Determining Magnitude: Leibniz's Legacy in the Eighteenth Century

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This essay gives a critical account of eighteenth-century attempts to distinguish between indeterminate and determinate magnitudes. For decades in the wake of Leibniz, articulating this distinction was seen as crucial for getting a correct epistemology and metaphysics for mathematics. However, this project ran into a series of internal difficulties, arising from tensions within post-Leibnizian epistemology, metaphysics, and philosophy of mathematics. I also aim to show that these problems are a neglected part of the background to Kant's Critical philosophy; it is an open question, which I do not try to resolve here, whether Kant convincingly addressed all of them.

Leibniz motivates attention to the indeterminate–determinate magnitude distinction with his suggestion that magnitudes cannot be known in isolation, but only through comparison and co-perception. If a metaphysics of magnitude is directly read off of this epistemological account, then mathematical truths—insofar as they are truths about magnitudes—look to be contingent.

Whether or not this reading is correct, it had a wide influence in the eighteenth century. Christian Wolff and Alexander Baumgarten seek to preserve much of Leibniz's epistemology of magnitude, while also avoiding the *problem of contingency*, that is, the threat that mathematical truths turn out contingent. Wolff and Baumgarten try to exploit a distinction between indeterminate and determinate magnitude here. They propose that some basic properties of indeterminate magnitude are metaphysically independent of comparison and co-perception. But when it comes to epistemology, a full grasp of magnitude as determinate does require these activities, and therefore some further contribution from the subject. This distinction is also taken up by philosophers outside the Leibnizian tradition, notably Crusius.

However, trying to articulate the determinate–indeterminate magnitude distinction leads to further difficulties. First, Wolff appeals to relations to help work out the determination of magnitude. However, he vacillates between more realist and idealist accounts of the relevant relations (and thus of magnitudes themselves). In the end, neither option seems consistent with his restrictive metaphysics of relations, which only countenances dependence relations among genuine substances, but also denies direct dependence relations between mind and matter. This *problem of relations* has not been fully appreciated. On standard readings Wolff is seen as ruling out, on logical grounds, any appeal to relations.

Next, Baumgarten takes up the same basic strategy as Wolff. His account is unambiguously a realist one, however, to such an extent that his epistemology and metaphysics seem to leave no room for indeterminate magnitude. Baumgarten's direct realist account of perception entails that we are in epistemic contact with fully determinate magnitudes. On the metaphysical side, although he makes a well-known distinction between the determinable and the determined, he thinks that all actual individuals are fully determined, and also that mathematical objects are actual individuals. So Baumgarten faces the *problem of the determinable*: he loses any workable distinction between determinable and determinate magnitudes.

Christian Crusius's eclectic theory of magnitude makes some of the same epistemological assumptions as the post-Leibnizian rationalist tradition. He breaks with them on a number of metaphysical points, however. Crusius commits to absolute space and holds that all magnitudes result from combining more basic essences. However, these two commitments are in serious tension with one another, since they respectively suggest whole-to-part and part-to-whole determination of magnitudes. This *part–whole priority problem* also surfaces in Kant, but to my knowledge, its prehistory in Crusius has not been discussed.

Lastly, I turn to Kant. I aim to show three things. First, Kant was well aware of his predecessors' struggles with these problems. Second, he engages with them and even endorses some of their views. Finally, important questions remain open about the success of Kant's own responses.

For the problem of contingency, Kant's answer is well-known—and strong, if one is willing to countenance his transcendental idealism. Regarding relations, Kant also would seem to be on firmer ground than his predecessors, but I contend that he faces internal tensions regarding the modal status of specifically mathematical relations. As for the part–whole priority problem, Daniel Sutherland has recently offered a sophisticated solution on Kant's behalf, drawing on Baumgarten's earlier conception of determination. However, I suggest that this does not fully settle the issue. Instead, the question shifts to whether Kant and Baumgarten can adequately distinguish determinate and indeterminate magnitudes. The answer to this remains unclear.

A word about the decision to treat a number of philosophers in one essay. The tangles of post-Leibnizian metaphysics can, I think, obscure structural similarities that become visible from a broader view. In this case, we'll see that not only Wolff and Baumgarten but also Crusius offer related responses to shared problems. In turn, this better reveals Kant's relationship to his predecessors. He developed his own philosophy of mathematics in part through adjudicating the complex debates *between* Wolffians and opponents such as Crusius. To appreciate his point of view, then, we should not just consider these earlier thinkers individually.

To preview some results of taking this broader standpoint, let me note two ways in which I diverge from most of the literature in understanding Kant's relationship to these figures. First, Kant's predecessors do not, in practice, attempt to reduce geometrical propositions to analytic or conceptual truths. Instead, a persistent theme is that perception or sense is a necessary condition for properly cognizing magnitudes.[1] The key inspiration is not Leibniz's work in logic and combinatorics, but his attempts to found geometry on relations of situation (*situs*). The post-Leibnizians insist that for finite beings like us, the epistemic role played by perception in geometry *cannot* be played by the understanding. They refuse to reduce determination to the genus–species relations of Aristotelian logic, even if it's unclear what alternative they develop. This contrasts with Lanier Anderson's (2015) influential reading of Wolff and Baumgarten as proto-logicists who seek to transform all propositions into analytic truths—and of Kant as countering them by using mathematical knowledge to establish the synthetic a priori. Meanwhile, I find that Crusius takes a more conceptualist line on mathematics, putting less emphasis than the Wolffians on direct perception.

A second lesson is that Kant's Copernican Turn does not provide quick solutions to most of these problems. His account does plausibly resolve the problem of contingency by abandoning the reliance on perceptual observation in earlier accounts, in favour of pure intuition. But that solution turns on a distinction *among* types of intuition (empirical versus pure), rather than on his contention that mathematics contains not only synthetic but also analytic propositions. The verdict is anyhow less clear for the other three problems, where Kant appears to face the same root difficulties around relations and the determination and

[1] I'm much indebted to De Risi (2007) and Sutherland (2005, 2022), who stress the role of perception in geometry among these thinkers. But they mainly focus, respectively, on Leibniz and Kant. I aim at a broader and more systematic treatment of the metaphysics and epistemology of the intervening figures. This in turn sheds new light on how Kant engaged with their ideas.

measurement of quantities. Therefore, it is rewarding to consider Kant as a participant in these pre-Critical debates, rather than assuming he transcends them by way of his idealism.

1. Leibniz and the Problem of Contingency

Leibniz was not satisfied with the existing foundations of geometry: Euclid left too much without proof and relied excessively on imagination, while the algebraic geometry of Descartes and Viète failed to express geometrical properties directly. So Leibniz sought a new foundational analysis of geometry based on situation. Despite not publishing most of his writings on these topics, Leibniz discussed this *analysis situs* in letters to Christian Wolff, and revealed still more in private conversation.[2]

Wolff in turn tries to expound an epistemological account of continuous magnitudes inspired by his predecessor's authoritative analysis of geometry, while jettisoning what he sees as its objectionable metaphysical consequences. Wolff may well have misread Leibniz. His reading was highly influential, however, and it is not without textual support. So it will be worth laying out in some detail the problem Wolff takes to be raised by Leibniz's new foundations of geometry.

A chief question Wolff faced was that some Leibnizian epistemological commitments push towards regarding geometrical facts as *contingent*. Consider these passages from across Leibniz's career:

> (A) I have found that two things are perfectly similar when they cannot be distinguished except *by compresence*, for example, two unequal circles of the same matter cannot be distinguished except by seeing them together, for then you can see that one is bigger than the other. You might say: I will measure one today and the other tomorrow and thus I will be able to discern them even without seeing the two of them together. But I maintain that this is still to distinguish them, *not by memory, but by compresence*, because you have the measure of the first one present, not in your memory, for magnitudes cannot be retained in memory, but in a material measure engraved in a ruler or some other thing. In fact, if all the things in the world affecting us were diminished by one and the same proportion, it is evident that nobody could make out the change. (GM I:180 [1677])

[2] See Wolff (1715), Leibniz and Wolff (1860/1963, 163) and note 15 below.

> (B) Those things are *similar* which cannot be distinguished when observed individually. Quantity can be grasped only when the things are actually present together or when some intervening thing can be applied to both. Quality represents something to the mind which can be known in a thing separately and can then be applied to the comparison of two things without actually bringing the two together either immediately or through the mediation of a third object as measure. (GM V:179–80 [1693])

> (C) Quantity or [*seu*] magnitude is that in things which can be known only through their simultaneous compresence—or [*seu*] by their simultaneous perception. Thus it is impossible for us to know what a foot or yard is unless we actually have something to serve as a measure which can be applied to successive objects after each other….Quality, on the other hand, is what can be known in things when they are observed singly, without requiring any compresence. (GM VII:18–19 [1715])

Leibniz has several aims in these passages, including giving a definition of geometrical similarity. I want to focus on the conditions he gives for knowledge of continuous magnitudes, such as lengths and angles in geometry.

Two initial points of clarification on the notion of magnitude. First, these texts can be read as distinguishing between concrete things, such the ruler mentioned in passage (A), and the magnitudes "in" concrete things. However, Leibniz does not emphasize the distinction here, and it is often blurred by Wolff and Baumgarten.[3] Second, unless otherwise noted I focus on *rational* magnitudes, defined in Book V of Euclid's *Elements* as magnitudes that can exceed one another when multiplied. This definition of rational magnitude excludes infinitesimals and infinitely large magnitudes.

Based on passages (A)–(C), then, Leibniz's criteria for our knowledge of continuous (rational) magnitudes can be given the following initial analysis:

(1) x's magnitude Q can only be known if Q is compared with magnitude R of some y, such that

[3] Kant would later use *quantum* to refer to concrete magnitude-bearers and *quantitas* for shareable magnitudes (Sutherland 2022, 76–85).

(a) $x \neq y$,

(b) x has Q at the same time that R has y, and

(c) either x and y are co-perceived, or each is co-perceived with some z that serves as a measure.

