Gaussian Rényi-2 correlations in a nondegenerate three-level laser

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Abstract

Quantum correlation is a key component in various quantum information processing tasks. Decoherence process imposes limitations on achieving these quantum tasks. Therefore, understanding the behavior of quantum correlations in dissipative-noisy systems is of paramount importance. Here, on the basis of the Gaussian Rényi-2 entropy, we analyze entanglement and quantum discord in a two-mode Gaussian state ρ_{AB}. The mode $A(B)$ is generated within the first(second) transition of a nondegenerate three-level cascade laser. Using realistic experimental parameters, we show that both entanglement and discord could be generated and enhanced by inducing more quantum coherence. Under thermal noise, entanglement is found more fragile having a tendency to disappear rapidly. While, quantum discord exhibits a freezing behavior, where it can be captured within a wide range of temperature. Surprisingly, we find that entanglement can exceed quantum discord in contrary to the expectation based on the assumption that the former is only a part of the later. Finally, we show numerically as well as analytically that optimal quantum discord can be captured by performing Gaussian measurements on mode B. The obtained results suggest that nondegenerate three-level lasers may be a valuable resource for some quantum information tasks, especially, for those do not require entanglement.

1 Introduction

Quantum systems are correlated in manners inaccessible to classical ones [1]. A peculiar quantum characteristic of correlations is quantum entanglement [2], which was defined as a nonclassical physical property that cannot be prepared by means of local operations and classical communication [2]. Entanglement is undoubtedly the key ingredient in most applications of quantum science [3], where it has been recognized as the fundamental resource for, e.g., quantum teleportation [4], dense coding [5], quantum computation [6] and quantum algorithm [7]. However, it is shown that some quantum tasks, e.g., quantum key distribution [8] can be carried out by unentangled states that nevertheless possess quantum correlations [9]. In fact, it has been proven theoretically [9] as well as experimentally [10] that some quantum tasks may be speed-up over their classical counterparts exploiting unentangled states with residual correlations that cannot be described by any classical probability distribution. Such residual correlations are quantified by the so-called quantum discord [11].

For bipartite quantum states endowed with finite-dimensional Hilbert spaces, the concept of discord was first introduced and defined as the mismatch between total and classical correlations [12, 13]. The definition of quantum discord involves a nontrivial optimization task which can be accomplished solely for very simple states, including X-states [14] and two-mode Gaussian states [15]. Restricting the minimization—implicated in the definition of quantum discord [12, 13]—to the set of Gaussian positive operator-valued measures [16], the optimization problem has been fully solved in [17, 18]. Quantum discord was predicted to play the main role in miscellaneous protocols, e.g., quantum state merging [19], remote state preparation [20], security in quantum key distribution [21] and quantum channel discrimination [22]. In this regard, quantum discord has been investigated in different systems including two-qubit states [14, 23], two resonant harmonic oscillators [24], photonic crystal cavity array [25], optomechanical Fabry-Pérot cavities [26]. The experimental exploration of quantum discord is accomplished in [10].

Notice that steering [27] and Bell nonlocality [28] are also two other incarnations of quantum correlations that can be, especially, used to implement secure quantum information processing tasks, e.g, quantum secret sharing protocol [29] and unconditionally secure quantum key distribution [30]. Another fundamental aspect that marks the departure of the quantum science from the classical one is quantum coherence [31]. Such purely quantum property constitutes a powerful resource for quantum information processing [31, 32], and plays a fundamental role in emergent fields like quantum biology [33] and thermodynamics [34]. In recent years, the topic of Gaussian quantum correlations has received a significant amount of attention as it plays a crucial role in quantum computation and communication protocols [15]. While the efficiency of quantum information schemes is strongly depends on the degree of quantum correlations, three-level lasers have been theoretically predicted to be a good candidate as a source of light in a highly entangled state [35]. Scully and Zubairy [35] have established the basic tools of a really complete theory of two-photon laser emitted by three-level atoms in a cascade configuration. In such lasers, the major role is played by the atomic coherence [35], which can be induced either by the injected coherence process [36] or by the driven coherence process [37]. Importantly, two-mode light generated within the cascade transition of a three-level laser is proven to evolve in a two-mode Gaussian state [38]. A two-photon laser, with a gain media constituted by a set of three-level atoms in a cascade configuration, is shown to display several quantum effects such as quenching of quantum fluctuations [39], quantum squeezing effect [40], as well as anomalous optical bistability [41].

Over the past 2 decades, miscellaneous works focused only on inseparable quantum correlations (steering, entanglement, and Bell nonlocality) in three-level lasers coupled to either vacuum or squeezed reservoirs. For instance, Ping *et al.* [42] and Alebachew [43] have studied Bell nonlocality, where the atomic coherence is induced via the driven coherence process and the injected coherence process, respectively. They found that violation of Bell's inequality of the entangled states is possible even in the presence of cavity losses. Recently, El Qars [44] has investigated Gaussian quantum steering, where the atomic coherence is induced by initially preparing the three-level atoms in a coherent superposition of the upper and lower levels. It is found that due the positivity of the intensity difference of the two emitted laser modes, one-way steering behavior can be detected only in one direction. Opposed to these works, the entanglement properties in three-level lasers have been extensively examined. Proposals include, but not limited to, [38, 40, 45, 46, 47], where the sufficient and necessary inseparability criterion proposed in [48, 49] is employed as an entanglement witness.

