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Tables Requisite to be used with the Nautical Ephemeris for Finding the Latitude and Longitude at sea

Maskelyne, Nevil

London, 1781

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Explanation of the tables.

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EXPLANATION of the TABLES.

TABLE I.

IT is certain, both from experience and the laws of dioptrics, that the rays of light are bent out of their rectilineal course on passing obliquely out of one medium into another of a different density: and if the density of this latter medium continually increases, rays of light, as they pass through it, will be bent more and more from their first direction towards a line which is perpendicular to the surface of the medium on which they fall. Hence it is that all the heavenly bodies, except when they are in the zenith, appear higher than they ought to do; and the more so, the nearer they are to the horizon; because they then pass through a greater portion of the earth's atmosphere, as well as more obliquely to the refracting surface. This apparent elevation of the heavenly bodies above their true height is called their refraction, and the effect of it is contained in Table I. All observed altitudes of the sun, moon, and stars must be lessened by the numbers in this table, which are to be taken out with the observed altitude of the object.

TABLE II.

In observing altitudes with Hadley's quadrant, the image of the object is brought down to the visible horizon; that is, to the edge of the water: but, on account of the convexity of the earth's surface, and the elevation of the observer's eye above it, a line drawn from the observer's eye to the water's edge will fall below the true horizontal line, drawn through the eye of the observer; consequently, by bringing the image of the object down to the former line, instead of the latter, the altitude is made too great, and the more so, the higher the observer is raised above the surface of the sea; and the quantity of this error, to every probable height that the observer may be raised to, is contained in Table II. and is to be taken out with the height of the observer's eye in feet. This correction is evidently subtractive from the observed altitude.

TABLE III.

The parallax of any celestial object is the distance between its place in the heavens, as seen from the surface of the earth, and that in which it would be seen from the center: the last is called its true place, and is that which is given directly from astronomical tables. On this account all objects (except when they are in the zenith) appear lower than they really are; and the quantity of this error depends

pends jointly on the diſtance of the object from the center of the earth, and its altitude above the horizon; conſequently, when the diſtance of the object is continually the ſame, or nearly ſo, as is the caſe of the ſun, it will depend wholly on its altitude; being greateſt when the ſun is in the horizon, where it is about 9″, and leſſening gradually as the altitude increaſes, until it arrives at the zenith, where it is nothing.

The correction of the ſun's altitude on this account is contained in Table III. out of which it muſt be taken with the ſun's apparent altitude, and is always to be added to the apparent altitude to obtain the true altitude.

TABLE IV.

It is well known that objects appear greater or leſs, as they are at a greater or leſs diſtance from the obſerver; conſequently, as the moon is nearer to the obſerver by a ſemi-diameter of the earth when ſhe is in the zenith than when ſhe is in the horizon, and as the earth's ſemi-diameter bears a very ſenſible proportion to the moon's diſtance from its center, it is manifeſt that the ſemi-diameter of the moon muſt appear greater to a ſpectator on the earth's ſurface, when ſhe is in his zenith than when ſhe is in his horizon: and as this augmentation of her diameter is variable, increaſing all the way from the horizon to the zenith, it has been thought proper to give her horizontal ſemi-diameter in the Nautical Almanac to every noon and mid-night; and the augmentation of it, according to her altitude, is contained in Table IV. out of which it is to be taken with her altitude, and added to her horizontal ſemi-diameter, found in the Almanac, for the given time.

TABLE V.

The numbers in Table II. expreſs the dip of the viſible horizon, below the true, when it is entirely open, and free from all incumbrances of land or other objects that might hide it from the obſerver. But as it frequently happens, eſpecially in harbours, and when ſhips are running along ſhore, where, nevertheleſs, an obſervation may be very deſirable, that the ſun is over the land at the time when it is wanted, and the ſhore nearer to the ſhip than the viſible horizon would be, if it was unconfined; and as in that caſe the dip will be different from what it would otherwiſe have been, and greater, the nearer the ſhip is to that part of the ſhore which the ſun is brought down to; it has been judged proper to inſert the dip of the ſea to different heights of the eye, and different diſtances from the ſhip, in Table V. to be uſed inſtead of the numbers in Table II. when occaſions require. This Table is to be entered in the top column with the height of the eye above the ſurface of the ſea in feet, and in the left hand ſide column with the diſtance of the ſhip from the land in ſea miles; and, directly under the former, and oppoſite to the latter, ſtands the dip of that point in minutes of a degree; which is to be ſubtracted from the obſerved altitude inſtead of the numbers in Table II.

Moſt ſeamen can eſtimate, nearly, the diſtance of any object from the ſhip, eſpecially when that diſtance is not greater than five or ſix miles, which is the greateſt diſtance that the viſible horizon can be from an obſerver on the quarter deck of any ſhip whatſoever. But if any perſon wiſhes for a method of determining that diſtance by actual meaſurement, the following one, if executed carefully, will give it ſufficiently near the truth for the purpoſe of taking the dip out of Table V.

Let two obſervers, one placed as high up the main-maſt as he can conveniently be, and the other on the deck, directly underneath him, each obſerve the altitude

tude of the ſun, or other object that may be wanted, at the ſame inſtant of time; and let the height of each obſerver's eye, above the ſurface of the water, be carefully meaſured. Take the ſum and difference of theſe two heights, in feet, and alſo the difference of the two altitudes (of which that obſerved by the upper perſon will always be greateſt) and ſay, as the difference of the heights of the two obſervers is to the ſum of them, ſo is the ſine of the difference of the two obſerved altitudes to the ſine of an angle: take half the ſum of this angle and the difference of the obſerved altitudes; and ſay, as the radius is to the co-tangent of the half ſum, ſo is the height of the upper obſerver above the ſea, in feet, to the diſtance of the ſhip from the land in feet; which being divided by 6120, the feet in a ſea mile, gives the diſtance in miles. Or, to ſave this laſt diviſion, the obſerver may write out the table, putting the number of feet in the ſide column that correſponds to the miles and parts of a mile that are there now.

EXAMPLE.

Admit that the height of an obſerver's eye on the deck of a ſhip be 22 feet, that of another obſerver at the main-top-maſt-head 90 feet; and that the difference of the altitudes of the ſun's limb when brought down to the water's edge, by theſe two obſervers, is 12 minutes: how far were they from the land according to this obſervation?

Height of the maſt-head	90 feet,			
Deck — —	22 ditto,			
Difference —	68	Log. Ar. Co.	——	8,16749
Sum — —	112	Log.	—— ——	2,04922
Diff. obſerved altitudes	0° 12′ 0	Sine	— — —	7,54291
	0 19 46	Sine		7,75962
Sum — —	0 31 46			
Half ſum — —	0 15 53	Co-tang.	—	2,33533
Height of the maſt-head	— 90 feet —	Log.	— —	1,95424
Diſtance of the land	— 19479 feet	Log.	—	4,28957
Or — 3,69 Miles.				

TABLE VI.

Is intended to facilitate the reduction of the ſun's declination from the noon at Greenwich, for which time it is given in the Nautical Almanac, to the noon under any other meridian, or to any other time under that meridian. It has been uſual in tables of this nature, to make one argument the longitude of the ſhip or place from the meridian of Greenwich; or the time from noon at Greenwich, and the other argument the daily difference of the ſun's declination. But it was conceived that if the day of the month could be ſubſtituted for this latter argument, it would not only render the reduction more ſhort and eaſy, but alſo anſwer ſome other uſeful purpoſes: particularly it would greatly facilitate the operation of correcting latitudes given in the journals of ſuch ſeamen as had not themſelves attended to this particular, which is abſolutely neceſſary to be done before ſuch latitudes can be uſed in the conſtruction of maps and charts, and in forming geographical tables. In conſtructing this Table, the daily difference of the ſun's declination was taken for every day throughout a period of four years, including leap

leap year, and the firſt, ſecond, and third years following it: a mean of the daily differences for every four correſponding days was made out from theſe; and the greateſt difference between any one of theſe means and any one of the four daily differences of which it was compounded was too trifling to be mentioned. The principal error aroſe from the ſun's unequal motion in the ecliptic, on which account he is not at equal diſtances from the equinoxes at equal intervals of time from the days on which they happen, and conſequently the daily change in the ſun's declination on the correſponding days in the ſeveral quarters is not the ſame: the difference, however, between any one of the numbers, here put down, which reſult from taking a mean of the four, and any one of the extremes of which it was formed, never exceeded 16 or 17 ſeconds, and therefore this Table is ſufficiently exact for all nautical purpoſes, for which alone it was intended. The uſe of this Table will be ſufficiently obvious from the following examples.

EXAMPLE I.

What the ſun's declination at noon, in longitude 143° W. on May 2d, 1780?

Declination for noon at Greenwich, May 2d — —	15° 38′ 20″ N.
143° W. long. May 2d, give in Table VI. — —	+ 7 20
Declination for noon at the given place — — —	15 45 40 N.

EXAMPLE II.

What is the ſun's declination September 24th 1780, at noon, in longitude 105° E.?

Declination for noon at Greenwich, Sept. 24th — —	0° 47′ 22″ S.
105° E. long. on Sept. 24th give in Table VI. —	— 6 51
Declin. for noon at the given place — — —	0 40 31 S.

EXAMPLE III.

What is the ſun's declination Auguſt 24th 1780, at 8h 20′ in the evening, long. 48° W.?

Declination for noon at Greenwich, Auguſt 24th —	10 50 45 N.
In Table VI. Auguſt 24th } 48° W. longitude give —	— 2 51
} 8h 20′ afternoon give — —	— 7 24
Declination at the given time and place — —	10 40 30 N.

EXAMPLE IV.

What is the ſun's declination January 22d 1780, at 18h 40′ in longitude 132° E.?

N. B. 18h 40′ is 5h 30′ before noon on the 23d.

Declination for noon at Greenwich on the 23d — —	19° 18′ 25″ S.
In Table II. Jan. 23d. } 132° E. longitude give — —	+ 4 57
} 5h 30′ before noon give —	— 3 0
Declination at the given time and place — —	19 20 22 S.

TABLE

TABLE VII.

Contains the right afcenfions in time, and the declinations of fixty of the principal fixed ftars, for the beginning of the year 1780, with their annual variations both in right afcenfion and declination. If the places of thefe ftars are wanted for any time after the beginning of the year 1780, multiply the annual variation both in right afcenfion and declination by the number of years that have elapfed fince that time; to the product add fuch part of the annual variation as is paffed of the current year, and the fum will be the variation from the beginning of 1780 to the given time. This variation muft always be added to the right afcenfion for 1780; but the variation in declination muft be added or fubtracted, according as the fign + or — is found againft the annual variation in the laft column of the Table, to give the right afcenfion and declination for fucceeding years. But if the places of the ftars be wanted for any time before the beginning of the year 1780, the variation in right afcenfion muft be fubtracted from the right afcenfion found in the Table, and the variation in declination muft be applied with a contrary fign to that which is put againft it.

TABLE VIII.

Contains the correction of the moon's apparent altitude for the joint effects of parallax and refraction. It is to be entered with the apparent altitude of the moon's center in the top column, and her horizontal parallax in the left-hand fide column, and directly under the former, and oppofite to the latter, ftands the correction fought; which is always to be added to the apparent altitude of the moon's center to obtain the true.

TABLE IX.

This Table contains certain logarithms which were contrived by the late Mr. Dunthorne to facilitate the computation of the effects of parallax and refraction on the diftance of the moon from the fun or a fixed ftar. As fome confiderable improvements have been made in this mode of reducing the diftance, it was thought proper to extend this Table, as well as Table VIII. which conduces alfo to the fame purpofe, to every tenth fecond of the moon's horizontal parallax. The logarithms in this Table are the arithmetical complements of the differences between the logarithmic co-fines of the moon's true and apparent altitudes, increafed by 120, which number is uniformly the difference between the logarithmic co-fine of the true and apparent altitudes of a fixed ftar, or any other celeftial object which is not fenfibly affected by parallax; that object being more than 25° high. At altitudes lefs than 25° this uniformity ceafes, and the difference of the fines is lefs than 120 by the numbers contained in Table XI. confequently the arithmetical complements in Table IX. muft be leffened by the numbers contained in that Table. Table IX. depends on the fame arguments, and the logarithms are taken out of it exactly in the fame manner as the numbers are out of Table VIII.

TABLE X.

The numbers in this Table are to be fubtracted from the logarithms taken out of Table IX. when the moon's diftance from the fun is obferved. The difference of the logarithmic co-fines of the true and apparent altitudes of the fun being lefs than 120 by by these differences, on account of the sun's altitude being sensibly affected by parallax, as well as refraction.

TABLE XII.

This Table contains the moon's parallax in altitude to every minute of her horizontal parallax. It is to be entered with the moon's horizontal parallax at the top, and her altitude in the left-hand side-column; and under the former, and opposite to the latter, stands the moon's parallax in altitude, to the nearest minute. It is of use in reducing the apparent distance of the sun and moon, or of the moon and a star to the true distance, by Mr. *Lyon*'s method, as given in the first edition of the Requisite Tables, but is not used in the improvement of that method, given in this edition; Table VIII. being used in its stead.

