The Macroeconomics of Irreversibility[*]

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Abstract

We study aggregate capital dynamics in an investment model with idiosyncratic productivity shocks, fixed capital adjustment costs, and irreversibility driven by a wedge between capital purchase and resale prices. We derive sufficient statistics capturing the role of investment frictions on aggregate capital fluctuations, measure these statistics with investment microdata, and exploit them to discipline the capital price wedge. Irreversibility doubles the persistence of capital fluctuations and is crucial for reconciling micro-level investment behavior with macroeconomic propagation.

JEL: D30, D80, E20, E30

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1 Introduction

Capital irreversibility, stemming from a wedge between buying and selling prices of capital, is a pervasive friction in firms' investment decisions. This wedge reflects factors like asset specificity (Ramey and Shapiro, 2001; Lanteri, 2018; Kermani and Ma, 2023), adverse selection and asymmetric information (Akerlof, 1970; Kurlat, 2013; Bigio, 2015; Li and Whited, 2015), intermediary fees (Nosal and Rocheteau, 2011), and obsolescence costs (Caunedo and Keller, 2020).

Exposed to a price wedge, firms adopt cautious investment strategies. During periods of high productivity, firms do not fully scale up their capital stock, fearing future adverse shocks that would force them to sell at a discount and instead make a sequence of gradual purchases. Conversely, during low productivity, firms avoid large sell-offs to limit capital losses and thus sell capital sequentially. The step-by-step nature of the investment process introduces path dependence: Positive investments beget future positive investments; negative investments beget future negative investments. At the aggregate level, investment becomes less responsive to productivity shocks, and business cycle fluctuations persist longer (Pindyck, 1991; Bertola and Caballero, 1994; Abel and Eberly, 1996).

We propose a new perspective for analyzing irreversibility's role in shaping macroeconomic dynamics. Our innovation is leveraging the Cumulative Impulse Response (CIR)—a measure of how firms' capital-productivity ratios respond to aggregate shocks—as both a diagnostic tool and a calibration target. Unlike traditional approaches, which focus on matching steady-state moments of investment rates (Cooper and Haltiwanger, 2006) and treat the CIR as an outcome, we use it as a lens to study irreversibility and as an input for disciplining investment frictions. Applying this framework to Chilean manufacturing data, we estimate a price wedge of 12%, which is necessary to simultaneously match both the CIR and the distribution of investment rates. The calibrated CIR is approximately 2, meaning a 1% aggregate productivity shock generates a 2% cumulative deviation in average capital-productivity ratios. Without irreversibility, the CIR collapses to 1, underscoring the wedge's role in amplifying the persistence of aggregate fluctuations.

To establish this framework, we begin with a parsimonious investment model with idiosyncratic productivity shocks, fixed capital adjustment costs, and a capital price wedge. The optimal investment policy consists of an inaction region and distinct reset points—levels to which firms reset their capital-productivity ratio after adjustments. This ratio would remain constant in a frictionless world, as capital perfectly tracks productivity. Fixed adjustment costs allow the ratio to drift during inaction, but firms reset to the same optimal ratio upon adjustment, erasing the history of shocks. In contrast, a price wedge creates two reset ratios: one for upsizing capital at the buying price and another for downsizing at the selling price. This dual-reset structure introduces persistent heterogeneity, as firms' timing and direction of their future investments differ based on their previous adjustment. We encode this heterogeneity in a tractable way by conditioning behavior on the previous reset point and characterizing a Markov chain across reset points.

Using this theoretical foundation, we define the CIR as the cumulative deviation of average capital-productivity ratios following an aggregate productivity shock. Two challenges arise in its characterization. The first challenge revolves around path dependence, as tracking firms until their first adjustment after an aggregate shock is insufficient. We resolve this by discovering a recursive formulation that splits the CIR into deviations before and after the first adjustment. With restored tractability, we characterize the CIR through three sufficient statistics: (i) the dispersion of capital-productivity ratios, (ii) the covariance of capital-productivity ratios and their age (i.e., the time elapsed since the last adjustment), and (iii) the covariance of the changes in capital-productivity ratios and the expected cumulative deviations when completing the first inaction spell.

The power of sufficient statistics lies in that responsiveness to idiosyncratic shocks, encoded in steady-state moments, informs about responsiveness to aggregate shocks, encoded in the CIR. The first two sufficient statistics describe steady-state behavior during periods of inaction and up to the first adjustment after an aggregate shock, extending insights from Baley and Blanco (2021). The dispersion of capital-productivity ratios reflects how far firms allow their capital to drift from its ideal level; the covariance of these ratios with their age captures how misalignment worsens the longer firms delay action. Irreversibility increases both statistics as firms tolerate larger misalignments and exhibit more significant delays when selling capital. The third statistic, unique to irreversibility, reflects whether firms ultimately choose to buy or sell and how that terminal behavior changes in response to aggregate shocks. For instance, after a negative shock to aggregate productivity, the mass of downsizing firms increases, and due to their sequential selling strategy, the shock's aggregate effects are significantly prolonged.

A second challenge is that capital-productivity distributions are unobservable, making it difficult to compute the CIR directly. To address this, we derive mappings from observable firm actions—such as the size and frequency of adjustments, conditioned on the direction of the previous adjustment—to the reset points and the CIR's sufficient statistics. We then extend the model to a generalized hazard framework, inspired by Caballero and Engel (1999) and Lippi and Oskolkov (2023), incorporating stochastic and asymmetric fixed costs for purchases and sales. This extension enhances the model's ability to replicate the observed investment rate distribution while preserving consistency with the CIR. Crucially, the generalized hazard with irreversibility captures all sufficient statistics, whereas without irreversibility, it fails to generate the second and third.

Our contributions go beyond investment. The CIR, its sufficient statistics, and its measurement in microdata provide a versatile framework for studying lumpy adjustments and path dependence in contexts like inventory management, durable goods consumption, and labor markets with sticky wages. We build on methodologies by Alvarez, Le Bihan and Lippi (2016) and Baley and Blanco (2021), incorporating path dependence (i.e., reinjection). By linking micro-level frictions to macroeconomic fluctuations, our framework provides a foundation for analyzing and quantifying irreversibility's impact on aggregate dynamics.

2 A Parsimonious Investment Model

We study a parsimonious investment model with idiosyncratic productivity shocks, a fixed capital adjustment cost, a wedge between capital purchase and resale prices, and a constant interest rate. We use this model to analyze how irreversibility shapes firms' optimal investment, derive sufficient statistics for aggregate capital dynamics (Section 3), and construct mappings from microdata to parameters and macro outcomes (Section 4). In the quantitative application (Section 5), we consider an extension with a general adjustment cost structure that matches the entire distribution of investment rates in the data that the baseline model misses.

2.1 Firm Investment Problem

Time is continuous, extends forever, and is denoted by s. The future is discounted at a rate of $r > 0$. For any stochastic process x_s, we use the notation $x_{s^-} \equiv \lim_{z \uparrow s} x_z$ to denote the limit from the left. We first present the problem of an individual firm and then consider a continuum of ex-ante identical firms to characterize the aggregate behavior of the economy.

Technology and shocks The firm produces output y_s using capital k_s according to a production function with decreasing returns to scale

$$y_s = u_s^{1-\alpha} k_s^{\alpha}, \quad \alpha < 1. \tag{1}$$

Idiosyncratic productivity u_s follows a geometric Brownian motion with drift $\mu > 0$ and volatility $\sigma > 0$,

$$\log u_s = \log u_0 + \mu s + \sigma W_s, \qquad W_s \sim Wiener. \tag{2}$$

The capital stock, if uncontrolled, depreciates at a constant rate $\xi > 0$.

Fixed adjustment cost The firm controls its capital stock by buying and selling investment goods. For every active investment, $i_s \equiv \Delta k_s = k_s - k_{s^-} \neq 0$, the firm must pay a fixed cost θ_s proportional to its productivity and measured in output units:

$$\theta_s = \theta u_s, \tag{3}$$

where $\theta > 0$ is a deterministic fixed cost equal for positive and negative investments. The fixed cost rationalizes establishment-level data on infrequent and sizeable investment spikes (Doms and Dunne, 1998) and reflects disruptions from installing or uninstalling capital, learning, time-to-build, and other factors independent of the investment size (Cooper and Haltiwanger, 2006).

Price wedge Capital is bought at price p and sold at a discount $p(1-\omega)$. We call ω the price wedge or $1-\omega$ the recovery rate. To simplify notation, we define the pricing function

$$p\left(\Delta k_s\right) \equiv p\mathbb{1}_{\{\Delta k_s>0\}}+p(1-\omega)\mathbb{1}_{\{\Delta k_s<0\}}. \tag{4}$$

To the extent that ω is a linear asymmetric cost, the price wedge allows for alternative interpretations, such as installation, transaction, or other fees that scale with the investment size and differ for capital purchases and sales. Moreover, setting $\omega=1$ eliminates the possibility of disinvesting (Sargent, 1980; Bertola and Caballero, 1994).

Investment problem Let $V(k,u)$ denote the value of a firm with capital stock k and productivity u. Given initial conditions (k_0,u_0), the firm chooses a sequence of adjustment dates $\{T_h\}_{h=1}^{\infty}$ and investments $\{i_{T_h}\}_{h=1}^{\infty}$, where h counts the number of adjustments, to maximize its expected discounted stream of profits. The sequential problem is

$$V(k_0,u_0) = \max_{\{T_h,i_{T_h}\}_{h=1}^{\infty}} \mathbb{E}\left[\int_0^{\infty} e^{-rs}y_s\,\mathrm{d}s-\sum_{h=1}^{\infty}e^{-rT_h}\left(\theta_{T_h}+p\left(i_{T_h}\right)i_{T_h}\right)\right], \tag{5}$$

subject to the production technology (1), the idiosyncratic productivity shocks (2), the fixed cost (3), the investment price function (4), and the law of motion for the capital stock

$$\log k_s = \log k_0 - \xi s+\sum_{h:T_h\leq s}\log\left(1+i_{T_h}/k_{T_h^-}\right), \tag{6}$$

which describes a period's capital as a function of its initial value k_0, the physical depreciation rate ξ, and the sum of all adjustments made at prior adjustment dates.

2.2 Capital-Productivity Ratio

To characterize the investment decision, we reduce the state space and recast the firm's problem using a new state variable, the (log) capital-productivity ratio:

$$\hat{k}_s \equiv \log\left(k_s/u_s\right). \tag{7}$$

Without investment frictions, $\hat{k}_s$ is a constant because firms are always at their optimal scale, which is proportional to productivity. Instead, investment frictions lead to prolonged misalignment between firms' capital and productivity levels. Between any two consecutive adjustment dates $[T_{h-1},T_h]$, the capital-productivity ratio $\hat{k}_s$ follows a stochastic process

$$\mathrm{d}\hat{k}_s = -\nu\,\mathrm{d}s+\sigma\,\mathrm{d}W_s. \tag{8}$$

The drift $\nu \equiv \xi + \mu$ includes the depreciation and productivity growth rates, and the volatility σ is inherited from productivity (the Wiener process is symmetric). At any adjustment date T_h, the capital-productivity ratio changes by the amount

$$\Delta \hat{k}_{T_h} \;=\; \log\left(1 + i_{T_h}/k_{T_h^-}\right), \tag{9}$$

where we use the continuity of the productivity process ($u_{T_h} = u_{T_h^-}$).

2.3 Tobin's q and Optimal Investment Policy

We characterize the optimal investment policy through Tobin's marginal q—the shadow value of installed capital. By definition, q is the marginal valuation of an extra unit of installed capital (the derivative of (5) to k_0) relative to the replacement cost (its purchase price p):

$$q(\hat{k}) \;\equiv\; \frac{1}{p}\frac{\partial V(k,u)}{\partial k}. \tag{10}$$

We pose that q is a function of the capital-productivity ratio $\hat{k}$, defined in (7), not of capital and productivity separately. The reason is that the derivative of the value function is a present discounted value of marginal products—as shown below—and the marginal product of capital can be written as $\alpha e^{\log(k/u)^{\alpha-1}} = \alpha e^{\hat{k}(\alpha-1)}$. The problem admits this reformulation because the production function is homothetic, adjustment costs are proportional to productivity, and idiosyncratic shocks follow a Brownian motion.

Four numbers characterize the optimal investment policy: $\{\hat{k}^- \leq \hat{k}^{*-} \leq \hat{k}^{*+} \leq k^+\}$. The smallest and largest numbers determine an inaction region $\mathcal{R} \equiv (\hat{k}^-, \hat{k}^+)$, which dictates a firm to leave its capital uncontrolled if $\hat{k}$ lies within this region. The two intermediate numbers $\hat{k}^{*-} < \hat{k}^{*+}$ are the reset points to which a firm sets $\hat{k}$ after hitting the corresponding border of inaction. Proposition 1 characterizes q and the optimal policy, defining the user cost of capital $\mathcal{U} \equiv r + \xi$.[1]

Proposition 1. ***(Optimal policy)*** *Marginal $q(\hat{k})$ and the optimal policy $\{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$ is characterized by the following sufficient optimality conditions:*

(i) Inside the inaction region $\mathcal{R}$, $q(\hat{k})$ solves the Hamilton-Jacobini-Bellman (HJB) equation:

$$\mathcal{U}q(\hat{k}) = \frac{\alpha e^{(\alpha-1)\hat{k}}}{p} \;-\; \nu q'(\hat{k}) \;+\; \frac{\sigma^2}{2}q''(\hat{k}), \qquad \forall\, \hat{k} \in (\hat{k}^-, \hat{k}^+). \tag{11}$$

[1] Proofs appear in Appendix A, B, and C. All theoretical results are proven for the generalized hazard model introduced in Section 5.

(ii) In the outer inaction regions, $q(\hat{k})$ satisfies the value-matching conditions:

$$\frac{\theta}{p} = \int_{\hat{k}^-}^{\hat{k}^{*-}} e^{\hat{k}} \left(q(\hat{k}) - 1\right) \mathrm{d}\hat{k}, \qquad \forall\, \hat{k} \in [\hat{k}^-, \hat{k}^{*-}], \tag{12}$$

$$\frac{\theta}{p} = \int_{\hat{k}^{*+}}^{\hat{k}^+} e^{\hat{k}} \left((1-\omega) - q(\hat{k})\right) \mathrm{d}\hat{k} \qquad \forall\, \hat{k} \in [\hat{k}^{*+}, \hat{k}^+]. \tag{13}$$

(iii) At the borders of the inaction region and reset points, $q(\hat{k})$ satisfies the optimality conditions:

$$q(\hat{k}) = 1, \qquad \hat{k} \in \left\{\hat{k}^-, \hat{k}^{*-}\right\}, \tag{14}$$

$$q(\hat{k}) = 1-\omega, \qquad \hat{k} \in \left\{\hat{k}^{*+}, \hat{k}^+\right\}. \tag{15}$$

From these conditions, q's stopping-time formulation is given by

$$q(\hat{k}) = \mathbb{E}\left[\int_0^\tau \frac{\alpha e^{-\mathcal{U}s + (\alpha-1)\hat{k}_s}}{p} \mathrm{d}s + e^{-\mathcal{U}\tau} q(\hat{k}_\tau)\right]. \tag{16}$$

The optimal policy satisfies (i) the HJB equation in (11) that describes q's evolution during inaction as the flow marginal product of capital expressed in capital units; (ii) two value-matching conditions in (12) and (13) that equalize the value of adjusting to the value of not adjusting; and (iii) four optimality conditions in (14) and (15) at the borders of inaction region and reset points.[2] Later, we use the stopping-time formulation in (16) to estimate the reset points from the data.

Figure I illustrates $q(\hat{k})$ (solid black line) and the optimal investment policy (four values of $\hat{k}$ in the x-axis). Let us first consider the environment without frictions ($\theta = \omega = 0$). Since firms always have their optimal capital-productivity ratio, then $q(\hat{k}) = 1$ and $q'(\hat{k}) = q''(\hat{k}) = 0$ for all $\hat{k}$. Substituting these values into the HJB in (11), we get that capital's marginal product equals the user cost, and the frictionless optimal capital $\hat{k}^*$ is given by

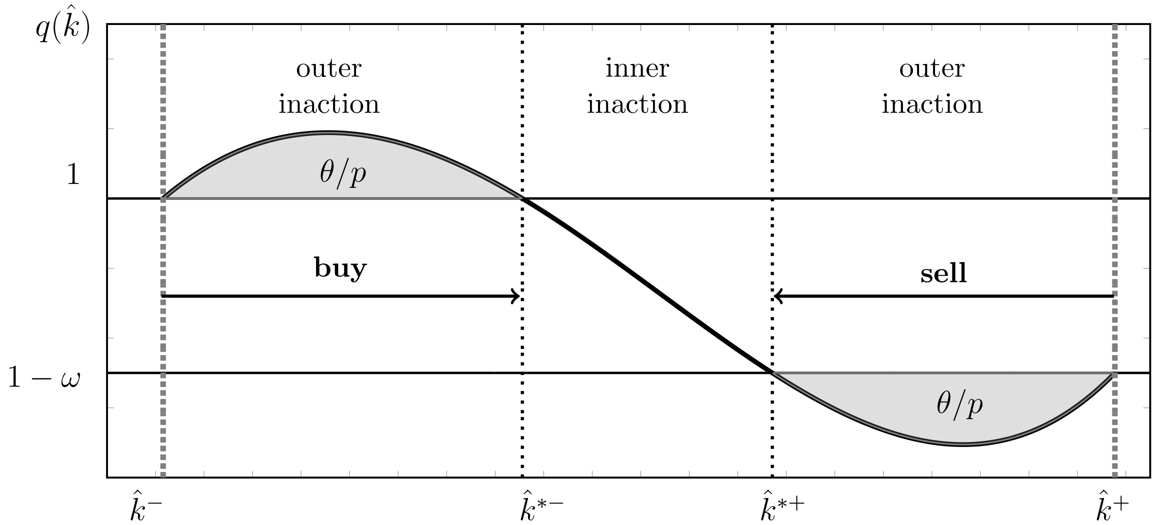
$$\frac{\alpha e^{(\alpha-1)\hat{k}^*}}{p} = \mathcal{U} \cdot 1 \iff \hat{k}^* = \frac{1}{1-\alpha} \log\left(\frac{\alpha}{p\mathcal{U}}\right), \tag{17}$$

as in the neoclassical investment theory.