Consider how these criteria apply for continuous magnitudes, as in the congruence of line segments. If two segments can be exactly superimposed, such that one fits exactly on the other, then they are congruent.[4] We can then conclude that the line segments have the same magnitude. To be superimposable, the segments must at least exist at the same time, thus satisfying Leibniz's condition (1b). Indirect comparison might also be possible if some third object is used as a measure. But in either case, condition (1c) must be satisfied: the two segments we wish to compare must be directly co-perceived, or else each of them must by co-perceived with third object that serves as a measure. Leibniz states in passage (A) that such a measure must be a material thing. For example, a non-collapsible compass can repeatedly draw circles of equal radii or cut a line into equal segments.[5] Leibniz portrays this measurement practice as essentially relying on co-perception.

By contrast, Leibniz tells us in passages (B) and (C), co-perception is not needed for knowledge of particular *qualities*, for example to distinguish hot from cold, or pain from pleasure. Likewise, qualities can be stored in memory and compared without being directly perceived. Magnitudes cannot be stored in memory, so they can only be compared with the help of direct perception. This affords a way of distinguishing between qualitative and quantitative properties.

Some interpreters read these passages as giving the rudiments of not just an epistemology but also a metaphysics of magnitude. Passage (C), after all, begins with a claim about what magnitude *is*. Vincenzo De Risi concludes that Leibniz's epistemological criteria for knowing magnitudes also provide a metaphysical account of magnitude, such that it is merely phenomenological, and the fundamental monads have only qualitative properties.

[4] For definitions of congruence in terms of superposition and co-perception, see CG 117; 172; GM VII, 263. Congruence is also foundational for Leibniz's attempts to define continuous magnitudes by the number of equal finite parts composing them, as the parts must be "congruent with each other" (GM VII, 53; Arthur and Rabouin 2024). Leibniz's reliance on superposition for segment congruence is a departure from Euclid, who mainly uses it to show the congruence of closed plane figures, such as triangles and parallelograms. For historical background on superposition in geometry, see De Risi (2007, 278–283), Arthur (2021), and Axworthy (2021). Also of contextual interest are seventeenth-century French debates about the proper object of geometry: see Descartes' *Method* (especially Parts Two and Four) and the works edited in Descotes (2009).

[5] Also see A VI 6 147 (*New Essays* II.xiii.4).

Richard Arthur concurs that passages such as (A) articulate what it is for a determination of a thing to be quantitative, though he draws different conclusions than De Risi.[6] On Arthur's view, it is only through the presence of a measure that the magnitude of a body becomes determinate. He thinks Leibniz denies that any two points suffice to define a distance relation between them. Some third thing, namely a measure, is needed to define the distance between two points.

Leibniz's texts do not force a metaphysical reading. They could be seen as merely giving epistemological criteria for grasping magnitudes, rather than stating what magnitudes are. There's no need to decide this here, since my focus is on Leibniz's reception. His rationalist successors did link this epistemology of magnitude to its metaphysical status. To understand them, we need to understand the metaphysics of magnitude they plausibly took themselves to find in Leibniz.

Naively reading off a metaphysical account from Leibniz's conditions on knowledge of magnitude would yield something like the following:

(2) All magnitudes are (a) wholly metaphysically grounded in relations, such that (b) every magnitude exists in virtue of a co-perception relation with at least one other magnitude.

The first part of this claim, (2a), may well have been accepted by Leibniz. He comes to think that the basic components of created reality are monads, which have quality but not magnitude.[7] Continuous magnitudes of bodies cannot be grounded in intrinsic magnitude properties of monads, since there are no such properties. Instead, Leibniz suggests that magnitude is relational, drawing on arguments that parallel his better-known case for relationalism about space and time. He uses an analogy with magnitudes to argue, against Clarke, that space and time can have quantity even though they consist in relations. Relations such as ratios and proportions possess quantity, showing that space and time can have

[6] Compare De Risi (2007, 146–47; 402) and Arthur (2021, 179; 228); also see Sutherland (2010, 161).

[7] See the opening of the "Monadology" (GP VI:608–9).

quantitative properties even though they consist in relations.[8] Privately, Leibniz goes further, writing that quantity in general just *is* a relation.[9]

This relationalism about magnitude need not be limited to relations among actual concrete things. As the basis for space and time, Leibniz sometimes appeals to relations as an order of possibilities.[10] A parallel move would allow Leibniz to give a relationalist account of magnitudes, where the relations in question are between possibilities, and not just between actual objects.

He might then be better placed to respond to some common objections to relationalism. One objection appeals to the possibility of worlds where only one object with a certain type of magnitude exists—say, just one object with a volume—so there are no relations among actual concrete objects to ground this magnitude. Another objection stems from the possibility that one magnitude could have been otherwise even if all other magnitudes of its type remain the same. Cologne Cathedral, for example, might have been slightly shorter than it actually is.[11] These cases are challenging to explain if one is limited to actual spatial relations, but if Leibniz can help himself to relations among *possibilia*, he might be able to account for them.

Claim (2b) is more problematic. I want to explore these problems while suspending judgment on whether Leibniz is actually committed to (2b).

One problem is that actual co-perception is world-bound: any agent is only able to co-perceive entities that also exist at her world. That threatens Leibniz's appeal to a relational order of possibilities to make sense of space, time, and other magnitudes.[12] Given (2b), Leibniz's relationalism looks to be limited to relations among actual bodies after all.

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[8] GP VII:404; VII:415; Arthur (2021, 76). Leibniz counts proportions as magnitudes and seems to regard them as second-order relations between relations, rather than four-place first-order relations (Mugnai 1992, 159; De Risi 2007, 411).

[9] See, e.g., GM V:12; C 8–9; De Risi (2006, 64). A potentially problematic consequence is that, since some magnitudes are composed of parts, some relations have parts.

[10] See GP VII:376, IV:568; V:136–37. An appeal to possibilities is also often seen as helping relationalists explain whether a given object can have an incongruent counterpart. Whether a counterpart exists depends on whether the object is in an orientable space, but the orientability of space is hard to capture in actualist relationalism. Kant, in 1768, raises incongruent counterparts as an objection to relationalism. Later, however, he seems to acknowledge that Leibnizian relationalism appeals to possibilities (A433/B461) and shifts his focus to the alleged idealist consequences of incongruent counterparts.

[11] Crusius will raise this sort of objection to relationalism (EVW §49).

[12] See further Wells (forthcoming). Leibniz defines extension in terms of actual co-perception (GP II:473). This hints at another problem I don't focus on here: co-perception provides scant conceptual resources for defining magnitude in general, or for distinguishing between kinds of magnitude. Leibniz takes types of magnitude, such as time, motion, or spatial magnitudes of different dimensions, to be mutually independent and not directly

A second problem, I'll argue, is even more important for Leibniz's eighteenth-century legacy. Co-perception seems to require presence to an observing subject, making determinate magnitudes dependent on the finite minds that perceive them. Finite minds like ours, and their activities, are contingent. Therefore, geometrical truths look to be contingent. Call this the *problem of contingency*.

As an example, consider again congruence for segments in classical geometry. If Leibniz is committed to (2b), then not only does grasping congruence require the method of superposition, but there are no facts about the lengths of segments apart from actual procedures of superimposition that depend on co-perception (Euclid, though silent about such facts, does not rule them out). Since (2b) means that congruence facts constitutively depend on contingent acts of co-perception—without the acts, the congruence facts would not obtain—the congruence facts themselves turn out to be contingent.

There are textual reasons to think Leibniz is not committed to (2b). He often states that geometrical truths are grounded in God's necessary attributes. They hold in all possible worlds, apart from any act of creaturely perception.[13] Admittedly, the epistemic upshot of these statements is not so clear. For example, he does not spell out how we can reliably know facts about distances, and some texts suggest that God could have actualized the same essences while shrinking all distances uniformly.[14] If so, spatial measurement facts cannot be read off from the divine understanding. Since my focus is on the reception of Leibniz, the key point again is that his successors tried to resolve a tension between this commitment to mathematical necessity and an epistemology of geometry that relies on co-perception.

In the next two sections, we'll see Wolff and Baumgarten unite in seeking to avoid the metaphysical commitments of (2a) and (2b), while still enthusiastically embracing relational criteria for distinct cognition of magnitudes. They do this by trying to pull apart what it takes for a magnitude to *exist* from what it takes to distinctly *cognize* a magnitude.

2. Wolff and the Problem of Relations

commensurable (VI:581). Mere co-perception determines neither what time and length have in common (as magnitudes), nor what distinguishes them (as different kinds or domains of magnitude).

[13] See GP II:49; II:305; V:210; VI:226; VII:184; VII:275–78. The issue is controversial. Levey (1999) and De Risi (2007) propose a reconstruction of Leibniz on which geometrical objects partly depend on contingent synthesizing operations carried out by finite minds. Arthur (2021) is among those who reject such a reading.

[14] See GM I:180; CG 182; Wells (forthcoming).

In the 1713 first edition of his *Elementa Matheseos*, Wolff directly adopts Leibniz's epistemological account of magnitude.[15] Differences in magnitude can only be discerned when two things are compresent, or when a third thing measures them by being compresent to each at different times. Two figures are geometrically similar, by contrast, when they cannot be discerned unless they are compresent. Subsequent editions attempt to ground this notion of magnitude in a definition of similarity in terms of identity and difference, but still draw the corollary that two similar things cannot be distinguished when considered in isolation.[16]

This appeal to compresence helps Wolff with a difficulty raised by his version of relationalism about space, time, and magnitude. He appeals to ordering relations among things, but mere ordering relations do not amount to a metric for continuous magnitudes, which Wolff needs to define basic physical concepts such as physical motion. Compresence allows Wolff to determine the equality and inequality of spatial magnitudes.

