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Understanding the inflation–output relationship across business cycle phases

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Abstract

We examine the state dependence of monetary policy transmission and the parameters of the Phillips curve, dynamic IS equation, and Taylor rule across four regimes defined by joint deviations of inflation from the Federal Reserve's target and output from potential. The analysis uncovers important regime-specific asymmetries. The Taylor principle holds across all four regimes. The systematic policy response to the output gap weakens when inflation is below target but output remains above potential, whereas the response to inflation is broadly similar across regimes. The size of monetary policy shocks is significantly larger when inflation exceeds its target. The Phillips curve steepens when inflation exceeds target and output is above potential, while output sensitivity to interest rate changes declines under high inflation and economic slack. This explains why monetary policy shocks are significantly larger in inflationary booms, but transmission becomes less effective when elevated inflation coincides with economic slack.

Keywords: Phillips curve, DIS curve, Taylor rule, monetary transmission, state dependence, threshold VAR.
JEL Classification: C32, E52.

Nontechnical summary

Monetary policy plays a crucial role in stabilizing the economy, and the Federal Reserve adjusts its policy rate in response to changes in inflation and the output gap. However, the way these adjustments are made may vary depending on the state of the economy. For example, the Federal Reserve tends to act more aggressively when inflation exceeds its target or when the economy experiences a large negative output gap. Additionally, the relationships between inflation, output, and interest rates, captured by the Phillips curve, the Dynamic Investment-Savings (DIS) curve, and the Taylor rule, may shift across different phases of the business cycle.

This study investigates these state-dependent dynamics using a nonlinear model that allows key parameters and shock transmission mechanisms to vary depending on whether inflation is above or below its target and whether the output gap is positive or negative. By analyzing four distinct economic regimes - inflationary boom, disinflationary boom, inflationary slack, and disinflationary slack - we assess how the slopes of the Phillips and DIS curves, the Federal Reserve's policy response, and the impact of economic shocks differ across economic states.

Our findings reveal significant differences across regimes. The Taylor rule, which describes how the Federal Reserve adjusts its policy rate in response to inflation and the output gap, remains consistent with economic theory across all regimes. The Taylor principle holds across all four regimes. This ensures that the real interest rate rises when inflation increases, thereby helping to stabilize the economy by controlling inflationary pressures. Monetary policy responses to the output gap weaken when inflation is below target and output is above potential, while reactions to inflation is broadly similar across regimes. We also show that the Federal Reserve is more data dependent during periods of inflationary slack. Moreover, we find that the size of monetary policy shocks is significantly larger when inflation exceeds its target, reflecting the Federal Reserve's attention to high inflation.

We find that the Phillips curve, which describes the positive relationship between inflation and the output gap, is relatively steeper when inflation is above target and output is above potential. Therefore, demand and monetary policy shocks have much more pronounced effects on the macroeconomy during inflationary booms.

The DIS curve, which measures the relationship between output, inflation, and interest rates, also exhibits state dependence. The sensitivity of the output gap to the interest rate in the DIS curve is flatter when inflation is above target and output is below potential. Consequently, monetary policy shocks have muted responses on the macroeconomy only in the inflationary slack regime, which account for 44% of the sample.

These findings highlight the limitations of traditional linear models, which assume constant relationships between variables across all economic conditions. By accounting for state dependence, our nonlinear model provides a more nuanced understanding of how monetary policy operates and how shocks are transmitted through the economy. These insights are particularly relevant for policymakers, as they suggest that the effectiveness of monetary policy depends on the prevailing economic regime. For example, during inflationary booms, restrictive monetary policy can effectively reduce inflation and close the output gap, while during inflationary slack, policy changes may have a more limited impact. This might imply a stronger determination to fight inflation.

1 Introduction

The Federal Reserve adjusts its policy rate in response to changes in economic conditions, conventionally summarized by deviations of inflation from target and output from potential (Taylor, 1993). The responsiveness of the policy rate to these deviations, however, is not invariant over time but depends on the broader economic environment (Clarida et al., 2000). For instance, the Federal Reserve may exhibit greater policy accommodation when faced with a pronounced negative output gap, or adopt a more restrictive stance when inflation persistently exceeds its target. The strength and transmission of monetary policy also depend on cyclical conditions, as variations in the behavior of households and firms can alter the slopes of key structural relationships such as the Phillips curve and the Dynamic Investment-Savings (DIS) curve (Benigno and Eggertsson, 2023; Fitzgerald et al., 2024; Furlanetto and Lepetit, 2024; Inoue et al., 2024; Karadi et al., 2024; De Santis and Tornese, 2025). Identifying whether the parameters governing these relationships and policy responses vary across economic states–such as periods of resource slack or inflationary pressure–is therefore central to understanding how shocks are transmitted through the economy and how monetary policy should be optimally designed across regimes.

Adopting the Federal Reserve's perspective, this paper investigates, within a unified empirical framework, whether the slopes of the Phillips curve and the Dynamic Investment-Savings (DIS) curve, the parameters governing the monetary policy rule, and the transmission of macroeconomic shocks exhibit state dependence. We define the state of the economy in a manner consistent with the Federal Reserve's policy assessment framework, based on the degree of resource utilization, proxied by the sign of the output gap (y_t), and on whether inflation (π_t) lies above or below its target (π^*). The interaction of these dimensions delineates four distinct and well-established phases of the business cycle.

We estimate the three core equations of the standard New Keynesian model and the underlying shocks using the structural vector autoregression (SVAR) approach of Baumeister and Hamilton (2018) but conditioned on the state of the economy. Specifically, we estimate a three-equation threshold vector autoregression model allowing both parameters and shock variances to be state-dependent.

The results indicate that the Taylor principle holds across all four regimes. This ensures that the real interest rate rises when inflation increases, thereby helping to stabilize the economy by controlling inflationary pressures. Monetary policy responses to the output gap weaken when $\pi_t \leq \pi_t^*$ and $y_t > 0$, while reactions to inflation is broadly similar across regimes. We also show that the Federal Reserve is more data dependent during periods of inflationary slack ($\pi_t > \pi_t^*$ and $y_t \leq 0$). Moreover, we find that the size of monetary policy shocks is significantly larger when inflation exceeds its target, reflecting the Federal Reserve's attention to high inflation.

We also find that the Phillips curve is relatively steeper when $\pi_t > \pi_t^*$ and $y_t > 0$. Therefore, demand and monetary policy shocks have much more pronounced effects on the macroeconomy during inflationary booms.

Our analysis further reveals that the sensitivity of the output gap to the interest rate in the DIS curve is flatter when $\pi_t > \pi_t^*$ and $y_t \leq 0$. Consequently, monetary policy shocks have muted responses on the macroeconomy only in the inflationary slack regime, which account for 44% of the sample. This might imply a stronger determination by the Federal Reserve to fight inflation in this regime. These muted responses are similar to those uncovered by Baumeister and Hamilton (2018) in their linear model.

Prior empirical research studied primarily the time variation of parameter estimates using linear models. Bergholt et al. (2024) employ a two-variable structural vector autoregression (SVAR) model, estimated before and after 1995, and found that the Phillips curve was relatively stable, while aggregate demand flattened in the period 1995-2019. Other studies suggest that the wage Phillips curve flattened over time (Galí and Gambetti, 2019). Others found that the sensitivity of inflation to shocks diminished over the years (Del Negro et al., 2020; Ascari and Fosso, 2024).

Much of the existing literature estimates the structural Phillips curve in a single equation framework using linear (Galí and Gertler, 1999; Sbordone, 2002; Kleibergen and Mavroeidis, 2009; Barnichon and Mesters, 2020, 2021; Inoue et al., 2024) and nonlinear models (Forbes et al., 2021; Ball et al., 2022; Benigno and Eggertsson, 2023; Cerrato and Gitti, 2023; Cecchetti et al., 2023; Blanco et al., 2024). Mavroeidis et al. (2014) and Barnichon and Mesters (2020) discuss the challenges to such estimation. Some studies find that inflation became less sensitive to real activity during the Great Inflation (Cogley and Sargent, 2005; Primiceri, 2006; Cogley and Sbordone, 2008). In contrast, using data from US states, Hazell et al. (2022) and Fitzgerald et al. (2024) observe a modest decline in the Phillips curve's slope over time. Other research indicates that while the Phillips curve's slope has generally decreased over time, it became steeper during the post-pandemic period (Inoue et al., 2024). Building on these findings, we argue that the Phillips curve exhibits relative stability in three out of four regimes, covering three-quarters of the sample period. However, in the inflationary boom regime, the curve becomes steeper. This observation is consistent with the argument put forth by Harding et al. (2023) and Karadi et al. (2024).

There are a limited number of studies that estimate the Euler equation for output, and these typically rely on structural single-equation models. Attanasio and Low (2004) estimate the elasticity of intertemporal substitution to be 0.7. In contrast, Fuhrer and Rudebusch (2004) and Cecchetti et al. (2023) find a very low elasticity of output or unemployment in response to changes in the interest rate. The range of our estimates, which varies according to the state of the economy, encompasses the elasticity estimated by Attanasio and Low (2004).

In the context of the Taylor (1993) rule, several studies have examined whether the Federal Reserve's estimated reaction function has changed over time (e.g., Judd and Rudebusch, 1998; Taylor, 1999; Clarida et al., 2000; Lubik and Schorfheide, 2004; Orphanides, 2004; Surico, 2007; Benati and Surico, 2009; Canova, 2009; Zhu and Chen, 2017; Carvalho et al., 2021) and whether interest rate persistence is due to policy inertia or persistent monetary shocks (e.g., Rudebusch, 2002; Coibion and Gorodnichenko, 2012). Clarida et al. (2000) found that the Taylor principle is not respected in the pre-Volker period. Our results suggests that the Taylor rules is state dependent and highly persistent, and that the Taylor principle holds across all four regimes.

The remainder of the paper is organized as follows. Section 2 introduces the 3 equation TVAR model. Section 3 details the empirical estimation strategy. Section 4 presents the empirical results. Section 5 concludes.

2 A three-equation threshold VAR

The 3-variable macro model. The nonpolicy block of the 3-variable macro model consists of a New Keynesian Phillips curve and a DIS curve, respectively:

$$\pi_t = \bar{\kappa} \left[\left(\tau^{-1} + \varphi \right) y_t + \xi_t^s \right] + \beta E_t \pi_{t+1} \tag{1}$$

$$y_t = E_t y_{t+1} - \tau (r_t - E_t \pi_{t+1}) + \xi_t^d \tag{2}$$

with π_t denoting price inflation, y_t the output gap, and r_t the nominal policy rate. The first equation expresses current inflation as a function of the output gap, expected future inflation, and supply-side shocks ξ_t^s, with the strength of these relationships determined by structural parameters.[1] Similarly, Equation (2) describes the evolution of the output gap in relation to expected future output, the real interest rate, and demand-side disturbances ξ_t^d.

Under a standard calibration, the New Keynesian Phillips curve (1) implies that in-

[1] $\bar{\kappa} = (1-\theta)(1-\theta\beta)\theta^{-1}$ represents the slope of the inflation equation (Galí, 2015), where $(1-\theta)$ is the firm's probability of resetting its price, β is the discount factor, τ denotes the intertemporal elasticity of substitution, and φ represents the inverse Frisch elasticity of labor supply.

flation responds positively to its expected future value and the output gap. Similarly, the DIS equation (2) describes the output gap as increasing with its expected future value and decreasing with the real interest rate.

The model is closed with a Taylor-type interest rate rule:

$$r_t - \bar{r} = \rho(r_{t-1} - \bar{r}) + (1 - \rho)\left(\psi_\pi(\pi_t - \pi^*) + \psi_y y_t\right) + \xi_t^m \tag{3}$$

where ψ_y and ψ_π describe the Federal Reserve's long-run response to output and inflation. The parameter $\rho < 1$ reflects the Federal Reserve's preference for adjusting the policy rate gradually over time, and u_t^m represents a monetary policy shock. The lower ρ, the more data-dependent the policymaker is on the current evolution of key economic indicators, as the policy rate is less constrained by its past values

Mapping the DSGE model to a SVAR. Baumeister and Hamilton (2018) map the system of equations (1)-(3) into a three-variable SVAR consisting of supply and demand schedules and an interest rate rule. We introduce state-dependence into the model by allowing parameters and shock variances depend on a given state of the economy S:

$$\begin{aligned} y_t &= k_S^s + \alpha_{\pi,S}\pi_t + [\boldsymbol{b}_S^s]' \boldsymbol{x}_{t-1} + u_t^s \\ y_t &= k_S^d + \beta_{\pi,S}\pi_t + \beta_{r,S} r_t + \left[\boldsymbol{b}_S^d\right]' \boldsymbol{x}_{t-1} + u_t^d \\ r_t &= k_S^m + \zeta_{y,S} y_t + \zeta_{\pi,S}\pi_t + [\boldsymbol{b}_S^m]' \boldsymbol{x}_{t-1} + u_t^m \end{aligned} \tag{4}$$

where $\boldsymbol{x}_{t-1} = \left(\boldsymbol{y}_{t-1}', \boldsymbol{y}_{t-2}', \ldots, \boldsymbol{y}_{t-m}', 1\right)'$, with u_t^s, u_t^d, and u_t^m representing supply, demand, and monetary policy shocks, respectively. The state-dependent matrix of contemporaneous coefficients, A_S, is defined by

$$\boldsymbol{A}_S = \begin{bmatrix} 1 & -\alpha_{\pi,S} & 0 \\ 1 & -\beta_{\pi,S} & -\beta_{r,S} \\ -\zeta_{y,S} & -\zeta_{\pi,S} & 1 \end{bmatrix}. \tag{5}$$

This formulation can be equivalently expressed in terms of structural coefficients.[2] Importantly, Davig and Leeper (2007) and Barnett and Duzhak (2019) establish that this class of New Keynesian models in a nonlinear setting remains robust to bifurcations and indeterminacy.