TABLE XIII.

Is also useful in Mr. *Lyon*'s method of reducing the apparent distance of celestial objects to the true.

TABLE XIV.

This Table is very useful for converting degrees and minutes of the equator into time, and the contrary. The method of using it is too obvious to need pointing out.

TABLE XV.

This Table is analogous to the common tables of logistical logarithms; but continued up to three degrees, or hours, which are here made the radius of the Table, instead of one degree, or hour, as hath been usual in other tables of this kind. By this means it is peculiarly adapted to the purpose of finding the apparent time at Greenwich, by comparing the observed distance of the moon and sun, or of the moon and a fixed star, when reduced to the true, with the same distances, put down in the Nautical Almanac for every three hours, under the meridian of Greenwich. In taking the logarithms out of this Table, the degree, or hour, and the minutes to either, must be looked for at the top of the page, and the seconds in the left-hand side-column; under the former, and opposite to the latter, stands the logarithm sought.

These logarithms are also very useful in facilitating the computation of the effects of parallax and refraction upon the moon's distance from the sun or a star, either by Mr. *Lyon*'s method, or these two which were invented by the Rev. Dr. *Maskelyne*, Astronomer Royal, and Mr. *Witchell*, F. R. S. and inserted at the end of the Nautical Almanac for 1772; and also in every case where a proportion is to be worked, in which two or more of the terms are sexagesimals, and do not exceed three degrees, or three hours.

TABLE XVI.

Is intended to facilitate the solution of the problem for finding the latitude of a ship at sea, having the latitude by account, two observed altitudes of the sun, the time elapsed between the observations, measured by a common watch, and the sun's declination. The solution of this very useful problem, on these principles, was first invented by Mr. *Cornelis Douwes*, examiner of the sea officers and pilots by the appointment of the right honourable College of Admiralty at Amsterdam,

ſterdam, about the year 1740. They were ſome time ſince tranſmitted by him to the Lords Commiſſioners of the Engliſh Admiralty; and Mr. *Douwes* was rewarded with 50l. by the Commiſſioners of Longitude. It has ſince been found that they may be uſefully applied in the ſolution of other problems, for which purpoſe the column, intitled log. riſing, has been extended to 9 hours.

TABLE XVII.

Is a Table of natural ſines, which are wanted in computing the latitude of a ſhip at ſea by means of the preceding table: they will alſo be found uſeful on ſome other occaſions, as will be ſhewn in the courſe of the following rules and examples.

TABLE XVIII.

Contains the logarithms of natural numbers, from 1 to 10,000; and to five decimal places of figures, which is as far as they are generally wanted in the practice of navigation. The index muſt be prefixed by the computer, and is always leſs by unity than the number of figures in the natural number.

TABLE XIX.

The logarithmic ſines, tangents, and ſecants have been found abundantly ſufficient for the general purpoſes of navigation, when printed to five places of figures, beſides the index: accordingly the tangents and ſecants are exhibited to no greater length in this Table. But it was thought expedient to print the ſines to ſix places of figures, beſides the index, for the convenience of ſuch gentlemen as chuſe to uſe that improvement of Mr. *Dunthorne*'s method of reducing the apparent diſtance of the ſun and moon, which is inſerted in Problem X. of this book, becauſe the reduced diſtance cannot be had true to the neareſt ſecond by that method with fewer. Moreover, in order to facilitate the taking out of the ſines to ſingle ſeconds, the differences of thoſe ſines to 100″ are printed in two ſmall columns adjoining to them, and denominated Diff. 100″, and D. ſo that by multiplying this difference by the number of odd ſeconds, cutting off the two right hand figures of the product, and adding the remaining ones to the right hand figures of the ſines of the even minute, or ſubtracting them from the co-ſines of the even minute, will give the logarithmic ſine, or co-ſine, for the degrees, minutes, and ſeconds propoſed.

EXAMPLE I.

Suppoſe it were required to find the ſine of 24° 16′ 48″.

The diff to 100″ is	—	467
Multiply by	—	48
		3736
		1868
Add	—	224,16
Log. ſine of 24° 16′	—	9,613825
Log. ſine of 24° 16′ 48″		9,614049

EXAMPLE II.

Find the Log. co-ſine of 74° 16′ 34″.

The diff. to 100″ is	—	748
Multiply by	—	34
		2992
		2244
Subtract	—	254,32
Log. co-ſine of 74° 16′	-	9,433226
Log. co-ſine of 74° 16′ 34″		9,432972

On

On the contrary, if the degrees, minutes, and feconds be wanted to a given logarithmic fine, or co-fine: look for that fine which is next lefs, or the co-fine which is next greater than the given one; againft which ftand the degrees and minutes. Take the difference between the fine, or co-fine thus found, and the given one; add two cyphers to it, divide this number by the difference to 100″, and the quotient will be the feconds to be annexed to the degrees and minutes found before.

EXAMPLE I.

Find the degrees, minutes, and feconds correfponding to the log. fine 9.614049.

The given fine is —	9.614049
Sine next lefs (24° 16′)	9.613825
The difference is —	224

Two cyphers being added, makes 22400; and if this be divided by 467, the difference to 100″, the quotient will be 48″, to be annexed to 24° 16′: the anfwer is therefore 24° 16′ 48″.

EXAMPLE II.

Find the degrees, minutes, and feconds anfwering to the log. co-fine 9.432968.

The given co-fine is —	9.432968
Co-fi. next greater (74° 16′)	9.433226
The difference is —	258

Two cyphers being added, makes 25800; and this being divided by 748, the difference to 100″, the quotient will be 34″, to be annexed to 74° 16′: the anfwer is therefore 74° 16′ 34″.

But that this additional place of figures may not embarrafs thofe who want five places only, the fixth place is feparated from the others by a point; by which means the five firft places, after the index, are taken out as readily as if the fixth was not there: with this caution, however, that when the fixth figure exceeds 5, the preceding figure, or laft of the five, muft be increafed by unity.

TABLE XX.

An exact knowledge of the geographical fituation of places is of the utmoft importance, efpecially to fea-faring perfons: it has therefore been thought proper to add a table of thofe places, of which the fituations are fuppofed to be known with tolerable exactnefs; either from aftronomical obfervations made there, or from good geographical furveys.

The table is divided into feven columns; the firft contains the names of the feveral places, digefted in alphabetical order; the fecond contains the part of the world; the third, the country, coaft, or fea where they are; the fourth the latitude; the fifth and fixth the longitude, in degrees, and in time, reckoned from the meridian of Greenwich; and in the feventh are put down the times of high water on the days of the full and change of the moon, at thofe places where it has been obferved.

TABLE XXI.

As the moon paffes the meridian of any place later every day than fhe did the day before, by a number of minutes, which is equal to the difference of the variation of the fun and moon's right afcenfions in time, in the interval, it is obvious, that the moon muft pafs the meridians of fuch places as lie to the weftward of Greenwich later, and the meridians of fuch places as are to the eaftward of Greenwich fooner, than fhe paffes the meridian of Greenwich by a number of minutes, which is to the number of minutes in the above-mentioned difference, as the diſtance of that meridian from the meridian of Greenwich is to 360°. And becauſe it is frequently of uſe, at ſea, to know the time of the moon's paſſage over the meridian, uſually called her ſouthing; the number of minutes by which ſhe paſſes the meridian of any place, before or after the time at which ſhe paſſes the meridian of Greenwich, is inſerted in this table. The table is to be entered in the top column with the daily variation of the moon's paſſing the meridian, and in the left-hand ſide-column with the longitude of the ſhip or place; directly under the former, and oppoſite to the latter, ſtand a number of minutes, which being added to the time of the moon's paſſing the meridian of Greenwich, if the longitude be weſt; or ſubtracted from it, if the longitude be eaſt, will give the time of its paſſage over the meridian of the given place.

Note. The daily variation of the moon's paſſing the meridian is found by taking the difference between the time of the moon's paſſage over the meridian of Greenwich on the propoſed day and the day following, if the longitude of the ſhip or place be weſt; or between the time of her paſſage on the propoſed day and that preceding it, if the ſhip or place be in eaſt longitude.

T A B L E XXII.

This table is uſeful in finding the moon's declination, at a given place and time, from her declination given in the Nautical Almanac for noon and midnight at Greenwich. The manner of uſing it is this. Turn the longitude of the ſhip or place into time; and if it be weſt, add it to, but if it be eaſt, ſubtract it from the time at the given place, and it will give the time at Greenwich. Take the moon's declination out of the Nautical Almanac for noon or midnight on the given day, according as the time at Greenwich is leſs or greater than 12^h: enter the table with the *variation of the moon's declination in* 12^h in the top column, and the time at Greenwich in the right hand ſide column, if that time be leſs than 12^h, but with its exceſs above 12^h, if it exceeds that quantity: under the former, and oppoſite the latter ſtands the correction of the moon's declination, which muſt be added to her declination for noon, or midnight, at Greenwich, if the declination be encreaſing, but ſubtracted from it if the declination be decreaſing; and the ſum, or difference, will be the moon's declination at the given time.

E X A M P L E I.

What is the moon's declination April 16th 1783, at $7^h\ 22'$ in long. 57° weſt?

Time at the ſhip, or place		$7^h\ 22'$
Long. in time W. add	—	3 48
Time at Greenwich	—	11 10
Moon's declin. at noon	—	7° 18′ S.
Tab. XXII. under 3° 15′ and opp. $11^h\ 10'$ gives	+	3 2
Moon's declination required		10 20 S.

E X A M P L E II.

What is the moon's declin. April 25th 1783, at $17^h\ 47'$ in long. 162° weſt?

Time at the ſhip, or place		$17^h\ 47'$
Long. in time W. add	—	10 48
Time at Greenwich on 26th		4 35
Moon's declin. at noon		10° 24′ S.
Tab. XXII. under 3° 3′ and opp. $4^h\ 35'$ gives	—	1 10
Moon's declin. required	—	9 14 S.

EXAM-

EXAMPLE III.

What is the moon's declination July 4th, 1783, at 19^h in long. 67° eaſt?

Time at the ſhip, or place	19^h 0′
Long. in time E. ſubt. —	4 28
Time at Greenwich —	14 32
Moon's declination at midn.	8° 7′N.
Tab. XXII. under 2° 54′ and opp. 2^h 32′ gives	— 37
Moon's declin. required	7 30 N.

EXAMPLE IV.

What is the moon's declination July 20th, 1783, at 4^h 49′ in Long. 114° E.?

Time at the ſhip, or place	4° 49′
Long. in time E. ſubt. —	7 36
Time at Greenwich the 19th	21 13
Moon's declin. at midnight	6° 58′N.
Tab. XXII. under 2° 58′ and opp. 9^h 13′ gives	+ 2 17
Moon's declin. required	9 15 N.

The moon's right aſcenſion may alſo be found by the help of this table, if it be entered at the top with half the variation of her right aſcenſion in 12^h: the number found in the Table muſt be doubled, and added to the right aſcenſion at noon or midnight.

TABLE XXIII.

This table will be found very uſeful in finding the ſun's right aſcenſion for any given time, either before or after noon, under the meridian of Greenwich, from the right aſcenſions of the ſun, given in p. II. of the Nautical Almanac for noon at that place; and alſo in finding the ſun's right aſcenſion at noon under any other meridian. It will alſo greatly facilitate the finding the ſame thing for any time under any given meridian, by combining the two former problems together. The table muſt be entered at the top with *the daily variation of the ſun's right aſcenſion,* and in the left hand column with the given time from noon, or with the ſhip's longitude in the right-hand column; and directly under the former, and oppoſite to the latter, ſtands a number of minutes and ſeconds to be added to the ſun's right aſcenſion for noon at Greenwich, if the time be after noon, or the longitude of the ſhip be weſt; but to be ſubtracted from it, if the time be before noon, or the longitude of the ſhip be eaſt.

EXAMPLE I.

What is the ſun's right aſcenſion at noon May 24th, 1780, in longitude 124° eaſt?

Sun's R. A. for N. at Greenwich		4^h 7′ 7″
Tab. XXIII. with daily diff. 4′ 2″ and eaſt long. 124°	gives	— 1 23
Sun's R. A. at N. in long. 124° E.		4 5 44

EXAMPLE II.

What is the ſun's right aſcenſion on July 21ſt, 1780, at 9^h 42′ P. M. at Greenwich?

Sun's R. A. for N. at Greenwich		8^h 5′ 18″
Tab. XXIII. with daily diff. 3′ 59″ and 9^h 42′ P. M.	gives	+ 1 37
Sun's R. A. at 9^h 42′ P. M.	—	8 6 55

EXAMPLE III.

What was the ſun's right aſcenſion January 16th, 1780, at 6^h 48′ A. M. in longitude 68° weſt?

6^h 48′ A. M. is 5^h 12′ before Noon.

Sun's R. A. for Noon at Greenwich		19^h 52′ 21″
Tab. XXIII. & 4′ 17″ daily variation	68° W. gives	+ 49
	5^h 12′ gives	— 56
Sun's R. A. for giv. time and place		19 52 14

EXAMPLE IV.