In realistic quantum systems, quantum correlations are inevitably effected by the surrounding environment which leads generally to their degradation [24]. This is a challenging issue for generating and preserving quantum correlations in dissipative-noisy quantum optical systems, which are of great importance for quantum information processing [1]. Due to the increasing interest in the quantum properties of correlated emission lasers, we propose here to investigate, against decoherence effect, both entanglement and discord in a nondegenerate three-level laser where the atomic coherence is initially induced by the injected coherence process. To this aim, we consider a two-mode Gaussian state ρ_{AB} coupled to a common two-mode thermal reservoir. The modes A and B are generated, respectively, during the first and second transitions of a single three-level atom. We use the Gaussian Rényi-2 entanglement and the Gaussian Rényi-2 discord to quantify, respectively, entanglement and quantum discord between the two laser modes A and B. Finally, we emphasize that entanglement and Gaussian quantum discord defined via the Rényi-2 entropy have been studied in an optomechanical system subjects to dissipation and thermal noise by El Qars *et al.* [26].

The remainder of this paper is organized as follows. In Sect. 2, we introduce the system under consideration. Next, by applying the master equation governing the dynamics of the state ρ_{AB}, we derive the explicit expression of the stationary covariance matrix fully describing the two-mode Gaussian state ρ_{AB}. In Sect. 3, using realistic experimental parameters, we quantify and study the Gaussian Rényi-2 entanglement and the Gaussian Rényi-2 discord in the state ρ_{AB}. Finally, in Sect. 4, we draw our conclusions.

2 A nondegenerate three-level cascade laser

Inside a resonant cavity, we consider a set of three-level atoms in interaction with two bosonic modes of the quantized cavity radiation [35]. The jth bosonic mode can be characterized by its annihilation operator ς_j, decay rate κ_j and frequency ω_j. We suppose that the atomic system is injected in the cavity with a rate r [50]. As illustrated in Fig. 1, the notations $|l_1\rangle$, $|l_2\rangle$ and $|l_3\rangle$ are used to indicate, respectively, the upper excited, intermediate and ground levels of a single three-level atom.

The interaction between the two cavity modes ς_1 and ς_2 and a single atom can be described by the Hamiltonian [35]

$$\mathcal{H}_{\text{int}} = \mathrm{i}\hbar\left[\upsilon_{12}\varsigma_1|l_1\rangle\langle l_2| + \upsilon_{23}\varsigma_2|l_2\rangle\langle l_3| - \upsilon_{12}|l_2\rangle\langle l_1|\varsigma_1^\dagger - \upsilon_{23}|l_3\rangle\langle l_2|\varsigma_2^\dagger\right], \tag{1}$$

where υ_{12} are υ_{23} being the coupling constants corresponding to the first $|l_1\rangle \to |l_2\rangle$ and second $|l_2\rangle \to |l_3\rangle$ transitions, respectively [50]. In addition, we suppose that the three-level atoms are initially prepared in an arbitrary quantum coherent superposition of the excited upper level $|l_1\rangle$ and the ground level $|l_3\rangle$. Therefore, the initial state $|\Psi_{\text{sa}}\rangle$ as well as the density operator ρ_{sa} for a single atom read [51]

$$\begin{aligned} |\Psi_{\text{sa}}\rangle &= p_1|l_1\rangle + p_3|l_3\rangle, && (2)\\ \rho_{\text{sa}} &= \rho_{11}|l_1\rangle\langle l_1| + \rho_{13}|l_1\rangle\langle l_3| + \rho_{31}|l_3\rangle\langle l_1| + \rho_{33}|l_3\rangle\langle l_3|, && (3)\end{aligned}$$

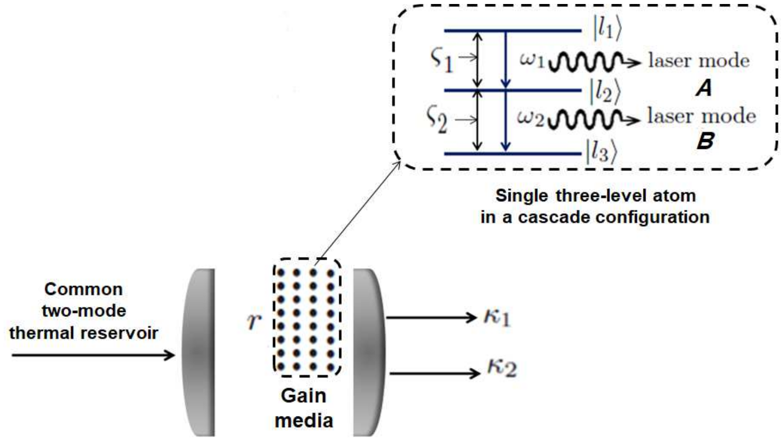

Figure 1: A three-level laser coupled to a common two-mode thermal reservoir. We adopt the notations $|l_1\rangle$, $|l_2\rangle$ and $|l_3\rangle$ for representing, respectively, the upper, intermediate, and ground levels for a single three-level atom. The first(second) transition $|l_1\rangle \to |l_2\rangle(|l_2\rangle \to |l_3\rangle)$ at the optical frequency $\omega_1(\omega_2)$ and spontaneous emission decay rate $\gamma_{12}(\gamma_{23})$, is assumed to be in resonance with the quantized cavity mode $\varsigma_1(\varsigma_2)$. While, the transition $|l_1\rangle \to |l_3\rangle$ is dipole forbidden [50]. The gain media of the laser is constituted by an ensemble of three-level atoms in a cascade configuration. We focus on the situation in which the atoms are initially prepared in a coherent superposition of the upper excited level $|l_1\rangle$ and the ground level $|l_3\rangle$. r denotes the rate at which the atoms are placed in the cavity. When a single atom makes a transition from the top level $|l_1\rangle$ to the bottom level $|l_3\rangle$ via the intermediate level $|l_2\rangle$, two strongly correlated photons are emitted with the frequencies ω_1 and ω_2. If $\omega_1 \neq \omega_2$, which we consider here, the laser is a nondegenerate three-level laser, and a degenerate three-level laser if $\omega_1 = \omega_2$.