Next, consider an environment with a price wedge ($\omega > 0$) but zero fixed costs ($\theta = 0$). The price wedge gives rise to an "inner" inaction region $[\hat{k}^{*-}, \hat{k}^{*+}]$ where the borders of inaction and reset points coincide, $\hat{k}^{*+} = \hat{k}^+$ and $\hat{k}^{*-} = \hat{k}^-$. Inside this region, $q(\hat{k})$ lies between the two prices,

[2] Given our assumption that the fixed cost scales with productivity ($\theta_s = \theta u_s$), the decision to invest or not encoded in the value matching conditions (12) and (13) depends only on the value of θ. If the fixed cost were scaled with the capital stock $\theta_s = \theta k_s$, as in Miao (2019), an appropriate rescaling of the fixed costs would generate a similar investment policy. The appropriate scaling requires defining $\theta^+ \equiv \theta e^{\hat{k}^+}$ and $\theta^- \equiv \theta e^{\hat{k}^-}$ and letting $\theta_s(i_s) = \theta^- u_s \mathbb{1}_{\{i_s>0\}} + \theta^- u_s \mathbb{1}_{\{i_s<0\}}$. See Baley and Blanco (2021) for the analysis of asymmetric fixed costs.

Figure I: Tobin's $q(\hat{k})$ and Optimal Investment Policy



Notes: This figure illustrates a firm's marginal $q(\hat{k})$ (solid line) and the investment policy $\{\hat{k}^-, \hat{k}^{*-}, \hat{k}^{*+}, \hat{k}^+\}$, expressed in terms of capital to productivity ratios, for $\omega > 0$ and $\theta > 0$. For illustration purposes, we do not rescale q by $e^{\hat{k}}$ in the shaded areas (as the value matching conditions require).

and it falls with $\hat{k}$ due to the decreasing marginal product of capital ($\alpha < 1$); thus, it is optimal to remain inactive. From (14) and (15), a firm purchases capital if $q(\hat{k}) \geq 1$ (or $\hat{k} \leq \hat{k}^{*-}$) and sells capital if $q(\hat{k}) \leq 1 - \omega$ (or $\hat{k} \geq \hat{k}^{*+}$) without any delay.

Finally, consider an environment with fixed costs ($\theta > 0$) and zero price wedge ($\omega = 0$). Without the price wedge, the "inner" inaction region collapses to a unique reset point k^*. However, the fixed cost creates an "outer" inaction region $[\hat{k}^-, \hat{k}^+]$ that prevents firms from adjusting, even if $q(\hat{k})$ lies above the purchase price or below the selling price. From (12) and (13), firms adjust to $\hat{k}^*$ if the value of adjusting (paying the fixed cost in capital units θ/p) is larger than the value of not adjusting (the cumulative deviations of q from one, weighted by capital-productivity ratios).

Investment and duration of inaction When both frictions are active, the investment policy features the "outer" and the "inner" inaction regions, two reset points, and a non-monotonic q function.[3] The firm purchases capital to bring $\hat{k}$ up to $\hat{k}^{*-}$ after hitting the lower border $\hat{k}^-$ and sells capital to bring $\hat{k}$ down to $\hat{k}^{*+}$ after hitting the upper border $\hat{k}^+$. Thus, adjustments occur at dates where the state falls outside the outer inaction region: $T_h = \inf \left\{ s \geq T_{h-1} : \hat{k}_s \notin \mathcal{R} \right\}$, with $T_0 = 0$. The duration of a complete inaction spell is the difference between consecutive adjustment dates $\tau_h = T_h - T_{h-1}$. Capital's age is the time elapsed since the last adjustment (the duration of an incomplete spell) $a_s = s - \max\{T_h : T_h \leq s\}$. Since the problem is recursive, we

[3] Although q monotonically decreases in the inner inaction region, it bends as it approaches the inaction thresholds because firms anticipate large adjustments. As $\hat{k}$ approaches the lower threshold $\hat{k}^-$, firms anticipate that a tiny reduction in the state $\mathrm{d}\hat{k} < 0$ would trigger a large positive investment $\Delta\hat{k} > 0$, lowering future and current $q(\hat{k})$ and bending the function down. A similar argument explains why $q(\hat{k})$ bends up as $\hat{k}$ approaches $\hat{k}^+$.

renormalize dates to 0 after each adjustment so that τ is both the subsequent random adjustment date and the duration of inaction and $\hat{k}_{\tau^-}$ the stopped capital-productivity ratio (immediately before adjusting).

For any τ, adjustments equal $\Delta\hat{k} = \hat{k}^*(\hat{k}_{\tau^-}) - \hat{k}_{\tau^-}$, where reset points depend on the stopped capital, denoted with $\hat{k}_{\tau^-}$:

$$\hat{k}^*(\hat{k}_{\tau^-}) \;=\; \begin{cases} \hat{k}^{*-} & \text{if} \quad \hat{k}_{\tau^-} = \hat{k}^- \\ \hat{k}^{*+} & \text{if} \quad \hat{k}_{\tau^-} = \hat{k}^+. \end{cases} \tag{18}$$

Crucially, optimal investment features a *positive serial correlation in the adjustment sign.* A firm is more likely to buy capital if it bought recently and is more likely to sell capital if it sold recently. Positive investments beget future positive investments; negative investments beget future negative investments. The positive serial correlation arises because the price wedge widens the distance between the two borders of inaction but shrinks the distance between each border of inaction and its corresponding reset point. Thus, it is more likely to reach $\hat{k}^-$ from the nearby $\hat{k}^{*-}$ than from the further $\hat{k}^{*+}$. Next, we will condition behavior on the previous reset point (i.e., if the last adjustment was up or down) to handle this path dependence.

2.4 Distribution of Capital-Productivity Ratios

To analyze the macroeconomic effects of irreversibility, we consider an economy populated by a continuum of ex-ante identical firms facing the same investment problem, in which idiosyncratic productivity shocks W_s are independent across firms. The economy has stationary cross-sectional distributions of capital-productivity ratios $\hat{k}$, adjustments $\Delta\hat{k}$, and durations of inaction τ.

Let $g(\hat{k})$ be the stationary density of firms' capital-productivity ratios. Also, let $\mathcal{N}^-$, $\mathcal{N}^+$, and $\mathcal{N} = \mathcal{N}^- + \mathcal{N}^+$ be the frequencies of upward, downward, and non-zero adjustments in the population, which are equal to the mass of firms that upsize to $\hat{k}^{*-}$, downsize to $\hat{k}^{*+}$, or adjust to either point. To avoid confusion with our notation, we emphasize that the sign in the exponent of an object refers to the previous reset point, not to the sign of the adjustment.

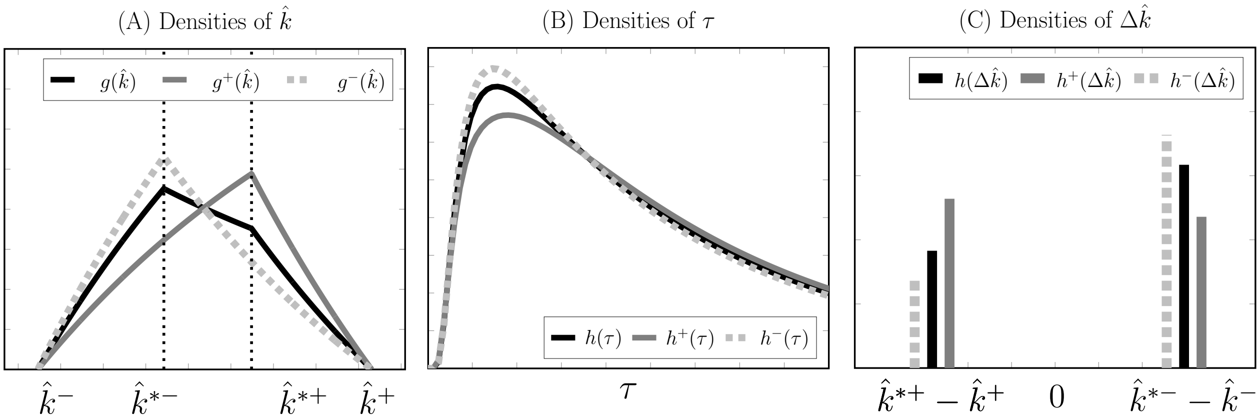
The density $g(\hat{k})$ and adjustment frequencies $\mathcal{N}^-$ and $\mathcal{N}^+$ solve the system that includes:

(i) A Kolmogorov forward equation (KFE) that describes the evolution of $g(\hat{k})$ in the interior of $\hat{k}$'s inaction region

$$\nu g'(\hat{k}) \;+\; \frac{\sigma^2}{2} g''(\hat{k}) \;=\; 0, \quad \text{for all} \quad \hat{k} \in (\hat{k}^-, \hat{k}^+) \setminus \{\hat{k}^{*-}, \hat{k}^{*+}\}; \tag{19}$$

(ii) Two border conditions that set the mass of firms at the inaction thresholds equal to zero

Figure II: Densities of $\hat{k}$, τ and $\Delta\hat{k}$



Notes: These figures plot the conditional and unconditional densities of capital-productivity ratios $\hat{k}$ (Panel A), duration of inaction τ (Panel B), and adjustments $\Delta\hat{k}$ (Panel C) for an illustrative parametrization.

and a condition ensuring that $g(\hat{k})$ is a density:

$$g(\hat{k}^{-}) \;=\; g(\hat{k}^{+}) \;= 0, \tag{20}$$

$$\int_{\hat{k}^{-}}^{\hat{k}^{+}} g(\hat{k})\,\mathrm{d}\hat{k} \;=\; 1; \tag{21}$$

(iii) Two resetting or reinjection conditions that relate the masses of upward and downward adjustments to the discontinuities in the derivative of g at the reset points:[4]

$$\mathcal{N}^{-} \;=\; \frac{\sigma^2}{2}\lim_{\hat{k}\downarrow\hat{k}^{-}} g'(\hat{k}) \;=\; \frac{\sigma^2}{2}\left[\lim_{\hat{k}\uparrow\hat{k}^{*-}} g'(\hat{k}) - \lim_{\hat{k}\downarrow\hat{k}^{*-}} g'(\hat{k})\right], \tag{22}$$

$$\mathcal{N}^{+} \;=\; -\frac{\sigma^2}{2}\lim_{\hat{k}\uparrow\hat{k}^{+}} g'(\hat{k}) \;=\; \frac{\sigma^2}{2}\left[\lim_{\hat{k}\uparrow\hat{k}^{*+}} g'(\hat{k}) - \lim_{\hat{k}\downarrow\hat{k}^{*+}} g'(\hat{k})\right], \tag{23}$$

and two continuity conditions at the reset points.

As anticipated, we study behavior conditional on the previous reset point throughout our analysis. We consider densities conditional on the previous reset point, after upsizing $g^{-}(\hat{k})$ and after downsizing $g^{+}(\hat{k})$, which satisfy the same KFE as $g(\hat{k})$ in (19), except that they only have one kink at the corresponding reset point. Panel A in Figure II plots the three densities g, g^{-}, and g^{+}; these are all proper densities and integrate to 1. We denote expectations computed with these distributions as $\mathbb{E}$, $\mathbb{E}^{-}$, and $\mathbb{E}^{+}$. Recall that the three distributions would be the same without irreversibility, as there would be a unique reset point and no dependence on past adjustments.

[4] In a small period $\mathrm{d}s$, the mass $\mathcal{N}^{-}$ that "exits" the inaction region by hitting the lower threshold—equal to $\frac{\sigma^2}{2}\lim_{\hat{k}\downarrow\hat{k}^{-}} g'(\hat{k})$—must coincide with the mass of firms that "enters" at the reset point $\hat{k}^{*-}$—equal to the jump in $\frac{\sigma^2}{2}g'$. This argument is analogous for $\mathcal{N}^{+}$.

It is important to note that, in the steady state, the average capital-productivity ratio of firms that last purchased capital is below the economy's average $\mathbb{E}_s^-[\hat{k}] - \mathbb{E}[\hat{k}] < 0$ (i.e., they don't purchase enough); similarly, the average capital-productivity ratio of firms that last sold capital is above the economy's average $\mathbb{E}_s^+[\hat{k}] - \mathbb{E}[\hat{k}] > 0$ (they do not sell enough). This fact will be critical for understanding why irreversibility amplifies aggregate capital fluctuations in Section 3.

2.5 Distributions of Actions

We denote the joint density over adjustments $\Delta\hat{k}$ and duration of inaction spells τ with $h(\Delta\hat{k}, \tau)$, and the densities conditional on the previous reset point with h^- and h^+, respectively. We also use h and $h^\pm$ to denote the marginal densities of τ and $\Delta\hat{k}$. Expectations conditional on taking action are denoted with bars: $\overline{\mathbb{E}}$, $\overline{\mathbb{E}}^-$, and $\overline{\mathbb{E}}^+$.

Panel B in Figure II plots the three densities of the duration of inaction.[5] We observe that $h^-(\tau)$ is more skewed toward shorter durations than $h^+(\tau)$, which is due to the negative drift. Panel C of Figure II plots the three distributions of adjustments $\Delta\hat{k}$.[6] The simple model generates almost no dispersion of investment rates—$h(\Delta\hat{k})$ only takes two values. However, the distributions conditional on the previous adjustment showcase the serial correlation of the adjustment sign. For instance, $h^-(\hat{k}^{*-} - \hat{k}^-) > h^+(\hat{k}^{*-} - \hat{k}^-)$ means that the probability of upsizing is higher after a previous upsize. Again, we emphasize that, without irreversibility, the three densities of τ and $\Delta\hat{k}$ would be identical. Figure VI in Section 4.3 plots their empirical counterparts.

2.6 Relationship Between Unconditional and Conditional Densities

Unconditional and conditional densities of capital-productivity ratios and adjustments relate in different ways.

Weighing with shares Adjusters' unconditional and conditional densities are related through a simple average using the relative shares of upward $\mathcal{N}^-/\mathcal{N}$ and downward $\mathcal{N}^+/\mathcal{N}$ adjustments: $h(\Delta\hat{k}, \tau) = \frac{\mathcal{N}^-}{\mathcal{N}} h^-(\Delta\hat{k}, \tau) + \frac{\mathcal{N}^+}{\mathcal{N}} h^+(\Delta\hat{k}, \tau)$. Thus unconditional and conditional expectations of adjustments relate according to

$$\overline{\mathbb{E}}[y] = \frac{\mathcal{N}^-}{\mathcal{N}} \overline{\mathbb{E}}^-[y] + \frac{\mathcal{N}^+}{\mathcal{N}} \overline{\mathbb{E}}^+[y], \qquad y \in \{\Delta\hat{k}, \tau\}. \tag{24}$$

Weighing with adjusted shares In contrast, the unconditional density of capital-productivity ratios is a weighted sum of the conditional distributions $g(\hat{k}) = r^- g^-(\hat{k}) + r^+ g^+(\hat{k})$, where the

[5] Let $h(\tau|\hat{k})$ be the stationary density of the duration of inaction given current $\hat{k}$. Using the formulas by Kolkiewicz (2002) evaluated at the two reset points, we obtain the densities of duration conditional on the previous reset point: $h^-(\tau) = h(\tau|\hat{k}^{*-})$ and $h^+(\tau) = h(\tau|\hat{k}^{*+})$.

[6] Since $\Delta\hat{k}$ only takes two values, its distribution consists of two mass points.

renewal weights r^- and r^+ rescale the shares by their relative average duration:

$$r^- \equiv \frac{\mathcal{N}^-}{\mathcal{N}} \frac{\overline{\mathbb{E}}^-[\tau]}{\overline{\mathbb{E}}[\tau]}, \quad \text{and} \quad r^+ \equiv \frac{\mathcal{N}^+}{\mathcal{N}} \frac{\overline{\mathbb{E}}^+[\tau]}{\overline{\mathbb{E}}[\tau]}. \tag{25}$$

Thus, unconditional and conditional expectations of capital-productivity ratios relate as follows:

$$\mathbb{E}[\hat{k}] = r^-\mathbb{E}^-[\hat{k}] + r^+\mathbb{E}^+[\hat{k}]. \tag{26}$$

Why do we rescale conditional densities by the duration of inaction? The answer is the *fundamental renewal property:* The average behavior in the economy is attributable to firms with more extended periods of inaction (which are observed less frequently). Adjusting the shares with their relative duration corrects this observational bias. Appendix A.5 illustrates the correction with an example.