Discussion on a nonlinear policy rule. The central bank's focus on output versus inflation stabilization can vary depending on the state of the economy. For example, in the 1970s, Federal Reserve Chairman Arthur Burns prioritized low inflation while remaining mindful of the costs of disinflation (Burns, 1979). In contrast, following the 2008 financial crisis, the Federal Reserve might have placed greater emphasis on output stabilization due to the depth of the recession and elevated unemployment. To capture such nonlinearities, we modify the monetary policy rule as follows:

$$r_t = k_S^m + (1-\rho_S)\psi_{y,S} y_t + (1-\rho_S)\psi_{\pi,S}\pi_t + \rho_S r_{t-1} + u_t^m, \tag{6}$$

where $(1-\rho_S)\psi_{y,S} \equiv \zeta_{y,S}$ and $(1-\rho_S)\psi_{\pi,S} \equiv \zeta_{\pi,S}$. This specification assumes that the Federal Reserve's long-run inflation target is invariant across economic states.[3] Instead, we posit that the Federal Reserve's responsiveness to inflation and output fluctuations varies with economic conditions. This framework suggests that, in certain states, the Federal Reserve may tolerate higher inflation to mitigate declines in output, consistent with the trade-off identified in Bianchi (2013).

[2] See Appendix Section B for a detailed derivation.

[3] This assumption is consistent with the findings of Liu et al. (2011). They used a variety of richly parameterized DSGE models within a unified framework incorporating regime switching both in shock variances and in the inflation target. They demonstrated that changes in the inflation target were not the primary driver of inflation, particularly during the high-inflation 1970s. Additionally, from a technical standpoint, estimating state-dependent Taylor rule parameters alongside a drifting inflation target becomes more cumbersome.

3 Model estimation

The TVAR in (4) is estimated using Bayesian methods and is expressed as:

$$\mathbf{A}_S \boldsymbol{y}_t = \sum_S \mathbf{B}_S \boldsymbol{x}_{t-1} \mathbb{I}\{z_{t-1} \in S\} + \boldsymbol{u}_t, \tag{7}$$

$$\boldsymbol{u}_t \sim N(0, \mathbf{D}_S), \quad \text{for } S \in \{R1, R2, R3, R4\} \tag{8}$$

where $\boldsymbol{y}_t = [y_t, \pi_t, r_t]'$ denotes the vector of endogenous variables. The indicator function $\mathbb{I}$ equals 1 when the state variable z_{t-1} is within one of the four regimes (discussed below). $\boldsymbol{B}_S$ is a matrix of lagged structural coefficients, and $\mathbf{D}_S$ is a diagonal variance-covariance matrix of the structural shocks with elements $d_{ii,S}$. Identification in this model is achieved by incorporating prior information on the structural parameters from the DSGE literature.

3.1 Priors on the contemporaneous structural coefficients

Prior information on the distributions of the contemporaneous coefficients are taken from Baumeister and Hamilton (2018), summarized in Table 2.

Taylor rule coefficients. We model the priors for ψ_y and ψ_π as Student's t-distributions with modes of 0.5 and 1.5, respectively.[4] Both distributions have a scale parameter of 0.4, have 3 degrees of freedom, and are truncated to be positive. Last, the smoothing parameter ρ is assumed to follow a Beta distribution with a mean of 0.5 and a standard deviation of 0.2, as in Lubik and Schorfheide (2004), Del Negro and Schorfheide (2004), and Baumeister and Hamilton (2018).

Other priors. We impose a prior mode of $\alpha_\pi = 1$, truncated to be positive, implying that an increase in inflation is associated with an increase in the output gap through the

[4] Taylor (1993) originally proposed values of $\psi_y = 0.5$ and $\psi_\pi = 1.5$.

Phillips curve. For β_π, we use a mode of 0.75 without imposing any sign constraints, reflecting our lack of strong prior beliefs about the correct specification for forecasting inflation. For β_r, we set a mode of -1, truncated to be negative, indicating that an increase in the interest rate is associated with a decrease in the output gap through the aggregate demand schedule. Additionally, we apply very loose priors with a scale parameter of 0.4 and 3 degrees of freedom (e.g. Baumeister and Hamilton, 2018).

3.2 Priors on the impacts of shocks

As noted by Baumeister and Hamilton (2018), the elements of the impact matrix $\mathbf{H} = \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\tilde{\mathbf{H}}$ can become infinite in certain regions of the parameter space before flipping signs due to the indeterminate sign of $\det(\mathbf{A})$. Following their approach, we incorporate prior information suggesting that the determinant is likely positive in all regimes S,

$$h_{1,S} \equiv \det(\mathbf{A}_S) > 0, \tag{9}$$

using an asymmetric Student's t prior with three degrees of freedom, a location parameter $\mu_{h_1} = 1.5$, a scale parameter $\sigma_{h_1} = 1.5$, and a shape parameter $\lambda_{h_1} = 4$.

Similarly, we hypothesize that the impact response of the output gap to a monetary contraction is negative:

$$h_{2,S} \equiv \frac{\partial \tilde{\mathbf{H}}_S(1,3)/\partial u_t^m}{\partial \tilde{\mathbf{H}}_S(3,3)/\partial u_t^m} < 0. \tag{10}$$

We again follow Baumeister and Hamilton (2018) by imposing an asymmetric prior with a location parameter $\mu_{h_2} = -0.3$, a scale parameter $\sigma_{h_2} = 0.5$, three degrees of freedom, a shape parameter $\lambda_{h_2} = -2$ for all states.

This additional information regarding $h_{1,S}$ and $h_{2,S}$ is integrated into the log-prior specification for the matrix of contemporaneous coefficients $\mathbf{A}_S$:

$$
\begin{aligned}
logp(\mathbf{A}_S) = logp(\alpha_{\pi,S}) + logp(\beta_{\pi,S}) \\
+ logp(\beta_{r,S}) + logp(\psi_{\pi,S}) + logp(\psi_{y,S}) \\
+ logp(h_{1,S}) + logp(h_{2,S}).
\end{aligned} \tag{11}
$$

3.3 Priors on structural variances and lagged structural coefficients

Following Baumeister and Hamilton (2018), we set priors for all estimated parameters and define the prior for $\boldsymbol{\theta}_S$, which includes the elements of $\boldsymbol{A}_S$, $\boldsymbol{B}_S$, and $\boldsymbol{D}_S$, such that $p(\boldsymbol{\theta}_S) = p(\boldsymbol{A}_S)p(\boldsymbol{D}_S|\boldsymbol{A}_S)p(\boldsymbol{B}_S|\boldsymbol{A}_S, \boldsymbol{D}_S)$. The functional form of $p(\boldsymbol{A}_S)$ is unrestricted, while $p(\boldsymbol{D}_S|\boldsymbol{A}_S)$ and $p(\boldsymbol{B}_S|\boldsymbol{A}_S, \boldsymbol{D}_S)$ are based on natural conjugate families. Specifically, $p(\boldsymbol{D}_S|\boldsymbol{A}_S)$ is modeled as a product of independent inverse-gamma distributions, and $p(\boldsymbol{B}_S|\boldsymbol{A}_S, \boldsymbol{D}_S)$ is a set of conditional Gaussian distributions $\boldsymbol{b}_{iS}|\boldsymbol{A}_S, \boldsymbol{D} \sim N(\boldsymbol{m}_{iS}, d_{iiS}\boldsymbol{M}_{iS})$. Here, $\boldsymbol{m}_{iS}$ and $\boldsymbol{M}_{iS}$ follow a Minnesota prior for the lagged coefficients in the Phillips curve and the aggregate demand equation. For the monetary policy rule, the first lag is set to ρ_S and the remaining lags set to zero, reflecting the belief that this equation should resemble Equation (6).

3.4 Dataset

Estimating the 3-equation TVAR model requires a sufficient number of observations for each regime. To achieve this, we use monthly data on $\mathbf{y}_t$ and estimate a twelfth-order VAR from January 1962 to December 2019. This approach allows us to capture various economic regimes in our nonlinear setting, including the inflation surges of the 1970s and mid-1980s, which were followed by a pronounced negative output gap, as well as the aftermath of the global financial crisis, marked by low inflation and a persistent negative output gap. As discussed in the next section, the use of monthly data is crucial for accurately estimating the Taylor rule parameters.

In our analysis, the output gap is defined as 100 times the logarithmic difference between real GDP and potential output. To convert real GDP to a monthly frequency, we apply the method proposed by Chow and Lin (1971), using monthly data from industrial production and real retail sales. While quarter-on-quarter real GDP growth rates align with observed values, intra-quarter dynamics follow those of industrial production and real retail sales. This monthly real GDP series serves as a coincident business cycle indicator, capturing both supply and demand factors.[5] Real potential output, provided by the Congressional Budget Office (CBO), changes slowly over time and is therefore linearly interpolated to a monthly frequency. Inflation is measured as 100 times the year-on-year logarithmic change in the personal consumption expenditures (PCE) price index on a monthly basis. The nominal interest rate follows the Federal funds rate until May 2009, after which it switches to the shadow rate provided by Wu and Xia (2016) during the zero lower bound period from June 2009 to November 2015 (see Appendix Figure A1.)

3.5 Definition of regimes

The Federal Reserve's monetary policy responses are not necesserely uniform over time but depend on the state of the economy (Sims and Zha, 2006b; Bianchi, 2013; Davig and Doh, 2014; Barthélemy and Marx, 2017). To explore this, we categorize the economy into four distinct regimes, defined by the interaction between the output gap and the deviation of inflation from its target. These regimes align with the Federal Reserve's policy assessment framework and capture key phases of the business cycle. By examining the responsiveness of the Phillips curve, the DIS curve, and the monetary policy rule across these regimes, we aim to uncover potential state dependence in the transmission of shocks and the design of optimal monetary policy.

The mandate of the Federal Open Market Committee's (FOMC) is maintaining stable inflation of 2 percent annually. Accordingly, we define two states: $\pi_t \leq 2\%$ and $\pi_t >$

[5]A similar procedure is undertaken by Bernanke and Mihov (1998) and Uhlig (2005).

2%. Simultaneously, the FOMC seeks to stabilize the economy by promoting maximum employment. Given that the Phillips curve may steepen under resource constraints, we establish two states also for the output gap: $y_t \leq 0\%$ and $y_t > 0\%$.[6] By combining these states, we construct a state variable z_t describing four distinct economic regimes based on inflation and the output gap:

- Regime 1 (R1): Disinflationary slack, where $\pi_t \leq 2\%$ and $y_t \leq 0\%$;
- Regime 2 (R2): Inflationary slack, where $\pi_t > 2\%$ and $y_t \leq 0\%$;
- Regime 3 (R3): Disinflationary boom, where $\pi_t \leq 2\%$ $y_t > 0\%$;
- Regime 4 (R4): Inflationary boom, where $\pi_t > 2\%$ and $y_t > 0\%$.

Table 1 presents the joint frequencies of the four regimes identified in the data. The inflationary slack regime (R2) is the most frequently observed, while the disinflationary boom regime (R3) is the least common. The other two regimes each account for approximately one-quarter of the observations. On average, the disinflationary slack regime (R1) lasts around 16 months, while the disinflationary boom regime (R3) persists for about 31 months.

Examining the variables' average values across the four different regimes, we observe that the output gap increases progressively from negative figures in the disinflationary slack (R1) regime to positive figures in the inflationary boom (R4) regime. Inflation displays a W-shaped pattern, while the policy rate follows an inverse W-shaped pattern, likely indicative of a policy response aimed at counteracting inflationary pressures.

The variables' standard deviation across regimes is also informative. Price inflation is relatively more (less) stable than the output gap only when inflation is below (above) target, regardless of whether the output gap is negative or positive. The largest standard deviation across all three variables is recorded during periods of inflationary slack.

[6] Okun's Law highlights the connection between unemployment and the output gap, indicating that when unemployment exceeds its natural rate, the economy operates below its potential and the output gap widens.

Figure 1 illustrates the distribution of these regimes over time. Prior to the year 2000, periods of inflationary slack (R2) were more common, whereas disinflationary slack (R1) became more prevalent after 2000. Observations associated with each regime are nevertheless scattered throughout the entire sample period from 1960 to 2019.