where $\rho_{11} = |p_1|^2$ and $\rho_{33} = |p_3|^2$ represent, respectively, the probabilities for a single three-level atom to be initially in the excited upper level and the ground level. While $\rho_{13} = \rho_{31}^* = p_1 p_3^*$ is the initial coherence of a single three-level atom [50]. For simplicity, we take the same spontaneous decay rates for the transitions $|l_1\rangle \to |l_2\rangle$ and $|l_2\rangle \to |l_3\rangle$, i.e., $\gamma_{12,23} = \gamma$, the same coupling transitions, i.e., $\upsilon_{12,23} = \upsilon$, and the same damping rates, i.e., $\kappa_{1,2} = \kappa$.

The dynamics of the reduced density operator ρ_{AB} for the two laser modes A and B—emitted during the first and second transitions, respectively—is described by the master equation [52, 53]

$$\frac{d\rho_{AB}}{dt} = \frac{-\mathrm{i}}{\hbar}\mathrm{Tr}_{\mathrm{sa}}[\mathcal{H}_{\mathrm{int}}, \rho_{\{\mathrm{sa}+AB\}}] + \sum_{j=1,2}\left(\frac{\kappa_j(n_{\mathrm{th},j}+1)}{2}\mathcal{L}\left[\varsigma_j\right]\rho_{AB} + \frac{\kappa_j n_{\mathrm{th},j}}{2}\mathcal{L}[\varsigma_j^\dagger]\rho_{AB}\right), \tag{4}$$

with $\rho_{\{\mathrm{sa}+AB\}}$ being the density operator describing the two laser modes A and B together with a single three-level atom, and $\mathrm{Tr}_{\mathrm{sa}}$ denotes the partial trace over the subsystem constituted by a single atom. In Eq. (4), the Lindblad operator $\mathcal{L}\left[\varsigma_j\right]\rho_{AB} = 2\varsigma_j\rho_{AB}\varsigma_j^\dagger - \left[\varsigma_j^\dagger\varsigma_j, \rho_{AB}\right]_+$ is added to take into account the coupling between the jth cavity mode and the jth thermal bath having mean thermal phonon number $n_{\mathrm{th},j}$ [35].

Applying the linear-adiabatic approximation [54] in the good cavity limit, i.e., $\kappa \ll \gamma$ [35] with common thermal bath, Eq. (4) would be [44, 55]

$$\begin{aligned}
\frac{d\rho_{AB}}{dt} \quad = \quad & \frac{\kappa(n_{\mathrm{th}}+1)}{2}[2\varsigma_1\rho_{AB}\varsigma_1^\dagger - \varsigma_1^\dagger\varsigma_1\rho_{AB} - \rho_{AB}\varsigma_1^\dagger\varsigma_1] + \frac{\kappa n_{\mathrm{th}}}{2}[2\varsigma_2^\dagger\rho_{AB}\varsigma_2 - \varsigma_2\varsigma_2^\dagger\rho_{AB} - \rho_{AB}\varsigma_2\varsigma_2^\dagger] + \\
& \frac{1}{2}\left(\mathcal{A}\rho_{11} + \kappa n_{\mathrm{th}}\right)[2\varsigma_1^\dagger\rho_{AB}\varsigma_1 - \varsigma_1\varsigma_1^\dagger\rho_{AB} - \rho_{AB}\varsigma_1\varsigma_1^\dagger] + \\
& \frac{1}{2}\left(\mathcal{A}\rho_{33} + \kappa(n_{\mathrm{th}}+1)\right)[2\varsigma_2\rho_{AB}\varsigma_2^\dagger - \varsigma_2^\dagger\varsigma_2\rho_{AB} - \rho_{AB}\varsigma_2^\dagger\varsigma_2] + \\
& \frac{\mathcal{A}\rho_{13}}{2}[\rho_{AB}\varsigma_1^\dagger\varsigma_2^\dagger - 2\varsigma_1^\dagger\rho_{AB}\varsigma_2^\dagger + \varsigma_1^\dagger\varsigma_2^\dagger\rho_{AB} - 2\varsigma_2\rho_{AB}\varsigma_1 + \varsigma_1\varsigma_2\rho_{AB} + \rho_{AB}\varsigma_1\varsigma_2],
\end{aligned} \tag{5}$$

with $\mathcal{A} = 2r\upsilon^2/\gamma^2$ being the linear gain coefficient that quantifies the rate at which the atoms are injected into the cavity [35]. In Eq. (5), the term proportional to $\rho_{33}(\rho_{11})$ represents the losses(gain) of the mode $B(A)$, while that proportional to ρ_{13} represents the coupling between the two modes A and B [35].