2.7 A Markov Chain Between Reset Points

We define the remaining objects needed to handle irreversibility. Given a current $\hat{k}_0$, let $\mathbb{P}^-(\hat{k}_0)$ be the probability of a subsequent purchase ($\hat{k}_\tau = \hat{k}^{*-}$) and $\mathbb{P}^+(\hat{k}_0)$ the probability of a subsequent sale ($\hat{k}_\tau = \hat{k}^{*+}$):

$$\mathbb{P}^-(\hat{k}_0) = \Pr[\hat{k}_\tau = \hat{k}^{*-}|\hat{k}_0] \quad \text{and} \quad \mathbb{P}^+(\hat{k}_0) = \Pr[\hat{k}_\tau = \hat{k}^{*+}|\hat{k}_0]. \tag{27}$$

Using these probabilities, we define the transition probability matrix between reset points.[7] Given a current purchase ($\hat{k}_0 = \hat{k}^{*-}$), the probability of making a subsequent purchase is $\mathbb{P}^{--} \equiv \mathbb{P}^-(\hat{k}^{*-})$; and given a current sale ($\hat{k}_0 = \hat{k}^{*+}$), the probability of making a subsequent sale is $\mathbb{P}^{++} \equiv \mathbb{P}^+(\hat{k}^{*+})$. Analogously, we define the off-diagonal elements of the transition matrix between reset points $\mathbb{P}^{-+}$ and $\mathbb{P}^{-+}$. The transition matrix $\mathbb{P}$ has the following four entries:[8]

$$\mathbb{P}^{--} = \Pr[\hat{k}_\tau = \hat{k}^{*-}|\hat{k}_0 = \hat{k}^{*-}], \qquad \mathbb{P}^{-+} = \Pr[\hat{k}_\tau = \hat{k}^{*+}|\hat{k}_0 = \hat{k}^{*-}], \tag{28}$$

$$\mathbb{P}^{+-} = \Pr[\hat{k}_\tau = \hat{k}^{*-}|\hat{k}_0 = \hat{k}^{*+}], \qquad \mathbb{P}^{++} = \Pr[\hat{k}_\tau = \hat{k}^{*+}|\hat{k}_0 = \hat{k}^{*+}]. \tag{29}$$

The following section characterizes aggregate capital fluctuations with irreversibility by conditioning behavior on the previous reset point, which requires the following objects: the conditional densities $g^-(\hat{k})$ and $g^+(\hat{k})$, the adjusting shares $\mathcal{N}^-$ and $\mathcal{N}^+$ in (22) and (23), the renewal weights r^- and r^+ in (25), the upsizing and downsizing conditional probabilities $\mathbb{P}^-(\hat{k})$ and $\mathbb{P}^+(\hat{k})$ in (27), and the Markov chain between reset points $\mathbb{P}^{--}, \mathbb{P}^{-+}, \mathbb{P}^{+-}$ and $\mathbb{P}^{++}$ in (28) and (29).

[7] Our approach complements methodologies by Caballero and Engel (2007) and Lanteri (2018), which diagnose irreversibility through transitions between marginal products of capital.

[8] The adjustment shares in (22) and (23) can also be obtained from the eigenvector of the transition matrix $\mathbb{P}$. The share of current positive investments equals the probability of a future positive investment (analogously for negative investments): $\frac{\mathcal{N}^-}{\mathcal{N}} = \frac{\mathcal{N}^-}{\mathcal{N}}\mathbb{P}^{--} + \frac{\mathcal{N}^+}{\mathcal{N}}(1 - \mathbb{P}^{++})$.

3 Aggregate Fluctuations with Irreversibility

This section theoretically studies how irreversibility affects aggregate fluctuations. Our analysis has four steps. First, we define our notions of capital fluctuations: the impulse response (IRF) and the cumulative impulse response (CIR) of average capital-productivity ratios to an aggregate productivity shock. Second, we analyze the IRF from a time-series perspective, which shows how irreversibility affects aggregate fluctuations by changing the shares of upsizing and downsizing firms. This shift has a persistent effect because of the differential behavior conditional on the previous reset point. Third, we analyze the CIR from a cross-sectional perspective, a preliminary step in finding sufficient statistics for capital fluctuations. In particular, it establishes three groups of firms whose behavior should be tracked: inactive firms until their first adjustment, upsizers, and downsizers. In the fourth and last step, we derive sufficient statistics for the CIR and investigate special cases that illustrate how irreversibility impacts these sufficient statistics.

3.1 Capital Fluctuations

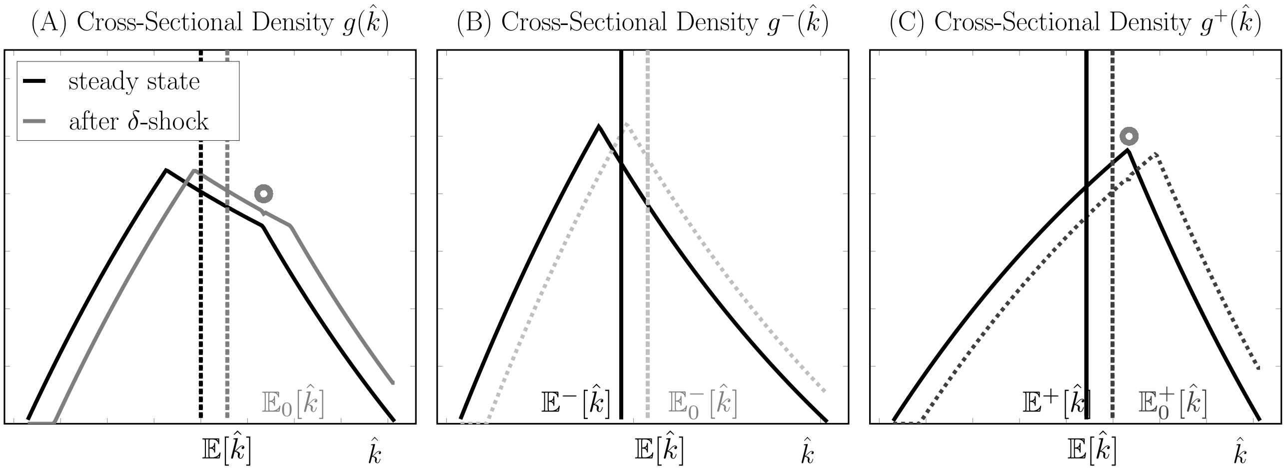
We measure capital fluctuations as the transitional dynamics of the average capital-productivity ratio that follow an aggregate productivity shock. Starting from the steady state at date $s = 0$, we introduce a small, permanent, and unanticipated decrease in the (log) level of productivity of size $\delta > 0$ to all firms. All firms' productivity falls, and capital-productivity ratios increase relative to their pre-shock levels u_{0^-} and $\hat{k}_{0^-}$, as follows:

$$\log(u_0) \;=\; \log(u_{0^-}) - \delta; \qquad \log(\hat{k}_0) \;=\; \log(\hat{k}_{0^-}) + \delta. \tag{30}$$

Panel A in Figure III plots the steady-state unconditional density $g(\hat{k})$ and the initial density following the productivity shock $g_0 = g(\hat{k} - \delta)$. Panels B and C plot the steady-state densities conditional on the previous reset point $g^{\pm}(\hat{k})$ and after the shock $g_0^{\pm} = g^{\pm}(\hat{k} - \delta)$. The negative productivity shock displaces all the steady-state distributions to the right. Without investment frictions, firms would immediately downsize their capital stock to restore their optimal capital-productivity ratio. With frictions, firms take time to absorb the shock, and the distribution of capital-productivity ratios remains away from the steady state distribution for some time. Until all firms have downsized their capital stock, there will still be positive deviations from the steady-state and persistent effects of the productivity shock. The thick circle in panels A and C corresponds to the mass of firms crossing $\hat{k}^+$ on the shock's impact and downsizing to reach $\hat{k}^{*+}$. Panel B has no thick circle in $g^-(\hat{k})$, as no mass of firms purchase capital after the shock.

Throughout the transition, the interest rate remains constant, and the steady-state investment policies hold, so there is no feedback from distributional changes in policies. Thus, our analysis

Figure III: Cross-Sectional Densities Before and After the Aggregate Shock



Notes: This figure illustrates the effects of an aggregate shock. Panel A shows the steady-state distribution $g(\hat{k})$ and the initial distribution following a productivity shock $g_0(\hat{k}) = g(\hat{k} - \delta)$. Panels B and C show analogous figures for the distributions conditional on the previous reset point, $g^{-}(\hat{k})$ and $g^{+}(\hat{k})$. Probability masses corresponding to entry points are shown in thick circles.

measures the strength of the partial equilibrium response to aggregate shock.[9]

IRF and CIR We define two measures of capital fluctuations. First, we define the impulse-response function, $\text{IRF}(\delta, s)$, as the deviation of the mean of $\hat{k}$ after s periods of the arrival of the aggregate productivity shock and the steady-state mean

$$\text{IRF}(\delta, s) \;\equiv\; \mathbb{E}_s[\hat{k}] - \mathbb{E}[\hat{k}], \tag{31}$$

where $\mathbb{E}_s[\cdot]$ denotes expectations with the time-s distribution. Second, we define the cumulative impulse response of the mean of $\hat{k}$, $\text{CIR}(\delta)$, as the area under the $\text{IRF}_s(\delta)$ across all dates $s \in (0, \infty)$:

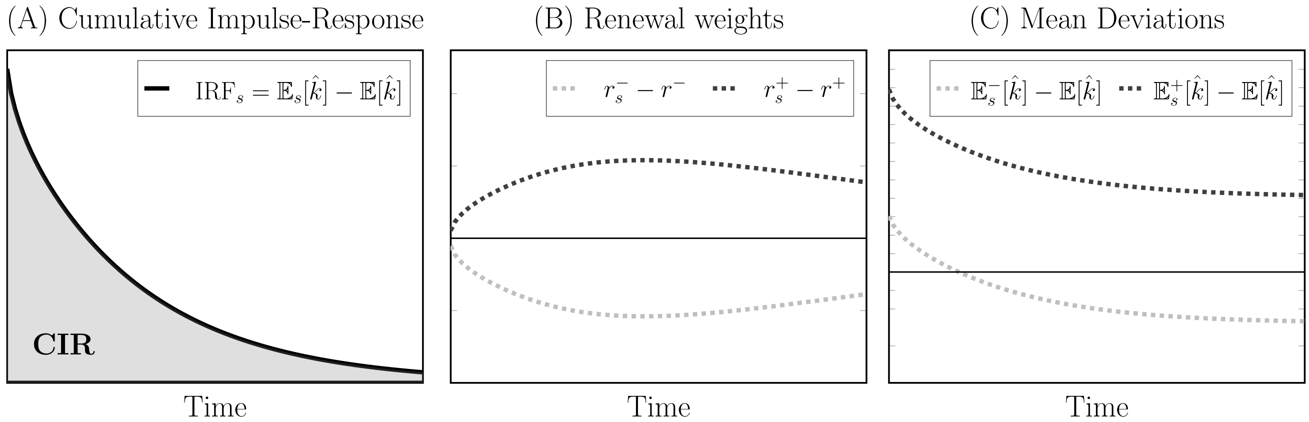
$$\text{CIR}(\delta) \;\equiv\; \int_0^\infty \text{IRF}(\delta, s)\; \mathrm{d}s \;=\; \int_0^\infty \Big(\mathbb{E}_s[\hat{k}] - \mathbb{E}[\hat{k}]\Big)\; \mathrm{d}s. \tag{32}$$

The CIR is a helpful metric of aggregate fluctuations, and it lies at the core of our strategy to discipline the price wedge and the aggregate effects of irreversibility. It summarizes the impact and persistence of the response in one scalar and eases comparison across different models.[10] Without investment frictions, firms respond instantly to the aggregate shock, and the CIR is zero. With investment frictions, the larger the CIR, the longer firms take to respond to the aggregate shock

[9] Appendix D presents a general equilibrium model that delivers constant prices as an equilibrium outcome.

[10] Alvarez, Le Bihan and Lippi (2016), Baley and Blanco (2019); Alvarez, Lippi and Oskolkov (2022); and Alexandrov (2021) use the CIR in the context of price-setting models to assess the real effects of monetary shocks.

Figure IV: Cumulative Impulse Response and Its Components



Notes: The figure shows paths after an aggregate productivity shock for an illustrative parametrization. Panel A shows the IRF(δ, s) and the CIR (area under the curve). Panel B shows the evolution of renewal weights r^- and r^+, relative to their steady-state values. Panel C shows average deviations from the steady-state mean, conditional on the previous reset point. In B and C, the lighter lines correspond to upsizing firms (previous reset to $\hat{k}^{*-}$); the darker lines correspond to downsizing firms (previous reset to $\hat{k}^{*+}$).

and the slower the transitional dynamics. Panel A in Figure IV plots the IRF (solid line) and the CIR (the area underneath the IRF).

3.2 The Time-Series Perspective

We begin analyzing aggregate fluctuations from a time-series perspective. This viewpoint teaches us that irreversibility's main contribution to aggregate fluctuations following a negative productivity shock is increasing the share of downsizing firms during the transition to the new steady state, precisely the set of firms that contribute most to persistent deviations from the steady state.

Conditional deviations and renewal weights We decompose deviations from the steady state into deviations conditional on the previous reset point. To do this, we use expectations conditional on the last rest point $\mathbb{E}_s^-$ and $\mathbb{E}_s^+$ and the renewal weights r_s^- and r_s^+ in (25) to rewrite the IRF in (31) as:

$$\mathrm{IRF}(\delta, s) \;=\; r_s^- \left(\mathbb{E}_s^-[\hat{k}] - \mathbb{E}[\hat{k}]\right) \;+\; r_s^+ \left(\mathbb{E}_s^+[\hat{k}] - \mathbb{E}[\hat{k}]\right). \tag{33}$$

Figure IV tracks the various components that give rise to deviations from the steady state. Panel B shows the path of the renewal weights r_s^- and r_s^+ relative to their steady-state values, which tell us the number of adjustments stemming from each reset point. After the negative aggregate productivity shock, the share of downsizing firms r_s^+ increases (darker line), whereas the share of upsizing firms r_s^- falls (lighter line).[11] Panel C shows the path of average capital-productivity ratios conditional on the last rest point relative to the economy's average ($\mathbb{E}_s^{\pm}[\hat{k}] - \mathbb{E}[\hat{k}]$). Both means go up after the shock.

The time-series decomposition in (33) highlights two channels that slow the shock absorption and increase the CIR. First, keeping the mean deviations at their steady-state values ($\mathbb{E}_s^-[\hat{k}] = \mathbb{E}^-[\hat{k}] < \mathbb{E}^+[\hat{k}] = \mathbb{E}_s^+[\hat{k}]$ for all s), consider the change in renewal weights. The population tilts toward more downsizing firms, which enter their typical persistent downsizing phase with capital-productivity ratios above the economy's average ($\mathbb{E}^+[\hat{k}] - \mathbb{E}[\hat{k}] > 0$). These firms, which are now dominant, increase deviations above the steady state and slow convergence. Upsizing firms also enter their typical upsizing phase with average capital-productivity ratios below the average ($\mathbb{E}^-[\hat{k}] - \mathbb{E}[\hat{k}] < 0$). However, since there are fewer of them, they also contribute to slowing the convergence back to the steady state. Thus, the change in renewal weights generates persistence even if the mean deviations remain at their steady-state values.

Second, besides the changes in weights, positive deviations $\mathbb{E}_s^+[\hat{k}] - \mathbb{E}[\hat{k}]$ become larger and converge slower, and negative deviations $\mathbb{E}_s^-[\hat{k}] - \mathbb{E}[\hat{k}]$ become smaller (and even positive on the shock's impact) and converge faster. This change in means brings an additional kick. In sum, the evolution in shares and conditional means following an aggregate shock increases the persistence of aggregate capital fluctuations when capital is partially irreversible.

3.3 The Cross-Sectional Perspective

Next, we formally analyze capital fluctuations from a cross-sectional perspective. The central insight is to express the aggregate dynamic response to the shock as the cross-sectional average of the expected cumulative deviations from the mean. Three steps lie behind this characterization: (i) exchanging the integrals over time and over firms, (ii) decomposing average cumulative deviations between the first and subsequent adjustments, and (iii) summarizing subsequent adjustments with two numbers reflecting upsizing and downsizing behavior.

Exchanging integrals Let us start from the CIR's definition in (32), which tracks the economy's average investment behavior along the transition, that is, integrating first over firms and then over time. We exchange orders of integration (i.e., integrating first over time and then over firms) to track each firm's investment and then average across them. The analytical gain stemming from exchanging the orders of integration arises because we can decompose the infinite horizon $[0, \infty]$ into two intervals: from the arrival of the aggregate shock to the (random) first adjustment $[0, \tau]$, and the first adjustment onward $[\tau, \infty]$.

