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Harmonious Hashing

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Abstract

Hashing-based fast nearest neighbor search technique has attracted great attention in both research and industry areas recently. Many existing hashing approaches encode data with projection-based hash functions and represent each projected dimension by 1-bit. However, the dimensions with high variance hold large energy or information of data but treated equivalently as dimensions with low variance, which leads to a serious information loss. In this paper, we introduce a novel hashing algorithm called Harmonious Hashing which aims at learning hash functions with low information loss. Specifically, we learn a set of optimized projections to preserve the maximum cumulative energy and meet the constraint of equivalent variance on each dimension as much as possible. In this way, we could minimize the information loss after binarization. Despite the extreme simplicity, our method outperforms superiorly to many state-of-the-art hashing methods in large-scale and high-dimensional nearest neighbor search experiments.

1 Introduction

In managing or mining massive databases with millions of data samples and thousands of features, fast approximate nearest neighbor (ANN) search is one of the most fundamental and important techniques in many machine learning and data mining problems, such as retrieval, classification and clustering [Muja and Lowe, 2009; Jégou *et al.*, 2011; Hajebi *et al.*, 2011]. Among various fast ANN search solutions, hashing-based technique has attracted great attention in past years. It has been actively studied, for both research and industry purposes, due to its substantially reduced storage cost and sub-linear query time.

Many existing hashing approaches encode data via projection-based hash functions and generate binary codes by thresholding. They try to preserve the pairwise distance (*e.g.*, Euclidean distance) of data samples on the original space by Hamming distance on the obtained binary feature space. Locality-Sensitive Hashing (LSH) [Indyk and Motwani, 1998; Charikar, 2002], and its kernelized version

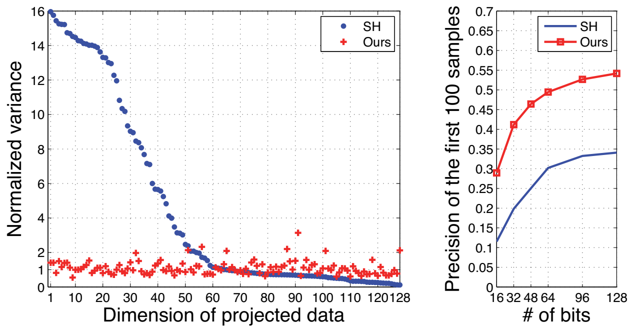


Figure 1: An illustrative example on a database of 1 million samples. Left: normalized variance (divided by the median value) on each projected dimension; Right: nearest neighbor search precision at top-100 samples on different number of bits. Spectral Hashing and our method are compared.

KLSH [Kulis and Grauman, 2009] are well known hashing algorithms which are fundamentally based on random projection. They hold asymptotic theoretical properties but in practice need a long binary code to achieve a satisfactory search accuracy, leading to additional hash table maintenance and look-up cost. In contrast, many recent hashing approaches attempted to *learn* more data-dependent projections to generate *compact* binary codes, *i.e.*, to encode data with short codes while retain comparable discrimination [Weiss *et al.*, 2008; Kulis and Darrell, 2009; Wang *et al.*, 2010a; Kumar and Udupa, 2011; He *et al.*, 2011; Zhang *et al.*, 2010; Yu *et al.*, 2010]. Spectral Hashing (SH) [Weiss *et al.*, 2008] proposed to find a code with balanced partitioned and uncorrelated bits as well as to map similar items to similar codes, but solved the problem with an approximate eigenfunction solution. BRE [Kulis and Darrell, 2009] constructed hash functions by explicitly minimizing the reconstruction error between original distance and Hamming distance. Semi-Supervised Hashing (SSH) [Wang *et al.*, 2010a] learned projections via the help of pairwise constraints. [Kumar and Udupa, 2011] learned hash functions from multiview data.