4 Slopes, policies and shocks

The model (7)-(8) is estimated using the algorithm outlined in Baumeister and Hamilton (2015). This algorithm generates $N = 10^6$ draws $\{\mathbf{A}_S^{(l)}, \mathbf{D}_S^{(l)}, \mathbf{B}_S^{(l)}\}_{l=1}^N$ from the posterior distribution $p(\boldsymbol{A_S}, \boldsymbol{D_S}, \boldsymbol{B_S} | \boldsymbol{Y_{T,S}})$, where $\mathbf{Y}_{T,S}$ represents the observed data within each regime. In the following analysis, we compare the posterior distributions of the structural coefficients across the four regimes. Additionally, we include results from the linear specification (i.e., the three-equation model of Baumeister and Hamilton (2018)) using the monthly frequency and over a longer sample period.

4.1 Contemporaneous coefficients using linear models

We begin the analysis by estimating the demand and supply curve slopes, along with the monetary policy response to business cycle conditions, using the linear model at different frequencies and time periods. This approach facilitates comparison with existing literature.

Previous work, using data from 1979 to 2007 and single equations, estimates the sensitivity of the federal funds rate to the output gap ψ_y to be 0.70 (Carvalho et al., 2021). That work also estimates the sensitivity of the policy rate to the year-on-year change in the personal consumption expenditures (PCE) index ψ_π to be 1.97. Our estimation of the Taylor rule parameters over the entire monthly sample period from 1962 to 2019 yields comparable average values of 0.75 for the output gap and 1.67 for the inflation rate (see Table 3). Conversely, the quarterly frequency model estimates the sensitivity of the policy rate to the output gap to be significantly greater than one. Similar results are observed when limiting the sample period to the Great Moderation period (1986-2008) analyzed by Baumeister and Hamilton (2018). These findings indicate that the frequency of the data significantly affects the estimation of Taylor rule parameters.

The slope of the Phillips curve $\alpha_{\pi,S}^{-1}$, derived from the monthly linear model is estimated to be 0.60 (1/1.67). Our estimate is very close to the 0.53-0.56 range reported in Bergholt et al. (2024) and similar to the estimate from the quarterly linear model. The slope of the dynamic IS curve, $\beta_{\pi,S}^{-1}$, derived from the linear model, is estimated to be -0.26 (-1/3.90). This falls between the two estimates (-0.09 and -0.41) for the two subperiods reported by Bergholt et al. (2024), while the quarterly model suggests a value close to -1. The estimates for the sensitivity of output growth to the interest rate are also similar across the two frequencies.

Overall, there is consistency between our linear estimates with monthly frequency and those from prior studies using linear methods.

4.2 Contemporaneous coefficients using nonlinear models

Figure 2 shows the posterior distributions for the five contemporaneous coefficients and the Taylor rule autoregressive term in our 3-equation TVAR model. The data offer substantial insights into parameter values across the four regimes, reveal significant differences between states, and suggest that the same shocks may have different effects on the business cycle depending on the current economic state. Table 4 presents the estimates with the median value and 68% credible interval.

4.2.1 Phillips curve coefficients.

The Phillips curve exhibits a high degree of state dependency. The coefficient $\alpha_{\pi,S}$, which measures the relationship between the output gap and inflation (the inverse of the Phillips curve), is consistently estimated as positive across all economic regimes, showing a rightward shift compared to the prior distribution. The Phillips curve is relatively flatter during periods of disinflationary boom, with a median slope coefficient of 0.47 (1/2.15), inflationary slack at 0.51 (1/1.96), and disinflationary slack at 0.58 (1/1.71). In contrast, the Phillips curve becomes relatively steeper during periods of inflationary boom, with a median slope coefficient of 0.73 (1/1.37).

However, the estimated Phillips curve may become steeper primarily due to a greater persistence parameter in the inflation equation, rather than changes in firms' price-setting behavior (see Equation B.5 in the Appendix). Assuming a consistent discount factor across regimes of $\beta = 0.99$, and using the median estimates of α_π and the persistence parameter ϕ^π (derived from the DIS curve as $\phi^\pi = -\beta^\pi/\beta^r$), the composite parameter representing firms' price-setting behavior (e.g., $\bar{\kappa}(\tau^{-1} + \varphi)$) averages 2.1 in the disinflationary slack regime, 2.5 in the disinflationary boom regime, 4.0 in the inflationary boom regime, and 5.9 in the inflationary slack regime.

These findings indicate that the Phillips curve becomes steeper during inflationary periods, largely due to the underlying price-setting behavior of firms. However, in the inflationary slack regime, this steepening effect is counteracted by the influence of inflation persistence, which ultimately flattens the empirical Phillips curve in that specific regime.

Notably, the inflationary boom regime was predominantly observed in the 1970s, during the dot-com bubble and in the years which culminated with the Global Financial Crisis (GFC). Therefore, our findings suggest two key insights: first, the Phillips curve was relatively flat after the GFC, implying a higher sacrifice ratio in the 2010-2019 period; second, the curve has the potential to become steeper if certain business cycle conditions arise.

4.2.2 DIS curve coefficients.

The responsiveness of the output gap to inflation ($\beta_{\pi,S}$) is rather homogeneous across economic states,[7] while that to the interest rate ($\beta_{r,S}$) varies across different states. The sensitivity of output to the interest rate varies between -0.31 and -0.79 during periods of inflationary slack and inflationary boom and -1.- and -1.1 during periods of disinflationary boon and disinflationary slack. These findings align with the -0.7 average value estimated by Attanasio and Low (2004) using the Euler equation and therefore focusing on consumption, while Fuhrer and Rudebusch (2004) report a low value. It is when inflation is below target that consumers can drastically cut spending today to save and consume much more tomorrow if the interest rate rise. When inflation is above target, savings are less protected due to the erosion of purchasing power by inflation; hence, the change in the policy rate has a smaller impact on the real economy.

4.2.3 Interest rate rule coefficients.

The Taylor rule parameters are moderately state dependent. The coefficients for the interest rate rule are aligned with economic theory across all four regimes, with the Taylor principle holding as $\psi_{\pi,S} > 1$. $\psi_{\pi,S}$ is on average larger when inflation is above target, indicating a strong commitment by the policymaker to achieve their inflation goal by adjusting their preferences in response to relatively high inflation.

The average Taylor rule parameter on inflation is 1.70 during the inflationary slack regime, 1.52 during the inflationary boom regime, 1.48 during the disinflationary boom regime and 1.23 during the disinflationary slack regime. However, as we will demonstrate in the differential test across regimes in the next subsection, the FED's response to inflation is generally similar across the different regimes and it is very close to the 1.5 estimate proposed by Taylor (1993), except for the two extreme policy parameters when

[7]The theoretical coefficient on inflation expectations in the DIS curve is expected to be positive; however, it is estimated to be strongly negative in all regimes. A similar finding is reported in Baumeister and Hamilton (2018). This result may stem from the assumption of adaptive expectations.

output is below potential.

The median long-term response to the output gap exhibits significant regime dependence with the lowest median value estimated in the disinflationary boom regime: 0.46, an estimate similar to the 0.5 coefficient proposed by Taylor (1993). This policy parameter in the other three regimes range between 0.72 and 0.79.

Lastly, the estimated interest rate smoothing parameter ρ_S indicates a strong preference for gradual monetary policy adjustments. Estimates range from 0.69 to 0.89 across economic regimes, with a slightly greater preference on data dependence observed in the inflationary slack regime. When inflation is above target but the economy is underperforming, the Federal Reserve tends to prioritize recent data over a forward-looking approach, reflecting caution aagainst a potentially harmful policy error in this senstive environment.

Our estimated coefficients are close to those of the inertial rule considered by the Federal Reserve, as modeled by English et al. (2003) and considered by the Federal Reserve: $\psi_\pi = 1.5$, $\psi_y = 1$ and $\rho = 0.85$.[8]

4.2.4 Testing the differential impact

The posterior parameters draws generated by the sampling algorithm are used to examine the posterior distribution of the differences in state-dependent parameters of the impact matrices across the four regimes. Specifically, for each parameter, we take (N) draws from one regime and subtract the corresponding (N) draws from an alternative regime, thereby constructing an (N $\times$ N) posterior draws of the differences.

The main findings, highlighting key differences, are presented in Figure 3, while a detailed comparison of parameters across the various regimes is provided in Figures A2 and A3 in the Appendix. For brevity, we compare the inflationary and disinflationary regimes in boom and slack periods by presenting the posterior distributions of the most

[8] See the Federal Reserve's chapter, "Policy Rules and How Policymakers Use Them".

significant differences, as identified in Table 4. In each plot, the vertical lines represent the 68% confidence intervals, and the dashed line indicates the reference point where the difference equals zero. The results demonstrate that the differences in contemporaneous parameters across regimes are highly significant for several key variables.

Focusing on the parameters of the Phillips curve ($\alpha_{\pi,S}$), a comparison between the inflationary boom (R4) regime and the disinflationary boom (R3) regime shows that the difference in the sensitivity of the output gap to inflation across these two regimes is strongly negative. The same conclusion can be derived comparing $\alpha_{\pi,S}$ across other regimes. The probability that $\alpha_{\pi,S}$ is lower in the inflationary boom (R4) regime is relatively higher. This indicates that the Philips curve is steeper in this regime also statistically.

Looking at parameters of the DIS curve ($\beta_{\pi,S}$ and $\beta_{r,S}$), a comparison between the inflationary slack (R2) regime and the disinflationary slack (R1) regime shows that the difference in the sensitivity of the output gap to the interest rate across these two regimes is strongly positive. $\beta_{r,S}$ in the inflationary slack (R2) regime is in absolute value lower also vis-à-vis other regimes. In contrast, the difference in the sensitivity of the output gap to inflation is, on average, close to zero. This indicates that changes in interest rates are expected to have a more limited impact on the output gap during the inflationary slack (R2) regime.

Examining the estimated Taylor rule parameters, specifically the inflation sensitivity ($\psi_{\pi,S}$), we find that the Federal Reserve exhibits small differential responsiveness across the business cycle. Although the credible interval of these differences includes zero, it is noteworthy that the estimated inflation sensitivity is higher in the inflationary slack regime (R2) than in the disinflationary slack regime (R1), suggesting that the Fed is more responsive to inflationary pressures than to disinflationary ones when economic slack is present. However, when comparing the inflation sensitivity of the inflationary slack regime (R2) with the inflationary boom (R4), and the the disinflationary slack regime

(R1) with the disinflationary boom (R3) regime, the evidence for a difference between the two parameters across regimes is inconclusive, as the posterior distribution is centered near zero and equally spans positive and negative values (see Figures A2 and A3 in the Appendix).

Instead, a comparison between the inflationary boom (R4) regime and the disinflationary boom (R3) regime reveals that the difference in the Federal Reserve's sensitivity to the output gap ($\psi_{y,S}$) across these two regimes has a large number of positive draws. This suggests that the Federal Reserve is less responsive to output gap during the disinflationary boom (R3) regime and more responsive during the inflationary boom (R4) regime. This finding applies also to the other two regimes compared with the disinflationary boom (R3) regime (see Appendix).

Finally, examining the persistence of the Taylor rule (ρ), a comparison between the inflationary slack (R2) regime and the disinflationary slack (R1) regime shows that the difference in the Federal Reserve's persistence parameter across these two regimes is strongly negative. The same conclusions can be drawn when comparing ρ in the inflationary slack (R2) regime with other regimes. This indicates that the Federal Reserve is more data dependent during the inflationary slack (R2) regime, reducing policy smoothing and giving more weight to current economic fundamentals in the policy decision making.

4.3 Structural shocks

What is the size of the shocks in the different regimes? $\mathbf{D}_S$ in (8) is a diagonal variance-covariance matrix of structural shocks, with elements $d_{ii,S}$. The median value and the 68% credible interval of $d_{ii,S}$ are provided in Table 5. The magnitudes of supply and demand shocks are generally similar within each type across different regimes; these shocks are marginally smaller during the disinflationary slack regime.

In contrast, the magnitude of monetary policy shocks varies significantly across regimes. Compared to periods when inflation is below target, these shocks are twice as large during inflationary boom periods and three times as large during inflationary slack periods compared to disinflationary periods.

As previously noted, the impact of interest rate changes on the output gap is expected to be more constrained during the inflationary slack (R2) regime. Consistently, this is also the regime in which the Federal Reserve enacts its most substantial policy changes, aiming to influence the business cycle.

4.4 Transmission of shocks along the curves

To construct the impulse response functions (IRFs), we must account for the possibility of shifts in economy across regimes. The nonlinear response of the endogenous variable $\boldsymbol{y}_t$ depends on both the information set at time $t-1$ (denoted Γ_{t-1}) and the sign and magnitude of the structural shocks u_t at time t. This information set includes the state variable z_{t-1} and the system history prior to the shocks, represented by $\bar{\boldsymbol{y}}_{t-1}$. Importantly, feedback from future changes in the information set is fully incorporated into the macroeconomic system dynamics when constructing the structural response functions. This approach enables the model to transition between different regimes based on the evolving dynamics of the state variable z_{t+k}, which is in turn endogenously determined by the output gap y_{t+k} and inflation π_{t+k} across all horizons $k = 0, \ldots, K-1$.