Wolff expands on this co-perception condition in his 1720 German *Metaphysics*:

> (D) Similar things cannot be distinguished from each other unless one either actually brings them together or does so in thought by means of a third thing…But when one brings together things that are to have a similarity, one distinguishes them either through their magnitude [*Grösse*] or through their position. Although magnitude is indeed an internal difference, it cannot be reckoned among those things by means of which one cognizes and distinguishes things, because it cannot be comprehended as such by the understanding, but rather is only given [*geben*] and thus must be grasped merely by the senses…. If I am supposed to tell someone how large something is, I must tell him what relation it has to a certain measure that he is familiar with. (DM § 20)

[15] On Leibniz's direct influence on the *Elementa* and the young Wolff's knowledge of Leibniz's theory of *analysis situs*, see Favaretti Camposampiero (2019) as well as the broader surveys by Poser (1979) and Sutherland (2010). The two philosophers had a number of conversations on these topics. Wolff states in a published article that during a 1712 meeting, Leibniz explained the compresence condition on congruence to him (Wolff 1715, 214). Upon learning that Wolff's 1713 *Elementa* took up some of his own ideas on magnitude and similarity, Leibniz apparently worried that he had been misrepresented. For he drafted an essay—now usually labeled "Metaphysical Foundations of Mathematics"—clarifying some ideas "mentioned" in the *Elementa* (GM VII:17; Sutherland 2010, 163–9; Favaretti Camposampiero 2019). This text is the source of quotation (C) above. Though Leibniz did not live long enough to publish the essay, it indicates his interest in how Wolff appropriated his ideas, and his intent to publish his compresence account of congruence.

[16] EM, *Arithmeticæ* §§ 25–26.

This passage insists that magnitudes are given and *must* be grasped by the senses, rather than comprehended by the understanding. This may come as a surprise. Wolff is often read as holding that we can resolve all sensory representations into distinct analytic propositions.[17] A main source of evidence for this reading is his conviction that all scientific knowledge can be recast in syllogistic form.

However, Wolff's official definition of syllogism is more inclusive than Aristotle's: it does not even mention logical consequence relations, let alone analyticity.[18] His syllogistic reconstructions of geometrical proofs include minor premises that are not purely conceptual, but irreducibly intuitive. These premises are based on the direct intuition of a diagrammatic figure ("*das Anschauen der Figur*"), not conceptual analysis—even if they are ultimately subsumed under universal geometrical principles.[19] One of his examples is the premise, justified by direct inspection of a Euclidean diagram, that two angles *a* and *b* are alternate interior angles. Predicating the concept <alternate interior angle> of a subject requires distinguishing interior angles as alternate. In turn, this requires characterizing them as on opposite sides of a line intersecting parallels, which Wolff thinks can be done only with reference to a diagram. One reason for this is that insofar as it is merely perceived, magnitude cannot be explained in words or distinctly grasped by the understanding.[20]

Passage (D) also makes a metaphysical claim, namely that magnitude is an internal property in virtue of which things differ from one another. Wolff often repeats this point later on. For example, quantity is an "internal distinction of similar things," even though "if nothing else is assumed, quantity cannot be understood by itself but can only be given."[21] That is, two qualitatively similar things are metaphysically distinguished by their differences in quantity. And in 1736, quantity is defined as the "internal" property by which qualitatively

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[17] For this reading see Anderson (2015, 30–31; 77; 222–24); for a critical response, see Dunlop (2019). Anderson emphasizes that for Leibniz and Wolff, it is in principle possible to carry out this resolution (2015, 222). Strictly speaking that is correct, but they take this task to be impossible for humans, so its in-principle possibility is irrelevant to human mathematics. This means the Wolffians have more in common with Kant than Anderson suggests. Kant could also grant that the concept–intuition divide is in principle resolvable, though not resolvable by humans. He attributes the distinction between concepts and intuitions to *finite* thinkers, not to thinkers in general (B145; KGS V:401–2).

[18] Wolff (1740/1983, § 332). Mancosu and Mugnai (2023, 89–90) detail how Wolff's reductions to so-called syllogisms often do not meet the criteria of Aristotelian logic.

[19] Wolff (1725, 97); see also DM § 346 and Mancosu and Mugnai (2023, 95).

[20] Wolff (1716/1965, 1279); see further Rusnock and George (1995, 262).

[21] EM, *Arithmeticæ* § 26 (1742). See further Sutherland (2010, 165) and Favaretti Camposampiero (2019). For Wolff, quantity is a broad genus that includes continuous, rational magnitude. His statements about quantity therefore apply to continuous magnitude.

similar things can be "intrinsically" distinguished.[22] Wolff therefore maintains that in metaphysical strictness, quantity is an inner or intrinsic property of things that they would have even if nothing else existed. At least based on these passages, Wolff seems committed to denying metaphysical claims (2a) and (2b), while trying to keep some of Leibniz's epistemology of geometry.

For further evidence that Wolff responds to Leibniz's ideas here, consider how he recycles one of his mentor's examples to illustrate the requirement of bringing magnitudes together in order to cognize them distinctly. The example, found just before passage (D) in the German *Metaphysics*, is of a man led blindfolded into two different houses that have all the same qualitative properties. Once inside each house, the blindfold is removed. Wolff thinks that the man will be unable to identify any quantitative differences between the two houses, and so will lack grounds to determine whether he has visited two houses or just the same house twice. Even if the quantitative differences between the houses are later explained to him, he cannot match these "two descriptions" to his experiences.[23]

One lesson Wolff draws from the example is clear.[24] Quantitative differences between the two houses can only be discerned if they are seen side by side, or co-perceived. The exact quantitative difference can be more exactly expressed by an *aliquot* measure, that is, some smaller magnitude that can be multiplied by an integer to yield the difference in question. But ensuring that *aliquot* parts are equal to one another also requires co-perception.[25] All measurement of continuous magnitudes therefore requires co-perception and co-presence. Moreover, his criterion for equality is that some *a* can be put in place of *b* in a way that makes no quantitative difference from *b*'s remaining in its original place.[26] In geometry, this

[22] O § 348.

[23] DM § 19; also see § 66. For Leibniz's use of the example, by which he defines *x* and *y* as *similar* in case they are indistinguishable when observed apart—see *De Analysi Situs* (GM V:178–83) as well as the earlier *De Rebus in Scientia Mathematica Tractandis* (A VI 4 380). While neither was published, it is extremely unlikely that the shared example is a mere coincidence, and plausible that the pair discussed it in person. Leibniz may even have provided Wolff a copy of *De Analysi Situs*: as Vincenzo De Risi has informed me, there is watermark evidence suggesting that this text actually dates from around the start of their correspondence in 1705.

[24] Wolff's handling of the example is otherwise problematic. He runs together the truism that two qualitatively identical things can be numerically different with the contestable claim that two qualitatively identical things can always be quantitatively different. The latter is traditional—compare Aristotle, *Metaphysics* Δ.15—but not clearly supported by his example. The blindfolded visitor does bring along a measure that can be applied to both houses, since his own body can be used to demonstratively measure sizes. Leibniz is aware of this problem: he bids us abstract from the visitor's body, so as to consider a "seeing mind concentrated at a point…without any magnitude about him" (GM V:181). By contrast, Wolff does not mention the issue—though on a charitable reading, one might assume he silently follows his mentor here.

[25] O § 440; see further DM § 62; EM *Arithmeticæ* § 13.

[26] DM § 22.

involves the superposition of *a* over *b*, which—given his assumption that discerning quantitative differences requires co-perception—must involve co-perception. As such, he appears to endorse Leibniz's epistemological criteria, at least when it comes to distinct cognition of magnitudes, while remaining cautious about the metaphysical conclusions suggested by some of Leibniz's texts.[27]

However, Wolff's metaphysical commitments are unstable. Some texts suggest that magnitudes are mind-independent. Spatial relations such as distances cannot depend on qualitatively identical geometrical points, he maintains, but must be grounded in substances with unique intrinsic properties. Geometrical space is abstracted from material composites, and some passages ground its metric properties in numerically distinct yet connected material parts.[28] In turn, the metric properties of material composites are determined by "nothing but" the "quantity of parts" (*Menge der Theile*) that compose them: this is a sufficient condition for matter's having a determinate, "measured" magnitude.[29] Here one might expect him to appeal to the congruence of composite, finite-sized parts of matter, but he thinks this can't give the full story, as it would lead to a vicious regress of finite parts. Instead, unobservable simple substances ultimately determine the magnitudes of material composites.[30] Some texts even suggest that finitely many simple elements compose a given extended magnitude. If so, equal lengths could be defined in terms of an equal finite number of composing simples.[31]

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[27] Wolff's later works complicate but don't alter this picture. The 1736 *Ontologia* reaffirms that quantity is "given" to us yet "cannot be understood in itself" (*per se intelligi non potest*) (O § 196). The complication is that Wolff now complains that compresence cannot define similarity *in general* (O § 201; see also Sutherland 2022, 225). He does not, however, give another way to distinguish geometrical magnitudes by quantity. And six years later, he once again defines congruence through co-perception (EM, *Geometrœ* §§ 2–3). Lisa Shabel (2003, 54–55) proposes that this work instead defines similarity in terms of two figures being *determined* the same way (compare EM, *Geometrœ* §120; O § 112). But as Sutherland (2010, 165–66) points out, Wolff never properly defines the determination of a particular figure. Nor does Wolff pick up Leibniz's account of unique determination relations among mathematical objects (on which see De Risi 2007, 221–24). Moreover, Shabel's reading does not respect the order of Wolff's axiomatic exposition of geometry. Similarity and congruence are defined prior to and independently of determination (EM, *Arithmeticœ* §§ 27; 154–55; O § 420). As I see it, Wolff continues to affirm co-perception's necessary role in geometry, but now seeks a definition of similarity that also holds for arithmetic and algebra.

[28] See DM §§ 46–53. Specifically, Wolff tries to reason from the qualitative distinctness of real parts and their serial ordering—which is cashed out in terms of dependence or grounding relations (§§ 188–98)—to three-dimensional space with a well-defined metric.

[29] DM §§ 63; 74; see further O §§713–14, 880–2.

[30] DM §§ 77; 92–93: if this were not so, Wolff suggests, the principle of sufficient reason would be violated.

[31] The German *Metaphysics* does not explicitly decide whether the elements composing concrete objects are finite in number. There are minimal "real" parts of space and time, but I take this to be an epistemic claim about smallest *perceivable* parts, not a commitment to discrete space and time (DM §§ 96–97). Wolff's later *Ontologia* forbids infinite numbers, however, implying that any material composite must be composed of a finite number of simples (O § 797; Favaretti Camposampiero 2021, 252–59).