[11] Naturally, the change in shares also happens without a price wedge and inaction purely generated by fixed costs; however, in that case, firms become identical *after their first adjustment* ($\mathbb{E}_s^-[\hat{k}] = \mathbb{E}_s^+[\hat{k}]$), deviations from the economy's average can be ignored and there is no additional persistence (Baley and Blanco, 2021).

To implement this decomposition, in the first step, we use the law of iterated expectations to condition on the initial capital-productivity ratio $\hat{k}_0$ (right after the δ shock) and integrate over the initial distribution of agents $g_0 = g(\hat{k} - \delta)$. Then, we exchange the order of integration over agents and over time:

$$(34)\quad \text{CIR}(\delta) \;=\; \mathbb{E}_{g_0}\left[\int_0^\infty \mathbb{E}_s\left[(\hat{k} - \mathbb{E}[\hat{k}])\right] \mathrm{d}s \middle| \hat{k}_0\right] \;=\; \mathbb{E}_{g_0}\left[\mathbb{E}\left[\int_0^\infty (\hat{k}_s - \mathbb{E}[\hat{k}])\, \mathrm{d}s \middle| \hat{k}_0\right]\right] + \mathbb{C}.$$

The constant $\mathbb{C}$ arises because Fubini's theorem fails (the double integral of the absolute value of deviations is infinite); thus, exchanging the order of integration does not give the same result.[12]

First and subsequent adjustments In the second step, we define $m(\hat{k}_0)$ as the expected cumulative deviations from the steady-state mean for a firm with current $\hat{k}_0$ as

$$(35)\qquad m(\hat{k}_0) \;\equiv\; \mathbb{E}\left[\int_0^\infty (\hat{k}_s - \mathbb{E}[\hat{k}])\, \mathrm{d}s \middle| \hat{k}_0\right] \;+\; \mathbb{C}.$$

We decompose $m(\hat{k}_0)$ into two intervals $[0, \tau]$ and $[\tau, \infty]$, where τ is the firm's first stopping time after the aggregate shock:

$$(36)\qquad m(\hat{k}_0) \;=\; \underbrace{\mathbb{E}\left[\int_0^\tau (\hat{k}_s - \mathbb{E}[\hat{k}])\, \mathrm{d}s \middle| \hat{k}_0\right]}_{\text{up to first adjustment}} + \underbrace{\mathbb{E}\left[\int_\tau^\infty (\hat{k}_s - \mathbb{E}[\hat{k}])\, \mathrm{d}s \middle| \hat{k}_0\right] \;+\; \mathbb{C}.}_{\text{subsequent adjustments}}$$

Without irreversibility, firms fully incorporate the aggregate productivity shock with the first adjustment (to a unique reset point $\hat{k}^*$) and then return to their "steady-state behavior"; this implies that the second term in (36) is independent of $\hat{k}_0$ and thus also independent of δ (Alvarez, Le Bihan and Lippi, 2016; Baley and Blanco, 2021). This is no longer the case with irreversibility, as the initial state $\hat{k}_0$ affects subsequent adjustments. The second term in (36) differs from zero because firms only partially adjust to the aggregate shock. In principle, one should keep track of firms until they fully incorporate the aggregate shock. This is a problem "with reinjection", as labeled by Alvarez and Lippi (2021). Nevertheless, it is enough to remember the first reset point and then condition future behavior because the first adjustment cleans from all heterogeneity *except for the adjustment sign.*

Summarizing subsequent adjustments In the third step, we summarize investment behavior after the first adjustment (the second term in (36)) with two numbers: average behavior after

[12] The constant $\mathbb{C}$ does not affect the sufficient statistics' characterization of the CIR in the next section because we make a first-order approximation for small shocks $\delta \approx 0$, and it drops out. Alexandrov (2021) considers large shocks where the constant becomes relevant.

upsizing $m(\hat{k}^{*-})$ and average behavior after downsizing $m(\hat{k}^{*+})$:

(37)
$$\mathbb{E}\Big[\mathbb{E}\Big[\int_\tau^\infty (\hat{k}_s - \mathbb{E}[\hat{k}])\,\mathrm{d}s\Big|\hat{k}_\tau\Big]\Big|\hat{k}_0\Big] = \mathbb{E}\Big[m(\hat{k}^*(\hat{k}_\tau))\Big|\hat{k}_0\Big] - \mathbb{C} = \mathbb{P}^-(\hat{k}_0)m(\hat{k}^{*-}) + \mathbb{P}^+(\hat{k}_0)m(\hat{k}^{*+}) - \mathbb{C}.$$

where the probabilities $\mathbb{P}^-(\hat{k}_0)$ and $\mathbb{P}^+(\hat{k}_0)$, defined in (27), depend on initial conditions after the aggregate shock. This expression shows that steady-state behavior is restored after the first adjustment—once a firm completes its first inaction spell.

Last, we join all results by substituting (37) into (36) and then into (34) and write the CIR as a cross-sectional average of cumulative deviations computed with the initial distribution g_0:

$$\text{CIR}(\delta) = \mathbb{E}_{g_0}[m(\hat{k}_0)], \tag{38}$$

where $m(\hat{k}_0)$ is defined recursively as follows:[13]

$$m(\hat{k}_0) \equiv \underbrace{\mathbb{E}\left[\int_0^\tau (\hat{k}_s - \mathbb{E}[\hat{k}])\,\mathrm{d}s\Big|\hat{k}_0\right]}_{\text{incomplete spells}} + \underbrace{\mathbb{P}^-(\hat{k}_0)m(\hat{k}^{*-})}_{\text{complete upsizing spell}} + \underbrace{\mathbb{P}^+(\hat{k}_0)m(\hat{k}^{*+})}_{\text{complete downsizing spell}}. \tag{39}$$

Proposition 2 organizes the previous results.

Proposition 2. ***(CIR)*** *Up to the first order, the CIR equals*

$$\frac{CIR(\delta)}{\delta} = -\int_{\hat{k}^-}^{\hat{k}^+} m(\hat{k})g'(\hat{k})\,\mathrm{d}\hat{k} + o(\delta), \tag{40}$$

where $m(\hat{k})$ is a continuously differentiable function equal to the average cumulative deviations of the capital-productivity ratio $\hat{k}$ from the economy's mean $\mathbb{E}[\hat{k}]$, satisfying the HJB

$$0 = \hat{k} - \mathbb{E}[\hat{k}] - \nu m'(\hat{k}) + \frac{\sigma^2}{2}m''(\hat{k}) \qquad \forall \hat{k} \in (\hat{k}^-, \hat{k}^+), \tag{41}$$

with two boundary conditions

$$m(\hat{k}^-) = m(\hat{k}^{*-}), \quad \textit{and} \quad m(\hat{k}^+) = m(\hat{k}^{*+}), \tag{42}$$

and a stationarity condition

$$\int_{\hat{k}^-}^{\hat{k}^+} m(\hat{k})g(\hat{k})\,\mathrm{d}\hat{k} = 0. \tag{43}$$

Expression (40) reexpresses the CIR in (38) using a first-order approximation to the initial

[13] $\mathbb{C}$ disappears as it is inside m.

distribution around a small shock δ, $g_0 = g(\hat{k}-\delta) \approx g(\hat{k}) - \delta g'(\hat{k})$, and the stationarity condition (43) which sets $\mathbb{E}_g[m(\hat{k}_0)] = 0$. We reexpress $m(\hat{k})$ in (39) recursively through the HJB in (41), inherited from $\hat{k}$'s law of motion, together the border conditions in (42). Due to continuity, the two boundary conditions equalize cumulative deviations at the inaction thresholds to their value at the corresponding reset point.

The astute reader will notice that equations (41) and (42) have infinite solutions: for any candidate solution $m(\hat{k})$, $m(\hat{k})+a$ is a solution $\forall a \in \mathbb{R}$. The stationarity condition (43) pins down the unique solution by requiring no fluctuations without shocks (i.e., CIR(0) = 0). Effectively, it imposes a linear relationship between the terminal values $m(\hat{k}^{*-})$ and $m(\hat{k}^{*+})$. Stationarity lies at the core of the steady state and plays a crucial role in the CIR's characterization with partial irreversibility. Next, we explain this condition in detail.

3.4 Stationarity: Balancing Complete and Incomplete Spells

The stationarity condition (43) sets the cross-sectional average of $m(\hat{k})$ in (39) to zero.[14] It implies that average deviations that follow a complete inaction spell (from upsizing or downsizing firms) should "balance" the average deviations from incomplete spells (from inactive firms):

$$\underbrace{\mathbb{C}ov[\hat{k}, a]}_{\text{avg. incomplete spells}} \quad + \quad \underbrace{\mathbb{E}[\mathbb{P}^-(\hat{k})]m(\hat{k}^{*-})}_{\text{avg. complete upsizing spells}} \quad + \quad \underbrace{\mathbb{E}[\mathbb{P}^+(\hat{k})]m(\hat{k}^{*+})}_{\text{avg. complete downsizing spells}} \quad = \; 0. \tag{44}$$

Let us discuss this condition in incrementally complex environments.

Symmetric environments With zero drift, symmetric inaction region, and zero price wedge, the unique reset point $\hat{k}^*$ equals the economy's average $\mathbb{E}[\hat{k}]$ and deviations above and below the mean for any age cancel out. The covariance between capital-productivity ratios and age is zero $\mathbb{C}ov[\hat{k}, a] = 0$, and consequently, stationarity requires $m(\hat{k}^*) = 0$.

Asymmetric environment without irreversibility In asymmetric environments (with non-zero drift or asymmetric fixed costs), the unique reset point differs from the economy's average. Deviations above and below do not cancel out and $\mathbb{C}ov[\hat{k}, a] \neq 0$. Stationarity requires:

$$m(\hat{k}^*) \; + \; \mathbb{C}ov[\hat{k}, a] \; = \; 0. \tag{45}$$

Deviations after complete inaction spells, $m(\hat{k}^*)$, and deviations during incomplete spells, $\mathbb{C}ov[\hat{k}, a]$, must sum up to zero in a steady state. For example, with negative drift $-\nu$, inactive firms have, on

[14] The integral of incomplete spells (the first term in (39)) equals the covariance between capital-productivity ratios and capital age: $\int_{\hat{k}^-}^{\hat{k}^+} \mathbb{E}\left[\int_0^\tau (\hat{k}_s - \mathbb{E}[\hat{k}])\,\mathrm{d}s \middle| \hat{k}_0\right] g(\hat{k}_0)\,\mathrm{d}\hat{k}_0 = \mathbb{C}ov[\hat{k}, a]$. The integral of the complete spells (the second term in (39)) becomes the sum of average probabilities times deviations.

average, capital-productivity ratios below the economy's mean, and more so the older their capital. In the inactive cross-section, $\mathbb{C}ov[\hat{k}, a] < 0$. Completed spells must revert this force to deliver zero deviations in steady-state, and thus, adjusting firms will overshoot their capital-productivity ratio: $m(\hat{k}^*) = -\mathbb{C}ov[\hat{k}, a] > 0$.

With irreversibility With irreversibility, the balancing argument is similar, but now we must consider two types of complete spells. We rewrite (44) as the weighted average of two numbers:

$$\underbrace{\left(m(\hat{k}^{*-}) + \mathbb{C}ov[\hat{k}, a]\right)}_{\equiv\, \mathcal{M}(\hat{k}^{*-})\, <\, 0} \mathbb{E}[\mathbb{P}^-(\hat{k})] \;+\; \underbrace{\left(m(\hat{k}^{*+}) + \mathbb{C}ov[\hat{k}, a]\right)}_{\equiv\, \mathcal{M}(\hat{k}^{*+})\, >\, 0}(1 - \mathbb{E}[\mathbb{P}^-(\hat{k})]) \;=\; 0, \tag{46}$$

where $\mathbb{E}[\mathbb{P}^+(\hat{k})] = 1 - \mathbb{E}[\mathbb{P}^-(\hat{k})]$. Recall that $\mathbb{C}ov[\hat{k}, a]$ reflects the deviations of inactive firms. For inaction spells ending in upsizing (with probability $\mathbb{E}[\mathbb{P}^-(\hat{k})]$), we have that average deviations are negative, $\mathcal{M}(\hat{k}^{*-}) \equiv m(\hat{k}^-) + \mathbb{C}ov[\hat{k}, a] < 0$, since resets fall short of the average. For inaction spells ending in downsizing (with probability $1 - \mathbb{E}[\mathbb{P}^-(\hat{k})]$), we have instead that average deviations are positive, $\mathcal{M}(\hat{k}^{*+}) \equiv m(\hat{k}^+) + \mathbb{C}ov[\hat{k}, a] > 0$, since resets succeed the average. The adjustments of upsizing and downsizing firms, weighted by their occurrence, must compensate for the deviations of inactive firms to ensure stationarity.

To better understand what $\mathcal{M}(\hat{k}^{*-})$ and $\mathcal{M}(\hat{k}^{*+})$ reflect, Proposition 3 provides alternative expressions, as a triple product of conditional deviations $(\mathbb{E}^{\pm}[\hat{k}] - \mathbb{E}[\hat{k}])$, the average duration of those deviations $\overline{\mathbb{E}}^{\pm}[\tau]$, and switching probabilities between reset points. These relationships will be exploited in Section 4 to recover sufficient statistics in the microdata, as m's are not observed, but the objects on the right-hand side are.

Proposition 3. ***(Expected sum of deviations)*** *The expected sum of deviations after upsizing $\mathcal{M}(\hat{k}^{*-}) \equiv m(\hat{k}^{*-}) + \mathbb{C}ov[\hat{k}, a]$ and after downsizing $\mathcal{M}(\hat{k}^{*+}) \equiv m(\hat{k}^{*+}) + \mathbb{C}ov[\hat{k}, a]$ are equal to*

$$\mathcal{M}(\hat{k}^{*-}) \;=\; (\mathbb{E}^-[\hat{k}] - \mathbb{E}[\hat{k}])\; \overline{\mathbb{E}}^-[\tau]\; \frac{\mathbb{E}[\mathbb{P}^+(\hat{k})]}{\mathbb{P}^{-+}} \;<\; 0 \tag{47}$$

$$\mathcal{M}(\hat{k}^{*+}) \;=\; (\mathbb{E}^+[\hat{k}] - \mathbb{E}[\hat{k}])\; \overline{\mathbb{E}}^+[\tau]\; \frac{\mathbb{E}[\mathbb{P}^-(\hat{k})]}{\mathbb{P}^{+-}} \;>\; 0, \tag{48}$$

where the average downsizing and upsizing probabilities are equal to

$$\mathbb{E}[\mathbb{P}^-(\hat{k})] = \frac{\overline{\mathbb{E}}\left[\tau' \mathbb{1}_{\{\hat{k}_{\tau'} = \hat{k}^-\}}\right]}{\overline{\mathbb{E}}[\tau]}, \qquad \mathbb{E}[\mathbb{P}^+(\hat{k})] = \frac{\overline{\mathbb{E}}\left[\tau' \mathbb{1}_{\{\hat{k}_{\tau'} = \hat{k}^+\}}\right]}{\overline{\mathbb{E}}[\tau]}. \tag{49}$$

Relative to inactive firms, upsizing firms (47) expect a negative deviation of size $\mathbb{E}^-[\hat{k}] - \mathbb{E}[\hat{k}]$ during the next inaction spell, which is expected to last $\overline{\mathbb{E}}^-[\tau]$ periods. Since the investment sign is serially correlated, upsizing firms remain in an upsizing phase and contribute to negative deviations for several periods; they would only leave this phase after a series of adverse shocks cause them to downsize. The ratio $\mathbb{E}[\mathbb{P}^{+}(\hat{k})]/\mathbb{P}^{-+}$ precisely reflects the expected time spent in the transient upsizing phase, where $\mathbb{E}[\mathbb{P}^{+}(\hat{k})]$ is the expected probability of downsizing and $\mathbb{P}^{-+}$ in (28) is the probability of switching phase from upsizing to downsizing. If they switch, they will enter a persistent downsizing phase with positive deviations as in (48). The explanation for downsizing firms is analogous, mutatis mutandis. The mappings in (49) show that average probabilities of upsizing and downsizing equal truncated expectations of durations.

3.5 Latent Deviations

Finally, we state the last ingredient needed to derive the CIR's sufficient statistics. The two numbers, $\mathcal{M}(\hat{k}^{*-})$ in (47) and $\mathcal{M}(\hat{k}^{*+})$ in (48), summarize the behavior of adjusting firms. However, to study the effects of aggregate shocks, we must consider that some firms will switch their investment strategy after the aggregate shock, from purchasing to selling or vice versa. To do that, we construct a new function $\mathcal{M}(\hat{k})$ that takes these two values in the outer inaction regions (where irreversibility plays no role) and then extends it to the inner inaction (where irreversibility plays a significant role) by imposing continuity. Concretely, we consider $\mathcal{M}(\hat{k}) \in \mathbb{C}^2$ to be *any* twice continuously differentiable function in the domain $[\hat{k}^{+}, \hat{k}^{-}]$ that takes two values in the outer inaction regions:

$$\mathcal{M}(\hat{k}) \;\equiv\; \begin{cases} m(\hat{k}^{*-}) \;+\; \mathbb{C}ov(\hat{k}, a) \;<\; 0 & \text{if } \hat{k} \in [\hat{k}^{-}, \hat{k}^{*-}] \\ m(\hat{k}^{*+}) \;+\; \mathbb{C}ov(\hat{k}, a) \;>\; 0 & \text{if } \hat{k} \in [\hat{k}^{*+}, \hat{k}^{+}]. \end{cases} \tag{50}$$

In the inner inaction region, $\mathcal{M}(\hat{k})$ takes values that ensure continuity up to the second derivative.