Though various projection-based hashing algorithms have been proposed, most of them ignore a crucial fact for the projected data – those dimensions with largest variance contain the most energy or information of source data, while some other dimensions are less informative. However, each dimension is encoded into 1-binary bit and the Hamming distance has equivalent weight on each bit, no matter what the original dimension is, *i.e.*, there is a serious information loss for the dimensions with high variance. A few recent researches have considered this unbalanced problem. In [Wang *et al.*, 2010b], the authors firstly claimed that this problem decreased the search performance substantially. To address it, they learned the bit-correlated hash functions sequentially and each new bit was to correct the errors made by previous bits. However such a greedy algorithm would be dependent on the selection of true (or pseudo) labels and how to get the labels is also a problem. Liu *et al*. designed a two-layer hashing function to represent each top dimension (eigenvector) by 2 bits to avoid the influence of low quality dimensions [Liu *et al.*, 2011]. However the performance of the heuristic setting of 2-bit depends on the data distribution and using 2-bit means that we could only use half of the projections, losing the information in the rest.

Different with the previous works, in this paper we propose a novel hashing method called *Harmonious Hashing* (HamH), which aims at learning hash functions with minimum information loss. The main idea is to adjust the projected data for uniform energy distribution on each dimension. Specifically, we learn a set of optimized projections which hold the maximum cumulative energy and meet the constraint of equivalent variance on each dimension as much as possible. In this way, we could preserve the maximum energy and minimize the information loss after binarization. Our method has a *non-iterative* and closed form solution.

More detailedly, our Harmonious Hashing algorithm is derived from the original eigenvector definition of Spectral Hashing but outperforms it a lot in our large-scale and high-dimensional nearest neighbor search experiments. Fig.1 is an illustrative example on GIST1M database with 1 million data samples. We plot the normalized variance (divided by the median value) on each (continuous) projected dimension. For Spectral Hashing (blue dot, sorted), the variance varies in a large range, while for our method, it is more uniform – the normalized variance is distributed around 1. As a result, the performance of our method is much better. Besides Spectral Hashing, our method is comparable or outperforms to many state-of-the-art hashing methods in our experiments.

2 Preliminaries

Our work comes of the eigenvector formulation of Spectral Hashing. In this section, we briefly review and analyze it.

2.1 Spectral Hashing

Given a data set $X = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n]^T \in \mathbb{R}^{n\times m}$ with n samples and each row $\mathbf{x}_i^T \in \mathbb{R}^m$ is a m-dimensional data vector. Let $Y = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n]^T \in \mathbb{R}^{n\times k}$ be the k-bit Hamming embedding (binary codes of length k) for all the samples and $A_{n\times n}$ be the symmetric weight matrix. To seek a qualified code, Spectral Hashing requires each bit to have half number of $+1$ or -1 and the bits to be uncorrelated to each other, as well as maps similar samples to similar binary codes. Its objective function can be written as:

$$\begin{aligned} \min_{Y} \quad & \text{tr}(Y^T L Y) \\ \text{s.t.} \quad & Y(i,j) \in \{-1, 1\} \\ & Y^T \mathbf{1} = \mathbf{0} \\ & Y^T Y = I \end{aligned} \tag{1}$$

where $L = D - A$ is the graph Laplacian [Chung, 1997] and D is a diagonal matrix with $D_{ii} = \sum_j A_{ij}$ and $\mathbf{1}$ ($\mathbf{0}$) is a column vector with all ones (zeros).

It was stated that even for a single bit ($k = 1$), solving problem (1) is equivalent to balanced graph partitioning and is NP hard [Weiss *et al.*, 2008]. For k-bits the problem is even harder, but by removing the binary constraint $Y(i,j) \in \{-1, 1\}$ to make Y continuous, the optimization becomes a well-studied problem whose solutions are the k eigenvectors of L with minimal eigenvalues. Then simply thresholding these eigenvectors to generate balanced binary codes. Though this approach is very intuitive, it can only compute the codes of items in the database (training data) but not for an out-of-sample, *i.e.*, a new query sample. Instead of solving the eigenvector solution, Spectral Hashing adopted an approximate eigenfunction solution with the assumption of uniform data distribution. The eigenfunction solution is not the focus of our work, thus we skip it here.

Beside those very recent related work in Section 1, we also find some interesting clues in Spectral Hashing (SH) [Weiss *et al.*, 2008]. The authors said 'using PCA to align the axes, and assuming an uniform distribution on each axis works surprisingly well...'. Although this discovery is very interesting and useful, the authors failed to offer further analysis to it but just make an eigenfunction extension for learning out-of-sample codes. Many later methods were inspired by Spectral Hashing's original eigenvector formulation or the eigenfunction extension, but none of them have analyzed this issue theoretically. In the analysis of our model (Section 3.4), We find that the idea of uniform (energy) distribution is *hidden* in the original definition of Spectral Hashing.