Now, utilizing Eq. (5) and the formula $\langle\frac{d\mathcal{O}}{dt}\rangle = \mathrm{Tr}\left[\left(\frac{d\rho_{AB}}{dt}\right)\mathcal{O}\right]$, we get the dynamics of the first and second moments of the variables associated with the laser modes A and B, i.e.,

$$\begin{aligned}
\frac{d}{dt}\langle\varsigma_j\rangle &= \frac{-\emptyset_j}{2}\langle\varsigma_j\rangle + \frac{(-1)^j\mathcal{A}\rho_{13}}{2}\langle\varsigma_{3-j}^\dagger\rangle \text{ for } j=1,2, && (6)\\
\frac{d}{dt}\langle\varsigma_j^2\rangle &= -\emptyset_j\langle\varsigma_j^2\rangle + (-1)^j\mathcal{A}\rho_{13}\langle\varsigma_1^\dagger\varsigma_2\rangle, && (7)\\
\frac{d}{dt}\langle\varsigma_j^\dagger\varsigma_j\rangle &= -\emptyset_j\langle\varsigma_j^\dagger\varsigma_j\rangle + \frac{(-1)^j\mathcal{A}\rho_{13}}{2}\left[\langle\varsigma_1^\dagger\varsigma_2^\dagger\rangle + \langle\varsigma_1\varsigma_2\rangle\right] + (2-j)\mathcal{A}\rho_{11} + \kappa n_{\mathrm{th}}, && (8)\\
\frac{d}{dt}\langle\varsigma_1\varsigma_2\rangle &= -\frac{\emptyset_1+\emptyset_2}{2}\langle\varsigma_1\varsigma_2\rangle + \frac{\mathcal{A}\rho_{13}}{2}\left[\langle\varsigma_1^\dagger\varsigma_1\rangle - \langle\varsigma_2^\dagger\varsigma_2\rangle + 1\right], && (9)\\
\frac{d}{dt}\langle\varsigma_1\varsigma_2^\dagger\rangle &= -\frac{\emptyset_1+\emptyset_2}{2}\langle\varsigma_1\varsigma_2^\dagger\rangle + \frac{\mathcal{A}\rho_{13}}{2}\left[\langle\varsigma_1^2\rangle - \langle\varsigma_2^{\dagger 2}\rangle\right], && (10)
\end{aligned}$$

where $\emptyset_1 = \kappa - \mathcal{A}\rho_{11}$ and $\emptyset_2 = \kappa + \mathcal{A}\rho_{33}$.

By introducing the population inversion η defined by $\rho_{11} = (1-\eta)/2$ with $-1 \leqslant \eta \leqslant 1$ [35], and using both $\rho_{11}+\rho_{33}=1$ and $|\rho_{13}| = \sqrt{\rho_{11}\rho_{33}}$ we get $\rho_{33} = (1+\eta)/2$ and $\rho_{13} = \sqrt{1-\eta^2}/2$.

Finally, by using the steady-state condition, i.e., $\frac{d\langle . \rangle}{dt} = 0$ in Eqs. (6)-(10), we get the non-zero correlations

$$\begin{aligned}
\langle\varsigma_1^\dagger\varsigma_1\rangle &= \frac{n_{\mathrm{th}}(\eta+1)}{2\eta^2} + \frac{[\mathcal{A}\eta(\eta-1)+2\kappa n_{\mathrm{th}}](1-\eta)}{4(\kappa+\mathcal{A}\eta)\eta^2} + \frac{(\eta^2-1)(4\kappa n_{\mathrm{th}}-\mathcal{A}\eta)}{2(2\kappa+\mathcal{A}\eta)\eta^2}, && (11)\\
\langle\varsigma_2^\dagger\varsigma_2\rangle &= -\frac{n_{\mathrm{th}}(\eta-1)}{2\eta^2} + \frac{[\mathcal{A}\eta(\eta-1)+2\kappa n_{\mathrm{th}}](1+\eta)}{4(\kappa+\mathcal{A}\eta)\eta^2} + \frac{(\eta^2-1)(4\kappa n_{\mathrm{th}}-\mathcal{A}\eta)}{2(2\kappa+\mathcal{A}\eta)\eta^2}, && (12)\\
\langle\varsigma_1\varsigma_2\rangle &= \frac{n_{\mathrm{th}}\sqrt{1-\eta^2}}{2\eta^2} + \frac{[\mathcal{A}\eta(\eta-1)+2\kappa n_{\mathrm{th}}]\sqrt{1-\eta^2}}{4(\kappa+\mathcal{A}\eta)\eta^2} - \frac{\sqrt{1-\eta^2}(4\kappa n_{\mathrm{th}}-\mathcal{A}\eta)}{2(2\kappa+\mathcal{A}\eta)\eta^2}, && (13)
\end{aligned}$$

which are physically meaningful only if $\eta \geqslant 0$, then $0 \leqslant \eta \leqslant 1$. The case $\eta = 0$ or equivalently $\rho_{11} = \rho_{33} = \rho_{13} = 1/2$ corresponds to maximum injected initial atomic coherence in the cavity. While, for $\eta = 1$, we have $\rho_{11} = \rho_{13} = 0$ and $\rho_{33} = 1$, thus no initially atomic coherence are injected.