Following an approach similar to that of Koop et al. (1996), we compute the state-dependent IRFs of $\boldsymbol{y}_t$ as the difference between the expectation of $\boldsymbol{y}_{t+k}$ at horizon k, conditional on the shock to the jth variable $u_{j,t}$ and the history Γ_{t-1}, and the expectation of $\boldsymbol{y}_{t+k}$ given only Γ_{t-1}:

$$IRF_{\boldsymbol{y}_t,S}(k, u_{j,t}, \Gamma_{t-1,S}) \equiv \mathbb{E}(\boldsymbol{y}_{t+k} |\, u_{j,t}, \Gamma_{t-1,S}) - \mathbb{E}(\boldsymbol{y}_{t+k} |\, \Gamma_{t-1,S}). \tag{12}$$

Here, $S \in \{R1, R2, R3, R4\}$ indicates the regime in which the shock at time t impacts the economy. We compute these conditional expectations by simulating the model forward, using the average of all in-sample observations within each regime as initial values for the endogenous variables. This method ensures that we capture the most representative dynamics for each regime and allows for endogenous shifts across regimes. The state-dependent IRFs are normalized to allow a comparison across regimes. The IRFs generated by the linear model are also provided for comparison.

Demand shocks that cause a reduction in output gap by 1% have a more pronounced effect on inflation during inflationary boom periods (see Figure 4), due to the Phillips curve being steeper. Consequently, the larger is the interest rate response.

Supply shocks that reduce the output gap by 1% have a somewhat marginally weaker impact on inflation during disinflationary booms compared to other regimes (see Figure 5). This can be attributed to the Federal Reserve's more muted response to changes in the output gap. Consequently, the reduction in policy rates is significantly less pronounced, further dampening the overall effect on inflation.

Following a contractionary monetary policy shock, the output gap and inflation exhibit stronger responses during economic booms (R3 and R4) and periods of disinflationary slack (see Figure 6). Specifically, an unexpected 1-percentage-point exogenous increase in the policy rate reduces the output gap by approximately 0.4% at through in these three regimes, a response three to four times greater than that observed in the inflationary slack regime, with the effect remaining persistent throughout economic booms. Similarly, year-on-year inflation initially declines by 0.1-0.2 percentage points, with a more sustained peak response ranging from -0.2 to -0.4 percentage points during economic booms. The effect is long lasting.

These results are driven by distinct underlying mechanisms. During the inflationary boom period, the Phillips Curve is relatively steeper, which magnifies the effects of monetary policy shocks on inflation. Conversely, during disinflationary boom and disinflationary slack periods, monetary policy shocks continue to have a strong impact on output and inflation given the large elasticity of intertemporal substitution.

In contrast, during the inflationary slack regime, the business cycle response is relatively muted, with a short-lived impact on inflation and a modest but persistent decline in the output gap. These dynamics can be attributed to the lower sensitivity of the output gap to the policy rate in the DIS curve in this regime.

While one might argue that the stronger impact on the output gap and inflation during boom periods is driven by the persistence of the policy rate, our estimates suggest otherwise. The policy rate returns to zero after 15-20 months during boom periods, compared to 48 months during periods of economic slack, indicating that persistence cannot explain the observed differences.

It is worth noting that the muted business cycle response observed during the inflationary slack regime aligns closely with predictions from linear models, as also found by Baumeister and Hamilton (2018). This similarity can be attributed to the fact that this regime accounts for 44% of the observations, which heavily influences the linear model's behavior. However, our findings suggest that the results derived from linear models are not universally applicable, as they fail to capture the heterogeneity across different regimes, particularly during periods of economic booms and disinflationary slack.

4.5 How should the Federal Reserve respond to a demand boom?

To assess the macroeconomic relevance of monetary policy decisions, analyzing the implications of monetary policy mistakes serves as an appropriate and insightful exercise.[9]

During periods of inflationary boom or disinflationary slack, policymakers experience fewer trade-offs, as restrictive or expansionary monetary policy can effectively

[9] An alternative approach involves examining the contribution of monetary policy shocks to the forecast error variance decomposition (FEVD) of the output gap and inflation. However, if the policymaker consistently adheres to a monetary policy rule for the majority of the time, the portion of the FEVD attributable to monetary policy shocks is likely to be relatively modest.

bring inflation closer to target and return the output gap to zero.

To explore this further, we conduct two counterfactual exercises, drawing on the methods of Sims and Zha (2006a), then applied among other by Bachmann and Sims (2012), Baumeister and Benati (2013), and Aastveit et al. (2023). Specifically, we model a demand boom that results in a 1% increase in the policy rate across all four regimes and two counterfactual scenarios obtained through a fully counteracting expansionary monetary policy shocks to assess the potential economic impact of a central bank policy mistake. In the first scenario, we assume that the policy rate is initially unchanged (at $k = 0$) to assess the impact of a one-period policy error. In the second scenario, we assume that the policy rate remains unchanged for 2 quarters, representing a series of consecutive policy errors.[10]

Our nonlinear model provides insights into whether the effects of such errors vary across different economic regimes and in which regimes the costs of policy mistakes are particularly severe.

As indicated by the solid red lines in Figure 7, the median response of the endogenous variables to a demand boom varies significantly across regimes. The largest expansion in output gap and inflation occurs during disinflationary periods, followed by expansions during inflationary boom and inflationary slack regimes. The central bank gradually raises the policy rate to temper demand expansion, thereby moderating economic growth, preventing economic overheating, and limiting inflation. Consistently, the increase in the policy rate is more pronounced during disinflationary periods.

[10]In the first exercise, the policymaker's delayed response occurs on impact, making the counterfactual fully robust to the Lucas critique. The critique is potentially more relevant in the second exercise, where policy mistakes persist for 2 quarters. However, given the inherent information lag (such as the four-month delay that may occur between monetary policy decisions and the release of the advance estimate for real GDP from the previous quarter), we find it plausible that agents can remain surprised by policy actions for a few months. Importantly, our analysis shows that shortening the horizon of repeated policy mistakes (e.g., two or four months) does not alter the qualitative implications of our findings, namely that such policy mistakes can be detrimental to the economy. McKay and Wolf (2023) propose a method for constructing counterfactuals that are robust to this Lucas critique. Their approach requires combining two policy shocks, a conventional monetary policy shock and a policy news shock. This combination enables the construction of a counterfactual in which the policy rate is closer to the desired path. We cannot apply their method within our framework with one monetary policy shock.

A one-month delay in the central bank's interest rate response (i.e., $r_{0,S} = 0$) generally leads to slight but positive increases in the output gap and inflation, and a slightly more aggressive policy response in subsequent periods, as shown in Panel A.

In the second counterfactual, we assume that the policy rate remains unchanged for 6 months (i.e., $r_{k,S} = 0$ for $k = 1, \ldots, 6$), representing a series of consecutive policy errors. As shown in Panel B, maintaining a steady policy rate over an extended period could lead to economic overheating. Inflation and output responses are substantial across all regimes, albeit less so in the inflationary slack regime. The output gap in this counterfactual increases more than in the baseline case. Peak differences are 0.5% higher on average after about one year during periods of disinflationary slack and disinflationary booms. Similarly, inflation spirals upward across all regimes as compared to baseline, with extraordinary large peak differences in the disinflationary slack, disinflationary boom, and inflationary boom regimes. When inflation falls below target and there is a negative output gap, maintaining the policy rate below the expected path for an extended period may help achieve desired economic conditions. Once these conditions are reached, future policy responses must be aggressive to address high inflation rates. The most significant trade-offs occur in the inflationary slack and disinflationary boom regimes. In the former, increasing the policy rate is necessary to avoid higher inflation, but carries the cost of a more negative output gap, which however is contained. In the latter, the delayed increase in the policy rate, required to lift inflation towards target, leads to an overheating economy with a larger positive output gap.

5 Conclusion

We study the complexity between inflation and the output gap by estimating a three-equation threshold VAR model with the Phillips curve, a dynamic investment–savings (DIS) curve, and the Taylor rule. Our estimation framework allows all parameters and shock variances to depend on the state of the economy. Regimes are defined by whether the output gap is positive or negative and inflation is above or below the central bank's target.

Results reveal significant differences in slope coefficients and policy responses across regimes. The Phillips curve is steeper during inflationary booms, meaning demand and monetary policy shocks have a stronger impact on inflation. The DIS curve also exhibits state dependence, with monetary policy having a more limited effect on output during inflationary slack, indicating that the transmission of policy shocks weakens in such periods.

The Taylor rule remains consistent with economic theory across all regimes, adhering to the Taylor principle. However, the Federal Reserve's response to the output gap is weaker when inflation is below target and the output gap is positive, while its reaction to inflation is broadly similar across regimes. Moreover, during inflationary slack, policy adjustments are more gradual, reflecting greater caution to avoid destabilizing the economy. In such cases, policymakers prioritize recent data over forward-looking considerations to mitigate the risks of policy missteps.

Monetary policy shocks exhibit varying degrees of effectiveness across regimes. They have a stronger impact on the output gap and inflation during economic booms and disinflationary slack periods, but elicit muted responses in the inflationary slack regime. Finally, the size of monetary policy shocks is significantly large when inflation exceeds its target.

These findings highlight the limitations of traditional linear models, which assume constant relationships between variables across all economic conditions. By accounting for state dependence, our nonlinear model provides a more nuanced understanding of how monetary policy operates and how shocks are transmitted through the economy. These insights are particularly relevant for policymakers, as they suggest that the effectiveness of monetary policy depends on the prevailing economic regime. For example, during inflationary booms, restrictive monetary policy can effectively reduce inflation and close the output gap, while during inflationary slack, policy changes may have a more limited impact. This might imply a stronger determination to fight inflation.

References

Aastveit, Knut Are, Hilde C. Bjornland, and Jamie L. Cross (2023) "Inflation Expectations and the Pass-Through of Oil Prices," *The Review of Economics and Statistics*, 105 (3), 733–743.

Ascari, Guido and Luca Fosso (2024) "The international dimension of trend inflation," *Journal of International Economics*, 148, 103896.

Attanasio, Orazio P. and Hamish Low (2004) "Estimating Euler Equations," *Review of Economic Dynamics*, 7 (2), 405–435.

Bachmann, Ruediger and Eric Sims (2012) "Confidence and the transmission of government spending shocks," *Journal of Monetary Economics*, 59 (3), 235–249.

Ball, Laurence, Daniel Leigh, and Prachi Mishra (2022) "Understanding US Inflation during the COVID-19 Era," *Brookings Papers on Economic Activity*, 53 (2 (Fall)), 1–80.

Barnett, William A. and Evgeniya A. Duzhak (2019) "Structural Stability of the Generalized Taylor Rule," *Macroeconomic Dynamics*, 23 (4), 1664–1678.

Barnichon, Regis and Geert Mesters (2020) "Identifying Modern Macro Equations with Old Shocks," *The Quarterly Journal of Economics*, 135 (4), 2255–2298.

——— (2021) "The Phillips multiplier," *Journal of Monetary Economics*, 117 (C), 689–705.

Barthélemy, Jean and Magali Marx (2017) "Solving endogenous regime switching models," *Journal of Economic Dynamics and Control*, 77, 1–25.

Baumeister, Christiane and Luca Benati (2013) "Unconventional Monetary Policy and the Great Recession: Estimating the Macroeconomic Effects of a Spread Compression at the Zero Lower Bound," *International Journal of Central Banking*, 9(2), 165–212.

Baumeister, Christiane and James Hamilton (2015) "Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information," *Econometrica*, 83 (5), 1963–1999.

——— (2018) "Inference in structural vector autoregressions when the identifying assumptions are not fully believed: Re-evaluating the role of monetary policy in economic fluctuations," *Journal of Monetary Economics*, 100 (C), 48–65.

Benati, Luca and Paolo Surico (2009) "VAR Analysis and the Great Moderation," *American Economic Review*, 99 (4), 1636–52.

Benigno, Pierpaolo and Gauti Eggertsson (2023) "Itâs Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," NBER Working Papers 31197, National Bureau of Economic Research, Inc.

Bergholt, Drago, Francesco Furlanetto, and Etienne Vaccaro-Grange (2024) "Did Monetary Policy Kill the Phillips Curve? Some Simple Arithmetics," *The Review of Economics and Statistics*, 1–45.

Bernanke, Ben S. and Ilian Mihov (1998) "Measuring Monetary Policy," *The Quarterly Journal of Economics*, 113 (3), 869–902.

Bianchi, Francesco (2013) "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics," *Review of Economic Studies*, 80 (2), 463–490.

Blanco, Andrés, Corina Boar, Callum J. Jones, and Virgiliu Midrigan (2024) "The Inflation Accelerator," NBER Working Papers 32531, National Bureau of Economic Research, Inc.

Burns, Arthur F. (1979) "The Economics of the Recovery: Some Reflections," *Federal Reserve Bank of St. Louis Review*.

Canova, Fabio (2009) "What Explains The Great Moderation in the U.S.? A Structural Analysis," *Journal of the European Economic Association*, 7 (4), 697–721.

Carvalho, Carlos, Fernanda Nechio, and Tiago Tristão (2021) "Taylor rule estimation by OLS," *Journal of Monetary Economics*, 124 (C), 140–154.