But whether the number of simple elements is finite or infinite, simples' intrinsic "inner state[s]" ground magnitudes.[32] Co-perception relations seem to drop out of the picture entirely. Consider a lead sphere: there is no need to compare the sphere to anything external to determine its magnitude, since that is determined by the simple elements composing it. God could know the sphere's magnitude through the intrinsic states of its elements, with no need for co-perception.[33] In turn, Wolff apparently thinks the metric properties of matter suffice to determine the metric properties of geometrical space, even if geometrical space only *exists* in virtue of acts of abstraction, so the same point would hold for geometrical objects. Going from these texts, Wolff would deny both (2a) and (2b).

In other passages, however, Wolff seems to commit to the partial mind-dependence of material magnitudes, because the properties of geometrical magnitudes infuse the properties of material things. Geometrical space is an indeterminate whole that is not composed of geometrical points, but is prior to points, lines, and planes. While geometrical space is abstracted from matter, bodies are themselves *phenomena.* That is, extended and continuous bodies are confused perceptions of non-extended simple substances.[34] So bodies are partly mind-dependent. This calls into question whether magnitude is an internal property of things. Spatial properties, including size, look to be purely relational and irreducible to the intrinsic properties of the elements: this also holds for spatial properties of material things.[35] The fact that bodies depend on confused perception indicates that bodies' magnitude depends on actual relations to finite minds. Extension is only actually instantiated in virtue of confused perception, suggesting that in spite of various passages considered earlier, grasping magnitude requires confused perception. Consider the sphere again: based on these passages, for it to be extended, some simples must be confusedly perceived *as* extended by a mind like ours. The lead sphere's magnitude is not sufficiently determined by the simple substances composing it, because finite minds are a necessary condition for the existence of extended objects. So Wolff's God—whose representations of necessity lack all confusion—would be

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[32] DM §§ 188–89; see further §§ 594–95.

[33] Robert Grosseteste and other Scholastics had considered the question of whether God could know the magnitude of a solitary line (N. Lewis 2005, 170).

[34] On matter as a confused representation of simples, see DM §§ 83; 604; *Cosmologia Generalis* § 224 (1737); *Theologia Naturalis* § 694 (1737). Wolff seems to be led in this direction because of the challenge of the composition of the continuum: his simples are not extended, but have "in themselves…no size" (DM § 583; also see §§ 77–79). He does not assume that abstraction generates an infinite number of geometrical points. Rather, it generates an *indeterminate* representation of space, where this indeterminacy is due to confused perception.

[35] DM §§ 49; 113. He does think spatial relations can be reduced to *non-spatial* relations of ordering or ontological dependence.

unable to directly grasp the sphere's magnitude.[36] Based on these more idealist passages, Wolff could not deny (2a) and (2b).

The charitable response would be to choose one of these two readings. Either option faces problems, however.

The first reading, in making magnitude a property of mind-independent parts, seems to abandon the Leibnizian *epistemology* that Wolff sought to preserve. If it avoids the problem of determining magnitude, this is because magnitude is simply assumed to be mind-independently determined: matter contains basic measures for space, which are inherited by geometry through some process of abstraction. To put it crudely, each simple substance works like a tiny measuring stick, providing an integer measure for composite things. However, this would clash with Wolff's vehement denial that his simples are extended, and would also push him toward viewing space as independent of simples, rather than dependent on them. Further, even this realist view seems tacitly committed to relations. The size of a continuous magnitude, on this account, is metaphysically determined by adding up all of its parts. Wolff thinks composition is accidental and contingent.[37] So the magnitude's parts can add up to a whole only in virtue of contingent relations between them. Now a difficulty arises with Wolff's metaphysics of relations. While he grants that relations have an extramental basis, he takes them to hold exclusively between per se substances.[38] There are just three types of per se substance for Wolff: God, souls, and finite simple substances. But the parts of magnitudes are not substances, so by Wolff's own account they cannot stand in extramental relations. The relations they stand in must therefore be mind-dependent, contradicting the realist reading.

That leaves the second, idealist reading, which falls back into the *metaphysics* Wolff sought to avoid, on which mathematical truths seem to turn out contingent. This reading roots the determination of magnitude in relations between simple substance and finite minds, both of which count as genuine substances for Wolff. This makes all the interesting properties of continuous magnitudes dependent on actual relations to finite minds. These relations are

[36] DM §§ 1067–69.

[37] DM §§ 55–56; 59–60.

[38] DM § 116; O §851. He argues that dependence and independence relations are fully grounded in the intrinsic properties of substances, so there are no relations without intrinsic grounds (O §857). But Wolff does not rule out all real relations between substances, as Friedman (1992, 2–3) suggests. Nor is Anderson (2015, 99–107) right to hold that Wolff limits logic to monadic predicates. Wolff uses polyadic predicates such as 'is equal to,' notably in geometrical proofs (1725, 93–97). The problem is not that he forbids polyadic predicates, but that he never explains how they work (Mancosu and Mugnai 2023, 89).

contingent, so most properties of continuous magnitudes turn out contingent too, leaving the original problem of contingency unsolved—as also suggested by the idealist reading's commitment to (2a) and (2b).

This idealist reading has its own problem of consistency with Wolff's metaphysics of relations. In addition to holding that the fundamental relata are substances, Wolff thinks the only fundamental relations are metaphysical dependence relations. This assumption is important for the realist argument we considered above: because substances' intrinsic properties must determine relations of dependence or independence, all actual relations are in the end grounded in intrinsic properties. Wolff even tries to define relations in general as asymmetrical grounding relations between substances. A paradigm example is causation: Cain depends on Adam because Adam contains an immediate causal ground (*Grund*) for Cain's existence.[39] A problem is that in Wolff's system there are no direct grounding relations between minds and simple substances. These substances do not depend on one another for their existence, as attributes and modes depend on substances. Nor are the attributes and modes of different minds and simple substances metaphysically grounded in each other. This is because Wolff endorses pre-established harmony between souls and the simple substances underlying matter.[40] So in his quest to ground non-fundamental relations in fundamental ones, Wolff risks eliminating the fundamental relations needed for the idealist reading I've sketched.

In sum, whether we choose a realist or idealist reading, Wolff's account of relations does not provide sufficient resources to characterize magnitudes in the way he wants—namely as metaphysically 'internal' or *non-relational* properties that can only be determinately *cognized* through relations. We can call this the *problem of relations.*

3. Baumgarten and the Problem of the Determinable

[39] For this example, see DM § 188. Also see §§ 545; 593–95; O §§ 851–57; Radner (1998, 419–21).

[40] See DM, "Vorrede zu der anderen Auflage"; § 765–67. His main rationale is that if souls had a causal influence on matter, e.g. if the will immediately caused motion, then moving force ("*bewegende Kraft*") would not be conserved (§ 761–62). Since Wolff partly follows Leibniz's pre-established harmony doctrine, it is open to him to adopt Leibniz's account of ideal influence: minds and bodies do not actually stand in real relations, but are in ideal dependence relations determined by their respective explanatory priority. But as we've seen, Wolff's definition of relation *only* gives an example of immediate causal dependence, as when Adam engenders Cain. He does not explain how pre-established harmony would fit with this account of relations and causation, and in particular, he does not seem to take up Leibniz's ideal-influence account (Watkins 2006; Wunderlich 2021).

Among Wolff's followers, I focus on Alexander Baumgarten, who was especially important for Kant. Baumgarten frames his work as an exposition of Leibnizian-Wolffian philosophy.[41] Baumgarten's discussion of quantity echoes his predecessors:

> (E) The distinguishing marks of a being are either external and relative, or internal…Internal distinguishing marks can be represented in a being considered in itself…and hence can be known in some way, or *given*. We can either *conceive* of and understand (i.e. know distinctly) given things without assuming or relating them to anything else (without the presence of anything else), or we cannot. If the first, then such an inner determination of a thing is a *quality* (*qualitas*) of the thing; if the second, it is a *quantity* (*quantitas*).[42]

In the first sentence, Baumgarten distinguishes between relational and internal (or non-relational) properties of beings. This contrast is drawn just in terms of how properties are *represented*. He holds that if we consider a being in itself, in abstraction from all its relations, we represent its internal properties or determinations.[43]

The second and third sentences in passage (E) introduce a privileged epistemic state, namely understanding or distinct representation. Baumgarten contrasts this with what he calls the mere givenness of an object. Understanding is a privileged cognitive state because it involves representation of its object with a higher degree of distinctness than what's merely given.[44]

The passage concludes by defining the difference between qualities and quantities. Quantity is distinguished by an epistemic criterion. The quantity Q belonging to some x cannot be known without relating x to some other y, whereas a quality can be so known.

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[41] Leading interpreters see Baumgarten's system as closer to Leibniz than to Wolff (e.g. Casula 1979; Schwaiger 2011). However, I have found no direct evidence that Baumgarten was aware of Leibniz's texts specifically on the foundations of mathematics. The direct influence was most likely Wolff—or Wolffians like Baumgarten's teacher Johannes Reusch.

[42] M §§ 68–69; see further M § 37.

[43] Baumgarten has been seen as conflating metaphysical and epistemological questions (Nuzzo 2018, 30; Sutherland 2022, 41). This need not be the case here: the representational distinctions he draws seem to be criteria for *identifying* different types of property—for example, internal marks are correlated with inner determinations—rather than equated with metaphysical distinctions among properties. Baumgarten's discussion of magnitude may, however, confuse concrete entities that bear magnitudes and the shareable magnitude properties they bear (or between what Kant calls *quantum* and *quantitas*).

[44] Degree of distinctness depends on the number of a thing's marks (*nota*) one is conscious of explicitly (M §§ 36; 67; 402; 510; 522; 531).