What was the ſun's right aſcenſion Auguſt 21ſt, 1780, at 9^h 17′ P. M. in longitude 167° eaſt?

Sun's R. A. for N. at Greenwich		10^h 4′ 12″
Tab. XXIII. & 3′ 42″ daily variation	167° E. gives	— 1 41
	9^h 17′ gives	+ 1 26
Sun's R. A for given time and place		10 3 57

THE

THE USE AND EXEMPLIFICATION OF THE TABLES.

PROBLEM I.

TO find the latitude of a ſhip from the obſerved meridional altitude of the ſun's upper or lower limb.

RULE.

Correct the obſerved altitude of the ſun's limb by ſubtracting the dip of the horizon (Tab. II.) and the refraction (Tab. I.) from it; and adding the ſemi-diameter (Naut. Almanac, p. III.) if the lower limb was obſerved, or by ſubtracting it, if the upper limb was obſerved, and by adding, if you pleaſe, the parallax in altitude (Tab. III.) This will give the true meridional altitude of the ſun's center. The refraction muſt be taken out with the altitude of the limb, corrected for the dip of the horizon. Take the true altitude from 90, and it will leave the true zenith diſtance; which is north, if the zenith was to the north of the ſun; or ſouth, if it was the contrary. Take the ſun's declination out of the Nautical Almanac (p. II.) by the help of Tab. VI. noting whether it be north or ſouth.

Then if the zenith-diſtance and declination are both north, or both ſouth, add them together; but if one be north, and the other ſouth, ſubtract the leſs from the greater, and the ſum or difference will be the latitude; of the ſame name with the greater,

EXAMPLE I.

July 24th 1783, longitude 54° weſt, the meridional altitude of the ſun's lower limb was obſerved to be 59° 16′, the zenith being north of the ſun, and the height of the obſerver's eye 24 feet above the ſurface of the ſea: what was the latitude?

Altitude of the ſun's lower limb — —	59°	16′	0″
Dip of the horizon from Tab. II. ſubtr. —		4	40
Refraction from Tab. I. ſubtr. — — —			34
Parallax in altitude Tab. III. add — — —			4
The ſun's ſemi-diameter (p. III. Naut. Alman.) add		15	48
True altitude of the ſun's center — —	59	26	38
	90	0	0
True zenith-diſtance — — — —	30	33	22 N.
The ſun's declination (Naut. Alman. p. II.) —	19	51	0 N.
Latitude of the Ship — — —	50	24	22 N.

SCHOLIUM.

It has been uſual to divide the rule for this problem into different caſes; but the neceſſity for ſuch diviſion aroſe wholly from conſidering, improperly, the zenith of the place as a fixed point, inſtead of the ſun.

PROBLEM

PROBLEM II.

To find the latitude of a ſhip at ſea from the obſerved meridional altitude of a fixed ſtar.

RULE.

Correct the obſerved altitude of the ſtar by ſubtracting the refraction (Tab. I.) and the dip of the horizon (Tab. II.) from it; which will give the true altitude. Take the true altitude from 90 degrees, and it will leave the true diſtance from the zenith, which is north or ſouth according as the zenith is north or ſouth of the ſtar at the time of obſervation. Take the ſtar's declination out of Table VII. and note whether it be north or ſouth.

Then, if the zenith-diſtance and declination be both north, or both ſouth, add them together; but if one be north, and the other ſouth, ſubtract the leſs from the greater, and the ſum or difference will be the latitude; of the ſame name with the greater.

EXAMPLE.

March 29th 1783, the meridional altitude of Procyon was obſerved 77° 27¼′, the zenith being ſouth of the ſtar, and the height of the obſerver's eye above the ſurface of the ſea 22 feet; what was the latitude?

Meridional altitude of Procyon — —	77°	27′	15″
Refraction from Tab. I. ſubtract — —			13
Dip of the horizon from Tab. II. ſubt. — —		4	28
True altitude of Procyon — — —	77	22	34
	90	0	0
True zenith-diſtance of Procyon — —	12	37	26 S.
Declination of Procyon from Tab. VII. ——	5	46	17 N.
Latitude of the Ship — — — —	6	51	9 S.

SCHOLIUM.

If the meridional altitude of a circum-polar ſtar be obſerved when it is below the pole, or the meridional altitude of the ſun at midnight, in any place where it does not ſet: then, if to ſuch altitude, corrected as above, there be added the ſtar or ſun's polar-diſtance; that is, the complement of its declination, the ſum will be the latitude of the place; of the ſame name with the declination.

PROBLEM III.

To find the latitude of a ſhip at ſea from the obſerved meridional altitude of the moon's upper or lower limb.

RULE.

To the longitude of the given place, in time, add the number from Tab. XXI. correſponding to that longitude, and take the ſum from the time of the moon's ſouthing (p. VI. of Naut. Alman.) for the given day, if the longitude be eaſt; or add it to the time of ſouthing if the longitude be weſt; and the ſum or difference will be the time at Greenwich when the moon was on the meridian of the given place. To this time take the moon's horizontal parallax and ſemi-

diameter

diameter from p. VII. of the Naut. Alman. and her declination from p. VI. by the help of Tab. XXII. noting whether it be north or south.

Correct the observed altitude of the moon's limb by subtracting the dip of the horizon (Tab. II.) from it, and adding the correction of her altitude (taken out of Tab. VIII. with the altitude of the limb corrected for dip) and also her semi-diameter, if the lower limb was observed, or by subtracting it if the upper limb was observed, which will give the true altitude of her center. Take the true altitude from 90 degrees, and it will leave the distance from the zenith, which will be north or south according as the zenith was north or south of the moon.

Then if the zenith distance and declination be both north, or both south, add them together; but if one be north and the other south, take the less from the greater, and the sum or difference will be the latitude; of the same name with the greater.

EXAMPLE.

August 24th 1783, in longitude 107° E. the meridional altitude of the moon's upper limb was observed to be 67° 42½′, the zenith being north of the moon, and the height of the observer's eye 23 feet above the surface of the sea; what was the latitude?

The dip of the horizon of the sea is 4′ 34″.

To 7^h 8′, the longitude in time, add 14′, the number corresponding to it in Tab. XXI. take their sum, 7^h 22′, from 22^h 11′, the time of the moon's southing in the Nautical Almanac for the given day, and there will remain 14^h 49′, or 2^h 49′ past midnight; at which time the moon's horizontal parallax was 54′ 27″, her semi-diameter 14′ 51″, her declination 24° 0½′ N. and the correction from Tab. VIII. answering to horizontal parallax 54′ 27″ and altitude corrected for dip 67° 38′ is 20′ 20″.

Meridional altitude of the moon's upper limb	67°	42′	30″
Dip of the horizon (Tab. II.) subtract ——		4	34
Correction from Tab. VIII. add — — —		20	20
Semi-diameter (p. VII. Naut. Alman.) subtract		14	51
True altitude of the moon's center — —	67	43	25
	90	0	0
True zenith-distance — — — —	22	16	35 N.
The moon's declination — — —	24	0	30 N.
Latitude of the ship — — — —	46	17	5 N.

PROBLEM IV.

To find the latitude of a ship at sea, having the latitude by account, two observed altitudes of the sun, the time elapsed between the observations, measured by a common watch, and the sun's declination.

RULE.

To the log-secant of the latitude by account, add the log-secant of the sun's declination: their sum, rejecting 20 from the index, is the log-ratio.

From the natural sine of the greater altitude, taken out of Tab. XVII. subtract the natural sine of the least altitude; find the logarithm of the remainder, and write it under the log-ratio.

With

With half the *elapſed time* enter Table XVI. and, from the column of *half elapſed time*, take out the logarithm anſwering to it, which is alſo to be ſet down under the log. ratio.

Add theſe three logarithms together, and look for their ſum in Table XVI. in the column of *middle time*; and, having found the logarithm neareſt to it, take out the time correſponding, put it under half the elapſed time, and ſubtract the leſs from the greater: their difference will be the time from noon when the greater altitude was taken.

With this time enter the Table again, and, from the column of *log. riſing*, take out the logarithm correſponding to it: from this logarithm ſubtract the *log. ratio*, and the remainder will be the logarithm of a natural number, which being found in Table XVIII. and added to the natural ſine of the greater altitude, will give the natural ſine of the meridional altitude of the ſun.

From the meridional altitude of the ſun the latitude of the ſhip is to be found by the latter part of Problem I.

Scholium.

If the latitude found by the preceding rule differ conſiderably from the latitude by account, the operation muſt be repeated, uſing the latitude laſt found, inſtead of the latitude by account, until the reſult gives a latitude which agrees nearly with the latitude uſed in the computation.

Example I.

July 20th 1779, being at ſea, in latitude 39° 28′ N. by account, at 11h 30′ 15″ by my watch, the altitude of the ſun's lower limb was obſerved to be 68° 18¾′; and at 12h 26′ 28″, it was 70° 58′, the height of my eye above the ſurface of the ſea being 21 feet: what was the true latitude of the ſhip?

Alt. ſun's L. L. 1ſt obſ.	68 18 45	—	Alt. ſun's L. L. 2d obſ. -	70 58 0
Dip of the horizon ſubt.	4 22			4 22
Refraction, ſubtract —	23			19
Sun's ſemi-diameter, add	15 48			15 48
True altitude ſun's center	68 29 48			71 9 7

Time by watch.	Alt. ſun's center.	Natural ſine.		
		—	Lat. by acc. - 39° 28′ ſecant -	0.11239
11h 30′ 15″	- 68° 30′	- 93042 —	Sun's declin. - 20 41 ſecant -	0.02893
12 26 28	- 71 9	- 94637 —	Log. ratio — —	0.14132
0 56 13 elapſ. time.		1592 —	— Logarithm — —	3.20194
0 28 6½	— —	Logarithm of half the elapſed time	— —	0.91154
0 20 36½	—	Logarithm of the middle time	— —	4.25480
0 7 30	Time from noon	———	Log. riſing —	1.72869
			Log. ratio — —	0.14132
Natural number	——	———	39 - -	1.58737

Natural number — — —	39	—	1.58737
Natural ſine of greater altitude —	94637	— —	
		90° 0′	
Natural ſine of merid. altitude —	94676	71 13	
Meridional zenith diſtance —		18 47	
The ſun's declination — —		20 41	
Latitude of the ſhip — —		39 28 North.	

Example II.

Nov. 21ſt 1779, being at ſea, in latitude 50° 40′ N. by account, and longitude 48° Weſt, at 10^h 17′ 30″ by watch, the altitude of the ſun's lower limb was obſerved to be 17° 4¼′, and at 11^h 17′ 30″ it was 19° 31¾′, the height of the obſerver's eye above the ſurface of the ſea being 21 feet: what was the latitude of the ſhip?

Alt. of ſun's L. L. at 1ſt ob.	17° 4′ 15″	Alt. of ſun's L. L. at 2d ob.	19° 31′ 45″
Dip of the horizon, ſub.	4 22		4 22
Refraction, ſub. —	3 4		2 39
Semi-diameter, add —	16 15		16 15
True alt. ſun's cent.	17 13 4		19 40 59

Time by watch.	Alt. ſun's center.	Natural ſine.		
			Lat. by acc. 50° 40′ ſec. -	0 19803
10° 17′ 30″ -	17° 13 -	29599	Sun's declin. 20 0 ſec. -	0.02701
11 17 30 -	19 41 -	33682	Log. ratio -	0.22504
1 0 0 elapſ. time.		4083 - -	Logarithm -	3.61098
0 30 0 half elapſed time		—	Logarithm -	0.88430
1 0 50 middle time		— — —		4.72032
0 30 50 time from noon		— —	Log. riſing —	2.95599
			Log. ratio —	0.22504

Nat. number —	538	— — —	2.73095
Nat. ſine gr. alt. —	33682 -	90° 0′	
Nat. ſine mer. alt. -	34220 -	20 1	
Meridional zenith diſtance -		69 59	
The ſun's declination - -		20 0	
Latitude of the ſhip —		49 59 N.	

As the latitude reſulting from this computation differs 41 miles from that by account, the operation muſt be repeated, uſing the laſt found latitude inſtead of that by account.

Laſt

Laſt found latitude 49° 59′ ſec.		-	0.19178
Sun's declination 20 0 ſec.		-	0.02701
		Log. ratio -	0.21879
Diff. N. ſines 4083	——	Logarithm -	3.61098
Half elapſed time 0ʰ 30′ 0″	——	Logarithm -	0.88430
1 0 0	——	Middle time -	4.71407
Time from noon - 0 30 0	——	Log. riſing -	2.93223
		Log. ratio -	0.21879
Natural number — 517	—— ——	Logarithm -	2.71344
Natural ſine gr. altitude 33682	- -	- 90 0	
Nat. ſine mer. altitude 34199	- -	- 20 0	
		70 0 zenith diſt.	
		20 0 ſun's declin.	
		50 0 latitude N.	

The latitude laſt found differing only one minute from that uſed in the operation may be relied on as the true latitude.