3.6 The CIR's Sufficient Statistics

To recap, we have transformed the CIR of capital-productivity ratios following an aggregate productivity shock in an environment with irreversibility—a complicated dynamic object—into a steady-state cross-sectional average of the recursive function $m(\hat{k})$—a static object. We have also discussed how active and inactive firms must balance in a steady state and defined the function $\mathcal{M}(\hat{k})$ that reflects latent deviations. With all these elements, Proposition 4 derives sufficient statistics—cross-sectional steady-state moments of $\hat{k}$ and a—that characterize the CIR.

Proposition 4. ***(Sufficient statistics)*** *Up to the first order, the CIR of average capital-productivity ratios equals the sum of three steady-state cross-sectional moments:*

$$\frac{CIR(\delta)}{\delta} \;=\; \underbrace{\frac{\mathbb{V}ar[\hat{k}]}{\sigma^2} \;+\; \frac{\nu \mathbb{C}ov[\hat{k}, a]}{\sigma^2}}_{\text{up to first adjustment}} \;+\; \underbrace{\frac{1}{\sigma^2}\,\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s[\mathrm{d}(\hat{k}_s \mathcal{M}(\hat{k}_s))]\right]}_{\text{subsequent adjustments}} \;+\; o(\delta). \tag{51}$$

According to (51), three sufficient statistics determine the CIR: (i) the cross-sectional variance of capital-productivity ratios $\mathbb{V}ar[\hat{k}]$, (ii) the covariance of capital-productivity ratios and age $\mathbb{C}ov[\hat{k}, a]$, and (iii) the average local drift of the product $\mathcal{M}(\hat{k})\hat{k}$, all divided by the idiosyncratic volatility σ^2. These cross-sectional moments are indeed sufficient statistics because an econometrician equipped with the joint distribution of $(\hat{k}, a)$ could back out the CIR from it.[15]

The three sufficient statistics reflect how, in a steady state, idiosyncratic productivity shocks shape incomplete spells and complete spells. Since idiosyncratic shocks $\mathrm{d}W_s$ and aggregate shocks δ enter $\mathrm{d}\hat{k}$ symmetrically, responsiveness to idiosyncratic shocks, encoded in steady-state moments, informs about responsiveness to aggregate shocks, encoded in the CIR. We apply this "responsiveness" principle to dissect the sufficient statistics.

Insensitivity of incomplete spells to productivity shocks The first two statistics indicate firms' willingness to tolerate deviations from their frictionless optimum and thus remain inactive. We follow Baley and Blanco (2021) to explain why this is the case. At any date s, consider the cumulative productivity shock (sum of innovations) received by a firm while inactive, normalized by volatility $\tilde{W}_s \equiv (W_s - W_{s-a_s})/\sigma$. Define the economy's *insensitivity of incomplete spells to productivity shocks* as the covariance of capital-productivity ratios and cumulative shocks: $\mathbb{C}ov[\hat{k}, \tilde{W}]$. Intuitively, if firms are extremely sensitive to productivity shocks, they continuously adjust their capital-productivity ratio to the reset points, yielding $\mathbb{C}ov[\hat{k}, \tilde{W}] = 0$ in the cross-section. Instead, if firms are insensitive, they allow $\hat{k}$ to move with shocks $\tilde{W}$, and this covariance becomes prominent.

Let us link the insensitivity to productivity shocks to the CIR. For simplicity, let us assume away the price wedge. The capital-productivity ratio of any firm at date s can be written as $\hat{k}_s = \hat{k}^* - \nu a_s + \sigma^2 \tilde{W}_s$. Subtracting the mean $\mathbb{E}[\hat{k}]$ on both sides, multiplying by $(\hat{k}_s - \mathbb{E}[\hat{k}])$, and taking the cross-sectional average, we obtain $\mathbb{V}ar[\hat{k}] = -\nu \mathbb{C}ov[\hat{k}, a] + \sigma^2 \mathbb{C}ov[\hat{k}, \tilde{W}]$. Rearranging yields $\mathbb{C}ov[\hat{k}, \tilde{W}] = (\mathbb{V}ar[\hat{k}] - (-\nu \mathbb{C}ov[\hat{k}, a]))/\sigma^2$, which is exactly the expression for the CIR's first two statistics.

It is tempting to claim that the variance of capital-productivity ratios is a sufficient indication of pervasive investment frictions and insensitivity to productivity shocks. But there is a caveat. The variance also reflects dispersion in capital-productivity ratios generated by the drift, which is unrelated to productivity shocks. The second statistic, the covariance of capital-productivity ratios and age, is, in effect, a bias correction term that ensures we accurately measure the reaction to the Brownian shocks by eliminating the drift effects (clearly, when $\nu = 0$, there is no need for correction and the first statistic is sufficient.)

How does the price wedge affect these two statistics? The price wedge ω increases $\mathbb{V}ar[\hat{k}]$ by

[15] Importantly, $\hat{k}$ is not directly observable since it depends on productivity, but under certain assumptions over the production technology, it can be recovered using revenue data (Hsieh and Klenow, 2009). Section 4 proposes an alternative to measure these statistics by exploiting exclusively investment data.

introducing a new layer of heterogeneity linked to the distinct reset points, and it may dramatically change $\mathbb{C}ov[\hat{k}, a]$. By imposing a downward rigidity, unproductive firms tend to have older capital stock and have capital-productivity ratios above the average, which pushes $\mathbb{C}ov[\hat{k}, a]$ to be positive. The sign of the covariance eventually depends on the relative size of the drift (a negative drift favors $\mathbb{C}ov[\hat{k}, a] < 0$) and the price wedge (a positive wedge favors $\mathbb{C}ov[\hat{k}, a] > 0$). When the price wedge dominates, $\mathbb{C}ov[\hat{k}, a] > 0$, the CIR rises, and aggregate fluctuations persist longer.[16]

Insensitivity of complete spells to productivity shocks The third sufficient statistic, exclusive to the irreversibility case, indicates how productivity shocks change the anticipated ending of a complete spell. In other words, if firms anticipated ending their inaction by purchasing $\mathcal{M}(\hat{k}^{*-})$ or selling $\mathcal{M}(\hat{k}^{*+})$ and then a shock makes them anticipate a different ending. If idiosyncratic shocks do not change the anticipated ending in a steady state, then an aggregate shock wouldn't either, and we wouldn't see the dynamics of renewal weights shown in Panel B of Figure IV.

While the expression for the third statistic appears complicated at first sight, we shed light on it by mirroring the explanation of the first two. At any date s, consider a firm's anticipated terminal condition $\mathbb{E}_s[\mathcal{M}(\hat{k}_\tau)]$. We define the economy's *insensitivity of complete spells to productivity shocks* as the covariance between the *change* in capital-to-productivity ratios $\mathrm{d}\hat{k}_s$ and the *change* in anticipated terminal conditions normalized by volatility and averaged across the population: $(1/\sigma^2)\mathbb{E}\left[\mathbb{C}ov_s\left[\mathrm{d}\hat{k}_s, \mathrm{d}\mathbb{E}_s[\mathcal{M}(\hat{k}_\tau)]\right] / \mathrm{d}s\right]$. At the individual level, the covariance is time-varying as it depends on the state. Intuitively, if terminal values are insensitive (e.g., firms always expect to end up buying), productivity shocks do not alter the spell's ending, and this covariance is zero. In contrast, if terminal values are very sensitive, a small $\mathrm{d}\hat{k}$ triggers a big change in the anticipated ending. In such sensitive cases, the renewal weights react strongly to an aggregate shock, generating additional persistence.

As before, there is a caveat. We must be careful in only capturing sensitivity to productivity shocks, not mechanical drift effects. The drift affects both variables, as it reduces capital-productivity ratios and makes upsizing endings at $\mathcal{M}(\hat{k}^{*-})$ more likely. To account for this, we must correct our definition of insensitivity. Let $D_s \equiv \mathbb{E}_s[\mathcal{M}_\tau] - \mathcal{M}_s$ be the expected change in $\mathcal{M}_s$ until the next adjustment. Subtracting the drift of the product $\mathrm{d}(\hat{k}_s D_s)$ from the covariance, which includes $\hat{k}$'s drift ν and D_s's drift, and rearranging, we can express the third sufficient statistic as the unbiased insensitivity of complete spells to productivity shocks in the population:

$$\frac{1}{\sigma^2}\mathbb{E}\left[\frac{\mathbb{E}_s[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))]}{\mathrm{d}s}\right] = \mathbb{E}\left[\frac{\mathbb{C}ov_s[\mathrm{d}\hat{k}_s, \mathrm{d}\mathbb{E}_s[\mathcal{M}_\tau]] - \mathbb{E}_s[\mathrm{d}(\hat{k}_s D_s)]}{\mathbb{V}ar_s[\mathrm{d}\hat{k}_s]}\right]. \tag{52}$$

[16]In Baley and Blanco (2021), we showed that higher fixed costs for downward than upward adjustments also generate a positive covariance and hence increase the CIR. However, asymmetric fixed costs imply one reset point, and the third sufficient statistic remains zero.

Finally, if we write idiosyncratic volatility in the denominator as $\sigma^2\,\mathrm{d}s = \mathbb{V}ar_s[\mathrm{d}\hat{k}_s]$, expression (52) is just the coefficient of an OLS regression of $\mathrm{d}\mathbb{E}_s[\mathcal{M}_\tau]$ onto $\mathrm{d}\hat{k}_s$, with a bias correction.

3.7 Irreversibility's Role for Sufficient Statistics

Before shifting gears to empirical and quantitative applications, we discuss how irreversibility shapes the CIR's sufficient statistics and establish connections with the literature. Irreversibility plays two different roles in the propagation of aggregate shocks. First, it directly impacts the CIR by shifting the masses of upward and downward adjustments, affecting investment behavior *after* the first adjustment captured by the third sufficient statistic. Second, as a source of downward rigidity, irreversibility indirectly impacts the CIR by turning positive the covariance of capital-productivity ratios and age, affecting investment behavior *before* the first adjustment captured by the first sufficient statistics. We highlight each channel by exploring extreme cases.

Irreversibility's direct effect To showcase irreversibility's direct impact on the CIR, Proposition 5 considers a zero-drift environment where the direct impact is maximal (the covariance is inherently zero in cases (i) and (ii) below) and an infinite-drift limit where the direct impact is minimal (the covariance is negative and largest in absolute value in case (iii)). We derive analytical expressions for the CIR in terms of investment frictions rescaled by the user cost $\mathcal{U}$ and other parameters.

Proposition 5. ***(Extreme cases)*** *Up to the first order, the CIR's sufficient statistics as a function of investment frictions are as follows.*

(i) No drift and only fixed cost: If $\nu = \omega = 0$ and $\theta > 0$, then

$$\text{(53)} \qquad \frac{CIR(\delta)}{\delta} = \frac{\mathbb{V}ar[\hat{k}]}{\sigma^2} = \left(\frac{12\tilde{\theta}}{(1-\alpha)\sigma^6}\right)^{1/4}, \quad \text{where} \quad \tilde{\theta} = \frac{\theta}{\alpha}\left(\frac{p\mathcal{U}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}.$$

(ii) No drift and only partial irreversibility: If $\nu = \theta = 0$ and $\omega > 0$, then

$$\text{(54)} \qquad \frac{CIR(\delta)}{\delta} = 2\times\frac{\mathbb{V}ar[\hat{k}]}{\sigma^2} = \left(\frac{12\tilde{\omega}}{(1-\alpha)\sigma^4}\right)^{1/3}, \quad \text{where} \quad \tilde{\omega} = \frac{\omega/2}{\mathcal{U}(1-\omega/2)}.$$

(iii) Large drift: If $\sigma^2 > 0$ and $\nu \to \infty$, then the price wedge is irrelevant and

$$\text{(55)} \qquad \mathbb{E}\left[\frac{\mathbb{E}_s[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))]}{\mathrm{d}s}\right] = 0, \qquad \nu\mathbb{C}ov[\hat{k}, a] = -\mathbb{V}ar[\hat{k}], \qquad \frac{CIR(\delta)}{\delta} = 0.$$

As a baseline, case (i) assumes no drift and only a fixed cost so that the only sufficient statistic is the ratio $\mathbb{V}ar[\hat{k}]/\sigma^2$. Expression (53) shows that the rescaled fixed cost $\tilde{\theta}$ increases the inaction region, the cross-sectional variance, and the CIR with an elasticity of 1/4, resembling the results by Barro (1972) and Dixit (1991).

Case (*ii*) assumes no drift and only a price wedge. Expression (54) shows that the rescaled price wedge $\tilde{\omega}$ affects aggregate fluctuations with an elasticity of 1/3, as in Abel and Eberly (1999) and Miao (2019). The CIR equals two times the ratio $\mathbb{V}ar[\hat{k}]/\sigma^2$—obtained in case (*i*)—since the first and third sufficient statistics in (51) are identical. This means that *irreversibility doubles the persistence of aggregate fluctuations* for a given cross-sectional dispersion $\mathbb{V}ar[\hat{k}]$. In other words, the source of inaction (fixed costs vs. price wedge) matters for aggregate fluctuations.

At the other extreme, case (*iii*) considers a huge drift. As the drift unwaveringly depletes capital relative to productivity, firms exclusively make and anticipate positive investments. The price wedge becomes irrelevant as firms never face it and have no bite in the aggregate. This case was first studied by Caplin and Spulber (1987) in a price-setting context. In our framework, this mechanism gets captured in (55), where the covariance becomes so negative that it completely unwinds the variance, and the local drift is zero. Aggregate shocks are immediately absorbed, there are no deviations from the steady state, and the CIR equals zero.[17]

Irreversibility's indirect effect The extreme cases above highlight that irreversibility's direct effect on the CIR depends on the drift. However, irreversibility's indirect impact on the CIR is muted in those cases. Next, we numerically explore cases with a moderate drift of $\nu = 0.07$ and price wedges in the range $\omega \in [0.0, 0.25]$ to showcase both the direct and indirect effects.[18] For a consistent comparison across economies, in the spirit of Hsieh and Klenow (2009), we fix the cross-sectional dispersion of capital-productivity ratios $\mathbb{V}ar[\hat{k}]$ to the value obtained for $\theta = 0$ and $\omega = 0.25$. Then, for other values of the wedge, we find the fixed cost θ that delivers the same level of dispersion. Fixing $\mathbb{V}ar[\hat{k}]$ implicitly fixes the adjustment frequency and the dispersion of adjustments across configurations.[19]

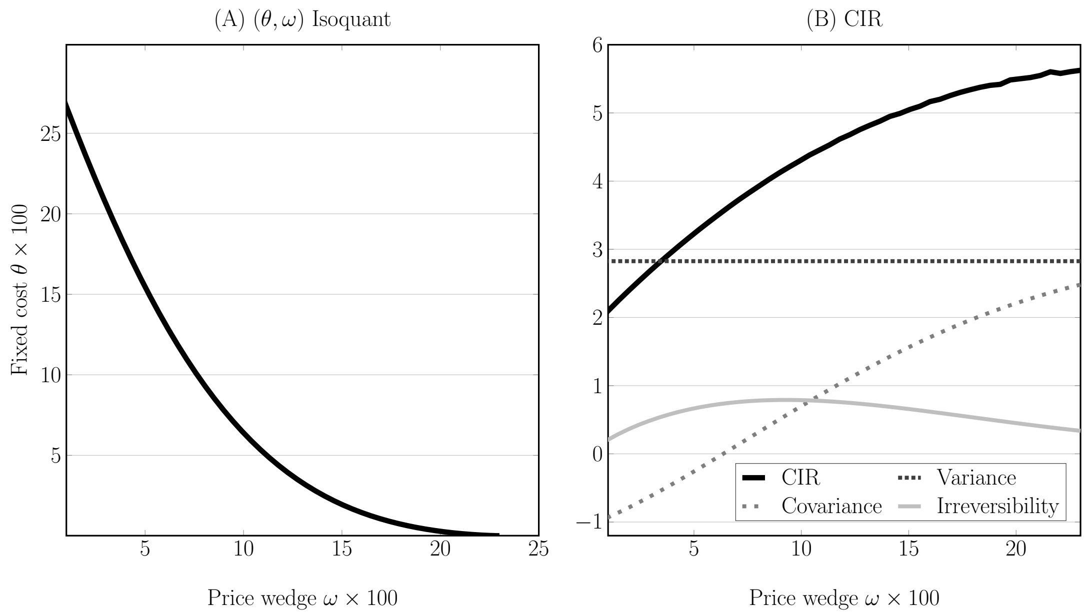
Panel A of Figure V plots the (θ, ω)-isoquant, which is convex. Going from left to right increases the relative importance of the price wedge vis-à-vis the fixed cost while delivering the same cross-sectional dispersion $\mathbb{V}ar[\hat{k}]$. Panel B of Figure V plots the CIR and its three sufficient statistics against the price wedge ω, computed along the isoquant. The CIR (solid black line) increases as the price wedge dominates—capital fluctuations are more persistent. Since $\mathbb{V}ar[\hat{k}]/\sigma^2$ remains fixed (flat dotted line), changes in the other two sufficient statistics generate all the action. The covariance $\mathbb{C}ov[\hat{k}, a]$ (gray dotted line) starts negative at $\omega = 0$ (as case (*iii*) of Proposition 5) as the drift makes old capital-productivity ratios negative. It becomes positive for $\omega > 0.08$ as the downsizing constraints imposed by the price wedge kick in (recall the discussion in Section

[17] Baley and Blanco (2021) shows an analogous result for asymmetric fixed costs and zero price wedge.