Cecchetti, Stephen, Michael Feroli, Peter Hooper, Frederic S Mishkin, and Kermit L. Schoenholtz (2023) "Managing Disinflations," CEPR Discussion Papers 18068.

Cerrato, Andrea and Giulia Gitti (2023) "Inflation Since COVID: Demand or Supply," Unpublished manuscript.

Chow, Gregory C and An-loh Lin (1971) "Best Linear Unbiased Interpolation, Distribution, and Extrapolation of Time Series by Related Series," *The Review of Economics and Statistics*, 53 (4), 372–375.

Clarida, Richard, Jordi Galí, and Mark Gertler (2000) "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, 115 (1), 147–180.

Cogley, Timothy and Thomas J. Sargent (2005) "The conquest of US inflation: Learning and robustness to model uncertainty," *Review of Economic Dynamics*, 8 (2), 528–563, Monetary Policy and Learning.

Cogley, Timothy and Argia M. Sbordone (2008) "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98 (5), 2101–2126.

Coibion, Olivier and Yuriy Gorodnichenko (2012) "Why Are Target Interest Rate Changes So Persistent?" *American Economic Journal: Macroeconomics*, 4 (4), 126–62.

Davig, Troy and Taeyoung Doh (2014) "Monetary Policy Regime Shifts and Inflation Persistence," *The Review of Economics and Statistics*, 96 (5), 862–875.

Davig, Troy and Eric M. Leeper (2007) "Generalizing the Taylor Principle," *American Economic Review*, 97 (3), 607â635.

De Santis, Roberto A. and Tommaso Tornese (2025) "Energy supply shocksâ nonlinearities on output and prices," *European Economic Review*, 176, 105037.

Del Negro, Marco, Michele Lenza, Giorgio E. Primiceri, and Andrea Tambalotti (2020) "What's up with the Phillips curve?" *Brookings Papers on Economic Activity*.

Del Negro, Marco and Frank Schorfheide (2004) "Priors from General Equilibrium Models for VARS," *International Economic Review*, 45 (2), 643–673.

English, William B., William R. Nelson, and Brian P. Sack (2003) "Interpreting the Significance of the Lagged Interest Rate in Estimated Monetary Policy Rules," *The B.E. Journal of Macroeconomics*, 3 (1), 1–18.

Fitzgerald, Terry, Callum Jones, Mariano Kulish, and Juan Pablo Nicolini (2024) "Is There a Stable Relationship between Unemployment and Future Inflation?" *American Economic Journal: Macroeconomics*, 16 (4), 114â42.

Forbes, Kristin J., Joseph E. Gagnon, and Christopher G. Collins (2021) "Low inflation bends the Phillips curve around the world: Extended results," Working Paper Series WP21-15, Peterson Institute for International Economics.

Fuhrer, Jeffrey and Glenn Rudebusch (2004) "Estimating the Euler equation for output," *Journal of Monetary Economics*, 51 (6), 1133–1153.

Furlanetto, Francesco and Antoine Lepetit (2024) "The Slope of the Phillips Curve," Finance and Economics Discussion Series 2024-043, Board of Governors of the Federal Reserve System (U.S.).

Galí, J. (2015) *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications - Second Edition*: Princeton University Press.

Galí, Jordi and Luca Gambetti (2019) "Has the U.S. Wage Phillips Curve Flattened? A Semi-Structural Exploration," NBER Working Papers 25476, National Bureau of Economic Research, Inc.

Galí, Jordi and Mark Gertler (1999) "Inflation dynamics: A structural econometric analysis," *Journal of Monetary Economics*, 44 (2), 195–222.

Harding, Martín, Jesper Lindé, and Mathias Trabandt (2023) "Understanding post-COVID inflation dynamics," *Journal of Monetary Economics*, 140 (S), 101–118.

Hazell, Jonathon, Juan Herreno, Emi Nakamura, and Jón Steinsson (2022) "The slope of the Phillips curve: evidence from US states," *The Quarterly Journal of Economics*, 137 (3), 1299–1344.

Inoue, Atsushi, Barbara Rossi, and Yiru Wang (2024) "Has the Phillips Curve Flattened?," CEPR Discussion Papers 18846, C.E.P.R. Discussion Papers.

Judd, John P. and Glenn Rudebusch (1998) "Taylor's rule and the Fed, 1970-1997," *Economic Review*, 3–16.

Karadi, Peter, Anton Nakov, Galo Nuno, Ernesto Pasten, and Dominik Thaler (2024) "Strike while the Iron is Hot: Optimal Monetary Policy with a Nonlinear Phillips Curve," mimeo.

Kleibergen, Frank and Sophocles Mavroeidis (2009) "Weak instrument robust tests in GMM and the New Keynesian Phillips curve," *Journal of Business & Economic Statistics*, 27 (3), 293–311.

Koop, Gary, M Pesaran, and Simon Potter (1996) "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74 (1), 119–147.

Liu, Zheng, Daniel F. Waggoner, and Tao Zha (2011) "Sources of macroeconomic fluctuations: A regimeâswitching DSGE approach," *Quantitative Economics*, 2 (2), 251–301.

Lubik, Thomas and Frank Schorfheide (2004) "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 94 (1), 190–217.

Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H. Stock (2014) "Empirical evidence on inflation expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, 52 (1), 124–188.

McKay, Alisdair and Christian K Wolf (2023) "What can time-series regressions tell us about policy counterfactuals?" *Econometrica*, 91 (5), 1695–1725.

Orphanides, Athanasios (2004) "Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches," *Journal of Money, Credit and Banking*, 36 (2), 151–75.

Primiceri, Giorgio E. (2006) "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy," *The Quarterly Journal of Economics*, 121 (3), 867–901.

Rudebusch, Glenn D. (2002) "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics*, 49 (6), 1161–1187.

Sbordone, Argia M. (2002) "Prices and unit labor costs: a new test of price stickiness," *Journal of Monetary Economics*, 49 (2), 265–292.

Sims, Christopher A. and Tao Zha (2006a) "Does Monetary Policy Generate Recessions?" *Macroeconomic Dynamics*, 10 (2), 231–272.

——— (2006b) "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 96 (1), 54–81.

Surico, Paolo (2007) "The Fed's monetary policy rule and U.S. inflation: The case of asymmetric preferences," *Journal of Economic Dynamics and Control*, 31 (1), 305–324.

Taylor, John (1999) "A Historical Analysis of Monetary Policy Rules," in *Monetary Policy Rules*, 319–348: National Bureau of Economic Research, Inc.

Taylor, John B. (1993) "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 39 (1), 195–214.

Uhlig, Harald (2005) "What are the effects of monetary policy on output? Results from an agnostic identification procedure," *Journal of Monetary Economics*, 52 (2), 381–419.

Wu, Jing Cynthia and Fan Dora Xia (2016) "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 48 (2-3), 253 â 291.

Zhu, Yanli and Haiqiang Chen (2017) "The asymmetry of U.S. monetary policy: Evidence from a threshold Taylor rule with time-varying threshold values," *Physica A: Statistical Mechanics and its Applications*, 473 (C), 522–535.

Regimes	Disinflationary Slack (R1)	Inflationary Slack (R2)	Disinflationary Boom (R3)	Inflationary Boom (R4)
Number of observations	156	300	63	164
Share of total observations	23%	44%	9%	24%
Duration in months (mean)	15.6	20.0	31.5	18.2
Correlation (output, inflation)	0.51	−0.15	0.30	−0.15
Mean				
Output gap	−2.46%	−2.22%	1.49%	1.76%
Y-o-Y PCE inflation	1.28%	4.33%	1.31%	4.09%
Federal funds rate	0.79%	6.40%	4.51%	6.44%
Standard deviation				
Output gap	1.32%	1.81%	1.27%	1.55%
Y-o-Y PCE inflation	0.73%	2.33%	0.34%	2.09%
Federal funds rate	2.37%	4.12%	0.84%	2.56%

Table 1: Summary statistics across regimes

Notes: The output gap is defined as 100 times the logarithmic difference between real GDP and potential output. To convert real GDP to a monthly frequency, we apply the method proposed by Chow and Lin (1971) using monthly data from industrial production and real retail sales. Real potential output is linearly interpolated to a monthly frequency. Inflation is measured as 100 times the year-on-year logarithmic change in the PCE price index. The federal funds rate is adjusted using the shadow rate provided by Wu and Xia (2016) during the zero lower bound period (June 2009 to November 2015). Sample period: Jan. 1962 - Dec. 2019.

	$\alpha_{\pi,S}$	$\beta_{\pi,S}$	$\beta_{r,S}$	$\psi_{y,S}$	$\psi_{\pi,S}$	ρ_S
$\mu_{h,S}$	1	0.75	-1	0.5	1.5	0.5
$\sigma_{h,S}$	0.4	0.4	0.4	0.4	0.4	0.2
$\nu_{h,S}$	3	3	3	3	3	-
Sign	$+$	?	$-$	$+$	$+$	$+$

Table 2: Priors for contemporaneous coefficients

Notes: A Student's t prior characterizes all coefficients of matrix $\boldsymbol{A_S}$, where $\mu_{h,S}$ is the location parameter, $\sigma_{h,S}$ the scale parameter, and $\nu_{h,S}$ the degrees of freedom parameter. Sign indicates if and where the distribution is truncated. ρ_S is modelled using a Beta distribution with mean 0.5, standard deviation 0.2, $\alpha = 2.6$, and $\beta = 2.6$.

Parameters (Priors)	Monthly (1962:M1 - 2019:M12)	Monthly (1986:M1 - 2008:M9)	Quarterly (1962:Q1 - 2019:Q4)	Quarterly (1986:Q1 - 2008:Q3)
$\alpha_\pi \sim t(1, 0.4)$	1.67 [1.21, 2.48]	1.54 [1.16, 2.13]	1.75 [1.28, 2.60]	1.46 [1.04, 2.24]
$\beta_\pi \sim t(0.75, 0.4)$	-3.90 [-5.72, -2.48]	-2.92 [-4.21, -1.96]	-1.10 [-1.87, -0.48]	-0.99 [-1.53, -0.53]
$\beta_r \sim t(-1, 0.4)$	-0.41 [-0.78, -0.14]	-0.77 [-1.14, -0.39]	-0.65 [-1.01, -0.29]	-0.72 [-1.10, -0.30]
$\psi_y \sim t(0.5, 0.4)$	0.75 [0.54, 1.03]	0.79 [0.51, 1.16]	1.32 [0.97, 1.86]	1.61 [1.12, 2.27]
$\psi_\pi \sim t(1.5, 0.4)$	1.67 [1.28, 2.15]	1.59 [1.21, 2.04]	1.45 [1.13, 1.79]	1.63 [1.30, 2.02]
$\rho \sim B(0.5, 0.2)$	0.78 [0.69, 0.86]	0.93 [0.89, 0.95]	0.52 [0.38, 0.64]	0.69 [0.59, 0.78]

Table 3: Contemporaneous parameters using linear models

Notes: This table shows the median value and 68% credible interval of the posterior distributions for contemporaneous coefficients estimated at different frequencies and time periods using linear models. The quarterly sample 1986:Q1 - 2008:Q3 is used by Baumeister and Hamilton (2018). A random walk Metropolis-Hastings algorithm, provided by Baumeister and Hamilton (2015), is used to generate draws of the unknown element of $\boldsymbol{A}$ from the distribution.

Parameters (Priors)	Linear	Disinflationary Slack (R1)	Inflationary Slack (R2)	Disinflationary Boom (R3)	Inflationary Boom (R4)
$\alpha_{\pi} \sim t\,(1, 0.4)$	1.67 [1.21, 2.48]	1.71 [1.31, 2.30]	1.96 [1.35, 3.06]	2.15 [1.38, 3.90]	1.37 [1.02, 1.88]
$\beta_{\pi} \sim t\,(0.75, 0.4)$	−3.90 [−5.72, −2.48]	−2.96 [−4.25, −1.96]	−3.32 [−5.46, −1.86]	−4.47 [−6.74, −2.47]	−3.52 [−4.63, −2.62]
$\beta_{r} \sim t\,(-1, 0.4)$	−0.41 [−0.78, −0.14]	−1.10 [−1.50, −0.75]	−0.31 [−0.66, −0.10]	−1.00 [−1.39, −0.63]	−0.79 [−1.13, −0.46]
$\psi_{y} \sim t\,(0.5, 0.4)$	0.75 [0.54, 1.03]	0.79 [0.51, 1.16]	0.72 [0.47, 1.03]	0.46 [0.25, 0.71]	0.73 [0.48, 1.04]
$\psi_{\pi} \sim t\,(1.5, 0.4)$	1.67 [1.28, 2.15]	1.23 [0.75, 1.64]	1.70 [1.30, 2.23]	1.48 [1.07, 1.89]	1.52 [1.14, 1.92]
$\rho \sim B\,(0.5, 0.2)$	0.78 [0.69, 0.86]	0.89 [0.85, 0.93]	0.69 [0.55, 0.80]	0.84 [0.79, 0.89]	0.80 [0.72, 0.87]

Table 4: State-dependent contemporaneous parameters

Notes: The table shows the median value and 68% credible interval of the posterior distributions for contemporaneous coefficients. A random walk Metropolis-Hastings algorithm provided by Baumeister and Hamilton (2015) is used to generate draws of the unknown element of A_S from the distribution. See notes in Figure 1 for definitions of each regime.