In the two preceding examples it has been ſuppoſed that both altitudes were taken at the ſame place; but as that can ſeldom happen at ſea, it is neceſſary to ſhew how to correct one of the altitudes ſo as to make it what it would have been if obſerved at the ſame place where the other was; and this may readily be done as follows:

Let the bearing of the ſun be obſerved by the compaſs at the inſtant of the firſt obſervation: take the number of points between it and the ſhip's courſe, corrected for lee-way, if ſhe makes any; with which, if leſs than eight, or with what it wants of 16 points, if more than eight, and the diſtance run between the obſervations, enter the *Traverſe Table*, and take out the *difference of latitude* correſponding to them. Add this *difference of latitude* to the firſt altitude, if the number of points between the ſun's bearing and ſhip's courſe were leſs than eight; but ſubtract it from the firſt altitude, if the number of points were more than eight, and it will be reduced to what it would have been if obſerved at the ſame place where the ſecond was.

Note, The reſult of the operation will be the latitude of the ſhip at the time when the ſecond altitude was taken, and muſt be reduced to noon by means of the log.

Example III.

November 19th 1779, latitude by account 47° 34′ N. longitude 30° E. at 9ʰ 55′ 30″ by watch, the altitude of the ſun's lower limb was obſerved to be 17° 24′, and the bearing of its center by compaſs, S. b. E ¼ E. and at 12° 54′ 10″ the obſerved altitude of its lower limb was 21° 45½′, the height of the obſerver's eye being 20 feet. The ſhip's courſe, by compaſs, was E. ½ S. at the rate of ſeven knots, and ſhe made no lee-way: what was the true latitude of the ſhip at the time of the latter obſervation?

Ship's courſe E. ½ S.	—	7½ points
Sun's bearing S. b. E. ¾ E.		1¾ points
Angle between	—	6¼ points, diſtance run 21 miles.
Difference latitude (add)	—	7′ 0″
Alt. ſun's low. limb, 1ſt obſ.		17 24 0

Firſt altitude reduced -	17 31 0	Alt. ſun's lower limb, 2d obſ.	21° 45′ 30″
Dip of the horizon, ſubt.	4 16	— —	4 16
Refraction, ſub. —	3 0	— —	2 22
Sun's ſemi-diam. add —	16 15	— —	16 15
True alt. ſun's center —	17 39 59	—	21 55 7

Time by watch.	Alt. ſun's center.	Natural ſine.			
9ʰ 55′ 30″	17° 40′	30348	Lat. by acc. 47° 34′ ſec.	-	0.17087
			Sun's decl. 19 30 ſec.	-	0.c2565
12 54 10	21 55	37326	— Log. ratio	—	0.19652
2 58 40 elapſ. time		6978	— Logarithm	— —	3.84373
1 29 20 half elapſ. time			— Logarithm	—	0.42022
0 33 10 middle time			— Logarithm	—	4 46047
0 56 10 time from noon			— Log. riſing	—	3.47539
			Log. ratio	—	0.19652

Natural number	-	1901	- -	3.27887
Natural ſine of greater alt.	-	37326		
				90 0
Natural ſine of mer. alt.	-	39227	=	23 6
Meridional zen. diſt.	-			66 54
Sun's declination	- -			19 30
Latitude of the ſhip	-			47 24

EXAMPLE IV.

October 28th 1766, latitude by account, at the time of the latter altitude 47° 50′ N. at 11ʰ 28′ 20″, A. M. by watch, the altitude of the ſun's lower limb was obſerved 28° 18′, and the azimuth of his center by compaſs, S. b. W. At 2ʰ 58′ 20″ by watch, the altitude of his lower limb was found 16° 40′; the height of the obſerver's eye being 20 feet; moreover the ſhip's courſe was N. E. with her larboard-tacks on board, at the rate of ſix knots, and ſhe made half a point lee-way: what was the latitude of the ſhip when the latter altitude was taken?

Ship's co. N E. with ½ a point lee-way on the larboard-tack makes N. E. ½ E. which is twelve points and a half from S. b. W. the ſun's bearing at the firſt obſervation; which being taken from 16 points, becauſe it is above eight, leaves 3½ points.

$3\frac{1}{2}$ points. This, as a course, with 21 miles, the distance run between the observations, gives diff. of lat. sub. 0° 16′ 0″
Alt. sun's low. limb at 1st obs. 28 18 0

First altitude reduced —	28 2 0	Alt. sun's l. limb, 2d obs.	16° 40′ 0″
Dip of the horizon, sub. -	4 16	- - - -	4 16
Refraction, sub. -	1 46	- - - -	3 8
Semi-diameter, add -	16 10	- - - -	16 10
True alt. of the center - -	28 12 8	— — —	16 48 46

Time by watch.	Alt. sun's center.	Natural sine.		
			Lat. by acc. 47° 50′ sec. - -	0.17309
			Sun's decl. 13 17 sec. - -	0.01178
11h 28′ 20″	28° 12′	47255	Log. ratio — -	0.18487
14 58 20	16 49	28931		
3 30 0 elapsf. time		18324	— — Logarithm —	4.26302
1 45 0 half elapsed time		———	Logarithm —	0.35430
1 14 0 middle time		—	— Logarithm —	4.80219
0 31 0 time from noon		—	— Log. rising —	2.96067
			Log. ratio —	0.18487
Natural number - - -		597	- - -	2.77580
Natural sine of greater altitude -		47255		
			90° 0′	
Natural sine merid. altitude		47852 =	28 35	
The sun's meridional zen. distance -			61 25	
The sun's declination - -			13 17	
The lat. of the ship —			48 8 N.	

Remark I. The operation is the same whether the sun hath north or south declination; and also whether the ship be in north or south latitude.

Remark II. When the sun hath no declination, the secant of the latitude will be the *log. ratio.*

Remark III. The observations must always be taken between nine o'clock in the morning and three in the afternoon; and the nearer the greater altitude is to noon, the better.

Remark IV. If both observations are in the forenoon, the interval must not be much less than half the distance of the first observation from noon.

Remark V. If both observations are in the afternoon, the interval between them must not be much less than the distance of the first observation from noon.

Remark VI. If one observation be in the forenoon, and the other in the afternoon, the interval must not exceed four hours and an half.

Remark VII. The above limitations are founded on a supposition that the sun's meridional zenith distance is not less than the latitude of the place; but if the latitude of the place should be double the sun's meridional zenith distance, the first of two altitudes taken in the forenoon must not be before half past nine, nor the second before three quarters past ten. The first of two taken in the afternoon must not be later than a quarter past one, nor the second after half past two.

If

If one be taken in the morning and the other in the afternoon; that in the morning muſt not be taken before half paſt nine o'clock, and the interval between them muſt not exceed $3\frac{1}{2}$ hours.

Remark VIII. If the latitude of the place be three times the ſun's meridional zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before ten o'clock, nor the ſecond before eleven. The firſt of two taken in the afternoon muſt not be later than one o'clock, nor the ſecond after two. If one obſervation be taken in the forenoon, and the other in the afternoon; that in the morning muſt not be before ten, and the interval between them muſt not exceed 3 hours.

Remark IX. If the latitude be five times the ſun's meridional zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before half paſt ten o'clock, nor the ſecond before a quarter after eleven. The firſt of two taken in the afternoon muſt not be later than three quarters paſt twelve, nor the ſecond later than half paſt one o'clock. If one be taken in the forenoon, and the other in the afternoon; the morning one muſt not be before half paſt ten, and the interval between them muſt not exceed two hours and a quarter.

Remark X. If the latitude be twelve times the ſun's meridional zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before eleven o'clock, nor the latter before half paſt eleven. The firſt of two taken in the afternoon muſt not be after half paſt twelve, nor the latter after one o'clock. If one be in the forenoon, and the other in the afternoon, the morning one muſt not be before eleven o'clock, and the interval between them not more than an hour and an half.

If the preceding remarks be attended to, the latitude found by the calculation will be, at leaſt, five times nearer the truth than the latitude by account; that is, the error in the computed latitude will not be above a fourth part of the difference between them: and hence a judgment may be formed whether it will be neceſſary to repeat the computation with the latitude laſt found or not.

PROBLEM V.

The apparent time, the ſhip's latitude and longitude, and the ſun's declination being given, to find its altitude.

RULE.

If the ſun's declination, and the co-latitude of the ſhip be both north or both ſouth, take their ſum*; but if one be north and the other ſouth, take their difference for the ſun's meridional altitude.

With the apparent time from noon enter Table XVI. and take the logarithm correſponding to it out of the column of *log. riſing*; to which add the co-ſine of the latitude, and the co-ſine of the ſun's declination; their ſum, rejecting 20 from the index, will be the logarithm of a natural number, which being ſubtracted from the natural ſine of the meridional altitude, will give the natural ſine of the ſun's altitude at the given time.

EXAMPLE I.

What is the true altitude of the ſun's center in latitude 49° 57′ N. on July 25th 1780, at 6^h 56′ 20″ in the morning?

* If this ſum exceed 90° take it from 180°, and uſe the natural ſine of the remainder.

Appa-

	12h 0′ 0″				
Apparent time —	6 56 20				
Time from noon	5 3 40	Its log. rising (Tab. XVI.) - -			4.87890
Co-latitude —	40° 3′ N.	log. sine (Tab. XIX.) - -			9.80852
Sun's declination	19 34 N.	log. co-sine (Tab. XIX) - -			9.97417
			45876	log. —	4.66159
Meridional altitude	59 37	Nat. sine	86266		
True alt. sun's center	23 49	Nat. sine	40390		

EXAMPLE II.

What was the true altitude of the sun, at London, November 24th 1779, at 3h 21′ 30″ apparent time, in the afternoon?

Apparent time from noon	3h 21′ 30″	—	Log. rising —		4.55900
Co-latitude —	38° 28′ N.	—	Log. sine —		9.79383
Sun's declination —	20 38 S.	—	Log. co-sine —		9.97121
			21088	Log. -	4.32404
Meridional altitude	17 50	Nat. sine	30625		
True alt. sun's center	5 28	Nat. sine	09537		

PROBLEM VI.

The apparent time, and the latitude and longitude of the ship being given, to find the altitude of any known-fixed star.

RULE.

Turn the longitude of the ship into time; and, if it be west, add it to, but if it be east, subtract it from the apparent time at the ship, and it will give the time at Greenwich. Take the sun's right ascension for that time out of the Nautical Almanac, by the help of Tab. XXIII. and add it to the apparent time at the ship, which will give the right ascension of the mid-heaven. Take the star's declination and right ascension out of Tab. VII. and take the difference between its right ascension and the right ascension of the mid-heaven, which will be the distance of the star from the meridian.

With the distance of the star from the meridian take the log. rising out of Table XVI. to which add the co-sine of the ship's latitude, or the sine of its co-latitude, and the co-sine of the star's declination; their sum, rejecting 20 from the index, will be the logarithm of a natural number, which being subtracted from the natural sine of the meridional altitude of the star (found as in the preceding problem) will give the natural sine of the star's altitude at the given time.

EXAMPLE

EXAMPLE.

What was the true altitude of Aldebaran, at London, on April the 11th 1780, at $5^h\ 56'\ 20''$ apparent time?

Apparent time —	$5^h\ 56'\ 20''$		
Long. in time W.	22		
Time at Greenwich	5 56 42	Sun's Æ $1^h\ 22'\ 53''$ by Tab. XXIII.	
	App. time	5 56 $20''$	
Right ascension of the mid-heaven		7 19 13	
The star's right ascension (Tab. VII.)		4 23 20	
Distance of the star from the meridian		2 55 53	Log. rising 4.44763
Declination of Aldebaran (Tab. VII.)		16 3 N.	Co-sine — 9.98273
Co-latitude of London — —		38 28 N.	Sine — 9.79383
Meridional altitude of the star		54 31	Nat. sine 81428
Natural number		—	16757 log. 4.22419
The star's true altitude —	$40°\ 18'$	Nat. sine	64671

PROBLEM VII.

The apparent time, and the latitude and longitude of the ship being given, to find the true altitude of the moon's center.

RULE.

Turn the longitude of the ship into time; and if it be west, add it to, but if it be east, subtract it from the apparent time at the ship, and it will give the time at Greenwich. By the help of Tab. XXIII. take the sun's right ascension out of the Nautical Almanac, p. II. and add it to the apparent time at the ship, which will give the right ascension of the mid-heaven. To the time at Greenwich, take the moon's declination and right ascension out of the Nautical Almanac (p. VI.) by the help of Tab. XXII. Turn the right ascension into time, and take the difference between it and the right ascension of the mid-heaven, which will be the distance of the moon from the meridian.

With the distance of the moon from the meridian take the log. rising out of Tab. XVI. to which add the co-sine of the ship's latitude, or the sine of its co-latitude, and the co-sine of the moon's declination; their sum, rejecting 20 from the index, will be the logarithm of a natural number, which being subtracted from the natural sine of the moon's meridional altitude (found as in Problem V.) will give the natural sine of the moon's true altitude at the given time.

EXAMPLE.

What was the true altitude of the moon's center, August the 26th 1774, at $19^h\ 16'\ 52''$, apparent time, in latitude $14°\ 45'$ S. and longitude $167°$ E.?