[18] Miao (2019) studies the case with full irreversibility ($\omega = 1$) for any drift $\nu \in \mathbb{R}$.

[19] In the quantitative model of Section 5, we match the empirical distribution of investment for each choice of ω. However, the key messages from this section continue to hold.

Figure V: Sufficient Statistics for Different Price Wedges and Fixed Allocation



Notes: Panel A shows the (θ, ω) isoquant, and Panel B the CIR and its components for an illustrative calibration.

3.4). The local drift (dotted gray line) neatly encapsulates the direct irreversibility effect, which is non-monotonic. The local drift dominates the (negative) covariance for low wedges; the (positive) covariance dominates the local drift for high wedges. Overall, we see that the price wedge impacts the CIR's level and the relative contribution of each sufficient statistic. In Section 5 we exploit this result to calibrate the price wedge.

4 Measuring Sufficient Statistics with Microdata

The challenge to analyzing the role of irreversibility on the CIR's sufficient statistics stems from the fact that these moments cannot be directly computed from the data as the distributions of capital-productivity ratios $g(\hat{k})$ and $g^{\pm}(\hat{k})$ are not directly observed. As economists, however, we have available detailed panel data with information on the actions of adjusters: the size of adjustments $(\Delta\hat{k})$ and the duration of complete (τ) and incomplete (a) inaction spells. In this section, we derive mappings from microdata to parameters, optimal investment policies, and sufficient statistics, assuming the price wedge ω as given. Section 5 combines these mappings with a fully-fledged quantitative investment model to discipline the price wedge.

Two-state strategy To measure sufficient statistics from panel microdata, we proceed in two steps. In Stage I, we assume we know the two reset points. We then apply structural relationships to the data linking the behavior of adjusting and non-adjusting firms to reverse-engineer the parameters of the productivity process and steady-state cross-sectional moments.

In Stage II, we use the data and the model's structure to reverse-engineer the reset points. To obtain these mappings, we condition adjusters' behavior on the direction of the previous adjustment so that their actions remain Markovian. Throughout, we exploit the properties of Markov processes and the fact that the two reset points are constant.

4.1 Stage I: Mappings Given the Reset Points

We take the two reset points $\{\hat{k}^{*-}, \hat{k}^{*+}\}$ as given and use the adjusters' expectations conditional on the previous reset point $\overline{\mathbb{E}}^{\pm}[\cdot]$ to back out parameters and cross-sectional moments. For this, we require information on consecutive inaction spells $\{(\Delta\hat{k}, \tau), (\Delta\hat{k}', \tau')\}$ to condition future behavior on the previous reset point.

For each firm's completed inaction spell $(\Delta\hat{k}, \tau)$, if the firm upsized its capital stock $\Delta\hat{k} > 0$ we record the reset point as $\hat{k}^* = \hat{k}^{*-}$ and construct the stopped capital as $\hat{k}_\tau = \hat{k}^{*-} - \Delta\hat{k}$; if the firm downsized its capital stock $\Delta\hat{k} < 0$, we record the reset point as $\hat{k}^* = \hat{k}^{*+}$ and the stopped capital as $\hat{k}_\tau = \hat{k}^{*+} - \Delta\hat{k}$. Similarly, we record the future reset point $\hat{k}^{*\prime}$ and the future stopped capital $\hat{k}_{\tau'}$ using the information from the subsequent spell $(\Delta\hat{k}', \tau')$.

We recover the parameters of the capital-productivity process in Proposition 6. Proposition 7 recovers conditional and unconditional means of $\hat{k}$. Proposition 8 recovers the cross-sectional variance of $\hat{k}$ and their covariance with age. Finally, Proposition 9 recovers the irreversibility's term in the CIR. We present the mappings for these objects separately to facilitate exposition, but they should be recovered simultaneously through a non-linear system of equations (details below).

Throughout, we exploit the relationship between conditional and unconditional moments of adjustments presented in (24): $\overline{\mathbb{E}}[y] = \frac{\mathcal{N}^-}{\mathcal{N}}\overline{\mathbb{E}}^-[y] + \frac{\mathcal{N}^+}{\mathcal{N}}\overline{\mathbb{E}}^+[y]$, where $y \in \{\hat{k}_{\tau'}, \tau'\}$.

Proposition 6. ***(Recovering parameters)*** *The drift ν and volatility σ^2 of capital-productivity ratios implied by investment microdata are recovered through the following mappings:*

$$\nu = \frac{\overline{\mathbb{E}}[\Delta\hat{k}]}{\overline{\mathbb{E}}[\tau]}, \tag{56}$$

$$\sigma^2 = \frac{\overline{\mathbb{E}}[(\hat{k}_{\tau'} + \nu\tau')^2 - (\hat{k}^*)^2]}{\overline{\mathbb{E}}[\tau]}. \tag{57}$$

Expression (56) recovers the drift $\nu = \xi + \mu$ using the average adjustment size $\overline{\mathbb{E}}[\Delta\hat{k}]$ times the adjustment frequency—the inverse of the expected duration of inaction $\overline{\mathbb{E}}[\tau] = \mathcal{N}^{-1}$. It uses the fact that, in a stationary environment, the average adjustment size $\overline{\mathbb{E}}[\Delta\hat{k}]$ must compensate for the average drift between adjustments, i.e., $\nu\overline{\mathbb{E}}[\tau]$. Expression (57) recovers idiosyncratic risk σ^2 from the difference in future and past resets squared, also scaled by the adjustment frequency. This difference reflects dispersion in adjustment size, accounting for potential shifts in reset points.[20] In Baley and Blanco (2021), we obtained related mappings from the data to the parameters without irreversibility. Irreversibility does not change the drift mapping, but it changes the volatility mapping because it affects transitions across reset points.

Proposition 7. ***(Recovering means)*** *Let $r^{\pm}$ be the adjusted shares in (25). The unconditional mean $\mathbb{E}[\hat{k}]$ and means conditional on the previous reset $\mathbb{E}^{\pm}[\hat{k}]$ are recovered as:*

$$\mathbb{E}[\hat{k}] \;=\; r^{-}\mathbb{E}^{-}[\hat{k}] \;+\; r^{+}\mathbb{E}^{+}[\hat{k}], \tag{58}$$

$$\mathbb{E}^{\pm}[\hat{k}] \;=\; \overline{\mathbb{E}}^{\pm}\left[\left(\frac{\hat{k}^{*\pm}+\hat{k}_{\tau'}}{2}\right)\left(\frac{\hat{k}^{*\pm}-\hat{k}_{\tau'}}{\overline{\mathbb{E}}^{\pm}[\hat{k}^{*\pm}-\hat{k}_{\tau'}]}\right)\right] \;+\; \frac{\sigma^2}{2\nu}. \tag{59}$$

The unconditional mean in (58) is the weighted average of the conditional means using adjusted shares $r^{\pm}$. The conditional means $\mathbb{E}^{\pm}[\hat{k}]$ in (59) are recovered from the middle point between the departing and the ending points of an inaction spell $(\hat{k}^{*\pm}+\hat{k}_{\tau'})/2$, weighed by relative adjustment size.[21] The term $\sigma^2/2\nu$ corrects for the accumulated drift between adjustments.

Proposition 8. ***(Recovering the variance and covariance)*** *The variance $\mathbb{V}ar[\hat{k}]$ and the covariance $\mathbb{C}ov[\hat{k},a]$ are recovered from the microdata as:*

$$\mathbb{V}ar[\hat{k}] \;=\; \frac{1}{3}\frac{\mathbb{E}\left[(\hat{k}^{*}-\mathbb{E}[\hat{k}])^3\right]-\overline{\mathbb{E}}\left[(\hat{k}_{\tau'}-\mathbb{E}[\hat{k}])^3\right]}{\hat{k}^{*}-\overline{\mathbb{E}}[\hat{k}_{\tau'}]}. \tag{60}$$

$$\mathbb{C}ov[\hat{k},a] \;=\; \frac{1}{2\nu}\left(\mathbb{V}ar[\hat{k}] \;+\; \sigma^2\mathbb{E}[a] \;-\; \frac{\overline{\mathbb{E}}\left[(\hat{k}_{\tau'}-\mathbb{E}[\hat{k}])^2\tau'\right]}{\overline{\mathbb{E}}[\tau]}\right). \tag{61}$$

The variance in (60) is recovered from the difference in the *cubes* of the departing and the ending points of an inaction spell, which reflects skewness in adjustments, divided by the difference between departing and ending points to express it in variance units. The covariance $\mathbb{C}ov[\hat{k},a]$ in (61) is recovered from the sum of the variance $\mathbb{V}ar[\hat{k}]$, average age $\mathbb{E}[a]$, and the product between stopped capital squared and stopping times. We stress that, for a model to match the covariance, it must match that latter dynamic moment, which is critical to correctly identifying the dynamic effects of irreversibility.

[20] Note that $\overline{\mathbb{E}}[(\hat{k}^{*})^2] = \frac{\mathcal{N}^{-}}{\mathcal{N}}(\hat{k}^{*-})^2 + \frac{\mathcal{N}^{+}}{\mathcal{N}}(\hat{k}^{*+})^2$.

[21] The second term in the product inside the expectation relates to the renewal measure again. Without irreversibility, this term collapses to $\Delta k'/\overline{\mathbb{E}}[\Delta \hat{k}']$, the adjustment size relative to the average adjustment. Midpoints of firms with larger adjustments receive a more prominent weight. With irreversibility, the numerator and denominator consider the different reset points, but effectively, it increases the weight on larger adjustments.

Proposition 9. ***(Recovering the irreversibility term)*** *The CIR's irreversibility term is recovered from the microdata as*

$$\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s\left[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))\right]\right] \;=\; \frac{\overline{\mathbb{E}}[\hat{k}_{\tau'}\mathcal{M}(\hat{k}_{\tau'})] - \overline{\mathbb{E}}[\hat{k}^*\mathcal{M}(\hat{k}^*)]}{\overline{\mathbb{E}}[\tau]}, \tag{62}$$

where departing deviations $\mathcal{M}(\hat{k}^{*\pm})$ and ending deviations $\mathcal{M}(\hat{k}_{\tau'})$ are recovered in Proposition 3.

According to (62), the third sufficient statistic equals the difference in the expected deviations between departing and ending points. If there was one reset point, both numbers equal $\overline{\mathbb{E}}[\hat{k}^*\mathcal{M}(\hat{k}^*)]$ and the statistic is zero. If there were no fixed costs, then $\hat{k}_{\tau'} = \hat{k}^{*\prime}$, the numerator becomes $\overline{\mathbb{E}}[\hat{k}^{*\prime}\mathcal{M}(\hat{k}^{*\prime})] - \overline{\mathbb{E}}[\hat{k}^*\mathcal{M}(\hat{k}^*)]$, and only consecutive adjustments that switch their ending point (from upsizing to downsizing or vice versa) matter. The larger the difference in resets, transition probabilities, or deviations, the more irreversible the investment, and the larger this statistic.

4.2 Stage II: Recovering the Reset Points

In Stage II, we recover the two reset points. We still take the price wedge ω as given. Evaluating q's stopping-time formulation in (16) at the reset points, we obtain the following conditions linking the optimal stopping policy τ^* and the optimal reset points $\{\hat{k}^{*-}, \hat{k}^{*+}\}$:

$$p \;=\; \overline{\mathbb{E}}^{-}\left[\int_0^{\tau^*} \alpha e^{-\mathcal{U}s-(1-\alpha)\hat{k}_s}\,\mathrm{d}s \;+\; p(\Delta\hat{k}')\; e^{-\mathcal{U}\tau^*}\right], \tag{63}$$

$$p(1-\omega) \;=\; \overline{\mathbb{E}}^{+}\left[\int_0^{\tau^*} \alpha e^{-\mathcal{U}s-(1-\alpha)\hat{k}_s}\,\mathrm{d}s \;+\; p(\Delta\hat{k}')\; e^{-\mathcal{U}\tau^*}\right]. \tag{64}$$

These expressions say that adjusting firms reset their capital-productivity ratios to equalize marginal costs and expected marginal benefits. The marginal cost is the investment price, either p if buying or $(1-\omega)p$ if selling. The expected marginal benefit is the cumulative marginal product of capital generated during the inaction spell (between date 0 and τ^*) plus the expected value of undepreciated capital upon adjustment. Expectations depend on the past reset. Proposition 10 uses these expressions to derive mappings from microdata to reset points. It extends the formula for the frictionless case in (17) to include a fixed cost and a price wedge.

Proposition 10. ***(Recovering reset points)*** *Let $\Phi \equiv \log\left(\alpha/(\mathcal{U} - (1-\alpha)\nu - (1-\alpha)^2\sigma^2/2)\right)$. The two reset points $\{\hat{k}^{*-}, \hat{k}^{*+}\}$ are recovered from the microdata as:*

$$\hat{k}^{*-} = \frac{1}{1-\alpha}\left(\Phi - \log p + \log \frac{1-\overline{\mathbb{E}}^{-}\left[e^{-\mathcal{U}\tau^{*} + (1-\alpha)(\hat{k}^{*-}-\hat{k}_{\tau'})}\right]}{1-\overline{\mathbb{E}}^{-}\left[\frac{p(\Delta\hat{k}')}{p}e^{-\mathcal{U}\tau^{*}}\right]}\right), \tag{65}$$

$$\hat{k}^{*+} = \frac{1}{1-\alpha}\left(\Phi - \log p(1-\omega) + \log \frac{1-\overline{\mathbb{E}}^{+}\left[e^{-\mathcal{U}\tau^{*} + (1-\alpha)(\hat{k}^{*+}-\hat{k}_{\tau'})}\right]}{1-\overline{\mathbb{E}}^{+}\left[\frac{p(\Delta\hat{k}')}{p(1-\omega)}e^{-\mathcal{U}\tau^{*}}\right]}\right). \tag{66}$$

Recalling $\mathcal{U} = r + \xi$ and $\nu = \mu + \xi$, the first constant term Φ reveals that reset points increase with productivity growth μ but decrease with the discount factor r and the depreciation rate ξ. Higher idiosyncratic risk σ^2 shifts reset points to the right, implying a larger average investment. This effect highlights the fact that firms can expand to exploit good outcomes and contract to insure against bad outcomes, making them potentially risk-loving (Oi, 1961; Hartman, 1972; Abel, 1983). The second term shows that reset points decrease with the corresponding investment price: Firms invest more the lower the purchasing price p and disinvest less the lower the selling price $p(1-\omega)$. Lastly, the third term reflects how irreversibility shapes the reset points through the expected marginal profits accrued during periods of inaction (in the numerator) and the expected user cost $\mathcal{U}$ (in the denominator).

As a measure of irreversibility, consider the difference between reset points: $(\hat{k}^{*+} - \hat{k}^{*-})$:

$$\frac{1}{1-\alpha}\left(\underbrace{\log\frac{1}{1-\omega}}_{\text{exogenous}} + \underbrace{\log\frac{1-\overline{\mathbb{E}}^{+}\left[e^{-\mathcal{U}\tau^{*} + (1-\alpha)(\hat{k}^{*+}-\hat{k}'_{\tau})}\right]}{1-\overline{\mathbb{E}}^{-}\left[e^{-\mathcal{U}\tau^{*} + (1-\alpha)(\hat{k}^{*-}-\hat{k}'_{\tau})}\right]} - \log\frac{1-\overline{\mathbb{E}}^{+}\left[\frac{p(\Delta\hat{k}')}{p(1-\omega)}e^{-\mathcal{U}\tau^{*}}\right]}{1-\overline{\mathbb{E}}^{-}\left[\frac{p(\Delta\hat{k}')}{p}e^{-\mathcal{U}\tau^{*}}\right]}}_{\text{endogenous irreversibility}}\right). \tag{67}$$

The constant Φ and the price p cancel out in the difference. The difference naturally increases in the exogenous price wedge ω, further amplified by the output-capital elasticity α.[22] The other two ratios reflect history dependence on the expected marginal product and the user cost. As long as the optimal policy depends on the previous reset, endogenous irreversibility arises.

4.3 Establishment-Level Manufacturing Data

We apply the mappings using yearly investment data on manufacturing establishments in Chile.