3 Harmonious Hashing

3.1 A Naive Extension of Spectral Hashing

We propose a naive out-of-sample extension of Spectral Hashing from its relaxed eigenvector formulation. This naive approach is the origin of our Harmonious Hashing algorithm. Without loss of generality, for the rest of this paper, we assume that the data samples are zero-centered, *i.e.*, $\sum_{i=1}^{n} \mathbf{x}_i = \mathbf{0}$, and the affinity matrix is calculated by its symmetric normalization, then we have:

$$\tilde{A} = D^{-1/2} A D^{-1/2} \tag{2}$$

$$\tilde{L} = I - \tilde{A} \tag{3}$$

where $\tilde{L}$ is the normalized graph Laplacian. For out-of-sample extension, let the k-dimensional embedding Y be calculated by k linear projections from original data X, *i.e.*, $Y = XW$, $W \in \mathbb{R}^{m\times k}$ and each column of W is a projection vector. Substituting the linear projection and the normalized graph Laplacian into the relaxed continuous version of problem (1) and replacing the orthogonal constraint, we could get a naive out-of-sample extension of Spectral Hashing as:

$$\begin{aligned} \max_{W} \quad & \mathrm{tr}(W^T X^T \tilde{A} X W) \\ \text{s.t.} \quad & W^T X^T \mathbf{1} = \mathbf{0} \\ & W^T W = I. \end{aligned} \tag{4}$$

The constraint $W^T X \mathbf{1} = \mathbf{0}$ always holds since the data is zero-centered. Thus the solutions of problem (4) are the k eigenvectors of the matrix $X^T \tilde{A} X$ with maximal eigenvalues. By this way, we could compute the binary code for both training data and query data in the same manner: projecting the data by $Y = XW$ and thresholding at zero for each bit. As this method is a naive extension of Spectral Hashing, we denote it by **nSH** for short.

However, there remains a critical problem for both SH and nSH – traditional constructing method for the affinity matrix A is costly for a large database, *e.g.*, building a kNN graph has a complexity of $O(n^2 \log k)$. Though it is computed offline, for millions of samples, the computation is too slow. Thus for the nSH algorithm, we adopt to build a landmark-based graph, or an anchor graph which has been proved to be efficient and effective in many recent work [Zhang *et al.*, 2009; Liu *et al.*, 2010; Xu *et al.*, 2011; Lin *et al.*, 2012; Lu *et al.*, 2012]. Let $U = \{\mathbf{u}_1, \ldots, \mathbf{u}_d\} \subset \mathbb{R}^m$ denote a set of d landmarks sampled from the database. To build the anchor graph based on the landmarks, we essentially find the sparse weight matrix $Z \in \mathbb{R}^{n \times d}$ so that each data sample can be approximated by its nearby landmarks:

$$\mathbf{x}_i \approx \sum_{j \in \mathcal{N}_{\langle i \rangle}} \mathbf{u}_j z_{ij}, \tag{5}$$

where $\mathcal{N}_{\langle i \rangle}$ denotes the index set of s nearest landmarks of $\mathbf{x}_i$. We compute z_{ij} by kernel regression method:

$$z_{ij} = \frac{K(\mathbf{x}_i, \mathbf{u}_j)}{\sum_{j' \in \mathcal{N}_{\langle i \rangle}} K(\mathbf{x}_i, \mathbf{u}_{j'})}, \tag{6}$$

where K is a pre-given kernel function [Xu *et al.*, 2011; Lin *et al.*, 2012]. Solving Z has a complexity of $O(nd \log s)$. Then the affinity matrix is calculated by a low-rank formulation as $A = ZZ^T$ and its normalization is

$$\tilde{A} = D^{-1/2} Z Z^T D^{-1/2} = HH^T, \tag{7}$$

where $H = D^{-1/2} Z$. As a result, we could re-write the objective function of nSH as:

$$\begin{aligned} \max_{W} \quad & \mathrm{tr}(W^T X^T H H^T X W) \\ \text{s.t.} \quad & W^T W = I. \end{aligned} \tag{8}$$

Through in-depth analysis of problem (8), we receive the main idea of our method. We describe it in the next section.