Apparent

Apparent time	$19^h\ 16'\ 52''$				
Ship's long. $167° = 11$	8 0 E.				
Time at Greenwich 8	8 52	☉'s Æ $= 10^h\ 21'\ 51''$. ☽'s Æ $= 37°\ 59'$ Dec. $10°\ 19'$ N.			
Apparent time at the ship —		19 16 52			
Right ascension of the mid-heaven		5 38 43			
Moon's right ascension in time —		2 31 56			
Distance of the moon from the meridian		3 6 47		Log. rising	4.49711
Moon's declination	$10°\ 19'$ N.	— — —		Co-sine	9.99292
Ship's co-latitude	75 15 S.	— — —		Sine —	9.98545
Moon's merid. alt.	64 56	Nat. sine	90582		
		Nat. number	29887	Log.	4.47548
True alt. moon's center	$37°\ 22'$	Nat. sine	60695		

Scholium.

The operations in the three last problems bring out the true altitude of the object; if, therefore, the apparent altitude be wanted, as is most commonly the case, the difference between the refraction and parallax in altitude must be added to the true altitude of the sun; the refraction must be added to the true altitude of a star; and the correction, taken out of Tab. VIII. must be subtracted from the true altitude of the moon, thus found, to obtain their respective apparent altitude.

Problem VIII.

The latitude of a place, the sun's declination, and its altitude being given to find the apparent time at that place.

Rule.

From the observed altitude subtract the dip of the horizon, and the refraction; and to the remainder add the sun's semi-diameter; the sum will be the true altitude of the sun's center. Subtract the natural sine of the altitude, thus corrected, from the natural sine of the meridional altitude, found by the directions given in Problem V. and to the logarithm of the remainder add the log. secant of the ship's latitude, and the log. secant of the sun's declination; their sum, rejecting 20 from the index, must be sought for in Tab. XVI. under log. rising, and the time corresponding to it is the apparent time from the nearest noon, when the sun's altitude was observed. Consequently, if the observation be made in the forenoon, the time, thus found, must be taken from 24 hours, and the remainder will be the apparent time from noon of the preceding day.

Example I.

March 5th 1780, about half past 2 P.M. in latitude $16°\ 24'$ N. longitude $138°$ E. the altitude of the sun's lower limb was observed to be $47°\ 13'$, the ob-

 server's

ſerver's eye being 20 feet above the ſurface of the ſea, what was the apparent time when this obſervation was made?

Refract. (Tab. I.)	0′ 53″	Sun's declin. N. A.	5° 41′ 30″ S.				
Dip (Tab. II.)	4 16	Ship's long. gives +	8 41	} Table VI.			
		Time from N. gives −	2 21				
Sum - -	5 9						
Sun's ſemi-diam.	16 9	Sun's declination	5 47 50 S.	ſecant	10.00223		
		Co-latitude —	73 36 0 N.	co-ſec.	10.01804		
Correct. ☉'s alt.	11 0						
Obſ. al. ☉'s l. l.	47 13 0	Mer. alt. —	79 34	Nat. ſi.	92587		
True alt. ſun	47 24 0	— —		Nat. ſine	73610		
					18977	log.	4.27823
Apparent time —	2ʰ 27′ 2″	—		Log. riſing			4.29850

EXAMPLE II.

July the 9th 1775, about 8 A. M. in latitude 34° 55′ N. longitude 40° W. the altitude of the ſun's lower limb was obſerved to be 36° 49½′; the obſerver's eye being 21 feet above the ſurface of the ſea: what was the apparent time when this obſervation was made?

Refr. (Tab. I.)	1′ 16″	Sun's decl. Naut. Al.	22° 23′ 15″ N.				
Dip (Tab. II.)	4 22	Ship's long. gives —	52	} Table VI.			
		Time fr. Noon gives +	1 18				
Sum -	5 38						
Sun's ſemi-dia.	15 47	Sun's declination	22 23 41 N.	log. ſec.	10.03407		
		Co-latitude —	55 5 0 N.	log. co-ſec.	10.08619		
Cor. ☉'s alt.	10 9						
Alt. ☉'s l. l.	36 49 30	Meridional alt.	77 28 41	nat. ſi.	97623		
☉'s true alt.	36 59 39	- - -		Nat. ſine	60181		
					37442	log.	4.57336
Time from Noon on the 9th - -	3ʰ 58′ 22″	-		Log. riſing			4.69362
	24 0 0						
Apparent time on the 8th -	20 1 38						

PROBLEM IX.

The latitude and longitude of a place, the right aſcenſion, declination, and altitude of a fixed ſtar being given, to find the apparent time at that place.

RULE.

Subtract the dip of the horizon, and the refraction, from the obſerved altitude of the ſtar; and let its right aſcenſion and declination be taken out of Tab. VII. for the given year, &c. Find alſo the meridional altitude of the ſtar by the direction given in Prob. V.; from the natural ſine of which take the natural ſine of the ſtar's corrected altitude, and find the logarithm of the remainder. To this

this logarithm add the logarithmic ſecant of the latitude of the ſhip or place, and the logarithmic ſecant of the ſtar's declination: their ſum, rejecting twenty from the index, muſt be ſought for in Table XVI. under *log. riſing*, and the time correſponding to it will be the diſtance of the ſtar from the meridian; which being added to the ſtar's right aſcenſion in time, if the ſtar was weſt of the meridian at the time of obſervation, or ſubtracted from it, if the ſtar was then eaſt of the meridian, will give the right aſcenſion of the mid-heaven. Find the ſun's right aſcenſion in time, by help of Table XXIII. for noon at the given place, and ſubtract it from the right aſcenſion of the mid-heaven; the remainder is the eſtimate time. Enter Table XXIII. a ſecond time, with the eſtimate time, and daily variation of the ſun's right aſcenſion, and ſubtract the minutes and ſeconds, thus found, from the eſtimate time; the remainder is the apparent time when the altitude of the ſtar was obſerved.

EXAMPLE.

April 14th 1780, latitude 48° 56′ N. longitude 66° W. the obſerved altitude of Aldebaran, weſt of the meridian, was 22° 24½′; the height of the obſerver's eye, above the ſurface of the ſea, 21 feet: what was the apparent time when that obſervation was made?

Sun's Æ for noon at Greenw.	1h 31′ 1″	Refraction, Table I. —	2′ 18″
Long. 66° W. Ta. XXIII. giv. +	41	Dip, Table II. — —	4 22
⊙'s Æ at noon given place	1 31 42	Correction ———	6 40
		Obſerved alt. ſtar —	22 24 30
Star's decl. Table VII. —	16 3 N.		
Co-latitude — —	41 4 N.	True alt. ſtar —	22 17 50

Star's meridian alt. ——	57 7	Nat. ſine	83978	
True alt. ſtar, ——	22 18	Nat. ſine	37946	
Difference of the nat. ſines —— —			46032 - Log.	4.66306
Latitude of the ſhip — —	48° 56′ 0″		- Log. ſecant —	10.18248
Star's declination — —	16 3 0		- Log. ſecant —	10.01727
Star weſt of the meridian —	4h 57 8		- Log. riſing —	4.86281
Star's right aſcen. Table VII. —	4 23 20			
Right aſcen. mid-heaven —	9 20 28			
Sun's right aſcen. at noon —	1 31 42			
Eſtimate time — ——	7 48 46			
Number from Table XXIII. ſubt.	0 1 12			
Apparent time — — —	7 47 34			

PROBLEM X.

Having the apparent, or obſerved, diſtance of the moon from the ſun, or a fixed ſtar, together with the obſerved altitude of each, to find their true diſtance.

Rule.

First method, or Mr. *Lyons*'s improved.

1st. To the proportional logarithm of the star's refraction, or the difference between the sun's refraction and its parallax in altitude, add the co-sine of the sun or star's apparent altitude, the sine of the apparent distance of the moon from the sun or star, and the co-secant of the moon's apparent altitude; their sum, rejecting 30 in the index, will be the proportional logarithm of the first arc.

2d. To the proportional logarithm of the star's refraction, or the difference between the sun's refraction and its parallax in altitude, add the co-tangent of the sun or star's altitude, and the tangent of the apparent distance of the moon from the sun or star; their sum, rejecting 20 in the index, will be the proportional logarithm of the second arc.

3d. If the apparent distance be less than 90°, take the difference between the first and second arcs, which must be added to the apparent distance, if the first arc be greater than the second, but subtracted from it, if the second arc be greater than the first: if the apparent distance be greater than 90°, the sum of the two arcs must be added to the apparent distance, to give the distance corrected for the refraction of the sun or star.

4th. Take the correction of the moon's altitude out of Table VIII. to the proportional logarithm of which add the co-sine of the moon's apparent altitude, the sine of the distance corrected for the sun or star's refraction, and the co-secant of the sun or star's true altitude; their sum, rejecting 30 in the index, will be the proportional logarithm of a third arc.

5th. To the proportional logarithm of the correction of the moon's altitude add the co-tangent of the moon's apparent altitude, and the tangent of the distance, corrected for the sun's or star's refraction; their sum, rejecting 20 in the index, will be the proportional logarithm of a fourth arc.

6th. If the distance, corrected for the sun or star's refraction, be less than 90°, take the difference between the third and fourth arcs, which difference must be subtracted from the distance, corrected for the sun or star's refraction, if the third arc be greater than the fourth; but it must be added to it if the fourth arc be greater than the third: if the distance, corrected for the sun or star's refraction, be greater than 90°, the sum of the two arcs must be subtracted from it to obtain the distance corrected for the sun or star's refraction and principal effect of the moon's parallax.

7th. Enter Table XIII. under the apparent distance, corrected for sun or star's refraction and principal effect of parallax in the top column, with the correction of the moon's altitude in the left-hand side column, and take out the number of seconds which stand under the former and opposite to the latter. Enter it again under the same corrected distance in the top column, and opposite to the principal effect of the moon's parallax in the left-hand side column, and do the like: the difference of these two numbers must be added to the distance, corrected for the sun or star's refraction and the principal effect of the moon's parallax, if the distance, so corrected, be less than 90°; but it must be subtracted from it, if that distance be greater than 90°, and the sum or difference will be the true distance of the objects.

Scholium.

It will greatly expedite the computation if all the logarithmic sines, tangents, &c. which fall at the same opening of the book, be taken out at the same time, whether

whether they relate to the firſt or ſecond parts of the operation: thus, the co-ſine and co-tangent of the ſtar's apparent altitude, and co-ſecant of its true altitude may all be taken out at the ſame time, and written down in different parts of the paper; and ſo alſo may the co-ſine, co-tangent, and co-ſecant of the moon's apparent altitude; the ſine and tangent of the apparent diſtance; and the ſine and tangent of the diſtance, corrected for the refraction of the ſun or ſtar.

EXAMPLE I.

Admit that the apparent altitude of a ſtar was 24° 48′, when that of the moon's center was 12° 30′, and their apparent diſtance 51° 28′ 35″; the moon's horizontal parallax being 56′ 15″: what was their true diſtance?

Star's apparent altitude	—	24° 48″	
Star's refraction	— —	2	3
Star's true altitude	—	24 45	57

Star's refraction	— 2′ 3″	P. L.	1.9435	— — —	1.9435
Star's apparent alt.	- 24° 48′	Co-ſine	9.9580	— Co-tangent	10.3353
Apparent diſt.	— 51 29	Sine	- 9.8934	— Tangent -	10.0991
Moon's apparent alt.	12 30	Co-ſec.	10.6647	Sec. arc 0′ 45½″ P.L.	2.3779
			2.4596	=P.L. 1ſt arc 0 37½	
Correction of the diſt. for the ſtar's refraction	—			0 8 ſub.	
Apparent diſtance	— — —			51 28 35	
Diſt. corrected for the ſtar's refraction	—			51 28 27	

Corr. moon's alt. Tab. VIII.	50′ 42″	P. L.	0.5502	— —	0.5502
Moon's apparent altitude	12° 30	Co-ſine	9.9896	- Co-tangent	10.6542
Diſt. corr. for ſtar's refrac.	51 28	Sine	- 9.8933	- Tangent -	10.0988
Star's true altitude	— 24 46	Co-ſec.	10.3779	4th arc 8′ 57″ P.L.	1.3032
			0.8110	=P.L. 3d arc 27 49	

Principal effect of the moon's parallax	—	18 52 ſubt.
Diſtance corrected for the ſtar's refraction	—	51 28 27
Diſt. corr. for ſtar's refract. and princip. effect of parall.		51 9 35
Corr. moon's altitude in Tab. XIII. gives 0′ 18″ } diff.		15 add
Second corr. diſt. in Tab. XIII. gives - 0 3 }		
True diſt. of the moon and ſtar	— —	51 9 50

EXAMPLE II.

Let the apparent altitude of the ſun's center be 84° 7′, that of the moon 5° 17′, their apparent diſtance 90° 21′ 13″, and the moon's horizontal parallax 61′ 48″: required the true diſtance of their centers?