Recalling Leibniz's criterion (1), Baumgarten invokes a specific relation: the "presence" of *y* to *x*.[45] Because extension must also be represented simultaneously, if *y* and *x* are extended, they must be simultaneous and present to one another, or compresent, in order for *Q* to be known distinctly.[46] He goes on to define compresence in terms of spatial proximity. Since *x* and *y* are as closely present to one another as possible when they are touching, *x* and *y* are more closely present to each other the less distance there is between them.[47] Though he does not explicitly mention co-perception, he takes distinct knowledge of *x* and *y* to require cognizing them as standing in some relation.[48] Given his simultaneity and co-presence requirements, this looks to be a perceptual relation.

The final sentence in passage (E) signals some important differences with Leibniz and also with the more idealist tendencies in Wolff. As an "inner determination of a thing," quantity for Baumgarten is not a relation. He would not accept Leibniz's criteria (2a) and (2b). This rejection of a relationalist metaphysics dovetails with Baumgarten's account of the composition of magnitudes, which he defines as continuous quantities.[49] Extension does not depend on relations to perceivers and their faculties of perception or imagination. Rather, extended beings are composed of unextended points. The number of points composing a line determines its extension. The shortest line between two points, for example, is defined as the line composed of the smallest number of points. Since he takes number to be discrete and finite by definition, he is committed to discrete, finite parts of continuous magnitudes.[50] This approach to the composition of the continuum is doomed to fail, but at least it is straightforward. Continuous magnitudes are grounded in discrete numbers, which provide unit-free answers to *how many?* questions. On standard accounts, by contrast, continuous

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[45] M § 69.

[46] M § 241.

[47] M § 223.

[48] M § 69.

[49] M § 159; also see Sutherland 2022, 225–26.

[50] For the definition of number, see M §159; for the claims about composition, see M §§ 235; 242–44; 286–87; 419; also see Watkins (2006, 293–98) and Pelletier (2013, 220–22). Baumgarten does not sufficiently clarify the relationship between his concepts of monad (*monas*) and point (*punctum*). He sometimes describes the concepts as coextensive, but that can't be true. Monads are non-extended substances, while points are components of extension. In Baumgarten's terminology, points are *moments of a quality* of some composite. This composite is not a substance "per se" but a mere "*phaenomenon substantiatum*" (§ 233). Baumgarten denies that Euclidean "mathematical" points could compose a line, since they are not impenetrable (§ 399; § 243). Adding impenetrability does not solve that problem, however.

magnitudes answer *how much?* questions. Baumgarten's continuous magnitudes have a built-in metric, namely the number of discrete points or monads composing them.[51]

So it is puzzling that Baumgarten follows Leibniz by requiring magnitudes to be *known* through relations. Consider again some *x* with continuous magnitude *Q*. For Baumgarten, *Q* is an internal, non-relational property of *x*. *Q* is fully grounded in properties of *x*'s parts. As we saw, he thinks every continuous magnitude is composed of a determinate, finite number of points, where the number of points determines the size of the magnitude. Since every continuous magnitude is composed by a finite number of points, it has an intrinsic metric. To get distinct knowledge of *Q*, in principle one need only count up the points: there's no need to look outside *Q* at all. Baumgarten may recapitulate criterion (1) more out of allegiance to what he calls the Leibnizian-Wolffian philosophy than for reasons internal to his system.

Given his debts to Wolff, we might expect Baumgarten to distinguish between indeterminate and determinate cognition of the compositional structure of magnitudes. Actually, he does not do this explicitly, and leaves little room for determinable magnitudes at all, either in epistemological or metaphysical contexts. Call this the *problem of the of determinable.*

On the epistemological side, we've seen that Baumgarten takes the distinctness of cognitive representations to come in degrees. The more marks or differentiating properties of a being one grasps, the more distinctly one cognizes it. While this is fine as far as it goes, he is what we'd now call a direct realist about magnitude properties. Magnitudes are known by direct acquaintance through the senses. Since Baumgarten's magnitudes have a built-in metric that is in principle accessible to us, it is unclear what more would be needed for distinct cognition. Put another way, if mind-independent magnitudes can be directly given to us such that they are measured, our role is purely receptive. No further cognitive determination of these magnitudes is required, and the distinction between indeterminate and determinate cognition is idle.

As for metaphysical determination, there's initial promise in Baumgarten's well-known distinction between determinable and determined. His idea is that *x* is determinable with respect to a property *F* if it is not posited whether *Fx* or *~Fx*. But if *x is* in fact posited as

[51] Kant understandably criticizes this proposal (A439/B467; IV:237–38). He does not name names, but Baumgarten is a likely target.

being *F* or as not being *F*, then *x* is determined with respect to *F*.[52] This definition threatens to be circular, absent some illuminating account of *positing as*, which Baumgarten does not seem to provide. But leaving that aside, there are specific problems with using it in the mathematical case.

Baumgarten affirms that each actual, particular individual is fully determined: that is, determined with respect to all possible properties whatsoever. Only non-actualized universals are determinable. Indeed, his main use of the determinable–determined distinction is precisely to pick out the (fully determined) actual world from (determinable) unactualized possible worlds.[53] But Baumgarten's mathematical objects are not mere *possibilia*, but actual particulars that exist somewhere and somewhen.[54] So mathematical objects, since they are actual existents, must be fully determined. There is no further work for a determinable–determined distinction. Baumgarten, despite his account of metaphysical determination, fails to articulate a helpful distinction between determinate and indeterminate magnitude.

4. Crusius and the Problem of Part–Whole Priority

Christian Crusius, another important influence on Kant, explicitly rejects the philosophical methods of Leibniz and Wolff, as well as many of their metaphysical commitments. For reasons that will become clear, it is especially important that Crusius rejects their relationalism and defends universal and absolute space and time, concluding that everything is spatial and temporal.[55] His epistemology of magnitude nevertheless has points in common with Leibniz, Wolff, and Baumgarten. He thinks that cognizing magnitude requires comparison and even co-perception. Also like them, he avoids committing to a relational metaphysics of magnitude and tries to maintain that some features of magnitudes are intrinsic and necessary.

Before proceeding, Crusius's idiosyncratic terminology needs some clarification. Crusius uses '*magnitudo*' both for token quantities and for relatively specific quantitative properties, whether continuous or discrete. A token ten-meter length is a magnitude in this sense, but so is the specific but repeatable property of being ten meters long. The term

[52] M § 34; compare O §§ 105; 112, and see further Nuzzo (2018); Sutherland (2022, 40–42). For pertinent remarks from the contemporary metaphysics literature on determination and quantity, see J. E. Wolff (2020, 15–21).

[53] M § 148. Baumgarten's possible worlds, it follows, are universals or kinds rather than completely determinate individuals.

[54] M § 281–82.

[55] EVW §§ 1–4; 48–51. On his arguments for this, see Messina (2015, 436–37).

'*quantitas*,' meanwhile, denotes either the property of having magnitude in general, or the property of having a certain type of magnitude, such as length. These generic, determinable properties are strictly speaking qualities (*Eigenschaften*), so they are distinct in kind from magnitudes as such.[56]

Odd as his usage may be, it's possible to make sense of the underlying point. Having some magnitude or other—or even having some length or distance—is a distributive property. Qualitative properties are also distributive in this sense. For example, the property of length in 'The tree's leaves have length' is semantically akin to the property of greenness in 'The tree's leaves are green'. In both cases, a property is distributed to each member of a collection. By contrast, Crusius understands magnitude in terms of distinct parts insofar as they are collected into a whole. The simple case is a natural number, which does not distribute to each thing numbered, but is predicated of a collection, as in 'The tree's leaves are 1000'.

Crusius agrees with Leibniz, Wolff, and Baumgarten that clear cognition of a magnitude requires comparing it with something else:

> (F) When we want to clearly represent [*vorstellen*] a magnitude and distinguish it from others, we must measure it; that is, we must hold it against another thing whose magnitude is known to us and determine how they relate to each other. (EVW § 159)

Crusius is discussing token magnitudes, such as concrete lengths and volumes. Passage (F) articulates a necessary condition for clearly and distinctly representing an unknown magnitude, namely holding this magnitude up against some other known measure. He means 'holding up' literally. The two unknown magnitudes must be compared at the same time, as well as co-perceived, as when one holds up a ruler or fixed compass to measure a length.

Co-perception is thus a necessary condition for clearly representing magnitudes. A magnitude can only be cognized if it is represented, so co-perception is a necessary condition for the cognition of magnitudes. However, co-perception is not a sufficient condition for the so-called "complete determination" of a magnitude.[57] This requires cognizing the ratio

[56] EVW § 159.

[57] If token magnitudes are held up against each other, we can order their sizes by greater-than and less-than relations. But this "incomplete determination" does not yet define "units" through which each magnitude can be assigned a "number" (EVW § 163; § 160). With the help of units one can, for any two unequal magnitudes, "say how much one is greater than the other" (§ 160). Following Aristotelian tradition, Crusius takes 1 to be a unit rather than a number: the whole numbers start at 2.

between a magnitude and its *aliquot* measure: the unit 1 provides all whole numbers with built-in measures. By contrast, continuous magnitudes lack a natural unit. So, despite other disagreements, Crusius would affirm Leibniz's criterion (1) as a necessary condition for cognizing magnitude.

Like Wolff and Baumgarten, Crusius resists the metaphysical conclusion that all magnitudes are relational. Relations can have magnitude, but it does not follow that they *are* magnitudes.[58] As seen in passage (F), Crusius sometimes treats magnitudes as concrete particulars, obscurely given to us in perception. But he officially defines magnitude as a repeatable property of concrete particulars.[59] Magnitudes, Crusius says, are properties that posit an essence (*Wesen*) more than once, thereby multiplying or composing one and the same essence. An essence is made up of properties, which would seem to make it repeatable and hence a universal. Only essences of the same type can be composed. Therefore, for each type of essence that can compose in this way, there is a corresponding type of magnitude. The magnitude-types Crusius mentions include powers, actions, and effects; space and extension as continuous quantities; and the "purely ideal" units making up whole numbers.[60]

When Crusius says that a single essence is multiplied, I take it that he means to capture the collective character of quantitative properties. To say the tree's leaves are green is to posit one essence: *greenness*. Since Crusius thinks that having a kind of quantity is itself a qualitative property, to say that each leaf on the tree has some length or other also requires positing only one common essence, namely *length*. But when I say the tree's leaves are 1000, I posit the very same essence—a unit—a thousand times over. Crusius seems to slide from taking the addition of units to characterize determinate magnitudes, to assuming it in all magnitudes, including indeterminate continuous magnitudes.