As mentioned above, it has been demonstrated in [38] that the two-photon light generated by a nondegenerate three-level laser evolves in a two-mode Gaussian state, so, the two-mode Gaussian state ρ_{AB} can be described by means of its covariance matrix defined as $[\mathcal{V}_{AB}]_{jj'} = \langle\mathcal{U}_j\mathcal{U}_{j'} + \mathcal{U}_{j'}\mathcal{U}_j\rangle/2$, where $\mathcal{U}^{\mathrm{T}} = (\mathcal{X}_1, \mathcal{Y}_1, \mathcal{X}_2, \mathcal{Y}_2)$, with $\mathcal{X}_j = (\varsigma_j^\dagger + \varsigma_j)/\sqrt{2}$ and $\mathcal{Y}_j = \mathrm{i}(\varsigma_j^\dagger - \varsigma_j)/\sqrt{2}$. Based on the results given by Eqs. [(11)-(13)], the covariance matrix $\mathcal{V}$ can be rewritten as

$$\mathcal{V}_{AB} = \begin{pmatrix} \mathcal{V}_A & \mathcal{V}_{A/B} \\ \mathcal{V}_{A/B}^{\mathrm{T}} & \mathcal{V}_B \end{pmatrix}, \tag{14}$$

where $\mathcal{V}_A = a\mathbb{1}_2$, $\mathcal{V}_B = b\mathbb{1}_2$, and $\mathcal{V}_{A/B} = \mathrm{diag}(c, c')$, which corresponds to an asymmetric two-mode squeezed thermal state [17], with $a = \langle\varsigma_1^\dagger\varsigma_1\rangle + 1/2$, $b = \langle\varsigma_2^\dagger\varsigma_2\rangle + 1/2$ and $c = -c' = \langle\varsigma_1\varsigma_2\rangle$. The submatrices $\mathcal{V}_A$ and $\mathcal{V}_B$ describe the two laser modes A and B, respectively, while $\mathcal{V}_{A/B}$ describes the correlations between them.

Equations (13) and (14) show that if $\mathcal{A} = 0$ or $\eta = 1$, $\det\mathcal{V}_{A/B} = -c^2 = 0$, which turns ρ_{AB} into a Gaussian product state, i.e., $\rho_{AB} = \rho_A \otimes \rho_B$ without any correlations (quantum and classical) [17]. Consequently, neither entanglement nor discord could be created between the modes A and B. This because $\det\mathcal{V}_{A/B} < 0$ is a necessary condition required for the inseparability of the bi-mode Gaussian state ρ_{AB} [48]. Next, using the Gaussian Rényi-2 entropy, we study two different kinds of quantum correlations, i.e., entanglement and quantum discord in the cavity radiation of the studied nondegenerate three-level laser.