Shocks	Linear	Disinflationary Slack (R1)	Inflationary Slack (R2)	Disinflationary Boom (R3)	Inflationary Boom (R4)
Supply	0.69	0.60	0.73	0.68	0.71
	[0.64, 0.81]	[0.55, 0.69]	[0.65, 0.90]	[0.60, 0.85]	[0.65, 0.80]
Demand	1.18	0.96	1.13	1.00	1.12
	[0.90, 1.59]	[0.78, 1.23]	[0.85, 1.59]	[0.77, 1.33]	[0.93, 1.38]
Monetary policy	0.48	0.20	0.66	0.18	0.38
	[0.47, 0.50]	[0.19, 0.22]	[0.63, 0.70]	[0.17, 0.20]	[0.36, 0.41]

Table 5: State-dependent structural shocks (standard deviations)

Notes: The table shows the median value and 68% credible interval of the posterior distributions for the standard deviation of shocks. A random walk Metropolis-Hastings algorithm, provided by Baumeister and Hamilton (2015), is used to generate draws of the unknown element of A_S from the distribution. See notes in Figure 1 for definitions of each regime.

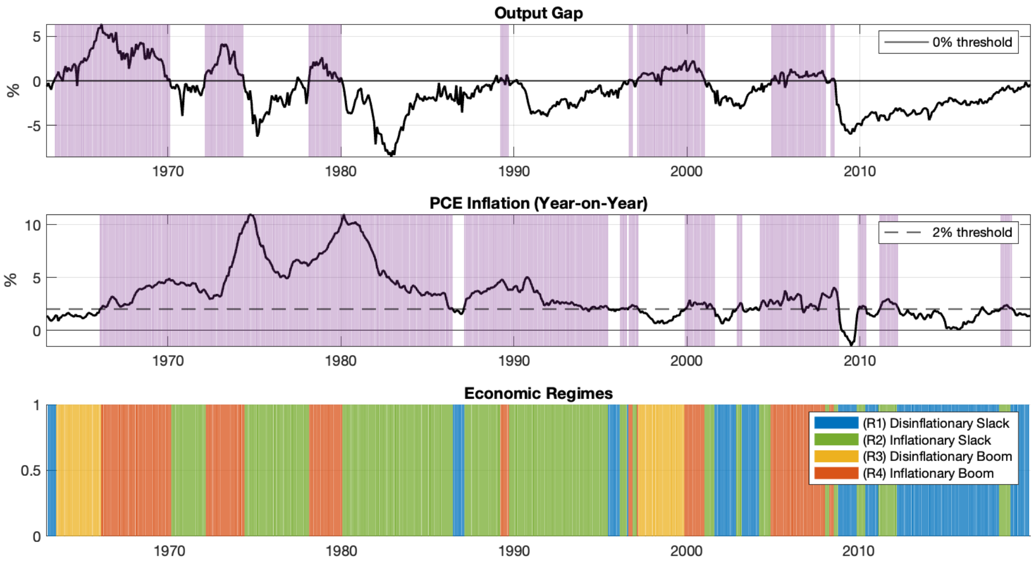


Figure 1: Four regimes

Notes: Regime 1 (R1): Disinflationary slack, where $\pi_t \leq 2\%$ and $y_t \leq 0\%$. Regime 2 (R2): Inflationary slack, where $\pi_t > 2\%$ and $y_t \leq 0\%$. Regime 3 (R3): Disinflationary boom, where $\pi_t \leq 2\%$ $y_t > 0\%$. Regime 4 (R4): Inflationary boom, where $\pi_t > 2\%$ and $y_t > 0\%$. The output gap is defined as 100 times the logarithmic difference between real GDP and potential output. To convert real GDP to a monthly frequency, we apply the method proposed by Chow and Lin (1971) using monthly data from industrial production and real retail sales. Real potential output is linearly interpolated to a monthly frequency. Inflation is measured as 100 times the year-on-year logarithmic change in the PCE price index.

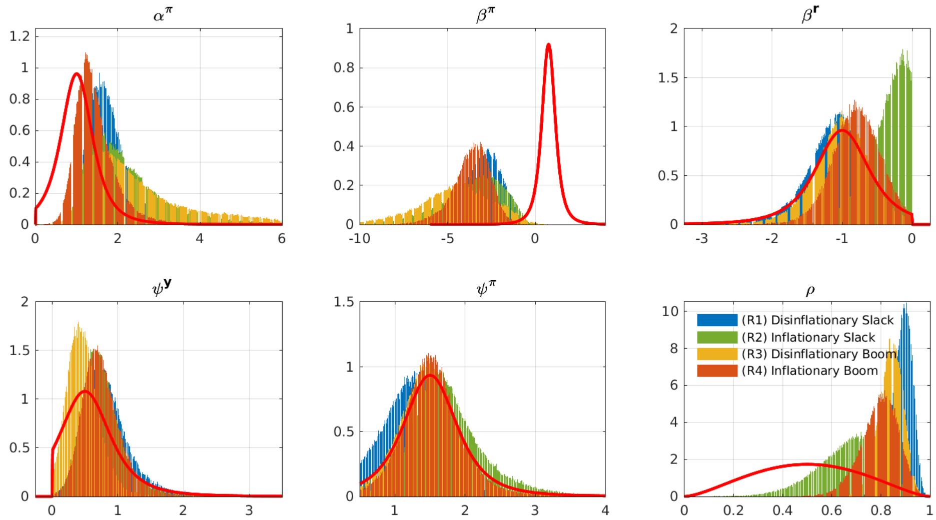


Figure 2: Contemporaneous parameters – posterior distributions

Notes: Prior distributions (red lines) and posterior distributions (histograms) for contemporaneous coefficients. A random walk Metropolis–Hastings algorithm, provided by Baumeister and Hamilton (2015), is used to generate draws of the unknown element of A_S from the distribution. See notes in Figure 1 for definitions of each regime.

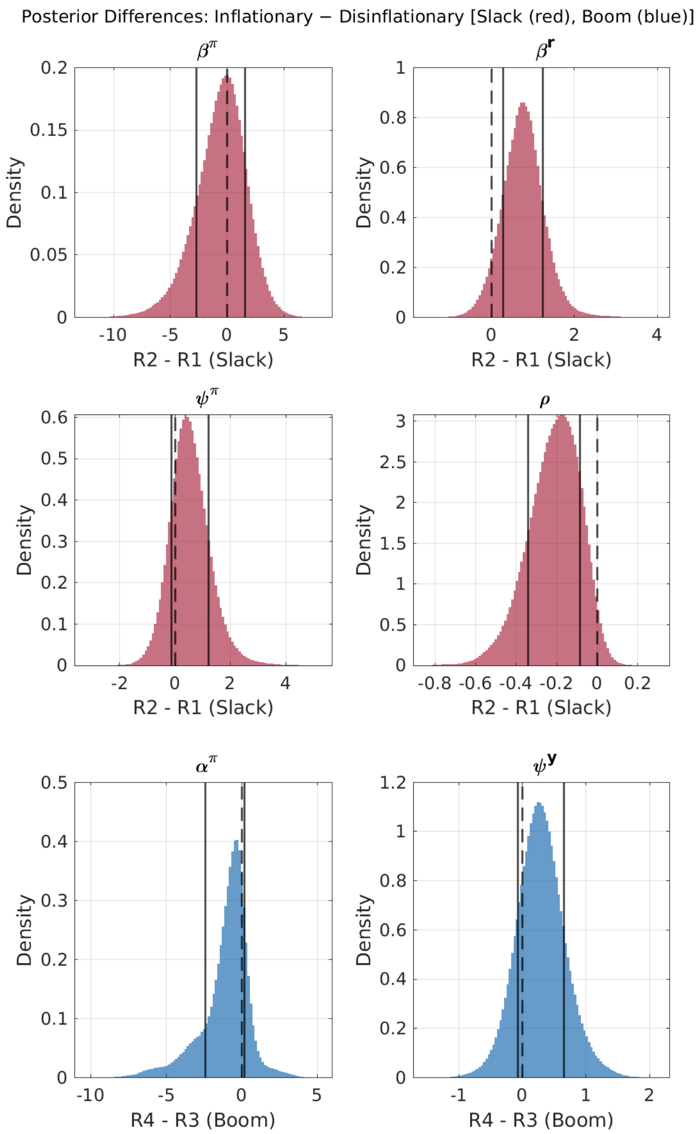


Figure 3: Differences across contemporaneous parameters: inflationary versus disinflationary periods – posterior distributions

Notes: The posterior parameters draws generated by the sampling algorithm are used to examine the posterior distribution of the differences in state-dependent parameters of the impact matrices across the four regimes. For each parameter, we take (N) draws from one regime and subtract the corresponding (N) draws from an alternative regime, thereby constructing an (N $\times$ N) posterior draws of the differences. The dashed vertical line is set to zero. The two vertical lines denotes the 68% credible set. Differences between the inflationary boom (R4) and disinflationary boom (R3) regimes are shown in blue, while differences between the inflationary slack (R2) and disinflationary slack (R1) regimes are shown in red. See notes in Figure 1 for definitions of each regime.

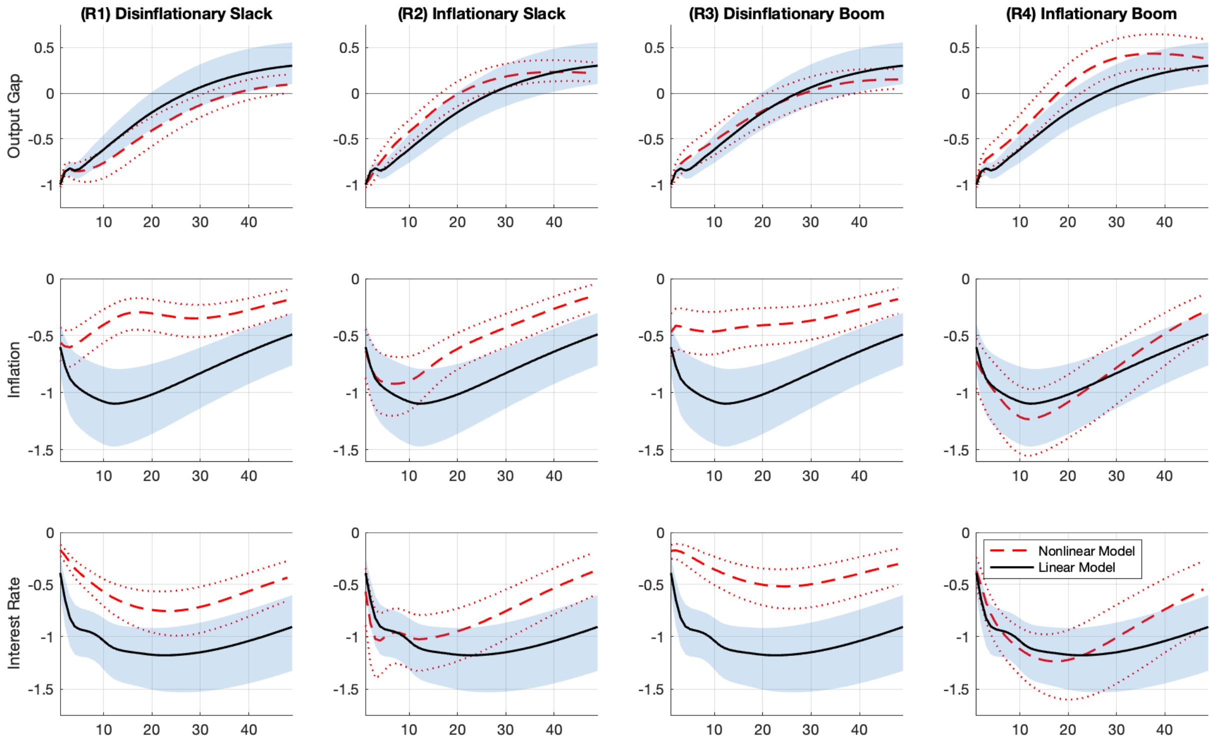


Figure 4: Response to a demand shock

Notes: The nonlinear IRFs (bold and dashed red lines) and linear IRFs (black line and blue shadow) are estimated using the 3-variable macro model featuring A_S (as in matrix 5) and A (as in matrix B.12), respectively. The shock is normalized such that the output gap declines by 1%. See notes in Figure 1 for definitions of each regime.