Crusius's talk of 'positing' essences might suggest that quantitative properties depend on acts of positing, and therefore on our minds. But he does not think the multiplication of essences is up to us. When we posit an essence more than once, if all goes well we truly describe features of the world. This is especially clear for discrete quantities. In one of Crusius's examples, a forest comes with a natural unit, namely a tree, for determining its quantity. Even if repeated positing is required for us to know the number of trees, their number is independent of these psychological acts. He thinks the point generalizes to

[58] EVW § 158.

[59] EVW § 157.

[60] EVW § 157; 158.

continuous magnitudes such as forces, actions, and effects, even if in such cases the unit of measure is arbitrary (*willkürlich*).[61] His idea seems to be that quantitative properties themselves are built up, in a mind-independent way, from more basic essential properties.

It's worth pausing to note the resemblance between this account, on which magnitudes are composed out of more basic essences, and recent views of magnitudes as *structural universals*. At least one recent objection to structural universals would apply to Crusius as well. In the paradigm case of whole numbers, each Crusian essence is a unit. So essences are exact duplicates of one another. This suggests that *x* and *y* can fall under numerically distinct but duplicate universals. Then, the objection goes, it becomes mysterious whether *x* and *y* have the same property. The possibility of duplicate universals undermines the work universals are supposed to do.[62]

Another problem is that given Crusius's account of essences, it is none too clear why clearly representing magnitudes should require comparing them in the first place. For as we saw, addition or multiplication is built into his definition of magnitude. If his account of essences is cogent, then every magnitude has a built-in metric, namely how many essences are multiplied or composed to form it. It's not clear what further work could be done by comparison. This is what I called the *problem of the determinable*, seen also in Baumgarten.

Crusius's epistemology does introduce some new twists. Baumgarten had thought direct perception gives us full cognitive access to magnitudes; perception is centrally important for Wolff as well. By contrast, Crusius appears to take the representation of magnitudes to be purely conceptual, such that the multiplication of essences somehow involves composing or concatenating concepts.[63] Later, Kant will object that composing concepts never yields a plurality, but only one thing "thought twice."[64] If this worry has force, then Crusius will never get magnitudes from his essences.

Crusius gets into a final difficulty because unlike the Leibnizians, he accepts absolute space and time. His paradigm conception of magnitude is the addition of discrete minimal units to form natural numbers. But an additive account won't work for continuous spatial and

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[61] EVW § 164.

[62] See e.g. D. Lewis (1999, 98).

[63] EVW § 163. Crusius's term is 'predicates,' but he means predicative *concepts*, not words. In the background is Crusius's view that veridical perceptual experience is propositional, hence irreducibly conceptual, even if it also involves sensation (1747, § 40–41). He sees mathematics as primarily syllogistic (§ 10; Mancosu and Mugnai 2023, 117).

[64] KGW XX:280; see further Sutherland (2022, 214).

temporal magnitudes. Space is not formed by adding up points, but is metaphysically prior to its parts, which only exist in thought. Continuous spatial magnitudes are carved out of an already existing continuous space. Crusius even thinks space is metaphysically prior to finite substances.[65]

The difficulty, then, is that Crusius tries to say that for all magnitudes, units are metaphysically prior to the wholes they compose. The forest, to use his example, depends on the trees. But this cannot be true for his continuous spatial magnitudes. Space and time as a whole are metaphysically prior to their parts, as if the trees depended on the forest. The natural numbers are not exempt from this tension. Crusius thinks they, like everything else, exist in and depend on absolute space and time. Following Daniel Sutherland, we can call this the *part–whole priority problem*. As we'll soon see, it is also faced by Kant.

5. Kant and the Post-Leibnizian Problems of Magnitude

In this final section, I first lay out some textual evidence that Kant was aware of these earlier debates on the metaphysics and epistemology of magnitude, and even partly sympathetic to the earlier positions. I then briefly survey how Kant responded to the problems I've discussed—without seeking to provide anything like a conclusive treatment of these issues or the extensive secondary literature. I aim only to call attention to links between Kant's work and the earlier debates, and to point out a few difficulties that deserve further research.

To begin with evidence for Kant's awareness of his predecessors: his lectures on metaphysics from the 1790s directly consider Wolff and Baumgarten's contention that quantity is an internal property of a being, but can only be understood with reference to some other thing. Although Kant denies that this affords a general definition of magnitude, he considers their claim correct and even "provable": reference to another thing is strictly required for a magnitude to be "given."[66] As he continues:

> (G) Through the comparison of the thing with itself and its parts one can clearly cognize that there is a quantum, but one can never determine, without comparison of a thing with other things...*how* large it is...The concept of magnitude [*Größe*] or the determination of a thing, that it has magnitude, is given, but the magnitude as magnitude, i.e., how large the object is, is impossible to cognize from the matter

[65] EVW §§ 115–17; 53. For discussion, see Messina (2015) and Carson (2019, 108–10).

[66] KGS XXIX:992.

> itself…In order to cognize a magnitude it is necessary that the concept of measure be connected with it at the same time...without measure it is impossible to imagine what kind of a quantum is produced through the composition…Measure is the unit [*unum*], which makes quantity cognizable [*cognoscibilem*] by counting.[67]

Properly cognizing the continuous magnitude of *x* requires a measure *y* that is distinct from *x*, and simultaneous and spatially adjacent to *x*. Kant distinguishes a mere indeterminate quantum or manifold from the relational determination of a quantity, or how large something is. Considering a thing's internal quantitative relations, such as ratios between the sizes of parts, cannot tell us how large it is: these relations could be the same for a planet as for a pea.[69] He thus agrees with Leibniz that if everything in the universe were to change its size uniformly, then at least "in regard to our subjective representation," we would be unable to detect the difference, because no comparison to a "third object" outside the universe is possible.[70] Based on this lecture and remarks in published works,[71] Kant concurs with Wolff and Baumgarten in endorsing the Leibnizian criteria (1a) and (1b) for the full cognition of magnitude.

As for (1c), Kant writes that there is a "fundamental measure" which underlies all measurement practices and requires that a magnitude be "measured by eye" through "mere intuition."[72] To measure by eye requires co-perception. In fact, Kant suggests that superposition in geometry allows for determining the equality of two continuous magnitudes when they have not been assigned a numerical measure. Superposition in turn requires the "immediate intuition" that two figures coincide.[73] This bears comparison to the co-perception condition from his predecessors.

A widely discussed case is Kant's criterion for detecting incongruent counterparts, such as two spherical triangles on opposite hemispheres. The difference between them cannot be found by conceptually describing each spherical triangle on its own, but "reveals itself" only when we *see* that one given triangle "cannot be put in the place of the other," that is, that

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[67] KGS XXIX:992–994. I have slightly modified the translation in Kant (1997).

[69] XXIX:992.

[70] XXIX:997; also see XXI:197 and, for discussion, Hogan (2024).

[71] Especially KGS V:248–51.

[72] V:251.

[73] IV:284. Also see IV:493: "Complete similarity and equality, insofar as it can be cognized only in intuition, is congruence. All geometrical construction of complete identity rests on congruence."

they cannot be superimposed.[74] Kant thus endorses a criterion that resembles (1c), though adding his distinctive account of pure intuition.

Recall two further metaphysical proposals about magnitude: that (2a) all magnitudes are wholly metaphysically grounded in external relations, and that (2b) every magnitude exists in virtue of a co-perception relation to some other magnitude. This raised the *problem of contingency* for mathematical truths.

In brief, Kant is sympathetic to (2a), but rejects (2b). Kant apparently accepts that all magnitudes of spatiotemporal things are constituted by external relations. For example, the mass of a physical body is "nothing but relations," and the body "is itself entirely a sum total of mere relations."[75] Although this differs from (2a) in stressing constitution rather than grounding, it is akin to (2a) in yielding a relational metaphysics of magnitude. Furthermore, Kant holds that the very identity of mathematical quantities depends on a given form of sensibility, since two equal and homogeneous units can only be individuated by occupying different spaces or times. Congruence facts in classical geometry are therefore not just epistemologically but also metaphysically dependent on forms of intuition. Kant can be seen as radicalizing the idea, found in Wolff and Baumgarten, that quantity must be sensibly given to us.

Kant's account of magnitude, however, ultimately dispenses with (2b)'s contingent relations of co-perception. Kant rejects any attempt to explain space and time through relations based on contingent acts of perception, a project he attributes to Wolff.[76] Doing so would sacrifice the necessity of mathematical truths, as well as the necessary harmony, in virtue of form, between concrete things and mathematical concepts.[77] If our only evidence for facts about geometrical congruence were empirical—like sensory perceptions of trees or clouds—then we could only have "empirical certainty" about mathematical facts.[78] These facts would be on an epistemic par with contingent inductive hypotheses. Kant rejects this: mathematical facts, such as facts about congruence, are necessary. Since congruence facts cannot be grounded in concepts alone, then by modus tollens, they must be given

[74] KGS IV:286. See among others Rusnock and George (1995); Sutherland (2005).

[75] A265/B321; see also A628/B656, KGS VIII:154–55.

[76] VI:208; also see A40/B56–57; IV:508; XI:51. On perception (*Wahrnehmung*) as representing contingent facts about concrete objects, see further A24/B39; B147; B207; A373–74. G. F. Meier, a Wolffian influenced by British empiricism, was particularly frank about grounding magnitude facts in contingent empirical relations (Meier 1765/2007, §29; Carson 2019).

[77] B41; B147; KGS IV:288; V:468.

