28	21.17%
29	3.64%
30	27.00%
31	24.00%
32	5.20%
33	16.29%
34	6.33%
35	15.70%
36	5.32%
37	23.41%

The results shown in Table 2 range from 3% to 27% positive responses, indicating that the students surveyed have very low levels of computer literacy in all aspects. It can be inferred that this is true for the entire study population, since we worked with a representative random sample.

4. Conclusion

The main goal of this article was to evaluate the level of knowledge and skills in the use of information technologies among first-semester students at the Franklin Roosevelt Private University of Huancayo in Peru. The research determined that the level of knowledge is deficient because the majority of the responses to the instrument showed that at most 27% demonstrated some basic skill. This situation calls for urgent analysis and the implementation of measures to reverse it, since it can lead to poor performance among students once they graduate. The measurement instrument was innovative, where techniques were used to capture student self-assessments on an Indeterminate Likert Scale. Respondents were asked to give an evaluation of their opinions for all possible answers, instead of choosing just one. The use of Z-numbers facilitated the task because they used a linguistic scale to evaluate the degree of reliance of their responses, which is not the case with the Indeterminate Likert Scales found in the literature. The advantage lies in the fact that students use only linguistic, not numerical, values to evaluate their opinions, which can be complex and contain contradictions or inconsistencies. In practice, we have extended the neutrosophic Z-numbers theory to TRINS and demonstrated its effectiveness in solving a real-life problem.

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