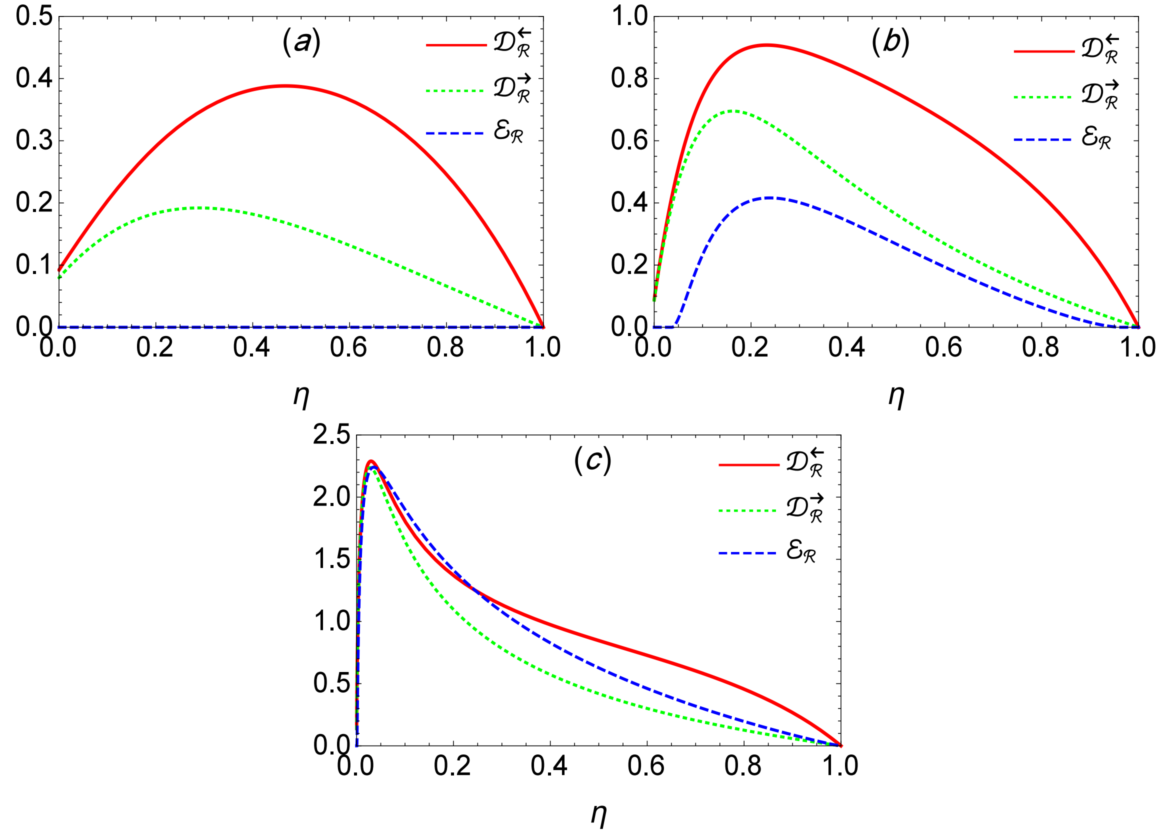

Figure 2: Plot of the Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ and the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ against the population inversion η using $n_{\rm th} = 5$ as value of the mean thermal phonon number and $\kappa = 3.85$ kHz as value of the cavity decay rate. For the linear gain coefficient, we used $\mathcal{A} = 100$ kHz in panel (a), $\mathcal{A} = 1$ MHz in panel (b), and $\mathcal{A} = 50$ MHz in panel (c). Remarkably, $\mathcal{E}_{\mathcal{R}} = \mathcal{D}_{\mathcal{R}}^{\leftarrow} = \mathcal{D}_{\mathcal{R}}^{\rightarrow} = 0$ for $\eta = 1$ regardless of the values of $\mathcal{A}$. However, optimal entanglement and quantum discord could be achieved by preparing the three-level atoms in a relatively stronger coherent superposition ($\mathcal{A} \gg 1$ with $\eta \to 0$). Panel (a) shows that quantum discord could be manifested without entanglement, while panel (c) shows that entanglement may exceed discord, which clearly indicates that quantum discord can not be viewed as a sum of entanglement and some other nonclassical correlations.

3 Gaussian Rényi-2 quantum correlations

3.1 Gaussian Rényi-2 entropy

The set of Rényi-α entropies have been first introduced by Alfred Rényi to generalize the concept of entropy [56]. These entropies encompass the usual entropic measures, i.e., the Shannon entropy and the von Neumann entropy [57]. Mathematically, the Rényi-α entropies of a quantum state ρ read as [18]

$$\mathcal{S}_{\alpha}(\rho) = \frac{1}{1-\alpha} \ln\left[\mathrm{Tr}\left(\rho^{\alpha}\right)\right] \text{ with } \alpha \in (0,1) \cup (1,+\infty). \tag{15}$$

The $\mathcal{S}_{\alpha}(\rho)$-entropies satisfy the following series of important mathematical properties: they are continuous, invariant under the unitary operations, and additive on tensor-product states [18]. Notice that the Rényi α-entropies $\mathcal{S}_{\alpha}(\rho)$ of discrete probability distributions are always positive. However, they can be negative in continuous case [58]. In the limit $\alpha \to 1$, the entropies $\mathcal{S}_{\alpha}(\rho)$ reduce to the conventional von Neumann entropy $\mathcal{S}(\rho) = -\,\mathrm{Tr}\,(\rho \ln \rho)$ [57]. We notice that such entropy is intensively used in various fields of sciences as well as in quantum information theory (see [26] for more details).

In pure quantum bipartite states, the von Neumann entropy—commonly known as the entropy of entanglement—is the only quantifier of entanglement [3]. Whereas, for mixed states, entanglement can be discriminated from separability by means of miscellaneous measures, which can be distinguished to each other due to their operational meaning and their mathematical properties [59]. Using $\alpha = 2$ in Eq.

(15), we simply obtain $\mathcal{S}_2(\rho) = -\ln\left[\mathrm{Tr}\left(\rho^2\right)\right]$, which corresponds to the Rényi-2 entropy [18]. For an arbitrary tripartite state ρ_{ABC}, it has been proven that the entropy $\mathcal{S}_2(\rho)$ satisfies the so called strong subadditivity inequality, i.e., $\mathcal{S}_2(\rho_{AB}) + \mathcal{S}_2(\rho_{BC}) \geqslant \mathcal{S}_2(\rho_{ABC}) + \mathcal{S}_2(\rho_B)$ [18], which allows to develop various measures of quantum correlations including entanglement and quantum discord.

3.2 Gaussian Rényi-2 entanglement

For a bipartite Gaussian state with covariance matrix $\mathcal{V}_{AB}$ (14), the Gaussian Rényi-2 entanglement is defined as [18]

$$\mathcal{E}_{\mathcal{R}}(\mathcal{V}_{AB}) := \inf_{\{\sigma_{xy}|0<\sigma_{xy}\leq\mathcal{V}_{AB},\ \det\sigma_{xy}=1\}} \mathcal{S}_2(\sigma_x), \tag{16}$$

where $\mathcal{S}_2(\varrho) = \frac{1}{2}\ln[\det\Theta]$ is the Gaussian Rényi-2 entropy of the state ϱ having the covariance matrix Θ [18]. In Eq. (16), the optimization is taken over a pure two-mode Gaussian state with covariance matrix σ_{xy} smaller than $\mathcal{V}_{AB}$, with σ_x being the marginal covariance matrix of party x obtained from σ_{xy} by partial tracing over party y.

For generally mixed two-mode Gaussian states ρ_{AB}, Eq. (16) admits a complicated expression [18]. In particular, for the two-mode Gaussian squeezed thermal state ρ_{AB} with the covariance matrix (14), Eq. (16) reads [59]

$$\mathcal{E}_{\mathcal{R}} = \begin{cases} \frac{1}{2}\ln\left[\frac{(g+1)s-\sqrt{[(g-1)^2-4d^2][s^2-d^2-g]}}{4(d^2+g)^2}\right]^2 & \text{if} \quad 2|d|+1 \leq g < 2s-1, \\ 0 & \text{if} \quad g \geqslant 2s-1, \end{cases} \tag{17}$$

where $s = (a+b)/2$, $d = (a-b)/2$ and $g = \sqrt{\det\mathcal{V}_{AB}}$.

It is interesting to notice here that a comparative analysis performed in [55], between the Gaussian Rényi-1 entanglement (that is the entanglement of formation defined via the von Neumann entropy) and the Gaussian Rényi-2 entanglement given by Eq. (17), showed that the former may not be the best choice for characterizing entanglement even when restricted to the simple case of two-mode Gaussian states.