[78] KGS IV:284; also see A40/B57; A718/B746; V:52; VIII:391–92; XX:279

immediately in space. He famously adds that it is only possible for mathematical truths to be necessary and universal if space and time are *subjective* forms of intuition, which do not apply to things in themselves.[79] The cost is that mathematical truths hold "only from the human standpoint," and need not be true for thinkers with other forms of intuition.[80]

Arguably, this is a better solution to the problem of contingency than can be found in Wolff or Baumgarten. Yet there is some historical irony here. As we saw, Leibniz himself did not seem willing to let all mathematical truths be contingent. Rather, in many texts he classifies the truths of arithmetic and geometry as necessary and grounded in the divine intellect. Leibnizian mathematics is then applicable to nature because in creation, God "uses the most perfect geometry."[81]

Another problem we've seen concerns *relations*. The challenge is to give a satisfactory relationalist account of magnitude if one's official theory of relations seeks to reduce them to dependence and independence among basic substances. Compared to Wolff or Baumgarten, Kant has the advantage of typically not seeking to reduce or eliminate relations. Three examples should suffice. First, space and time for Kant contain principles of relations among objects. Such relations are necessary and irreducible.[82] Second, Kant gives a detailed account of real substance–accident, cause–effect, and interaction relations among phenomena. These relations also appear irreducible. Third, he often states that there are causal relations between the phenomenal world and things in themselves, because the latter causally "affect us."[83] Puzzling though these relations may be, he is staunchly committed to them.

A closer look, however, reveals a neglected difficulty for mathematical relations. Kant holds that quantitative relations are by definition structurally different from real relations such as causation. Quantitative relata never stand in a relation necessarily. He gives the example of two triangles put together to form a square. The composition of the triangles is contingent, and not based in the nature of either triangle. The contingency of composition

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[79] See A48/B65–66; KGS IV:281–85.

[80] A26/B42; see also KGS IV:282.

[81] GP IV:375–77 = L 398 (1692); also see IV:568–69. Against this picture of mathematics, Kant might complain that we lack "criteria" to determine which mathematical propositions are really grounded in the divine intellect, and which have a "spurious origin" (KGS IV:320). But this problem—which Kant may not avoid, given the possibility of error even in pure mathematics—can be separated from the modal status of mathematical truths.

[82] A26–27/B43; cf. A20/B34, IV:286, and for time, A33–34/B50–51. Geometrical truths about relations are a priori, so these relations hold with (non-logical) necessity (A39/B55).

[83] KGS IV:451; see also A494/B522; VIII:215. He may also countenance real relations *among* things in themselves, though this is more controversial.

also applies in the case of adding whole numbers.[84] By contrast, actually existing substances can stand in necessary relations, such as causal relations.

This doctrine of the contingency of geometrical relations is in apparent conflict with other passages. Ratios between magnitudes are necessary relations, as are the relations of geometrical congruence and incongruence. For other examples, consider the assumptions that any two points define a unique straight line, and that any three points not on a straight line define a unique plane. These axioms, which Kant calls "self-evident," assert necessary relations between points, lines, and planes. They have the same necessity as 2 + 2 = 4, which asserts what Kant calls a necessary numerical relation.[85] More generally:

> (H) Although…geometry does not have to do with the existence of things but only with their determination a priori in a possible intuition, it nevertheless passes, just as through the causal concept, from one determination (A) to another altogether different one (B) as still necessarily connected with the former. (KGS V:52)

Mathematical and causal relations are both described as strictly necessary connections here. It is not obvious how to square these passages with Kant's official theory of mathematical relations, which excludes necessary connections.

Now consider the *part–whole priority problem* for continuous spatial and temporal magnitudes. We saw it in Crusius, but it is more famously associated with Kant. On the one hand, he holds that pure space and time—as infinite given magnitudes "in which all objects must be determined"—are prior to their parts rather than composed out of them.[86] But he also claims that all intuitions are extensive magnitudes, where by definition parts make possible and necessarily precede the whole.[87] So it appears that he sometimes considers space prior to its parts, but at other times considers the parts prior to the whole.

[84] KGS XX:420; for the triangle case see B201(n.). Kant links the special character of these relations to magnitudes' qualitative homogeneity. Sutherland suggests there is "nothing remotely like" Kant's homogeneity doctrine among the Leibnizians (2022, 229). I have doubts about this: compare Leibniz (GM VII:30; VII:274). I am also not sure if Kant has a suitable account of homogeneity, which on Sutherland's reading is defined in terms of contingent or even conventional features of concepts. I cannot further pursue these topics here, however.

[85] See B16, A24/B39, A164/B205, and KGS IV:370. On ratios as necessary see A480/B508.

[86] A26/B42; also see A24–25/B39, A31–32/B47, and A169/B211.

[87] A162/B203.

Sutherland has recently developed a response to this problem. This is a divide-and-conquer strategy that distinguishes between indeterminate and determinate magnitudes. An overarching Kantian principle governing magnitudes—the so-called principle of the Axioms of Intuition—states that "All intuitions are extensive magnitudes."[88] As Sutherland points out, Kant repeatedly refers to the determination of merely determinable space, thereby generating bounded spaces such as lines or triangles.[89] Given this distinction between indeterminate space and determinate spaces, Sutherland argues that there are good textual grounds for reading the principle of the Axioms of Intuition as implicitly restricted to the claim that all *determinate* intuitions are extensive magnitudes. Pure space and time are *indeterminate* intuitions. They are not required to be extensive magnitudes and need not depend on their parts. This avoids the part–whole priority problem.

Sutherland's reading, however, brings us to a final problem: the *problem of the determinable*. In section 3, we saw Baumgarten stress a distinction between determinate and indeterminate magnitude. Baumgarten fails, however, to give either an epistemological or a metaphysical account of indeterminate magnitude fit for this purpose.

Kant, to his credit, squarely faces the questions of how a merely determinable magnitude is possible, and of how finite minds like ours could determine it. Space and time are magnitudes, but they are also mere subjective forms of our sensible intuition. Sensible intuition per se is given as "merely determinable," and since space is subjective, we have the ability to determine it.[90]

Nevertheless, this does not entirely clarify the distinction between indeterminate and determinate magnitudes. To start with a simple point, Kant takes mathematical objects such as circles and triangles to be particulars that are determinate in some respects, yet indeterminate in others. Triangles lack a determinate colour or location, for example. Kant suggests the quantitative properties of mathematical objects are determinate in the following sense:

> (I) The determinate concept of a quantity is the concept of the generation of the representation of an object through the composition of the homogeneous. (KGS IV:489)

[88] B201.

[89] Sutherland (2022, 37). Compare B137–38; B147; B154; B201–203.

[90] A26/B42. See further B39; KGS V:241; V:364; XX:268.

This is one of many passages implying that a determinate magnitude just *is* an extensive magnitude: one in which the representation of the parts ground the representation of the whole.[91] An extensive magnitude, such as a line, is a singular, determinate intuition.[92] Now, recall that on Sutherland's solution to the part–whole priority problem, Kant's principle for magnitudes would claim only that all *determinate* intuitions are extensive magnitudes. But if Kant just defines a determinate intuition as an extensive magnitude, then his principle reduces to the tautology that *All extensive magnitudes are extensive magnitudes*. If so, Sutherland's reading would avoid the part–whole priority problem only at the cost of trivializing Kant's principle of the Axioms of Intuition.

Sutherland anticipates this worry and argues that Kant has an independent definition of *determinate intuition* that does not equate it with an extensive magnitude. There's textual evidence for this, since Kant sems to allow determinate intuitions that are intensive rather than extensive magnitudes. Then, showing all determinate intuitions to be extensive magnitudes would be more than a mere tautology. Sutherland sees Kant's general account of determination as following Baumgarten: some determinable x is determined with respect to a property F if either Fx or $\sim Fx$.[93] In Kant, the determinable x can be an intuition: x can range over pure intuitions of space and time, which are continuous magnitudes. Conceptual determination with respect to some property F is a *necessary* condition for x to become a determinate intuition.[94] It is not a sufficient condition, however, since generating determinate intuitions also requires the use of schemata and what Kant calls the productive imagination.

But which properties do concepts impart in order to determine pure intuition? It might be said these are properties of bounded concrete things that fill space. However, Kant makes clear that this sort of empirical determination cannot be involved in geometry, on pain of making geometry contingent.[95] Nor could this approach yield any non-circular explanation of distinctively quantitative determination, as it differs from other, non-quantitative determination.

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91 See also A162/B203; A142/B182; B288; KGS XVIII:322; XVIII:337; XVIII:629; XXVIII:637.

92 A162/B203; A713/B741.

93 Sutherland (2022, 42; 93); compare KGS IX:99.

94 See Sutherland (2022, 42; 50; 55).

95 A429/B457.

The temptation to invoke specific mathematical concepts, such as the concept <triangle>, should also be resisted, since Kant wants to prove a general point. He seeks to show not just that all triangles or all circles are extensive magnitudes, but that all determinate intuitions whatsoever are extensive magnitudes.

Finally, determinate intuitions can't be adequately defined through the concept of an integer.[96] Assigning an integer to a spatial magnitude presupposes some way to define equal units. So the congruence of units must be settled before numbers of units can be assigned to a spatial magnitude. If two spatial magnitudes are congruent, they are arguably already determinate, so the problem of determinate magnitude must be addressed before any appeal to numbers.

Instead, Sutherland proposes that the determination is effected by way of the a priori concept <magnitude>. Constructing this concept of magnitude allows for the a priori generation of a "representation of a determinate magnitude."[97] Moreover, <magnitude> is plausibly a necessary condition for representing magnitudes in general.

While this answer avoids the disadvantages of the three alternatives canvassed above, it may not be adequate. For it can still be asked how the concept <magnitude> serves as a necessary condition for *determinate* as opposed to indeterminate magnitudes, or in other words, how it helps explain the difference between indeterminate and determinate magnitudes. Without further analysis of the concept <magnitude>, no property F has been non-circularly identified whereby space can be determined as either F or $\sim F$. But it is necessary to identify such a property in order to explain how space is determined through the concept of <magnitude>. The problem is acute because space and time also fall under the concept <magnitude>, but are not determinate magnitudes.[98] Therefore, it is hard to see how the general concept of magnitude could be used to articulate the specific difference between determinate and indeterminate magnitudes. Analogously, to explain the specific difference between <mammal> and <non-mammal>, it is no use to appeal to the genus concept <animal>.