Refraction

Refraction of the sun	—			6″
Parallax in altitude	—			1
Correct. of the sun's alt.				5
Sun's apparent alt.	-	84°	7′	0″
Sun's true altitude	-	84	6	55

Corr. sun's alt.	— 10′ 5″	P. L.	3.3344	-	-	-	3.3344
Sun's app. alt.	— 84° 7′	Co-sine	9.0107	-	-	Co-tang.	9.0130
Apparent dist.	— 90 21	Sine	10.0000	-	-	Tangent	12.2140
Moon's app. alt.	- 5 17	Co-se.	11.0358	-	1st arc 0′ 0½″	P. L.	4.5614
			3.3809 P.L.		2d arc 0 4½		

Correction for the sun's refraction	—	—	0 5 add
Apparent distance	—	—	90 21 13
Distance corrected for sun's refraction		—	90 21 18

Corr. of ☽'s alt. T. VIII.	52′ 4″	P. L.	0.5387	-	-		0.5387
Moon's apparent alt. —	5° 17′	Cos.	9.9982	-	-	Co-tang.	11.0340
Dist. correct. sun's ref.	90 21	Sine	9.9999	-	-	Tangent	11.6277
Sun's true altitude —	84 7	Co-se.	10.0023		4th arc 0′ 7″	P.L.	3.2004
			0.5391 = P.L.		3d arc 52 1		

Principal effect of the moon's parallax	—	—	52 8 subt.
Distance corrected for the sun's refraction	—		90 21 18

Dist. correct. for ☉'s refract and princip. effect of parallax 89 29 10; which is the true distance in this case, the correction from Table XIII. being nothing.

Example III.

Suppose the apparent altitude of the star was 5° 6′, that of the moon's center 88° 46′, their apparent distance 89° 58′ 6″, and the moon's horizontal parallax 61′ 18″; what would the true distance of the star from the moon's center be?

Refraction of the star	—		9′	44″
Star's apparent altitude	—	5	6	0
Star's true altitude	—	4	56	16

Star's refaction -	9′ 44″	P. L.	1.2670	-	-	-	1.2670
Star's apparent alt.	5° 6′	Co-sine	9.9983	-	-	Co-tang.	11.0494
Apparent distance	89 58	Sine	10.0000	-	-	Tangent	13.2352
Moon's apparent alt.	88 46	Co-sec.	10.0001		2d arc 0′ 0″	P.L.	5.5516
			1.2654 = P. L.		1st arc 9 46		

Correction for the star's refraction	—	—	9 46 add
Apparent distance	—		89 58 6
Distance corrected for the star's refraction	—		90 7 52

Corr-

Corr. ☽'s alt. Tab. VIII.	1′ 17″	P. L. 2.1469	- - -	2.1469
Moon's app. alt. —	88° 46′	Co-sine 8.3329	- -	Co-tang. 8.3330
Dist. corr. for star's refr.	90 8	Sine 10.0000	- -	Tangent 12.6332
Star's true altitude —	4 56	Co-se. 11.0655	4th arc 0′ $8\frac{1}{2}$″	P.L. 3.1131
		1.5453 = P.L. 3d arc 5 $7\frac{1}{2}$		

Principal effect of the moon's parallax ——— — 5 16 subt.
Dist. corrected for the star's refraction ——— 90 7 52

Distance corrected for principal effect of parallax 90 2 36, and which is the true distance in this case, because the correction from Table XIII. is nothing.

EXAMPLE IV.

The apparent altitude of the sun's center was observed to be 19° 3′ 36″, that of the moon's center 71° 6′ 2″, the apparent distance of their centers 103° 29′ 27″, and the moon's horizontal parallax, at that time, was 58′ 35″: what was the true distance of their centers?

Refraction of the sun ———		2′	44″
Its parallax in altitude —			8
Correction of the sun's alt.		2	36
Sun's apparent altitude —	19	3	36
Sun's true altitude —	19	1	0

Corr. ☉'s alt.	2′ 36″	P. L. 1.8403	- - -	1.8403
☉'s app. alt.	19° 4′	Co-sine 9.9755	- -	Co-tangent 10.4614
App. distance	103 29	Sine 9.9879	- -	Tangent 10.6202
☽'s app. alt.	71 6	Co-sec. 10.0241	- - 2d arc 0′ $12\frac{1}{2}$″	P. L. 2.9219
		1.8278 = P. L. 1st arc 2 $40\frac{1}{2}$		

Correction for the sun's refraction ——— 2 53 add
Apparent distance - 103 29 27

Distance corrected for the sun's refraction — 103 32 20

Corr. ☽'s alt. T.VIII.	18′ 39″	P. L. 0.9846	- -	0.9846
Moon's apparent alt.	71° 6′	Co-sine 9.5104	- -	Co-tangent 9.5345
Dist. corr. ☉'s refr.	103 32	Sine 9.9878	- -	Tangent 10.6185
Sun's true altitude	19 1	Co-sec. 10.4870	- 4th arc 13′ $6\frac{1}{2}$″	P.L. 1.1370
		0.9698 = P. L. 3d arc 19 $17\frac{1}{2}$		

Principal effect of the moon's parallax ——— — 32 24 subt.

Prin-

		°	′	″	
Principal effect of the moon's parallax	——		32	24	subt.
Distance corrected for the sun's refraction	—	103	32	20	
Distance corrected for the principal effect of parallax		102	59	56	
Corr. moon's alt. gives in Tab. XIII. — 0; Second corr. distance Tab. XIII. — $1\frac{1}{2}$	diff.			$1\frac{1}{2}$	subt.
True distance of the sun and moon	———	102	59	$54\frac{1}{2}$	

Another METHOD, or Mr. *Dunthorne*'s improved.

1st. With the moon's apparent altitude and horizontal parallax, found in the Nautical Almanac, p. VII. take the logarithm out of Table IX. which reserve; and also the correction of her altitude out of Table VIII. to which add the refraction of the star, and call their sum *the correction of the moon's altitude*.

2d. If the altitude of the star be greater than that of the moon, take the above *correction* from the difference of their apparent altitudes; but let them be added together if the altitude of the moon be greatest, and you will have the difference of their true altitudes: of which take half.

3d. To the apparent distance of the moon and star add the difference of their apparent altitudes, and take half the sum: also, from the apparent distance subtract the difference of the apparent altitudes, and take half the remainder.

4th. Add together the logarithmic sine of this half sum, the logarithmic sine of the half remainder, and the logarithm above-reserved; reject radius from the sum, and half of what remains, will be the logarithmic sine of an arch.

5. Take the sum and difference of this arch and half the difference of the true altitudes, found by the second rule, and add together the logarithmic co-sines of this sum and difference: half the sum of these two logarithms will be the logarithmic co-sine of half the true distance.

EXAMPLE.

EXAMPLE I.

Admit that the apparent altitude of a ſtar was 24° 48′, when that of the moon's center was 12° 30′, and their diſtance 51° 28′ 35″, the moon's horizontal parallax being 56′ 15″; what was the true diſtance?

				App. alt. ✳	24°	48′	0″			
				App. alt. ☽	12	30	0			
Diff. app. alt. —	12°	18′	0″	—	12	18	0			
Cor. from Tab. VIII.	—	50	42	App. diſt.	51	28	35	Log. from Tab. IX. — — —		9.99865
Star's refract. Tab. I.	—	2	3	Sum —	63	46	35,	half is - 31° 53′ 18″	Log. ſine - -	9.72285
Diff. true alts. —	11	25	15	Difference	39	10	35,	half is - 19 35 17	Log. ſine - -	9.52538
Half — —	5	42	38					Sum, rejecting radius, — —		(2) 19.24688
Arch — —	24	50	46	—	—	—		— —	Log. ſine - -	9.62344

Sum — —	30	33	24	— Log. co-ſine —	9.93507
Difference —	19	8	8	— Log. co-ſine —	9.97531
					2) 19.91038
	25	34	56	— Log. coſine —	9.95519
			2		
True diſtance —	51	9	52		

EXAMPLE

EXAMPLE II.

Let the apparent altitude of the moon's center be 5° 17′, that of the ſun 84° 7′, and their apparent diſtance 90° 21′ 13″; the moon's horizontal parallax being 61′ 48″: what is the true diſtance of their centers?

				App. alt. ☉	84°	7′	0″		
				App. alt. ☽	5	17	0		
Diff. app. alts. —	78°	50′	0″	——	78	50	0		
Cor. from Tab. VIII.	—	52	3	App. diſtance	90	21	13	Log. from Tab. IX. — — —	9.99946
Sun's refract. Tab. I.		—	6	Sum —	169	11	13,	half is - 84° 35′ 36″ Log. ſine - -	9.99806
Diff. true alts. —	77	57	51	Difference	11	31	13,	half is - 5 45 37 Log. ſine - -	9.00159
Half — —	38	58	55					Sum, rejecting radius, — — (2)	18.99911
Arch — —	18	24	55	— —				— — — Log. ſine - -	9.49955
Sum — —	57	23	50	— Log. co-ſine —	9.73144				
Difference —	20	34	0	— Log. co-ſine —	9.97140				
				2)	19.70284				
	44	44	37	— Log. co-ſine -	9.85142				
			2						
True diſtance	89	29	14						

In the two preceding examples, five places only of figures, beſides the index, are uſed in the logarithmic ſines, no regard therefore is paid to the laſt place of the logarithmic numbers contained in Table IX. which is ſeparated from the others by a point for this purpoſe; and in this manner the diſtance may always be obtained true within 10″, if the obſerved diſtance exceed 14°, and generally within leſs than half that quantity. But if it be required to have the diſtance true to the neareſt ſecond, it will be neceſſary to uſe ſix places of the log. ſines, beſides the index; in which caſe the laſt place in Table IX. muſt alſo be taken in: and it will be farther neceſſary to diminiſh the logarithms in this Table by the numbers contained in Table X. if the moon's diſtance from the ſun be obſerved; or by the numbers in Table XI. if her diſtance be obſerved from a fixed ſtar, and the ſtar's altitude do not exceed 25 degrees. Moreover, when the moon and ſun are the objects, it may be proper to leſſen the ſun's refraction by his parallax in altitude, which is contained in Table III. Take an example or two computed this way.

EXAMPLE III.

Suppose the apparent altitude of the moon's center be 88° 46′, that of the star 5° 6′, the apparent distance 89° 58′ 6″, and the moon's horizontal parallax 61′ 18″; required the true distance?

App. alt. ☽		88°	46′	0″	
App. alt. ☉		5	6	0	
——		83	40	0	
App. dist.		89	58	6	
Sum	—	173	38	6,	
Difference		6	18	6,	

Diff. app. alts.	—	83°	40′	0″	
Cor. from Tab. VIII.	+		1	18	
Refraction of ✳	— +		9	44	
Diff. of true alts.		83	51	2	
Half	— —	41	55	31	
Arch	— —	13	25	42	

Log. from Tab. IX.	— —		9.992431
Log. from Tab. XI.	— —		— 13
Log. from Tab. IX. and XI.	— —		9.992418
half is - 86° 49′ 3″	Log. sine	-	9.999329
half is - 3 9 3	Log. sine	-	8.740083
Sum; rejecting radius,	— —	(2)	18.731830
— — — — — — —	Log. sine		9.365915

55	21	13	—	Log. co-sine —	9.754738
28	29	49	—	Log. co-sine —	9.943911
				2)	19.698649
45	1	16	—	Log. co-sine —	9.849325
		2			

True distance	—	90	2	32

EXAMPLE

EXAMPLE IV.

The apparent altitude of the ſun's center was obſerved to be 19° 3′ 36″, that of the moon's center 71° 6′ 2″, and the apparent diſtance of their centers 103° 29′ 27″, when the moon's horizontal parallax was 58′ 35″; what was the true diſtance?

				App. alt. ☽	71°	6′	2″		
				App. alt. ⊙	19	3	36	Log. from Tab. IX. — —	9.993176
Diff. app. alts. —	52°	2′	26″	—	52	2	26	Log. from Tab. X. — —	7
Cor. from Tab. VIII. —	+	18	39	App. diſt.	103	29	27	Log. from Tab. IX. and X. —	9.993169
Diff. of ⊙'s refr. and par.	+	2	35	Sum	155	31	53,	half is 77° 45′ 56″ Log. ſine	9.990023
Diff. of true alts. —	52	23	40	Difference	51	27	1,	half is 25 43 31 Log. ſine	9.637546
Half — —	26	11	50					Sum, rejecting radius, — (2)	19.620738
Arch — — —	40	15	21	— — — — —				Log. ſine	9.810369
Sum — — —	66	27	11	—				Log. co-ſine —	9.601517
Difference — —	14	3	31	—				Log. co-ſine —	9.986793
								2)	19.588310
	51	29	58	—				Log. co-ſine —	9.794155
			2						
True diſtance —	102	59	56						

By this mode of computation the operation is very ſhort, and the uſe of natural ſines is avoided, which is always troubleſome, and more eſpecially to perſons who are not accuſtomed to uſe them; but what conſtitutes the principal advantage of this new method is, that there is no diſtinction of caſes, as in every one elſe which has hitherto been offered to the public, and which embarraſſes ſeamen more than any thing elſe: in Mr. *Dunthorne*'s method, from which this is partly derived, every one of the preceding Examples falls under a different caſe; although ſome of thoſe caſes are not there pointed out.