Data sources Data comes from the Annual National Manufacturing Survey (*Encuesta Nacional Industrial Anual*) for the period 1980 to 2011. We use information on depreciation rates and price deflators from national accounts and Penn World Tables to construct the capital series. The sample considers plants that appear in the sample for at least ten years (more than 60% of the

[22] In the quantitative section, we discuss the challenge of identifying ω from α.

sample) and have more than ten workers. We keep all pairs of consecutive adjustments $(\Delta k, \tau)$ and $(\Delta k', \tau')$ for each firm. Appendix E presents all the data details.

Capital stock and investment rates We construct the capital stock series using the perpetual inventory method. We include structures, machinery, equipment, and vehicles. A plant's capital stock in year s evolves as

$$k_s \;=\; (1-\xi)k_{s-1} \;+\; \frac{I_s}{p(I_s)D_s}, \tag{68}$$

where ξ is the physical depreciation rate; I_s is the nominal value of the investment; $p(I_s)$ is the investment pricing function, which considers different prices for capital purchases p and sales $p(1-\omega)$, for a given wedge ω; D_s is the gross fixed capital formation deflator; and k_0 is a plant's self-reported nominal capital stock at current prices for the first year in which it is nonnegative. Note that the ratio $I_s/(p(I_s)D_s)$ is the real investment in capital units (the data counterpart to $i_s = \Delta k_s$ in the model), and it is affected by the price wedge.

Constructing investment rates We construct the gross nominal investment i_s with information on purchases, reforms, improvements, and fixed asset sales. We define the investment rate ι_s as the ratio of real gross investment to the capital stock:[23]

$$\iota_s \;\equiv\; \frac{I_s/(p(I_s)D_s)}{k_{s-1}}. \tag{69}$$

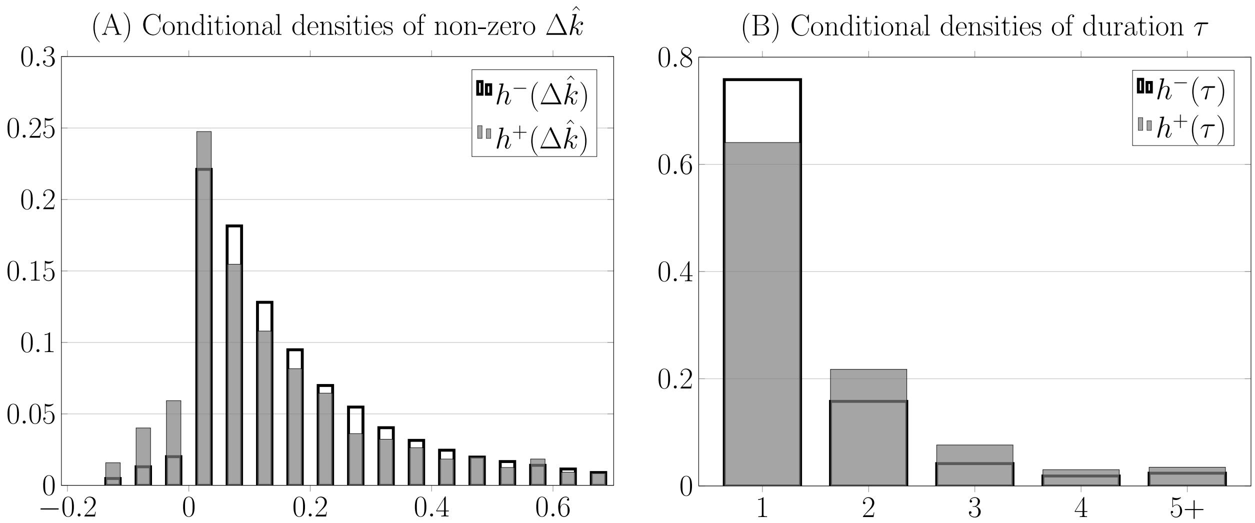
For each plant and each inaction spell h, we record the change in the capital-productivity ratio upon action $\Delta\hat{k}_h$ and the spell's duration τ_h. We construct $\Delta\hat{k}_h$ with investment rates from (69):

$$\Delta\hat{k}_h \;=\; \begin{cases} \log\left(1+\iota_h\right) & \text{if} \quad |\iota_h| > \underline{\iota}, \\ 0 & \text{if} \quad |\iota_h| < \underline{\iota}. \end{cases} \tag{70}$$

The threshold $\underline{\iota} > 0$ reflects the idea that small maintenance investments should be excluded. Following Cooper and Haltiwanger (2006), we set $\underline{\iota} = 0.01$, such that all investment rates below 1% in absolute value are considered inaction. We define an adjustment date T_h using $\Delta\hat{k}_{T_h} \neq 0$ and compute a spell's duration as the difference between two adjacent adjustment dates: $\tau_h = T_h - T_{h-1}$. Finally, we truncate the investment distribution at the 2^{nd} and 98^{th} percentiles to eliminate outliers.

[23] The investment rate equals $\iota_{T_h} \equiv i_{T_h}/k_{T_h^-} = (k_{T_h} - k_{T_h^-})/k_{T_h^-}$, where $k_{T_h^-} = \lim_{s\uparrow T_h} k_s$. In contrast to the continuous-time model, in which investment is computed as the difference in the capital stock between two consecutive instants, in the data, we compute it as the difference between two consecutive years. Potentially, a bias could arise from time aggregation as we take a continuous time model to annual data. We leave the assessment of this bias for further research.

Figure VI: Empirical Densities of Observable Actions



Notes: Panel A plots the distribution of non-zero changes in capital-productivity ratios, and Panel B plots the duration of inaction spells. Light bars = conditional on previous purchase ($\hat{k}_\tau = \hat{k}^{*-}$); Dark bars = conditional on previous sale ($\hat{k}_\tau = \hat{k}^{*+}$). Sample: Firms with at least ten years of data, truncation at the 2nd and 98th percentiles of investment rate distribution, and inaction threshold of $\underline{\iota} = 0.01$.

Figure VI plots the empirical cross-sectional distribution of non-zero changes in the capital-productivity ratios $\Delta\hat{k}$ in Panel A and completed inaction spells τ in Panel B, conditional on a past sale or purchase. We obtain an inaction rate of 40%.[24] The data shows the same qualitative patterns as in Section 2.5, consistent with irreversibility. Investment distributions have few negative investments, plenty of small positive investments, and few large positive investments, and are convex as they move away from zero. Moreover, the density of investment rates conditional on a sale $h^+(\Delta k)$ is more tilted toward negative values than $h^-(\Delta k)$, which means that the probability of a sale is higher after a sale, and vice versa. The expected duration of inaction is longer after a previous sale than after a purchase. The conditional durations are $\mathbb{E}^-[\tau] = 1.72$ and $\mathbb{E}^+[\tau] = 1.98$.

4.4 Putting the Theory to Work

Before applying the mappings, we need values for a few standard parameters and a price wedge. One period is a year. We set the real interest rate to 6.6% ($r = 0.066$) to match the average real interest rate in Chile reported by the IMF. The productivity growth rate is 2.0% ($\mu = 0.02$). To set the returns to scale α, we consider a Cobb-Douglas production function with frictionless labor input ℓ: $y = u^{1-\eta\tilde{\alpha}}\left(k^{\tilde{\alpha}}\ell^{1-\tilde{\alpha}}\right)^{\eta}$. Static maximization over labor implies $y \propto k^{\frac{\eta\tilde{\alpha}}{1-(1-\tilde{\alpha})\eta}}$. Assuming standard values $\eta = 0.90$ and $\tilde{\alpha} = 0.4$, the output-capital elasticity is $\alpha = (\eta\tilde{\alpha})/(1-(1-\tilde{\alpha})\eta) = 0.85$.[25] We

[24]Table E.1 in Appendix E.6 presents additional investment rate statistics.
[25]For robustness, we conduct comparative statics on α in Appendix E.7.

normalize the price level to $p = 6$. Finally, we set the price wedge to our preferred value $\omega = 0.12$, disciplined in the next section.

We apply the mappings to the Chilean investment data to recover the productivity parameters, the two reset points, and the cross-sectional moments behind the CIR's sufficient statistics. Since all these objects are simultaneously determined, we develop an iterative method to solve the non-linear system of mappings in (56), (57), (65) and (66), substituting the population moments with their sample counterparts. Appendix E.4 provides a step-by-step guide to recovering these objects.

Table I: Mappings from microdata for $\omega = 0.12$

Inputs from microdata		
Size, Duration and Age		
Avg. Size	$\overline{\mathbb{E}}[\Delta\hat{k}]$	0.200
Avg. Duration	$\overline{\mathbb{E}}[\tau]$	1.733
Avg. Age	$\mathbb{E}[a]$	1.677
Dynamic covariances		
Covariance I	$\overline{\mathbb{E}}[(\hat{k}_{\tau'} + \nu\tau')^2]$	0.801
Covariance II	$\overline{\mathbb{E}}\left[(\hat{k}_{\tau'} - \mathbb{E}[\hat{k}])^2\tau'\right]$	0.275
Probabilities		
Adjustment frequency	$\mathcal{N}$	0.482
Purchase frequency	$\mathcal{N}^-$	0.464
Sale frequency	$\mathcal{N}^+$	0.018
Purchase renewal weight	r^-	0.952
Sale renewal weight	r^+	0.047
Purchase – purchase	$\mathbb{P}^{--}$	0.958
Sale – sale	$\mathbb{P}^{++}$	0.124
Avg. purchase prob.	$\overline{\mathbb{E}}[\mathbb{P}^-(\hat{k})]$	0.947
Avg. selling prob.	$\overline{\mathbb{E}}[\mathbb{P}^+(\hat{k})]$	0.052

Outputs from mappings		
Parameters		
Drift	ν	0.115
Volatility	σ^2	0.057
Depreciation	ξ	0.095
User cost	$\mathcal{U}$	0.161
Reset points		
Reset after purchase	$\hat{k}^{*-}$	−0.854
Reset after sale	$\hat{k}^{*+}$	−0.041
Inner inaction	$\hat{k}^{*+} - \hat{k}^{*-}$	0.813
Moments of $\hat{k}$		
Average	$\mathbb{E}[\hat{k}]$	−0.841
Avg. cond on purchase	$\mathbb{E}^-[\hat{k}]$	−0.977
Avg. cond on sale	$\mathbb{E}^+[\hat{k}]$	−0.336
Variance	$\mathbb{V}ar[\hat{k}]$	0.097
Covariance with age	$\mathbb{C}ov[\hat{k}, a]$	0.152
Local drift	$\mathbb{E}\left[\frac{\mathbb{E}_s[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))]}{\mathrm{d}s}\right]$	0.035

Notes: Mappings assume a price wedge $\omega = 0.12$, output-capital elasticity $\alpha = 0.85$, real interest rate $r = 0.066$, productivity growth $\mu = 0.02$ and purchase price $p = 6$.

Data inputs The left part of Table I reports the cross-sectional moments of adjusting firms in the microdata, which are inputs into the theory mappings. The average inaction period lasts $\overline{\mathbb{E}}[\tau] = 1.733$ years and ends with an average adjustment of $\overline{\mathbb{E}}[\Delta\hat{k}] = 0.200$. Capital has an average age of 1.677 years.

We also report key dynamic moments, shares, renewal weights, transition probabilities, and average probabilities. On average, half of firms adjust every period ($\mathcal{N} = 0.482$), from which more than 96% of firms upsize and less than 4% downsize. Upsizing is quite persistent, as the likelihood of upsizing after an upsize is $\mathbb{P}^{--} = 0.958$, whereas the probability of downsizing following a downsize is only $\mathbb{P}^{++} = 0.124$.

Theory outputs The right part of Table I reports the values of various model objects. Regarding the productivity process, investment data implies a drift of $\nu = 0.115$, which includes capital depreciation and productivity growth. Given values for μ and r, the implied capital depreciation rate is $\xi = \nu - \mu = 0.095$ and the user cost is $\mathcal{U} = r + \xi = 0.161$. We recover idiosyncratic volatility of $\sigma^2 = 0.058$, consistent with the volatility of productivity used in quantitative models.[26]

The reset points are $\hat{k}^{*-} = -0.854 < -0.041 = \hat{k}^{*+}$ and the average capital-productivity ratio is $\mathbb{E}[\hat{k}] = -0.841$. They imply that firms' capital fluctuates between 0.42 and 0.96 times their idiosyncratic productivity, with an average ratio of 0.43. Conditional on upsizing, on average, firms reset their capital to 0.37 times their productivity ($\mathbb{E}^-[\hat{k}] = -0.986$), and conditional on downsizing, they do it to 0.75 times productivity ($\mathbb{E}^+[\hat{k}] = -0.287$). The width of the inner inaction region—a direct measure of irreversibility—is given by the difference $\hat{k}^{*+} - \hat{k}^{*-} = 0.813$, out of which 45% is generated by the exogenous price wedge and the remaining 55% is generated by the endogenous response to the wedge, according to equation (67).

The dispersion of capital-productivity ratios is $\mathbb{V}ar[\hat{k}] = 0.097$. In turn, the covariance of capital-productivity ratios with capital age is positive $\mathbb{C}ov[\hat{k}, a] = 0.152 > 0$, which means that the price wedge's positive impact on the covariance dominates the drift's negative effects. Finally, the local drift that captures history dependence equals 0.035. The positive covariance and the positive local drift amplify aggregate fluctuations.

Autocorrelation in adjustment sign We assess the serial correlation in the adjustment sign in the data as additional evidence of the effect of irreversibility on plants' investment, complementing the transition probabilities across reset points. Since the adjustment sign is a binary variable, computing a simple correlation is not recommended. The standard Pearson correlation coefficient fails to capture the degree of association between two binary variables meaningfully. Instead, we run a logistic regression of $sign(\Delta\hat{k}')$ on $sign(\Delta\hat{k})$, which yields an odds ratio of 3.3. This ratio suggests it is more than three times more likely to purchase after a previous purchase than after an earlier sale.

5 A Quantitative Investment Model

In this section, we extend the adjustment cost structure of the parsimonious model introduced in Section 2 to better match the empirical investment rate distribution, specifically its dispersion and the coexistence of both large and small investment rates. To achieve this, we incorporate a generalized adjustment hazard, initially proposed by Caballero and Engel (1999, 2007) and further developed in the price-setting context by Alvarez, Lippi and Oskolkov (2022) and in the investment context by Lippi and Oskolkov (2023). By recovering the generalized hazard directly from the data, we discipline the price wedge and quantitatively evaluate the role of irreversibility. As noted earlier, the sufficient statistics outlined in Section 3 and the data mappings discussed in

[26] Irreversibility increases the recovered volatility σ^2 from 0.049 (see Table I in Baley and Blanco (2021)) to 0.058.

Section 4 are valid within this generalized hazard framework.

5.1 The Generalized Hazard Model

The generalization focuses solely on the structure of fixed adjustment costs while preserving the assumptions regarding production technology, the productivity process, and the price wedge.[27] For every non-zero investment, $\Delta k_s \neq 0$, firms incur a *stochastic* fixed adjustment cost, θ_s, proportional to productivity:

$$\theta_s = \Theta_s(\Delta \hat{k})\, u_s. \tag{71}$$

The function $\Theta_s(\Delta \hat{k})$ follows a compound Poisson process, allowing for distributions of fixed costs and free adjustment opportunities (i.e., mass points at zero cost), which may differ between positive and negative adjustments. This adjustment cost structure can alternatively be expressed using an adjustment hazard function, $\Lambda(\hat{k})$, such that for a given capital-productivity ratio, $\hat{k}$, the probability of adjusting within a small interval $\mathrm{d}t$ is $\Lambda(\hat{k})\mathrm{d}t$ in the outer inaction region and zero in the inner inaction region. Unlike the baseline model, which assumes zero adjustments inside the inaction region, this extended model introduces a positive probability of adjustment within that domain.

The extended Kolmogorov Forward Equation characterizing the stationary distribution of $\hat{k}$ includes an additional term, absent in the baseline KFE from (19),

$$\Lambda(\hat{k})g(\hat{k}) = \nu g'(\hat{k}) + \frac{\sigma^2}{2} g''(\hat{k}), \qquad \forall \hat{k} \in (\hat{k}^-, \hat{k}^+) \setminus \{\hat{k}^{*-}, \hat{k}^{*+}\}, \tag{72}$$

together with the boundary and reinjection conditions that depend on the price wedge, ω.

Our generalized hazard model with irreversibility nests several existing investment models: the standard fixed-cost model (Scarf, 1959); random fixed costs, including those of Thomas (2002), Gourio and Kashyap (2007), and Khan and Thomas (2008); asymmetric fixed costs and time dependent adjustments (Baley and Blanco, 2021); and the generalized hazard model without irreversibility (Lippi and Oskolkov, 2023).

5.2 Recovering the Adjustment Hazard

To recover the hazard function that generates the empirical investment rate distribution shown in Figure VI, we follow the methodology outlined by Lippi and Oskolkov (2023).[28]

[27] See Baley and Blanco (2021) for an analysis of ex-ante heterogeneity in firms' production and adjustment technologies.