3.3 Gaussian Rényi-2 discord

In the spirit of the measures proposed in [17], where Gaussian quantum discord has been defined via the conventional von Neumann entropy, Adesso and the co-workers [18] have obtained closed formula of the Gaussian Rényi-2 discord for generally mixed two-mode Gaussian states.

In a two-mode Gaussian state ρ_{AB}, the Gaussian Rényi-2 discord $\mathcal{D}_{\mathcal{R}}$ is defined as the difference between the total correlations, quantified by the Gaussian Rényi-2 mutual information $\mathcal{I}_{\mathcal{R}}$, and the Gaussian Rényi-2 classical correlations $\mathcal{C}_{\mathcal{R}}$ [60, 61], i.e.,

$$\mathcal{D}_{\mathcal{R}}(\rho_{AB}) \doteq \mathcal{I}_{\mathcal{R}}(\rho_{AB}) - \mathcal{C}_{\mathcal{R}}(\rho_{AB}), \tag{18}$$

where $\mathcal{I}_{\mathcal{R}}(\rho_{AB})$ is defined by

$$\begin{aligned} \mathcal{I}_{\mathcal{R}}(\rho_{AB}) &= \mathcal{S}_2(\rho_A)+\mathcal{S}_2(\rho_B)-\mathcal{S}_2(\rho_{AB}), & (19) \\ &= \frac{1}{2}\ln\frac{\det\mathcal{V}_A\det\mathcal{V}_B}{\det\mathcal{V}_{AB}}. & (20) \end{aligned}$$

From an operational point of view, the Gaussian Rényi-2 mutual information (20) quantifies the phase space distinguishability between the Wigner function of the two-mode Gaussian state ρ_{AB} and the Wigner function of the product of the marginal $\rho_A \otimes \rho_B$.

Following [18, 60, 61], the Gaussian Rényi-2 mutual information $\mathcal{I}_{\mathcal{R}}(\rho_{AB})$ can be interpreted as the degree of extra-discrete information that requires to be transmitted among a continuous variable channel for reconstructing the complete joint Wigner function of the two-mode Gaussian state ρ_{AB} rather than just the two marginal Wigner functions of the two modes A and B. $\mathcal{I}_{\mathcal{R}}$ is always positive and vanishes only on product states, i.e., $\rho_{AB} = \rho_A \otimes \rho_B$. Besides, the Gaussian Rényi-2 classical correlations $\mathcal{C}_{\mathcal{R}}(\rho_{AB})$ are defined in terms of how much the ignorance about the state of a party, saying A, is reduced when the most informative local measurement is implemented on party B [17]. Operationally, we define $\mathcal{C}_{\mathcal{R}}^{\leftarrow}(\rho_{AB})(\mathcal{C}_{\mathcal{R}}^{\rightarrow}(\rho_{AB}))$ as the maximum decrease in the Gaussian Rényi-2 entropy of party $A(B)$, when a set

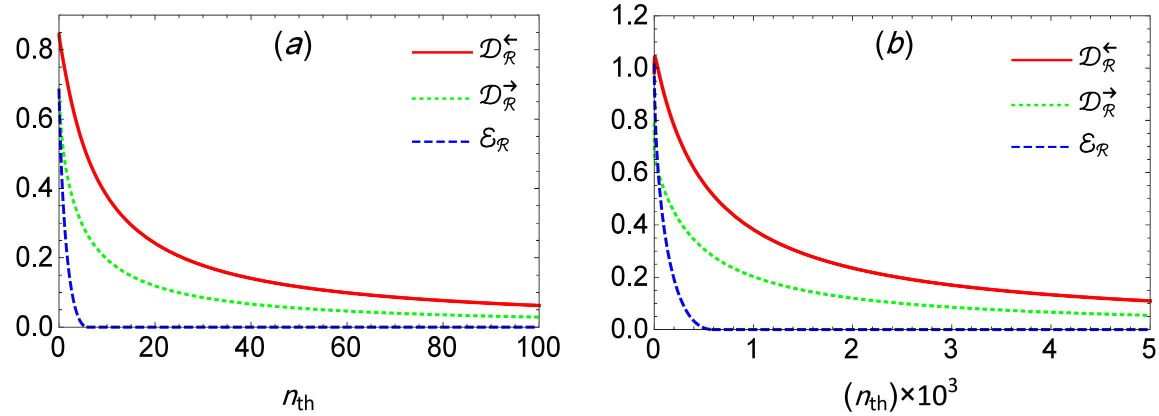

Figure 3: Plot of the Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ and the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ versus the mean thermal phonon number $n_{\rm th}$ using $\eta = 0.35$ and $\kappa = 3.85$ kHz. For the linear gain coefficient, we used $\mathcal{A} = 200$ kHz in panel (a) and $\mathcal{A} = 20$ MHz in panel (b). As shown, $\mathcal{E}_{\mathcal{R}}$ is more degradable under thermal noise, having a tendency to vanish rapidly. In contrast, the discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ are more robust against thermal noise, where they are significantly nonzero for $n_{\rm th} > 100$ in panel (a) and for $n_{\rm th} > 5 \times 10^3$ in panel (b). These observations entail that the decoherence effect induced by high values of $n_{\rm th}$ could be overcome via preparing the atoms with a higher degree of quantum coherence, i.e., $\mathcal{A} \gg 1$ and $\eta \to 0$.