PROBLEM XI.

Having the latitude of a ſhip and its longitude by account; alſo the obſerved diſtance of the neareſt limbs of the ſun and moon, together with the obſerved altitudes of their upper or lower limbs, to find the true longitude of the ſhip.

RULE.

1ſt. Turn the longitude of the ſhip, by account, into time, by means of Table XIV. and if it be weſt, add it to, but if it be eaſt, ſubtract it from the eſtimated time at the ſhip, when the obſervation was made, and it will give the time at Greenwich nearly.

2d. To this time take the moon's ſemi-diameter and her horizontal parallax out of p. VII. of the Nautical Almanac; alſo the ſun's ſemi-diameter for the day out of p. III. and augment the moon's ſemi-diameter by adding to it the number of ſeconds found in Table IV. with her obſerved altitude.

3d. Correct the obſerved diſtance by adding to it the ſemi-diameter of the ſun, and the augmented ſemi-diameter of the moon: correct alſo the obſerved altitudes by ſubtracting the dip of the horizon, taken out of Table II. with the height of the obſerver's eye above the ſurface of the ſea, and adding, or ſubtracting the ſemi-diameters of the objects, according as the altitudes of the lower or upper limb were obſerved; by which means the apparent diſtance and altitudes of the centers of the ſun and moon are obtained.

4th. With the apparent diſtance, and the two apparent altitudes, find the true diſtance by either of the methods given in Problem X. or by the Parallactic Tables publiſhed by order of the Commiſſioners of Longitude, or by either of the methods which are given at the end of the Nautical Almanac for 1772.

5th. Amongſt the diſtances of the moon's center from the ſun and fixed ſtars, put down on p. VIII. IX. X. and XI. of the Nautical Almanac, find thoſe two diſtances of the ſun and moon which are next leſs and next greater than the true diſtance, found from the obſervation: take the difference between them; alſo between that which ſtands firſt in the Ephemeris, and the true obſerved diſtance, and ſubtract the proportional logarithm of the former difference from the proportional logarithm of the latter; the remainder will be the proportional logarithm of a portion of time, to be added to the time which the diſtance, ſtanding firſt in the Ephemeris, was computed for, and the ſum will be the apparent time at Greenwich.

6th. To this time take the ſun's declination out of p. II. of the Nautical Almanac; and correct the apparent altitude of the ſun's center by ſubtracting from it the difference between the refraction of the ſun and its parallax in altitude, taken out of Table I. and II. with theſe, and the ſhip's latitude, find the apparent time at the ſhip by Problem VIII.

7th. Take the difference between the apparent time at Greenwich and the apparent time at the ſhip, and convert it into degrees and minutes by the help of Table XIV. and it will be the true longitude of the ſhip at the time of obſervation: eaſt, if the time at the ſhip be greater than the time at Greenwich, but weſt, if the time at the ſhip be leſs than the time at Greenwich.

EXAMPLE.

EXAMPLE.

July the 7th 177 , about a quarter paſt three, P. M. in latitude 33° 37′ N. longitude 40° W. by account, the following obſervations were taken; the height of the obſerver's eye being 21 feet, and the corrections for the errors of the ſeveral quadrants as underneath:

Altitude of ſun's low. lim	Altitude of Moon's U. L.	Diſt. ☽ & ☉'s neareſt limb.	
° ′	° ′	° ′ ″	
45 54	19 32	109 51 45	
45 45	19 52	52 45	
45 $18\frac{3}{4}$	20 5	53 30	
45 4	20 $17\frac{1}{2}$	53 45	
44 $48\frac{1}{2}$	20 34	54 15	
226 50 15	100 20 30	16 0	Sums to be divided by 5, the N° of obſ.
45 22 3	20 4 6	109 53 12	Means.
ſub. 48	ſub. 1 0	ſub. 2 37	Corrections for the errors of the quad.
ſub. 4 22	ſub. 4 22		Dip of the horizon.
add 15 47	ſub. 15 43	add 31 30	Semi-diameters.
45 32 40	19 43 1	110 22 5	Apparent diſtance and altitudes.

Eſtimated time at the ſhip	3^h 15′
Long. in time weſt - -	2 40
Time at Greenw. nearly	5 45
☽'s hor. par. (Nau. Al. p. VII.)	57′ 19″
Moon's ſemi-diameter —	15′ 37″
Augmentation (Table IV.) —	6
Moon's aug. ſemi-diam. —	15 43

Reduction of the diſtance by the firſt method in Problem X.

Refraction of the ſun's altitude — —	0′ 56″
The ſun's parallax in altitude — —	0 6
Correction of the ſun's altitude —	0 50
Apparent altitude of the ſun — —	45 32 40
True altitude of the ſun — —	45 31 50

Cor. of the ☉'s alt. 0′ 50″	P. L. 2.3344	- -	-	2.3344
The ☉'s app. alt. 45° 33	Co-ſine 9.8453	-	- Co-tang. —	9.9917
Apparent diſt. 110 22	Sine 9.9720	-	- Tangent -	10.4304
Moon's app. alt. 19 43	Co-ſec. 10.4719		2d arc 0′ 19″ P. L.	2.7565
	2.6236 = P. L. 1ſt arc 0 $25\frac{1}{2}$			

Correction for the ſun's refraction -	0 $44\frac{1}{2}$ add.
Apparent diſtance of the ſun and moon - -	110 22 5
Diſtance corrected for the ſun's refraction -	110 22 $49\frac{1}{2}$

Cor-

Cor. moon's alt. (Tab. VIII.)	51′ 22″	P. L. 0.5446	- - 0.5446
Moon's apparent alt.	19° 43′	Co-fine 9.9738	- Co-tang. 10.4457
Dift. cor. for fun's refrac.	110 23	Sine 9.9719	- Tang. 10.4300
The fun's true alt. —	45 32	Co-fec. 10.1465	4th arc 6′ 50″ P.L. 1.4203
		0.6368 = P.L. 3d ar.	41 32½

Principal effect of the moon's parallax	48 22½ fubt.
Diftance corrected for the fun's refraction —	110 22 49½
Dift. corr. for fun's refrac. and princ. effect of parallax -	109 34 27
Corr. moon's alt. 51′ 22″ in Table XIII. gives 8″ } diff. Parallax in dift. 48 22 in Table XIII. gives 7 }	1 fubt.
True diftance of the fun and moon — — —	109 34 26
Diftance at 3^h (Nautical Almanac, p. X.) —	108 5 58
Diftance at 6^h — — — — —	109 37 16
Difference between the firft and fecond — —	1 28 28 P.L. 0.3085
Difference between the fecond and third — —	1 31 18 P.L. 0.2948
	2^h 54′ 25″ P.L. 0.0137
	3 0 0
Apparent time at Greenwich — — —	5 54 25
Sun's declination at noon July 7th 1775 — • —	22 36 51 N.
5^h 54′ 25″ P. M. on July 7th in Tab. VI. gives — —	1 43 fub.
Sun's declination July 7th 1775 at 5^h 54′ 25″ P. M. —	22 35 8 N.

Co-latitude of the fhip -	56° 23′ N.	- -	Co-fecant 10.07948
The fun's declination -	22 35 N.	- -	Secant 10.03465
The fun's merid. alt. -	78 58	- Nat. fine 98152	
The fun's true alt. -	45 32 -	- Nat. fine 71366	
Difference of the natural fines —		26786	Log. 4.42791
Apparent time at the fhip -	3^h 17′ 21″	-	Log. rifing 4.54204
App. time at Greenwich —	5 54 25		
Longitude of the fhip —	2 37 4 equal to 39° 16′ W.		

PROBLEM

PROBLEM XII.

Having the latitude of a ſhip, and its longitude by account; alſo the obſerved diſtance of the moon's enlightened limb from a fixed ſtar, together with the obſerved altitude of each, to find the true longitude of the ſhip.

RULE.

1ſt. Turn the longitude of the ſhip, by account, into time, by means of Table XIV. and if it be weſt, add it to; or if it be eaſt, ſubtract it from the eſtimated time at the ſhip when the obſervation was made, and it will give the time at Greenwich nearly.

2d. To this time take the moon's ſemi-diameter and her horizontal parallax out of p. VII. of the Nautical Almanac, and augment the moon's ſemi-diameter by adding to it the number of ſeconds which ſtand in Table IV. againſt her apparent altitude.

3d. Correct the obſerved diſtance by adding to it the augmented ſemi-diameter of the moon, if the enlightened limb be that which is neareſt to the ſtar, or by ſubtracting the augmented ſemi-diameter of the moon from it, if the enlightened limb of the moon be that which is fartheſt from the ſtar: the reſult will be the apparent diſtance of the ſtar from the moon's center. Correct alſo the two altitudes, by ſubtracting the dip of the horizon from each, and by adding or ſubtracting the augmented ſemi-diameter of the moon to or from the moon's obſerved altitude, according as its lower or upper limb was obſerved; and the apparent altitude of each will be obtained.

4th. With the apparent diſtance and the two apparent altitudes find the true diſtance by any of the methods mentioned in Art. 4, of Problem XI.

5th. With the true diſtance, thus found, find the apparent time at Greenwich by the 5th Art. of Problem XI.

6th. Take the ſtar's right aſcenſion and declination out of Table VII. and correct its apparent altitude by ſubtracting its refraction, taken out of Table I. With theſe, and the latitude of the ſhip, find the apparent time at the ſhip by means of Problem IX. and thence the true longitude of the ſhip by Art. 7, of Problem XI.

EXAMPLE.

June the 12th 1775, about half paſt 9, P. M. in latitude 2° 26′ N. longitude by account 32° W. I obſerved the following diſtances of the moon's remote limb from α Aquilæ: the height of the obſerver's eye being 21 feet, and the errors of the quadrant as underneath:

Altitude of the ſtar.	Altitude of the moon's upp. limb.	Diſtance of the moon and ſtar.	
			Eſtimated time at the ſhip 9h 0′ 0″
			Longitude in time weſt 2 8 0
° ′	° ′	° ′ ″	Time at Greenw. nearly 11 8 0
18 30	55 24	50 26 0	
18 40½	55 47	26 15	Moon's hor. par. (p.VII. Nau. Al.) 60′ 5″
19 15	56 6	25 45	Moon's ſemi-diameter - 16′ 23″
19 37	56 27	24 45	Augmentation (Tab. IV. · 13
19 55	56 46	24 30	
20 17½	57 5	24 30	Moon's aug. ſemi-diameter 16 36
116 15	337 35	151 45	Sums, to be divided by 6.
19 22 30	56 15 50	50 25 17½	Means.
0 0	add 45	0 0	Errors of the quadrants.
4 22	4 22		Subt. dip of the horizon.
	16 36	16 36	Moon's ſemi-diameter, ſubtract.
19 18 8	55 55 37	50 8 41	Apparent diſtance and altitudes.

The ſtar's apparent altitude	—	19 18 8
Refraction	—	2 41
The ſtar's true altitude	—	19 15 27

Reduction of the diſtance by the ſecond method in Problem X.

Log. from Table IX. —	9.993887		
Log. from Table XI. —	1		
Log. from Tab. IX. and XI.	9.993886	Correction from Tab. VIII.	33′ 2″
Apparent alt. moon's cent.	55° 56′	Refraction of the ſtar —	2 41
Apparent alt. ſtar's center	19 18	Sum of the corrections -	35 43
Diff. apparent altitudes -	36 38	- - -	36 38 0
Apparent diſtance —	50 8 41	Diff. true altitudes —	37 13 43
		Half —	18 36 51

Sum — 86 46 41 ; half is 43° 23′ 20″ Log. ſi. 9.836923
Difference - 13 30 41 ; half is 6 45 20 Log. ſi. 9.070532
Log. from Tab. IX. and XI. 9.993886

Half diff. true alt. - 18° 36′ 51″ Sum rejecting rad. — 18.901341

Arch —	16 23 46	- -	Log. ſine -	9.450671
Sum —	35 0 37	- -	Log. co-ſine -	9.913310
Difference	2 13 5	- -	Log. co-ſine -	9.999674
				19.912984
	25° 13′ 14½″	- -	Log. co-ſine	9.956492
	2			

True diſt. moon and ſtar 50 26 29

Diſtance at 9 hours —	51 44 54			
Diſtance at 12 hours —	50 16 0			
Difference firſt and ſecond	1 18 25	P. L.	—	3609
Difference ſecond and third	1 28 54	P. L.	—	3064
	2 38 46	P. L.	—	0545
	9 0 0			
Time at Greenwich —	11 38 46			

The ſun's right aſcenſion for noon at Greenwich —	$5^h\ 21'\ 46''$
In Table XXIII. 32° W. long. and daily var. 4′ 9″ give -	22 add
Sun's right aſcen. for noon at the place of obſervation -	5 22 8

Co-latitude ſhip —	87° 34 N.	-	Co-ſecant -	10 00039
Star's declination, Tab. VII.	8 18 N.	-	Secant -	10.00457
Star's merid. altitude —	95 52	-	Natural Sine 99476	
Star's true altitude — —	19 15	-	Natural ſine 32969	
Difference of the Natural Sines —			66507 Log.	4.82287
Star eaſt of the meridian —	$4^h\ 43'\ 35''$	-	-	Log.riſ. 4.82783
Star's right aſcen. (Tab. VII.)	19 39 50			
Right aſcen. mid-heaven —	14 56 15			
Sun's right aſcen. at noon -	5 22 8			
Eſtimate time — —	9 34 7			
Numb. from Tab. XXIII. ſub.	1 39			
Apparent time at the ſhip -	9 32 28			
App. time at Greenwich -	11 38 46			
Longitude of the ſhip in time	2 6 18,	equal to 31° 34½′W.		