3.2 Observation and Motivation

We first introduce the definition of cumulative energy:

Definition Let $C \in \mathbb{R}^{m \times m}$ be the covariance matrix of any data set, its full eigen-decomposition can be written as $C = U \Lambda U^T$ where Λ is the diagonal matrix of sorted eigenvalues of C, *i.e.*, $\Lambda_{ii} = \lambda_i$ and $\lambda_1 \geq \lambda_2 \geq, \ldots, \geq \lambda_m$. The eigenvalues represent the distribution of the source data's energy among each of the eigenvectors. Then the **cumulative energy** content g for the k-th eigenvector is the sum of the energy content across all of the first k eigenvalues:

$$g_k(C) = \sum_{i=1}^{k} |\Lambda_{ii}| \tag{9}$$

Now going back to problem (8), let $\tilde{C} = X^T H H^T X$, which plays a very similar role as the covariance matrix but integrates more graph-based structure information. We name it as *graph covariance matrix*. The solution of problem (8) for k-bit embedding is the top-k eigenvectors of $\tilde{C}$ with maximal eigenvalues, forming the matrix $W^* \in \mathbb{R}^{m \times k}$. $\tilde{C}$ is a positive semi-definite matrix, *i.e.*, all of its eigenvalues are nonnegative, then the object of problem (8) is formulated as

$$\mathrm{tr}(W^{*T} \tilde{C} W^*) = \mathrm{tr}(\tilde{\Lambda}) = \sum_{i=1}^{k} \tilde{\Lambda}_{ii} = g_k(\tilde{C}), \tag{10}$$

which indicates that to solve the problem of (8), we essentially find a set of k eigenvectors holding the maximum cumulative energy. An intuitive next step would be projecting the source data by W^* , *i.e.*, $Y = XW^*$, and then thresholding at zero (or median) to obtain the binary codes, just as most traditional hashing methods did.

However this approach ignores a crucial fact of the projected data Y – the top dimensions hold the most energy of source data while the rest dimensions keep much less energy, since the eigenvalue decreases dramatically for many real data. From another point, we could observe the variance of the projected data: the top dimensions have large variances while the rest are much smaller. The variance of one dimension reflects its energy or information of the data. However, each dimension is represented by 1-binary bit equivalently. As a result, some bits are energy overloaded and some are less informative. Hamming distance has equivalent weight on each bit, *i.e.*, general projected data does not match the Hamming distance metric well.

To solve the problem above, we propose a novel approach which preserves the maximum cumulative energy, and at the same time allocates the energy into each projection uniformly by restricting the variance of each projected dimension equally. We adopt a *non-iterative* algorithm to solve it.

3.3 Learning Algorithm

It is observed that, if W is an optimal solution of problem (8), then so is $\tilde{W} = WE$ with $E \in \mathbb{R}^{k \times k}$ an arbitrary orthogonal matrix ($E^T E = EE^T = I$), since

$$\mathrm{tr}(E^T W^T \tilde{C} W E) = \mathrm{tr}(W^T \tilde{C} W) = g_k(\tilde{C}). \tag{11}$$

That is to say, there is an *infinite* set of (continuous) projected data in the form of $Y = XWE$, that can preserve the maximum cumulative energy. Our objective is to find some Y in this set with equivalent variance on each dimension to guarantee uniform energy distribution. We design a loss function as:

$$\begin{aligned} \mathcal{Q}(Y,E) \quad &= \|Y - XWE\|_F^2 \\ \text{s.t.} \quad & Y^TY = Diag(\sigma\mathbf{1}) \\ & E^TE = I, \end{aligned} \tag{12}$$

where $Diag(\sigma\mathbf{1})$ is a diagonal matrix with each diagonal element equal to the scalar σ and $\|\cdot\|_F$ is the Frobenius norm of a matrix. σ is a specific variance value but we just denote it by a symbol now and discuss its effect later in Section 3.4.

By observation, we find that given any arbitrary orthogonal matrix E_0 , we could get an optimal Y fitting the object by solving the following single problem:

$$\begin{aligned} \min_{Y} \quad & \|Y - XWE_0\|_F^2 \\ \text{s.t.} \quad & Y^TY = Diag(\sigma\mathbf{1}). \end{aligned} \tag{13}$$

That is to say, there is still an *infinite* candidate set of Y holding the maximum cumulative energy and equivalent variance. But in other words, any Y in this set is qualified for our object, thus we just pick one of them. Solving the single problem (13), we should use the following theorem.