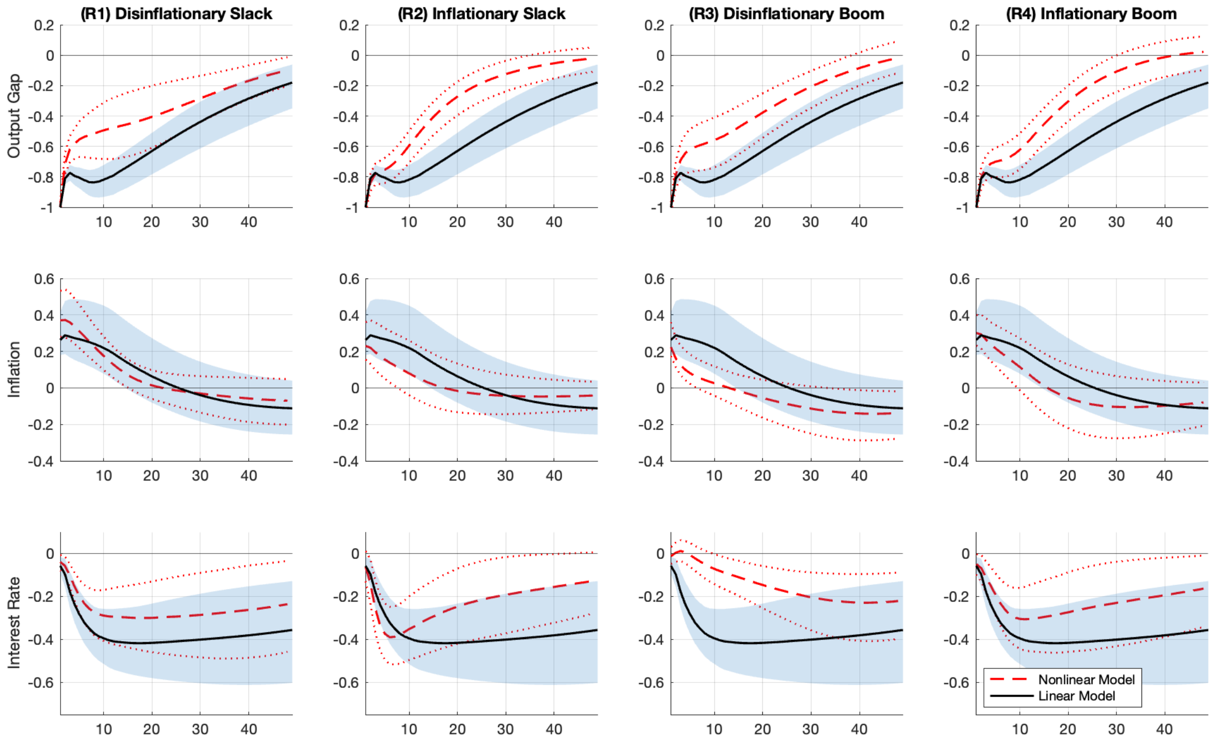


Figure 5: Response to a supply shock

Notes: The nonlinear IRFs (bold and dashed red lines) and linear IRFs (black line and blue shadow) are estimated using the 3-variable macro model featuring A_S (as in matrix 5) and A (as in matrix B.12, respectively. The shock is normalized such that the output gap declines by 1%. See notes in Figure 1 for definitions of each regime.

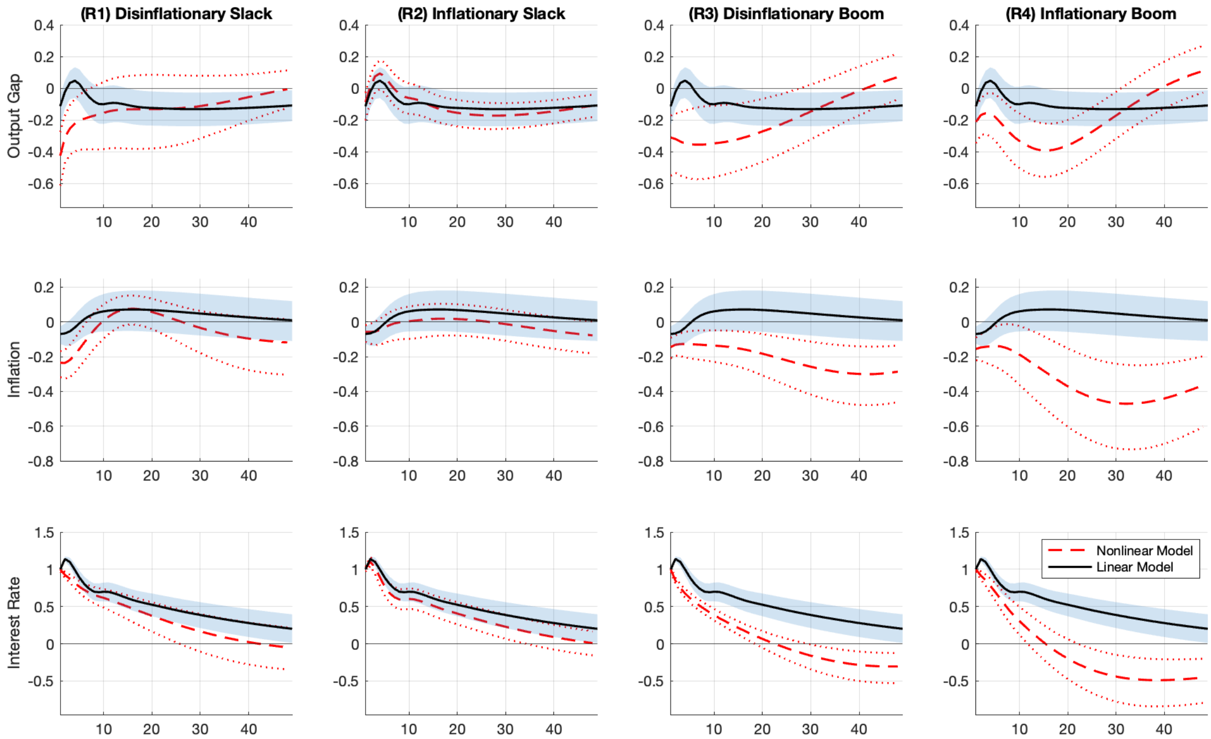
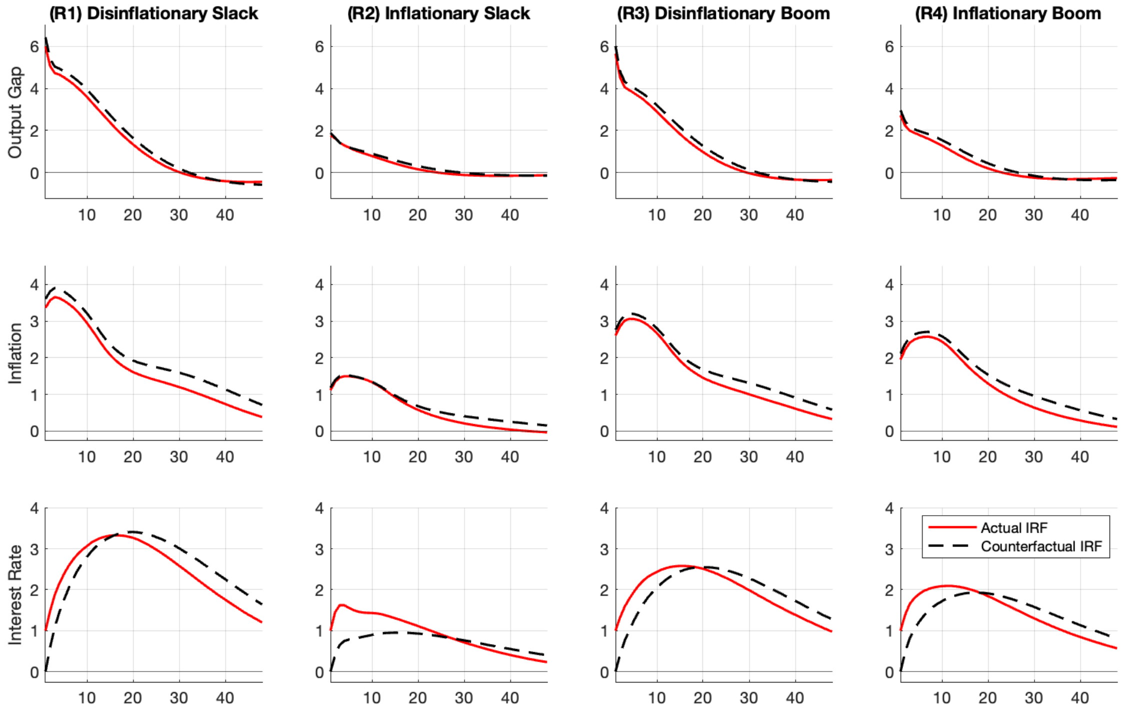
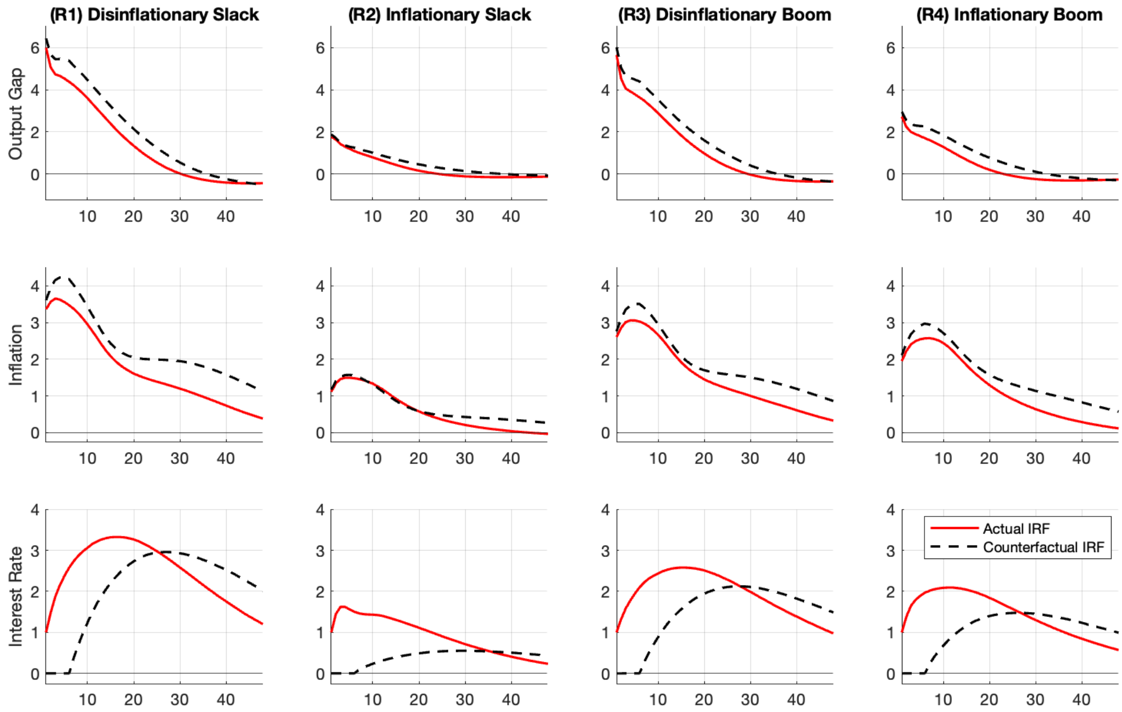


Figure 6: Response to a monetary policy shock

Notes: The nonlinear IRFs (bold and dashed red lines) and linear IRFs (black line and blue shadow) are estimated using the 3-variable macro model featuring A_S (as in matrix 5) and A (as in matrix B.12), respectively. The shock is normalized such that the federal funds rate increases by 1%. See notes in Figure 1 for definitions of each regime.



(a) Response to a demand boom if the interest rate is unchanged at $k = 0$



(b) Response to a demand boom if the interest rate is unchanged for 6 months

Figure 7: Response conditional on a policy rate path after a demand boom

Notes: The nonlinear IRFs (bold red lines) and nonlinear counterfactual IRFs (black dashed lines) are estimated using the 3-variable macro model featuring A_5 (as in matrix 5). The shock is normalized such that the federal funds rate increases by 1%. Counterfactual IRFs are obtained by imposing monetary policy shocks such that the response of the nominal interest rate is zero at different horizons k. See notes in Figure 1 for definitions of each regime.

Appendix

A Data

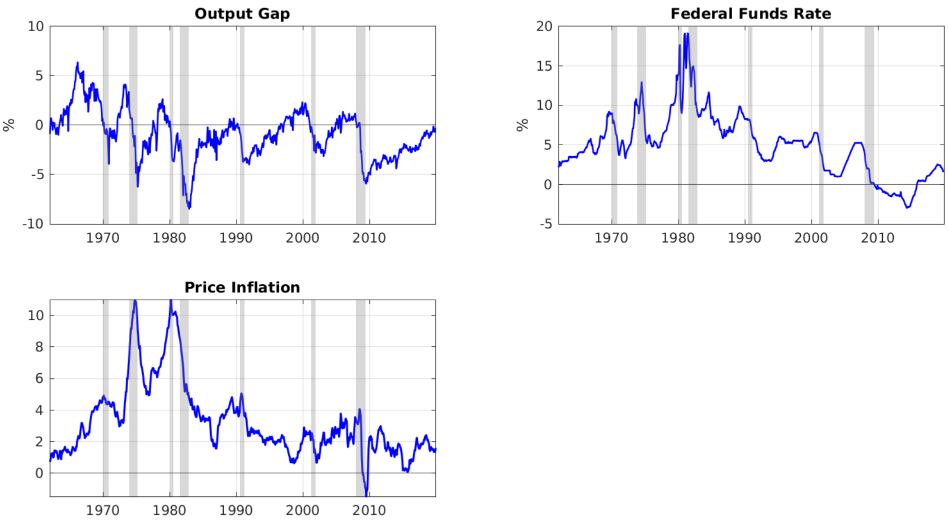


Figure A1: Data series

Notes: The output gap is defined as 100 times the logarithmic difference between real GDP and potential output. To convert real GDP to a monthly frequency, we apply the method proposed by Chow and Lin (1971) using monthly data from industrial production and real retail sales. Real potential output is linearly interpolated to a monthly frequency. Inflation is measured as 100 times the year-on-year logarithmic change in the PCE price index. The nominal interest rate follows the federal funds rate until May 2009, after which it switches to the shadow rate provided by Wu and Xia (2016) for the remainder of the sample period.