REMARK

In the two preceding Problems and Examples, the apparent time at the ſhip was found from the altitude of the ſun, or ſtar, which was taken at the ſame time with the diſtances: but if it ſhould ſo happen that the ſun, or ſtar, from which the moon's diſtance is obſerved be very near the meridian; or if, either through hazineſs of the atmoſphere, or badneſs of the horizon there be reaſon to ſuſpect that ſuch altitude is not exact enough for that purpoſe, which may be the caſe, and yet the altitude be ſufficiently accurate for the purpoſe of clearing the obſerved diſtance of the effects of parallax and refraction, then the times when thoſe diſtances and altitudes were taken muſt be noted by a watch, and other altitudes, either of the ſun, or a bright ſtar, muſt be taken at a greater diſtance from

from the meridian, or when the air or horizon is clearer, and the times noted by the ſame watch. By means of theſe laſt-mentioned altitudes the apparent time at the ſhip may be found by Problems VIII. or IX. and, of courſe, how much the watch is too faſt or too ſlow. Correct the mean of the times when the diſtances were taken by adding to it what the watch was too ſlow, or ſubtracting from it what the watch was too faſt, and the ſum or difference will be the apparent time at the ſhip when the diſtances were obſerved, reckoned from the meridian which the ſhip was under when the altitudes were taken for correcting the watch.

EXAMPLE I.

February 17th 1775, latitude 54° 25′ S. and longitude, by account, 10° eaſt, at about a quarter paſt four P. M. the following obſervations of the ſun's altitude were made; the error of the quadrant being 24″ to be added, and the height of the obſerver's eye, above the ſurface of the ſea, 21 feet.

Times by the watch.			Altitudes of the ſun's low. limb.			
h	′	″	°	′	″	
3	43	10	24	42		
	43	37		$39\frac{1}{2}$		
	43	53		$36\frac{3}{4}$		
	44	12		$33\frac{1}{2}$		
	44	31		31		
	45	7		27		
	264	30		209	45	Sums, to be divided by 6.
3	44	5	24	39	55	Means.
					24	Error of the quadrant, add.
				16	13	Semi-diameter, add.
				4	22	Dip of the horizon, ſubtract.
				2	4	Refraction, ſubtract.
			24	50	6	True altitude of the ſun's center.

Eſtimated time at the ſhip -	4^h 15′					
Long. in time eaſt ſubt. -	0 40					
Eſtimated time at Greenw.	3 35	in Table VI. Feb. 17th give		3′	5″	ſub.
		The ſun's declination at noon	11	55	41	S.
Sun's declination when the obſervation was made	—		11	52	36	S.

Co-latitude

Co-latitude of the ſhip	35° 35′ S.	-	-	Co-ſecant	10.235116
Sun's declination -	11 53 S.	-	-	Secant —	10.009441
Meridional altitude	47 28	- Nat. ſine	73688		
Sun's true altitude -	24 50	- Nat. ſine	41998		
			31690	- Log. —	4.500092
Apparent time at the ſhip	— —		4h 14′ 42″	- Log. riſing	4.745549
Time by the watch	— — —		3 44 5		
Watch too ſlow for apparent time	——		0 30 37		

About half paſt ten o'clock the ſame evening, the following obſervations were made of the diſtance of the ſtar Regulus from the moon's remote limb.

Times by the watch.	Altitude of Regulus.	Altitude of Moon's low. limb.	Diſtance of Moon and Regulus.	
h ′ ″	° ′ ″	° ′ ″	° ′ ″	
9 50 7	19 50½	18 6	28 27	
52 32	20 2	18 21	28¾	
55 7	20 15	18 39½	29¼	
57 11	20 29	18 55	30¼	
59 19	20 40	19 9	32	
274 16	101 16 30	93 10 30	147 15	Sums, to be divided by 5.
9 54 51	20 15 18	18 38 6	28 29 27	Means.
add 30 37		add 7 30	add 24	Errors of the quadrants.
10 25 28	ſub. 4 22	ſub. 4 22		Dip of the horizon.
		add 15 12	ſub. 15 12	Moon's ſemi-diameter.
	20 10 58	18 56 26	28 14 39	Apparent altitude and diſtance.
	2 34	The ſtar's refract. ſub.		
	20 8 24	The ſtar's true altitude.		

		h ′
Eſtimated time at the ſhip	——	10 30
Long. in time eaſt	—— ——	40
Time at Greenwich	—— ——	9 50
Moon's horizontal parallax	——	55′ 30″
Moon's ſemi-diameter	——	15 7
Augment. Table IV.	—— — —	5
Moon's augmented ſemi diameter	—	15 12

Reduction of the diſtance by the firſt method in Problem X.

Cor. of the ſtar's alt.	2′ 34″	P L. 1.8459	-	-	1.8459
The ſtar's app. alt.	20° 11′	Co-ſi. 9.9725	-	-	Co-tang. 10.4346
Apparent diſtance	28 15	Sine 9.6752	-	-	Tang. 9.7302
Moon's app. alt.	18 56	Co-ſec. 10.4888	-	2d arc 1′ 45½″	P.L. 2.0107
		1.9824 = P. L.		1ſt arc 1′ 52½″	

Correction for the ſtar's refraction	—— ——	0 7 add.
	Apparent diſtance —	28 14 39
Diſtance corrected for the ſtar's refraction	——	28 14 46

Corr. moon's alt. (Tab. VIII.)	49′ 45″	P.L.	0.5585	- - -	0.5585
Moon's apparent altitude	18 56	Co-ſi.	9.9758	- -	Co-tan. 10.4647
Diſt. cor. ſtar's refract.	28 15	Sine	9.6752	- -	Tang. 9.7302
Star's true alt. —	20 8	Co-ſec.	10.4632		4th arc 31′ 45½″ L.P. 07534
			0.6727 = P.L. 3d arc 38 14½		

Principal effect of parallax	—	6 29 ſubt.
Diſtance corrected for the ſtar's refraction	—	28 14 46
Corrected diſtance	—	28 8 17
Correct moon's alt. in Table XIII. gives 41″ ; Parallax in diſtance Table XIII. gives 1	add	40
True diſtance of the moon and ſtar	— —	28 8 57
Diſtance at nine hours (Nautical Almanac, p. X.)	—	27 43 39
Diſtance at midnight	— — — —	29 16 54
Difference between the firſt and ſecond	— —	0 25 18 P. L. 8522
Difference between the ſecond and third	— —	1 33 15 P. L. 2856
		h ′ ″
		0 48 49 P. L. 5666
		9 0 0
Apparent time at Greenwich	— — —	9 48 49
At the ſhip	—	10 25 28
Longitude of the ſhip in time	— — —	0 36 39 equal 9° 9¾′ E.

Example II.

December 6th 1774, latitude 53° 29′ ſouth, longitude 105° weſt, by account; about 20¼ʰ, or 8¼ʰ A. M. on the 7th, the following altitudes of the ſun's lower limb were obſerved; the error of the quadrant being 3′ 4″ to be ſubtracted, and the height of the obſerver's eye 21 feet above the ſurface of the ſea.

Times of the watch.			Altitude of the ſun's lower limb.			
h	′	″	°	′	″	
20	49	41	38	27¾		
	50	32		35		
	50	56		39		
	51	24		43		
	51	58		48		
	52	35		53		
	307	6		245	45	Sums, to be divided by 6.
20	51	11	38	40	57	Means.
				3	4	Error of the quadrant, ſubtract.
				4	22	Dip of the horizon, ſubtract.
				1	11	Refraction, ſubtract.
				16	18	Semi-diameter, add.
			38	48	38	True altitude of the ſun's center.

Eſtimated

Estimated time at the ship	—	20^h 15′		
Longitude in time west	——	7 0		
Time at Greenwich on the 7th		3 15	gives in Table VI. —	50″ add.
December 7th at noon the sun's declination was —				22 40 50 S.
Sun's declination when the observation was made	—— —			22 41 40 S.

Co-lat. of the ship —	36° 31′ S.	— —	Co-secant —	10.22544
Sun's declination ——	22 42 S.	—— ——	Secant ——	10.03502
Merid. alt. of the sun —	59 13	Nat. Sine 85911		
True altitude observed —	38 49	Nat. Sine 62683		
Diff. natural sines — —		23228	- Logarithm —	4.36601
Time from noon —— ——		3^h 39′ 4″	- Log. rising —	4.62647
		24 0 0		
Apparent time on the 6th —		20 20 56		
Time by the watch ———		20 51 11		
Watch too fast ———		0 30 15		

A few minutes before the sun was on the meridian, an opportunity offered of making the following observations.

		h ′
Estimated time at the ship	———	23 55
Longitude in time add	—— ——	7 0
Time at Greenwich 7th	———	6 55
Moon's horizontal parallax	—— ——	59′ 58″
Moon's horizontal semi-diameter	——	16 21
Augmentation	—— —— -	7
Moon's augmented semi-diameter	—	16 28

Time by the watch.	Altitude of the sun's low. limb.	Altitude of the moon's upp. limb.	Dist. moon's limb from the sun's.	
h ′ ″	° ′ ″	° ′ ″	° ′ ″	
0 23 6	59 $2\frac{3}{4}$	27 3	58 48	
24 10	$2\frac{1}{2}$	9	48	
24 58	$2\frac{1}{2}$	21	$48\frac{1}{2}$	
25 55	3	28	49	
26 48	$2\frac{3}{4}$	33	$49\frac{1}{4}$	
27 35	$2\frac{3}{4}$	40	$49\frac{1}{4}$	
152 32	16 15	134	292	Sums, to be divided by 6.
0 25 25	59 2 $42\frac{1}{2}$	27 22 20	58 48 40	Means.
sub. 30 15	sub. 2 $46\frac{1}{2}$	add 1 0	add 4 8	Errors of the quadrants, &c.
23 55 10	58 59 56	27 23 20	58 52 48	
	4 22	4 22		Dip of the horizon, subt.
	add 16 18	sub. 16 28	add 32 46	Semi-diameters.
	59 11 52	27 2 30	59 25 34	Apparent altitudes and distance.

Reduction

Reduction of the diſtance by the ſecond method in Problem X.

Log. from Table IX	—	9.996733	Sun's parallax in alt. –	0′ 4″
Log. from Table X.	——	16	Sun's refraction ——	0 34
Log. from Tab. IX. and X.		9.996717	Corr. of the ſun's alt.	0 30
Apparent alt. ſun's center		59° 12′ 0″	Corr. moon's alt. Tab. VIII.	51 33
Apparent alt. moon's center		27 2 0	Sum of the corrections —	52 3
Diff. apparent altitudes	-	32 10 0	—— —— —	32 10 0
Apparent diſtance	——	59 25 34	Diff. true altitudes —	31 17 57
			Half —	15 38 59

Sum —	91 35 34;	half is 45° 47′ 47″	Log. ſine	9.855438
Difference —	27 15 34;	half is 13 37 47	Log. ſine	9.372261
			Log. from Table IX. and X.	9.996717
Half diff. true altitudes	15° 38′ 59″		Sum rejecting radius —	19.224416
Arch —	24 10 15	— —	Log. ſine	9.612208
	8 31 16	— —	Log. co-ſi.	9.995179
	39 49 14	— —	Log. co ſi.	9.885392
				19.880571
	29° 21′ 44½″ 2	- -	Log. co-ſi.	9.940285

True diſtance — —	58 43 29		
Diſtance at ſix hours —	58 11 54		
Diſtance at nine hours —	59 51 59		
Diff. firſt and ſecond —	0 31 35	- P. L.	7558
Diff. ſecond and third —	1 40 5	- P. L.	2549
	0h 56′ 48″	- P. L.	5009
	6 0 0		
Apparent time at Greenwich	6 56 48	on the 7th.	
At the ſhip — — —	23 55 10	on the 6th.	
Longitude in time —	7 1 38	equal to 105° 24½′ W.	

REMARK.

That the longitude, thus found, is the longitude of the ſhip at the inſtant when the altitudes were obſerved for finding the time by the watch, is obvious; for the time being found at the meridian which the ſhip was then under, the watch, if it goes right, as it is ſuppoſed to do for a few hours, will continue to ſhew the time at that meridian, let the ſhip be where it will. Hence, therefore, it is the difference between the times by the meridian of Greenwich and that meridian which the ſhip was under when the altitudes were obſerved, which we take for the longitude of the ſhip; and, conſequently, it muſt be the longitude of that meridian from the meridian of Greenwich, and not the longitude of the meridian which the ſhip was under when the diſtances were obſerved.

THE END.