Theorem 3.1 *Given any real valued matrix $Y \in \mathbb{R}^{n\times k}$ and $V \in \mathbb{R}^{n\times k}$ ($n \geq k$), and any real valued vector $\mathbf{u} \in \mathbb{R}^k \geq 0$, suppose the singular value decomposition of V is $V = P\Delta Q$, then the optimal solution to the following problem is $Y^* = PDiag(\sqrt{\mathbf{u}})Q$, where $Diag(\cdot)$ is a diagonal matrix.*

$$\begin{aligned} \min_{Y} \quad & \|Y - V\|_F^2 \\ \textit{s.t.} \quad & Y^TY = Diag(\mathbf{u}). \end{aligned} \tag{14}$$

The proof of this theorem is presented in the Appendix. Let $V = XWE_0$ (any arbitrary orthogonal matrix E_0), the optimal solution of problem (13) is $Y^* = \sqrt{\sigma}PQ$.

With the fixed Y^*, we want to find an unique optimal E by minimizing the reconstruction error of problem (12), *i.e.*, to solve a second single problem as:

$$\begin{aligned} \min_{E} \quad & \|Y^* - XWE\|_F^2 \\ \text{s.t.} \quad & E^TE = I, \end{aligned} \tag{15}$$

whose optimal solution is $E^* = GS$, where G and S are the left and right singular vectors of the $k \times k$ matrix $W^TX^TY^*$, *i.e.*, $W^TX^TY^* = G\Sigma S$, which has been investigated in some previous works [Yu and Shi, 2003; Chen *et al.*, 2011; Gong and Lazebnik, 2011]

NOTE: since picking any Y in the infinite set meeting our object has almost the same effect to uniformly distribute the data energy along each dimension, our optimization algorithm for problem (12) is **non-iterative**. It stops when we solve E^* of problem (15) for the first time, which guarantees the light weight of the optimization.

By now, we have obtained a k-bit data embedding as $Y = XWE^*$ for the in-sample data X (training data), where W is the top-k eigenvectors of $\tilde{C}$ and E^* is the solution of problem (15), then we cut Y at zero to get the binary codes. For any out-of-sample query data x_q, we use the same strategy to generate its binary code b_q, *i.e.*,

$$b_q = (\text{sgn}(x_qWE^*) + 1)/2. \tag{16}$$

3.4 Analysis

Let's consider the value of variance σ for each dimension of Y, which represents a corresponding degree of energy. A natural conclusion is that: when the maximum energy of all the dimensions is fixed, asking each dimension's energy to be equal to each other has the same effect to constrain each energy to be a specific value. This can be observed from our optimization algorithm above. As $Y^* = \sqrt{\sigma}PQ$, the value of σ only does a scaling to Y^*, as well as to the value of Σ but not to G and S in the SVD of $W^TX^TY^* = G\Sigma S$. As a result, E^* remains the same no matter what the σ is. We could just set σ to 1 for simplicity, *i.e.*, $Y^TY = I$. Surprisingly, we find this constraint appeared in the relaxed eigenvector definition of Spectral Hashing – the authors said it is to enforce the bits to be uncorrelated, but a *hidden* effect is to make the variance on each dimension equally, as well as the energy, which conforms with the idea of our Harmonious Hashing model. However the eigenfunction solution of Spectral Hashing lost this constraint.

Two very recent hashing algorithms Iterative Quantization (ITQ) [Gong and Lazebnik, 2011] and Isotropic Hashing (IsoH) [Kong and Li, 2012] have close relationship to our model. ITQ balances the variance on each dimension by iteratively minimizing the quantization error, but its local optimum cannot guarantee equivalent variance. IsoH iteratively minimizes the reconstruction error of the the covariance matrix and a diagonal matrix, but their optimization on the small covariance matrix seems to be too restricted – the algorithm is unstable in large-scale and high-dimensional experiments. Moreover, both ITQ and IsoH focus on PCA-projections, while we derive our algorithm from Spectral Hashing, which utilizes more data structure information.