In sum, despite recent progress in interpreting Kant's views on magnitude, it remains unclear whether he has a convincing solution to the problem of determination that does not

[96] As Winegar (2022, 653) suggests.

[97] Sutherland (2022, 50); for related proposals see Shabel (2003, 103–106) and Longuenesse (2005, 104).

[98] Sutherland (2022, 74–82) grants that pure space and time fall under the concept <magnitude>. See also Messina (2015).

presuppose one of his own central conclusions about magnitudes, namely that all determinate intuitions are extensive magnitudes.

6. Conclusion

I have not aimed to rehabilitate the philosophies of magnitude of Wolff, Baumgarten, and Crusius. I've suggested that their views, even charitably reconstructed, face serious internal problems. A full appreciation of these problems, however, is vital for understanding Kant's position. Further, I've suggested that Wolff's approach to the epistemology and metaphysics of continuous magnitude—itself taken up by Baumgarten, and even partly accepted by Crusius—emerges from what Wolff takes to be Leibniz's ideas. Even if his portrayal of Leibniz is not fair or accurate, Wolff sees himself as carrying on his predecessor's project in the foundations of mathematics. While it is commonplace to cast this influence in terms of a logicist reduction of mathematics to syllogisms, I've defended an alternative story on which what is central is *perception.* This in turn explains Kant's initially surprising criticism of his Wolffian predecessors—that their approach to geometry relied too much on *empirical* perception—as a response to a largely neglected strand of thought in Leibniz and his successors.[99]

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References and abbreviations

Anderson, R. L. (2015). *The Poverty of Conceptual Truth*. Oxford: Oxford University Press.

Arthur, R. (2021). *Leibniz on Time, Space, and Relativity*. Oxford: Oxford University Press.

Arthur, R. and D. Rabouin (2024). On the Unviability of Interpreting Leibniz's Infinitesimals through Non-standard analysis. *Historia Mathematica*, *66*, 26–42.

Axworthy, A. (2021). *Motion and Genetic Definitions in the Sixteenth-Century Euclidean Tradition*. Cham: Birkhäuser.

Baumgarten, A. (1763). *Metaphysica*. Editio V. Halle. **(Abbreviated as M)**

Friedman, M. (1992). *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

Carson, E. (2019). Space and Time in Kant's Lectures on Metaphysics. In C. Fugate (Ed.), *Kant's Lectures on Metaphysics: A Critical Guide* (pp. 102–121). Cambridge: Cambridge University Press.

Casula, M. (1979). A.G. Baumgarten entre G.W. Leibniz et Chr. Wolff. *Archives de Philosophie*, 42, 547–574.

Crusius, C. (1747). *Weg zur Gewißheit*. Leipzig.

————. (1766). *Entwurf der nothwendigen Vernunft-Wahrheiten*. Third edition. Leipzig. **(EVW)**

De Risi, V. (2006). Leibniz around 1700: Three Texts on Metaphysics. *The Leibniz Review*, *16*, 55–69.

————. (2007). *Geometry and Monadology*. Basel: Birkhäuser.

Descotes, D. (2009). *Géométries de Port-Royal*. Paris: Honoré Champion.

Dunlop, K. (2019). Definitions and Empirical Justification in Christian Wolff's Theory of Science. *Logical Analysis and History of Philosophy*, *21*(1), 149–96.

Favaretti Camposampiero, M. (2019). Mereology and mathematics: Christian Wolff's foundational programme. *British Journal for the History of Philosophy*, *27*(6), 1151–1172.

————. (2021). Infinite Regress: Wolff's Cosmology and the Background of Kant's Antinomies. *Kant-Studien*, 112(2), 239–264.

Hogan, D. (2024). Kant's nutshell argument for idealism. *Noûs*. Online first. DOI: 10.1111/nous.12528

Kant, I. (1900–) *Gesammelte Schriften* (Preussische Akademie der Wissenschaften, Deutsche Akademie der Wissenschaften zu Berlin, and Akademie der Wissenschaften zu Göttingen, Eds.). Berlin: De Gruyter. **(KGS)**

Kant, I. (1997). *Lectures on Metaphysics*. (K. Ameriks and S. Naragon, Eds.). Cambridge: Cambridge University Press.

Leibniz, G. W. (1849–63). *Leibnizens mathematische Schriften*. (C. I. Gerhardt, Ed.). Halle: Schmidt. **(GM)**

————. (1875–90). *Die philosophischen Schriften*. (C. I. Gerhardt, Ed.). Berlin: Weidmann. **(GP)**

————. (1903). *Opuscules et fragments inédits*. (L. Couturat, Ed.). Paris: Alcan. **(C)**

————. (1923–). *Sämtliche Schriften und Briefe*. (Berlin-Brandenburg and Göttingen Academies of Sciences, Eds.). Darmstadt and Berlin: Akademie-Verlag. **(A)**

Leibniz, G. W. and C. Wolff. (1860/1963). *Briefwechsel zwischen Leibniz und Christian Wolff*. (C. I. Gerhardt, Ed.). Hildesheim: Olms Verlag.

Levey, S. (1999). Leibniz's Constructivism and Infinitely Folded Matter. In R. J. Gennaro and C. Huenemann (Eds.), *New Essays on the Rationalists* (pp. 134–162). Oxford: Oxford University Press.

Lewis, D. (1999). *Papers in Metaphysics and Epistemology*. Cambridge: Cambridge University Press.

Lewis, N. (2005). Robert Grosseteste and the Continuum. In L. Honnefelder et al. (Eds.), *Albertus Magnus & die Anfänge der Aristoteles-Rezeption* (pp. 159–187). Münster: Aschendorff.

Longuenesse, B. (2005). *Kant on the Human Standpoint*. New York: Cambridge University Press.

Mancosu, P. and M. Mugnai (2023). *Syllogistic Logic and Mathematical Proof*. Oxford: Oxford University Press.

Meier, G. F. (1765/2007). *Metaphysik: Erster Teil*. In *Christian Wolff Gesammelte Werke: Materiellen und Dokumente*, Band 108.1. Hildesheim: Olms Verlag.

Messina, J. (2015). Conceptual Analysis and the Essence of Space. *Archiv für Geschichte der Philosophie*, *97*, 416–57.

Mugnai, M. (1992). *Leibniz' Theory of Relations*. Stuttgart: Franz Steiner.

Nuzzo, A. (2018). Determination, Determinability, and the Structure of *Ens*. In C. D. Fugate and J. Hymers (Eds.), *Baumgarten and Kant on Metaphysics* (pp. 23–41). Oxford: Oxford University Press.

Pelletier, A. (2013). Monades sans monadologie? La “Métaphysique” de Baumgarten face à l’exposé monadologique de Leibniz. *Studia Leibnitiana*, *45*(2), 209–227.

Poser, H. (1979). Die Bedeutung des Begriffs ‘Ähnlichkeit’ in der Metaphysik Christian Wolffs. *Studia Leibnitiana*, *11*(1), 62–81.

Radner, M. (1998). Unlocking the second antinomy: Kant and Wolff. *Journal of the History of Philosophy*, *36*(3), 413–441.

Rusnock, P. and R. George. (1995). A Last Shot at Kant and Incongruent Counterparts. *Kant-Studien*, *86*, 257–77.

Shabel, L. A. (2003). *Mathematics in Kant’s Critical Philosophy*. New York: Routledge.

Schwaiger, C. (2011). *Baumgarten: Ein intellektuelles Porträt*. Stuttgart: Frommann-Holzboog.

Sutherland, D. (2005). Kant on Fundamental Geometrical Relations. *Archiv für Geschichte der Philosophie*, *87*, 117–158.

————. (2010). Philosophy, Geometry, and Logic in Leibniz, Wolff, and the Early Kant. In M. Domski and M. Dickson (Eds.), *Discourse on a New Method* (pp. 155–192). Chicago: Open Court.

————. (2022). *Kant’s Mathematical World*. Cambridge: Cambridge University Press.

Watkins, E. (2006). On the Necessity and Nature of Simples. In D. Garber and S. Nadler (Eds.), *Oxford Studies in Early Modern Philosophy, Volume 3* (pp. 361–314). Oxford: Oxford University Press.

Wells, A. (forthcoming). Situations, Congruence, and Leibniz’s Relationalism. *The Leibniz Review*.

Winegar, R. (2022). Kant’s Three Conceptions of Infinite Space. *Journal of the History of Philosophy*, *60*(4), 635–659.

Wolff, C. (1715). Meditatio de similitudine figurarum. *Acta Eruditorum*, May 1715, 213–218.

————. (1716/1965). *Mathematisches Lexicon*. Reprinted in *Christian Wolff Gesammelte Werke*, I. Abt. Bd. 11. Hildesheim: Olms Verlag.

————. (1725). *Vernünfftige Gedancken von den kräfften des menschlichen Verstandes*. Fourth edition. Halle.

————. (1730/2001). *Philosophia prima sive Ontologia*. Reprinted in *Christian Wolff Gesammelte Werke*, II. Abt. Bd. 3. Hildesheim: Olms Verlag. **(O)**

————. (1730/2003). *Elementa Matheseos Universæ. Tomus 1*. Reprinted in *Christian Wolff Gesammelte Werke*, II. Abt. Bd. 29. Hildesheim: Olms Verlag. **(EM)**

————. (1740/1983). *Philosophia rationalis sive logica. Pars II*. Reprinted in Christian Wolff Gesammelte Werke, II. Abt. Bd. 1.2. Hildesheim: Olms Verlag.

————. (1751). *Vernünfftige Gedancken von Gott, der Welt und der Seele des Menschen, auch allen Dingen überhaupt*. Halle. **(DM)**

Wolff, J. E. (2020). *The Metaphysics of Quantities*. Oxford: Oxford University Press.

Wunderlich, F. (2021). Wolff on Monadology and "*Materialisterey*". In S. d. F. Araujo et al. (Eds.), *The Force of an Idea* (pp. 105–120). Cham: Springer.