[28] Lippi and Oskolkov (2023) propose a detailed framework to recover the underlying distribution of fixed adjustment costs responsible for the observed investment patterns. This approach is critical for understanding heterogeneity and the origins of inaction. For our purposes, it suffices to recover the hazard function to compute the

First, we exploit the relationship between the model's hazard rate $\Lambda(\hat{k})$ and the density of capital-productivity ratios $g(\hat{k})$, and the data's adjustment frequency $\mathcal{N}$ and investment density $h(\Delta\hat{k})$. Specifically, for any $\hat{k}$, these objects satisfy the following relationship:

$$\underbrace{\Lambda(\hat{k})g(\hat{k})}_{\text{model}} = \underbrace{\mathcal{N}h(\Delta\hat{k})}_{\text{data}}, \quad \text{where} \quad \Delta\hat{k} = \hat{k}^*(\hat{k}) - \hat{k}. \tag{73}$$

Second, we parameterize the investment density, $h(\Delta\hat{k})$, using a Gamma distribution that accounts for asymmetries between positive and negative adjustments. This is achieved by introducing separate frequency (Υ), shape (ϱ^-, ϱ^+), and scale (ς^-, ς^+) parameters for positive and negative adjustments:

$$h(\Delta\hat{k} \mid \Upsilon, \varrho^-, \varrho^+, \varsigma^-, \varsigma^+) = \begin{cases} \frac{\Upsilon}{\Gamma(\varrho^-)(\varsigma^-)^{\varrho^-}}(\Delta\hat{k})^{\varrho^- - 1}\exp\left(-\frac{\Delta\hat{k}}{\varsigma^-}\right) & \text{if } \Delta\hat{k} > 0, \\ \frac{1-\Upsilon}{\Gamma(\varrho^+)(\varsigma^+)^{\varrho^+}}(-\Delta\hat{k})^{\varrho^+ - 1}\exp\left(\frac{\Delta\hat{k}}{\varsigma^+}\right) & \text{if } \Delta\hat{k} < 0. \end{cases} \tag{74}$$

We estimate these five parameters $(\Upsilon, \varrho^-, \varrho^+, \varsigma^-, \varsigma^+)$ using a simulated method of moments. Using the estimated values, we substitute the investment density, $h(\Delta\hat{k})$, into the KFE. We then solve the resulting system of equations for $g(\hat{k})$ at each $\hat{k}$, employing finite differences and incorporating the boundary conditions: $g(\hat{k}^-) = g(\hat{k}^+) = 0$ and $\int_{\hat{k}^-}^{\hat{k}^+} g(\hat{k})\,\mathrm{d}\hat{k} = 1$, where the boundaries depend on the price wedge ω. By varying the price wedge, we compute the corresponding hazards and distributions.[29]

Finally, with the estimated density $g(\hat{k})$, we recover the hazard function as:

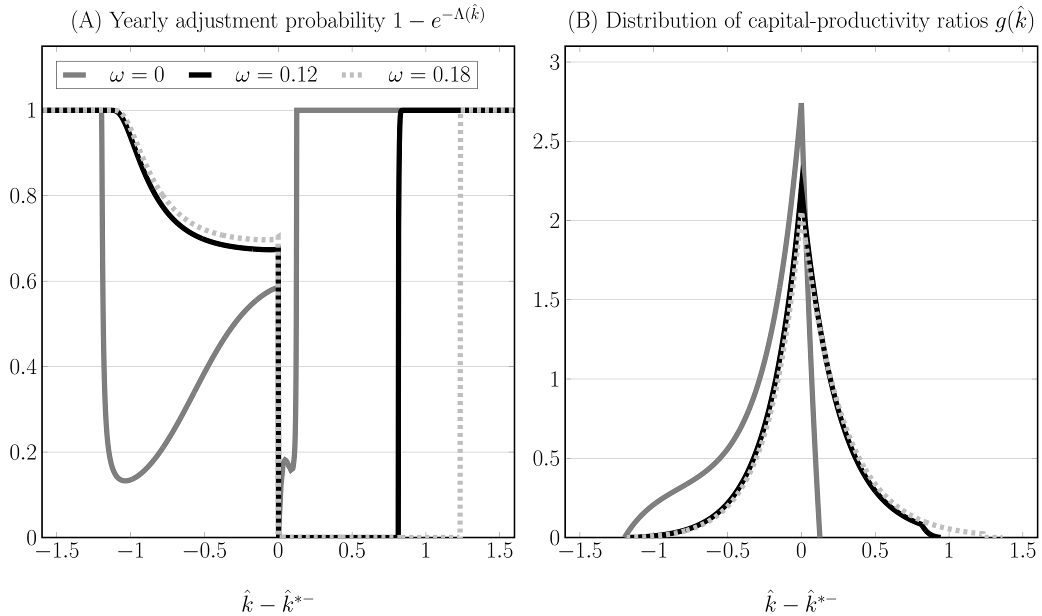
$$\Lambda(\hat{k}) = \begin{cases} \frac{\mathcal{N}h(\hat{k}^{*-} - \hat{k})}{g(\hat{k})} & \text{if } \hat{k} < \hat{k}^{*-}, \\ \frac{\mathcal{N}h(\hat{k}^{*+} - \hat{k})}{g(\hat{k})} & \text{if } \hat{k} > \hat{k}^{*+}. \end{cases} \tag{75}$$

Panel A of Figure VII displays the yearly adjustment probability, $1 - e^{-\Lambda(\hat{k})}$, derived as a transformation of the hazard function. Panel B shows the capital-productivity distribution, $g(\hat{k})$. The x-axes plot capital-productivity ratios relative to the lower reset point $\hat{k} - \hat{k}^{*-}$. We analyze three different wedges, $\omega \in \{0, 0.12, 0.18\}$. Notably, all three specifications align with the empirical investment distribution in Figure VI, despite substantial variations in adjustment probabilities and distributions across price wedges. In essence, given the adjustment frequency $\mathcal{N}$, the generalized hazard approach ensures the existence of an adjustment hazard $\Lambda(\hat{k})$ and a capital-productivity distribution $g(\hat{k})$ such that their product rationalizes the data, $\mathcal{N}h(\Delta\hat{k})$. However, as illustrated in Figure VII, these components can differ significantly, resulting in distinct implications for aggregate fluctuations.

CIR's sufficient statistics.

[29] See Appendix E.5 for additional technical details.

Figure VII: Adjustment Probability and Capital-Productivity Distribution



Notes: Panel A plots the yearly adjustment probability $1 - \exp(-\Lambda(\hat{k}))$ and Panel B plots the capital-productivity distribution $g(\hat{k})$ for price wedges $\omega \in \{0, 0.12, 0.18\}$. In both figures, the x-axis shows capital-productivity ratios relative to the lower reset point $\hat{k} - \hat{k}^{*-}$.

Through the lens of a model without irreversibility ($\omega = 0$, solid black line), the few negative investments arise by limiting $\hat{k}$'s growth and immediately correcting positive capital-productivity ratios through disinvestment. The large mass of small positive investment rates and the convexity of the investment distribution are driven by a decreasing hazard function: firms are more likely to adjust when $\hat{k}$ deviates slightly from the reset point than when the deviation is more considerable. As a result, the adjustment probability is non-monotonic, and the capital-productivity distribution is skewed toward lower ratios.

In contrast, a model with irreversibility ($\omega > 0$, gray lines) introduces an inner inaction region, breaking away from the single reset point. This allows capital-productivity ratios to grow and positively covary with firm age. In this case, firms with high capital-productivity ratios account for the few negative investments observed. The large mass of small positive investment rates and the convexity of the investment distribution is explained by an increasing hazard function, which is the natural outcome under profit optimization: the likelihood of adjustment rises as $\hat{k}$ deviates further from the optimal point.

The adjustment hazard is also higher under irreversibility than without for all $\hat{k}$ in the outer inaction region. Consequently, the density of capital-productivity ratios in these regions is lower, ensuring that the product of the hazard function and the capital-productivity distribution delivers consistent values. This mechanism highlights the distinct dynamics introduced by irreversibility in the model and its implications for micro-level investment behavior.

Table II: CIR's Sufficient Statistics: Data vs. Model

	$\omega = 0.00$				$\omega = 0.12$				$\omega = 0.18$			
	Data		**Model**		**Data**		**Model**		**Data**		**Model**	
CIR	2.54		0.92		2.60		1.93		**2.33**		**2.39**	
Suff. statistics:												
(i) Variance term	1.88	(74)	1.73	(188)	1.68	**(65)**	1.40	**(72)**	1.38	(56)	1.58	(66)
(ii) Covariance term	0.65	(26)	−0.81	(−88)	0.30	**(11)**	0.19	**(10)**	−0.13	(−5)	0.50	(21)
(iii) Irreversibility term	0.00	(0)	0.00	(0)	0.61	**(23)**	0.34	**(18)**	1.20	(49)	0.31	(13)

Notes: CIR computed using Chilean data and calibrated model. The relative importance of each sufficient statistic, expressed in %, is reported in parenthesis. Variance term: $\mathbb{V}ar[\hat{k}]/\sigma^2$, Covariance term: $\nu\mathbb{C}ov[\hat{k}, a]/\sigma^2$, and Irreversibility term: $\mathbb{E}\left[\frac{1}{\mathrm{d}s}\mathbb{E}_s[\mathrm{d}(\hat{k}_s\mathcal{M}(\hat{k}_s))]\right]/\sigma^2$.

This analysis demonstrates that fully capturing micro-level investment dynamics requires more than just matching the investment distribution. As the figures show, capital-productivity distribution changes with the price wedge. Extra information is necessary to identify the correct parameter configuration. The CIR provides this critical information. Next, we exploit the CIR's sufficient statistics as model discrimination tools to discipline the price wedge.

5.3 Sufficient Statistics for Aggregate Capital Fluctuations

With the adjustment hazard function recovered, we compute the CIR's sufficient statistics to evaluate aggregate capital fluctuations. Table II reports the CIR and its sufficient statistics for Chilean manufacturing plants between 1980 and 2011. We consider three price wedges: $\omega = 0$, $\omega = 0.12$, and $\omega = 0.18$. These wedges illustrate the model's performance across different degrees of irreversibility and provide a basis for selecting the value of ω that best matches the data.

The first case, $\omega = 0$, represents the baseline scenario where irreversibility is absent. While this is a valuable benchmark, it fails to match the CIR's level and decomposition into sufficient statistics. Specifically, the variance term is overstated, the covariance term is negative instead of positive, and the irreversibility term is absent. These discrepancies highlight the necessity of introducing irreversibility to explain the dynamics of aggregate capital fluctuations.[30]

At the other extreme, $\omega = 0.18$ achieves a CIR level (2.39) that matches the data (2.33) remarkably well. However, the decomposition into sufficient statistics diverges significantly from the observed contributions. The variance term dominates excessively, while the covariance and irreversibility terms deviate from their empirical counterparts. Although this wedge captures the overall CIR level, it fails to reflect the actual economic channels driving capital adjustment, making

[30](Baley and Blanco, 2021) show that even with a zero wedge, an extremely high fixed cost for downward adjustments can revert the sign of the covariance to positive. Nevertheless, that parametrization does not generate the observed investment rate distribution.

it unsuitable for accurately representing the underlying dynamics.

The wedge $\omega = 0.12$ represents the intermediate case and provides the best balance. While the CIR level (1.93) is slightly below the observed value, the decomposition into sufficient statistics aligns closely with the data. The variance term accounts for 72%, followed by irreversibility and covariance, mirroring the empirical structure. This wedge correctly captures the mechanisms driving aggregate fluctuations, making it the preferred choice.

5.4 Disciplining the Price Wedge

We select $\omega = 0.12$ as the preferred value for the price wedge because it maximizes consistency between the model and the data regarding sufficient statistics. While it does not perfectly match the CIR level, it accurately captures the decomposition, reflecting the relative importance of variance, covariance, and irreversibility in driving aggregate capital fluctuations. Prioritizing mechanisms over level ensures the model represents the actual economic dynamics.

At $\omega = 0.12$, we recover a CIR of 1.93, meaning a 1% decrease in aggregate productivity leads to a nearly 2% deviation of average capital-productivity ratios from their steady-state value. The first sufficient statistic, $\mathbb{V}ar[\hat{k}]/\sigma^2$, accounts for 72% of the CIR, highlighting the dominant role of variance. The second statistic, $-\nu\mathbb{C}ov[\hat{k}, a]/\sigma^2$, captures the covariance channel, which represents 10% of the CIR. Lastly, the third statistic, $\mathbb{E}\left[\mathbb{E}_s\left[\mathrm{d}(\mathcal{M}(\hat{k}_s)\hat{k}_s)\right]/\mathrm{d}s\right]/\sigma^2$, measures the contribution of irreversibility which is 18% of the CIR. Together, these statistics provide a clear decomposition of the CIR and validate $\omega = 0.12$ as the most reliable representation of capital adjustment dynamics. Besides the CIR, these sufficient statistics match the distribution of investment rates.

5.5 Price Wedges in the Literature

We compare our preferred price wedge value, $\omega = 0.12$, with estimates from existing literature and discuss alternative approaches. To express values in the same units, the price wedge is calculated as one minus the recovery rate—the liquidation value over replacement cost net of depreciation.[31]

Empirical studies provide direct evidence of price wedges, often based on observed recovery rates. Ramey and Shapiro (2001) analyze capital reallocation from closing aerospace plants in the United States, estimating a price wedge of 0.72, which reflects significant discounts during liquidation. Kermani and Ma (2023) document industry-level wedges of around 0.65 for plant, property, and equipment, consistent with the high levels of asset specificity in these sectors. These estimates are likely upward biased due to selection effects, as they are based on liquidating firms subject to fire-sale dynamics. Surveys offer complementary insights. Dibiasi, Mikosch and Sarferaz (2021) survey Swiss CEOs and CFOs and find an average price wedge of approximately 0.47.

[31] Appendix F summarizes the values reported in the literature.

Structural quantitative models estimate wedges by calibrating them to static features of the investment rate distribution. For instance, Bloom (2009) calibrates a wedge of 0.34, while Fang (2023) and Senga and Varotto (2024) report values between 0.30 and 0.41. These larger wedges typically reflect settings with substantial capital specificity or adjustment frictions. In contrast, models such as Cooper and Haltiwanger (2006), Khan and Thomas (2013), and Lanteri (2018) rely on smaller wedges, ranging from 0.025 to 0.07, indicating less severe frictions or more fluid capital adjustment.

Our estimate of $\omega = 0.12$, which relies on matching the CIR and the micro-dynamic moments, lies in between. This value likely captures heterogeneity across sectors and capital types and the effects of internal capital transfers, such as mergers and acquisitions, that may mitigate irreversibility for some firms (Bhandari, Martellini and McGrattan, 2024).

6 Final Thoughts

We investigate how partially irreversible investment shapes aggregate fluctuations. Our approach innovates by characterizing fluctuations with lumpy adjustments across two reset points, using (i) conditioning on prior resets, (ii) transition probabilities across resets, and (iii) microdata to discipline those transitions. Our flexible methodology can accommodate a finite number of reset points and applies broadly wherever sufficient microdata exist to discipline transitions. Extensions of our framework could study financial frictions by linking reset points to fund availability through firm-level financial data.

We outline four directions for future research. First, while we focus on aggregate productivity as a source of fluctuations, our framework applies to other aggregate shocks, such as changes in profitability, capital prices, or interest rates.[32] This opens avenues for studying corporate tax reforms (Altug, Demers and Demers, 2009; Winberry, 2021; Chen, Jiang, Liu, Suárez-Serrato and Xu, 2023) or monetary policy and their interaction with investment frictions (Fang, 2023; Baley, Blanco and Oviedo, 2024).

Second, our analysis assumes fixed price wedges and interest rates suited to small open economies. However, ample evidence shows that wedges vary across the business cycle and are endogenously determined in secondary markets (Lanteri, 2018; Gavazza and Lanteri, 2021). Incorporating time-variation in price wedges and interest rates are natural extensions in this direction, allowing the assessment of general equilibrium effects (Veracierto, 2002; Gourio and Kashyap, 2007).

Third, while we focus on first-moment shifts in the capital-productivity distribution, our methodology can analyze higher-order moments, such as dispersion or skewness, by adapting the CIR's sufficient statistics framework. We characterize the CIR for any continuous function

[32] Shocks to the price wedge or the stochastic process of capital-productivity ratios would entail changes in the sufficient statics, so we leave them for future study.

$f(\hat{k})$; thus, setting $f(\hat{k}) = \hat{k}^m$ allows characterizations of cross-sectional moments ($m = 2$ for variance, $m = 3$ for skewness), while $f(\hat{k}) = e^{\alpha \hat{k}}$ can characterize aggregate output.[33]

Finally, we focus on small aggregate shocks (δ) and first-order perturbations, potentially ignoring nonlinearities and the response to large shocks. Appendix G studies numerically the non-linearities in the generalized hazard model regarding the sign and magnitude of aggregate productivity shocks. We find tiny non-linearities and asymmetries for productivity shocks below $\delta = 5\%$.

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[33]In this case, the baseline model requires additional features to make the economy stationary (the baseline economy with Brownian idiosyncratic shocks does not feature a stationary distribution), such as stochastic firm exit or monopolistic competition with quality shocks (see Appendix D).

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