4 Experimental Results

In this section, we evaluate the performance of our method for nearest neighbor search task on two real world large-scale and high-dimensional databases **GIST1M** and **ImageNet**, and compare with many state-of-the-art hashing approaches. Our experiments are implemented on a computer with double 2.0 GHz CPU and 64GB RAM.

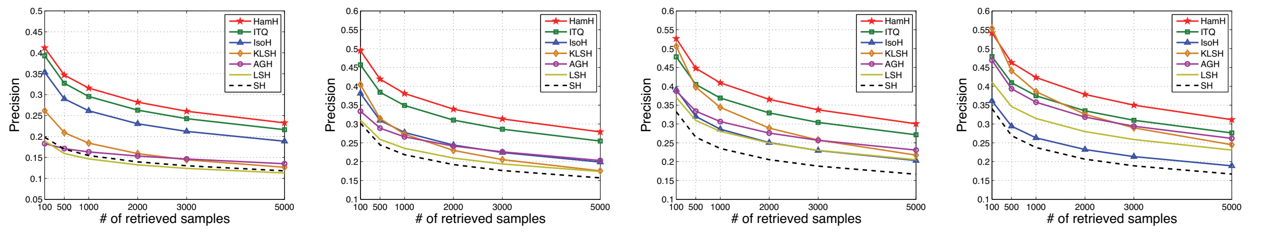
4.1 Date Statistics and Experimental Setup

- GIST1M database has 1 million GIST descriptors from the web[1]. It is a well known database for the task of approximate nearest neighbor search. Each GIST descriptor (one sample) is a *dense* vector of 960 dimensions.
- ImageNet is an image database organized according to the WordNet nouns hierarchy, in which each node of the hierarchy is depicted by hundreds and thousands of images[2]. We downloaded about 1.2 million images' BoW representations from 1,000 nodes. A visual vocabulary of 1,000 visual words is adopted, *i.e.*, each image is represented by a 1,000-length *sparse* vector.

For both of the two databases, we randomly select 1,000 data samples as out-of-sample test queries and the rest samples

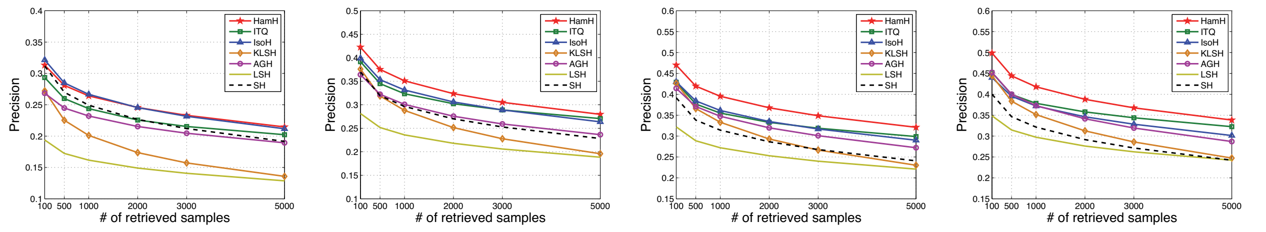
[1]http://corpus-texmex.irisa.fr/

[2]http://www.image-net.org/index



(a) 32 bits (b) 64 bits (c) 96 bits (d) 128 bits

Figure 2: Precision at different number of retrieved samples on GIST1M database with 32, 64, 96 and 128 bits respectively.



(a) 32 bits (b) 64 bits (c) 96 bits (d) 128 bits

Figure 3: Precision at different number of retrieved samples on ImageNet database with 32, 64, 96 and 128 bits respectively.

Table 1: Basic statistics of the two databases.

	GIST1M	ImageNet
# of samples	1,000,000	1,261,406
# of dimensions	960	1,000
is sparse	no	yes

are treated as training data. Some basic statistics of the two databases are listed in Table 1.

To evaluate the performance of nearest neighbor search, we should first obtain the groundtruth neighbors. Following the criterion used in many previous works [Weiss *et al.*, 2008; Wang *et al.*, 2010a; 2010b], the groundtruth neighbors are obtained by brute force search with Euclidean distance. A data sample is considered to be a true neighbor if it lies in the top-1 percent samples closest to the query.