B Three-variable macro model

The 3-variable macro model consists of the following aggregate supply equation, aggregate demand equation, and policy rule:

$$\pi_t = \bar{\kappa}\left[\left(\tau^{-1} + \varphi\right) y_t + \xi_t^s\right] + \beta E_t \pi_{t+1} \tag{B.1}$$

$$y_t = E_t y_{t+1} - \tau(r_t - E_t \pi_{t+1}) + \xi_t^d \tag{B.2}$$

$$r_t - \bar{r} = \rho(r_{t-1} - \bar{r}) + (1-\rho)\left(\psi_\pi(\pi_t - \pi^*) + \psi_y y_t\right) + \xi_t^m \tag{B.3}$$

where the dynamic relationship between price inflation π_t, the output gap y_t, and the policy rate r_t depends on structural parameters[11] as well as disturbances to supply (ξ_t^s), demand (ξ_t^d), and monetary policy (ξ_t^m).

Mapping the supply equation to the SVAR framework. Equation (B.1) is solved using the framework presented in Baumeister and Hamilton (2018). We assume that variables evolve according to an AR(1) process, such that $E_t \pi_{t+1} = c^\pi + \phi^\pi \pi_t$ where $|\phi^\pi| < 1$.

Using B.1,

$$\begin{aligned} \pi_t &= \bar{\kappa}\left[\left(\tau^{-1} + \varphi\right) y_t + \xi_t^s\right] + \beta c^\pi + \beta \phi^\pi \pi_t \\ \pi_t &= (1 - \beta\phi^\pi)^{-1} \bar{\kappa}\left[\left(\tau^{-1} + \varphi\right) y_t + \xi_t^s\right] + (1 - \beta\phi^\pi)^{-1} \beta c^\pi. \end{aligned} \tag{B.4}$$

Then,

$$\pi_t = \frac{\beta c^\pi}{1 - \beta\phi^\pi} + \frac{\bar{\kappa}(\tau^{-1} + \varphi)}{1 - \beta\phi^\pi} y_t + \frac{\bar{\kappa}}{1 - \beta\phi^\pi} \xi_t^s. \tag{B.5}$$

[11] In the nonpolicy block of the New Keynesian model, τ represents the intertemporal elasticity of substitution, φ the inverse Frisch elasticity of labor supply, $(1-\theta)$ the firm's probability of resetting its price, β the discount factor, and $\bar{\kappa} = (1-\theta)(1-\theta\beta)\theta^{-1}$ the slope of the inflation equation (Galí, 2015). The interest rate rule is governed by ψ_y and ψ_π, which describe the central bank's long-run response to output and inflation, while $\rho < 1$ reflects the Federal Reserve's preference for gradual adjustments over time.

Mapping the demand equation to the SVAR framework. Similarly to the New Keynesian Phillips curve, we let the output gap follow an AR(1) process of the type $E_t y_{t+1} = c^y + \phi^y y_t$. We can therefore simplify the dynamic demand equation as follows:

$$\begin{gathered} y_t = c^y + \phi^y y_t - \tau(r_t - E_t\pi_{t+1}) + \xi_t^d \\ y_t = (1-\phi^y)^{-1}c^y - (1-\phi^y)^{-1}\tau(r_t - c^\pi - \phi^\pi \pi_t) + (1-\phi^y)^{-1}\xi_t^d. \end{gathered} \tag{B.6}$$

Then,

$$y_t = \frac{c^y + \tau c^\pi}{1-\phi^y} + \frac{\tau\phi^\pi}{1-\phi^y}\pi_t - \frac{\tau}{1-\phi^y} r_t + \frac{1}{1-\phi^y}\xi_t^d. \tag{B.7}$$

Mapping the Taylor rule to the SVAR framework. The Taylor rule can be equivalently re-written as

$$r_t - \bar{r} = \rho(r_{t-1} - \bar{r}) + (1-\rho)\left(\psi_\pi(\pi_t - \pi^*) + \psi_y y_t\right) + \xi_t^m \tag{B.8}$$

$$r_t = (1-\rho)\bar{r} + \rho r_{t-1} + \underbrace{(1-\rho)\,\psi_y}_{\zeta_y} y_t + \underbrace{(1-\rho)\,\psi_\pi}_{\zeta_\pi}(\pi_t - \pi^*) + \xi_t^m. \tag{B.9}$$

In accordance with the literature, we assume that

$$\zeta_y > 0, \zeta_\pi > 0. \tag{B.10}$$

Contemporaneous impact matrix. Using (B.5), (B.7), and (B.9),

$$\begin{aligned} y_t =& \quad k^s + \alpha_\pi \pi_t + [\boldsymbol{b}^s]' \boldsymbol{x}_{t-1} + u_t^s, \\ y_t =& \quad k^d + \beta_\pi \pi_t + \beta_r r_t + \left[\boldsymbol{b}^d\right]' \boldsymbol{x}_{t-1} + u_t^d, \\ r_t =& \quad k^m + \zeta_y y_t + \zeta_\pi \pi_t + [\boldsymbol{b}^m]' \boldsymbol{x}_{t-1} + u_t^m, \end{aligned} \tag{B.11}$$

where $\boldsymbol{x}_{t-1} = \left(\boldsymbol{y}'_{t-1}, \boldsymbol{y}'_{t-2}, \ldots, \boldsymbol{y}'_{t-m}, 1\right)'$, u_t^s denotes the supply shock, u_t^d denotes the demand shock, and u_t^m denotes the monetary policy shock.

The contemporaneous effect matrix $\boldsymbol{A}$ is defined by

$$\boldsymbol{A} = \begin{bmatrix} 1 & -\alpha_{\pi} & 0 \\ 1 & -\beta_{\pi} & -\beta_{r} \\ -\zeta_{y} & -\zeta_{\pi} & 1 \end{bmatrix}, \tag{B.12}$$

which can be equivalently re-stated in terms of structural coefficients,

$$\boldsymbol{A} = \begin{bmatrix} 1 & -\frac{1-\beta\phi^{\pi}}{\bar{\kappa}\left(\tau^{-1}+\varphi\right)} & 0 \\ 1 & -\frac{\tau\phi^{\pi}}{1-\phi^{y}} & +\frac{\tau}{1-\phi^{y}} \\ -\left(1-\rho\right)\psi_{y} & -\left(1-\rho\right)\psi_{\pi} & 1 \end{bmatrix}. \tag{B.13}$$

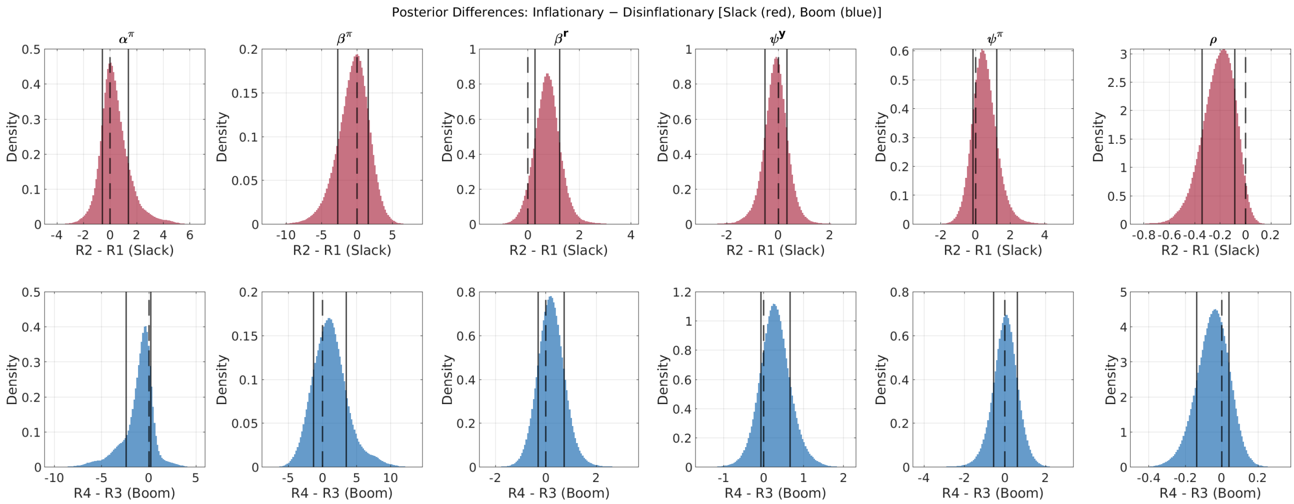


Figure A2: Differences across contemporaneous parameters: inflationary versus disinflationary periods – posterior distributions

Notes: The posterior parameters draws generated by the sampling algorithm are used to examine the posterior distribution of the differences in state-dependent parameters of the impact matrices across the four regimes. For each parameter, we take (N) draws from one regime and subtract the corresponding (N) draws from an alternative regime, thereby constructing an (N × N) posterior draws of the differences. The dashed vertical line is set to zero. The two vertical lines denotes the 68% credible set. Differences between the inflationary boom (R4) and disinflationary boom (R3) regimes are shown in blue, while differences between the inflationary slack (R2) and disinflationary slack (R1) regimes are shown in red. See notes in Figure 1 for definitions of each regime.

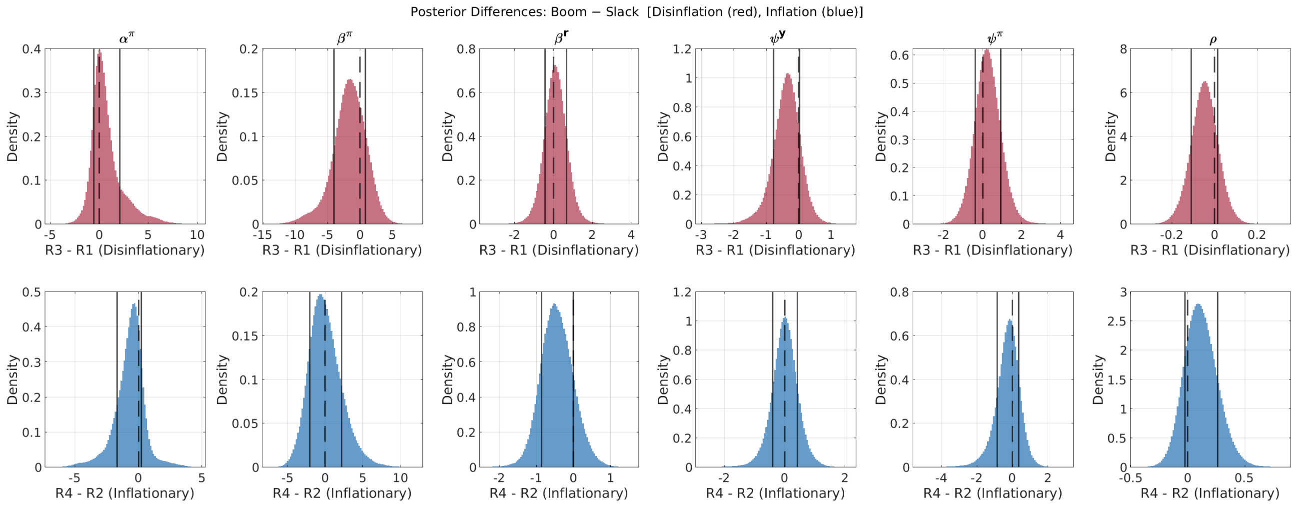


Figure A3: Differences across contemporaneous parameters: boom versus slack periods – posterior distributions

Notes: The posterior parameters draws generated by the sampling algorithm are used to examine the posterior distribution of the differences in state-dependent parameters of the impact matrices across the four regimes. For each parameter, we take (N) draws from one regime and subtract the corresponding (N) draws from an alternative regime, thereby constructing an (N × N) posterior draws of the differences. The dashed vertical line is set to zero. The two vertical lines denotes the 68% credible set. Differences between the inflationary boom (R4) and inflationary slack (R2) regimes are shown in blue, while differences between the disinflationary boom (R3) and disinflationary slack (R1) regimes are shown in red. See notes in Figure 1 for definitions of each regime.

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The views expressed in this paper are those of the authors and do not necessarily reflect those of the European Central Bank or the Eurosystem.

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