To run our model, we build a landmark-based graph. We should determine the number of landmarks, the local size s of nearby landmarks and the kernel function. However from experiments, we find that the model HamH is not sensitive to the parameters (since all the graph information is embedded in a small graph covariance matrix $\tilde{C}$). In all of our experiments below, we fix all the parameters the same as [Xu *et al.*, 2011], *i.e.*, we use the quadratic kernel and set $s = 5$. We select landmarks *randomly* form the training data and the number is fixed to be twice the number of binary bits, *i.e.*, for a 32-bit code, we use 64 landmarks. The random selection procedure and fixed number of landmarks can guarantee the efficiency as well as the robustness of our model.

4.2 Compared Approaches

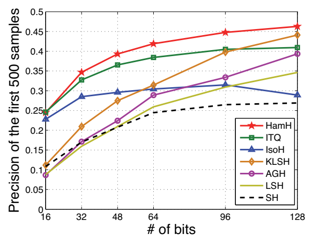
We compare the proposed Harmonious Hashing (**HamH**) algorithm with the following state-of-the-art hashing methods:

- **SH**: Spectral Hashing [Weiss *et al.*, 2008].
- **LSH***: Locality-Sensitive Hashing [Charikar, 2002]. We generated the random projections via a $(0, 1)$ normal distribution.
- **KLSH**: Kernelized Locality-Sensitive Hashing [Kulis and Grauman, 2009], constructing random projections using the kernel function and a set of examples.
- **AGH**: Anchor Graph Hashing [Liu *et al.*, 2011] with two-layer hash functions generates r bits actually use the $r/2$ lower eigenvectors twice.
- **ITQ**: Iterative Quantization [Gong and Lazebnik, 2011], minimizing the quantization loss by rotating the PCA-projected data.
- **IsoH***: Isotropic Hashing [Kong and Li, 2012], learning projection functions with isotropic variances for PCA-projected data. We implemented the Lift and Projection algorithm to solve the optimization problem.

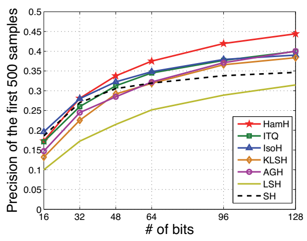
Among these approaches, LSH and KLSH are random projection methods and the others are learning-based methods. ITQ and IsoH are two very recent proposed hashing methods with close relationship to our method. The methods with $\star$ are implemented by ourselves strictly according to their papers and the others are provided by the original authors.

4.3 Results

We first compute the *precision* (percent of true neighbors) under the Hamming ranking evaluation, *i.e.*, the Hamming distance of the query and each database sample is calculated and sorted. The precision values at different number of retrieved samples are recorded in Fig.2 and Fig.3. We vary the length of binary codes from 32 bits to 128 bits to see the performance of all the methods on compact codes and relatively long codes. Despite the simplicity of our method, it



(a) GIST1M database



(b) ImageNet database

Figure 4: Precision at first 500 samples on different number of bits on (a) GIST1M and (b) ImageNet databases.

Table 2: Training time (in seconds) of all the methods.

	GIST1M			ImageNet		
	32 bits	64 bits	128 bits	32 bits	64 bits	128 bits
HamH	201.05	208.72	228.83	313.62	331.10	371.02
ITQ	128.76	205.47	377.43	201.47	327.19	573.49
IsoH	34.17	35.27	40.77	59.05	60.53	65.60
KLSH	104.09	108.14	109.71	133.89	136.98	136.08
AGH	267.80	313.53	492.07	352.94	409.90	630.26
LSH	2.60	3.93	5.52	3.77	6.17	9.67
SpH	137.06	242.08	662.17	194.28	339.65	950.24

performs comparably to or outperforms the other methods on each database and each code length, proving its effectiveness and stability. As discussed in section 1, random projection algorithms play well on a long code – KLSH achieves a high precision on 96 or 128 bits but performs poorly on a short code. Both GIST1M and ImageNet are large-scale and high-dimensional, but GIST1M has dense features and ImageNet has sparse (BoW) features. Compare Fig.2 to Fig.3, some methods performs better on the dense set while some others performs better on the sparse set, *i.e.*, they are sensitive to the data characteristic. However, our method HamH focusing on the data energy distribution is more robust – it performs well on both sets. On the ImageNet database, IsoH performs almost the second best on each code length. However, on the GIST1M database, it seems to be unstable: IsoH performs well on a short code but drops its precision on a long code.

In Fig.4, we plot the precision of the first 500 retrieved samples on different length of binary bits. It can be seen that, the algorithms perform differently on the dense and sparse sets, while our method performs well on both sets.

Finally, we record the training time of all the methods in Table 2. Our method HamH is at an intermediate level of training cost. However, considering the large scale of the databases and the training process running totally off-line, the time of each method is acceptable. On the query process, they have similar steps (projection and binarization), both very efficient, so we skip the comparison.

5 Conclusion

We have designed an extremely simple but effective binary code generating method Harmonious Hashing, which adjusts the projected dimensions via uniform energy distribution. With a non-iterative and efficient solution, our method outperforms or is comparable to many state-of-the art methods for nearest neighbor search experiments on large-scale and high-dimensional databases. However, we find that with a small code (32 or 64 bits), our method (as well as many other learning-based algorithms) has arrived at a competitive level of search accuracy, but improvement is limited when more bits are adopted. From the view of data energy, the most energy is kept in the top dimensions, so the later ones do not help much. But for random projection methods, the situation is totally opposite. How to combine learning projections and random projections is an interesting future work.

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A Appendix

A.1 Proof of Theorem 3.1

To solve the problem (14), we introduce the Lagrange function (with multipliers saved in $\Lambda \in \mathbb{R}^{k\times k}$) defined as:

$$L(Y,\Lambda) = \|Y - V\|_F^2 + \text{tr}(\Lambda(Y^TY - Diag(\mathbf{u}))). \quad (17)$$

Suppose Y^* is the optimal solution, according to the KKT conditions, we have

$$Stationarity: \quad \frac{\partial L}{\partial Y_d}|_{Y=Y^*} = 0, \quad d = 1,\ldots,k \quad (17a)$$

$$Primal\ Feasibility: \quad Y^{*T}Y^* = Diag(\mathbf{u}) \quad (17b)$$

By Eq.(17a), we get $V = Y^*\Theta$, where $\Theta = \Lambda + I$ is symmetric since $Y^TY - Diag(\mathbf{u})$ is symmetric. When $k \le n$, assume Y^* and V are full column rank, then Θ is invertible. Let the SVD of V be $V = P\Delta Q$, then $Y^* = P\Delta Q\Theta^{-1}$. Substituting V and Y^* into the original object function, the resulting objective is to maximize

$$\mathcal{O} = \text{tr}(Q\Theta^{-1}Q^T\Delta^2) = \text{tr}(H\Delta^2) = \sum_{d=1}^{k} h_{dd}\delta_{dd}^2, \quad (18)$$

where $H = Q\Theta^{-1}Q^T$. According to the Primal Feasibility:

$$Diag(\mathbf{u}) = Y^{*T}Y^* = QY^{*T}Y^*Q^T = H^T\Delta^2H. \quad (19)$$

That is to say, for any d, $\sum_{j=1}^k h_{dj}^2\delta_{jj}^2 = \mathbf{u}_d$ and then $h_{dd}^2\delta_{dd}^2 \le \sum_{j=1}^k h_{dj}^2\delta_{jj}^2 = \mathbf{u}_d$, thus

$$h_{dd}\delta_{dd} \le \sqrt{\mathbf{u}_d}. \quad (20)$$

As a result, the object value of Eq.(18) has a upper bound of

$$\mathcal{O} = \sum_{d=1}^{k} h_{dd}\delta_{dd}^2 \le \sum_{d=1}^{k} \sqrt{\mathbf{u}_d}\delta_{dd}. \quad (21)$$

To achieve the maximum, for any d, the equality of Eq.(20) holds, *i.e.*, $h_{dd} = \sqrt{\mathbf{u}_d}\delta_{dd}^{-1}$ and $h_{dj}|_{j\neq d} = 0$, or

$$H = Diag(\sqrt{\mathbf{u}})\Delta^{-1}. \quad (22)$$

Since $H = Q\Theta^{-1}Q^T$, we could get the optimal solution as

$$Y^* = P\Delta Q\Theta^{-1} = P\Delta HQ = PDiag(\sqrt{\mathbf{u}})Q. \quad (23)$$

This theorem can be seen as a generalized version of theorem 2.1 in [Chen *et al.*, 2011].

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