of Gaussian local measurements have been implemented on party $B(A)$, provided that the optimisation is over all possible Gaussian measurements [17]

$$\mathcal{C}_{\mathcal{R}}^{\leftarrow}(\rho_{AB}) = \sup_{\Gamma_B^M} \left[\frac{1}{2} \ln \frac{\det \mathcal{V}_A}{\det \tilde{\mathcal{V}}_A^M} \right], \tag{21}$$

where $\tilde{\mathcal{V}}_A^M = \mathcal{V}_A - \mathcal{V}_{A/B}(\mathcal{V}_B + \Gamma_B^M)^{-1}\mathcal{V}_{A/B}^{\rm T}$ is the covariance matrix of the mode A after an optimised Gaussian measurement is performed on mode B, with Γ_B^M being a positive operator valued measure [16]. $\mathcal{C}_{\mathcal{R}}^{\rightarrow}(\rho_{AB})$ can be obtained from Eq. (21) by performing the double exchange $\mathcal{V}_A \leftrightarrow \mathcal{V}_B$ and $\Gamma_B^M \to \Gamma_A^M$. For the two-mode Gaussian states ρ_{AB} with covariance matrix (14), Eq. (18) reads [18]

$$\begin{aligned} \mathcal{D}_{\mathcal{R}}^{\leftarrow}(\rho_{AB}) &\doteq \mathcal{I}_{\mathcal{R}}(\rho_{AB}) - \mathcal{C}_{\mathcal{R}}^{\leftarrow}(\rho_{AB}), & (22)\\ \mathcal{D}_{\mathcal{R}}^{\leftarrow}(\rho_{AB}) &= \inf \left[\frac{1}{2} \ln \frac{\det \mathcal{V}_B \det \tilde{\mathcal{V}}_A^M}{\det \mathcal{V}_{AB}} \right], & (23) \end{aligned}$$

with

$$\inf\left[\ln \det \tilde{\mathcal{V}}_A^M\right] = \begin{cases} a\left(a - \frac{c^2}{b}\right) \text{ if } \left[ab^2c'^2 - c^2\left(a + bc'^2\right)\right]\left[ab^2c^2 - c'^2\left(a + bc^2\right)\right] < 0, \\ \left[\frac{|cc'| + \sqrt{[a(b^2-1) - bc'^2][a(b^2-1) - bc^2]}}{b^2-1}\right]^2 \text{ otherwise.} \end{cases} \tag{24}$$

In general, the Gaussian Rényi-2 discord is nonsymmetric by exchanging the roles played by the modes A and B, i.e., $\mathcal{D}_{\mathcal{R}}^{\leftarrow} \neq \mathcal{D}_{\mathcal{R}}^{\rightarrow}$, where the discord $\mathcal{D}_{\mathcal{R}}^{\leftarrow}(\mathcal{D}_{\mathcal{R}}^{\rightarrow})$ is obtained after Gaussian measurements have been performed on mode $B(A)$. Moreover, form Eqs. (23) and (24), we remark that if $c = 0$ or $c' = 0$, the discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ vanish, which turns the state ρ_{AB} into a product state with no position or momentum correlations between the modes A and B.

Notice that a comparative study [23] between the entanglement of formation and quantum discord, in a two-qubit state, showed that these two measures of quantum correlations are not only quantitatively but also qualitatively different.

For realistic estimation of the Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ and the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ we use parameters from [45, 62]: the cavity decay rate $\kappa = 3.85$ kHz, the atomic decay rate $\gamma = 20$ kHz, the coupling constant $\upsilon = 43$ kHz, the rate at which the atoms are injected into the cavity $r = 22$ kHz. Using these values, we get $\mathcal{A} = \frac{2r\upsilon^2}{\gamma^2} \approx 200$ kHz for the linear gain coefficient.

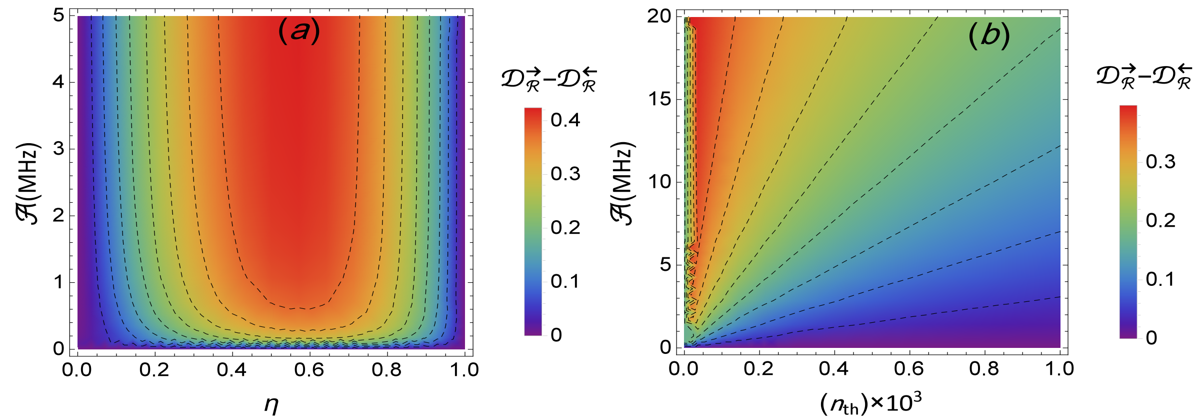

Figure 4: (a) Density plot of the difference $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ against η and $\mathcal{A}$ using $\kappa = 3.85$ kHz and $n_{\mathrm{th}} = 5$. (b) Density plot of the difference $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ against n_{th} and $\mathcal{A}$ using $\kappa = 3.85$ kHz and $\eta = 0.35$. Under various conditions, we remark that $\mathcal{D}_{\mathcal{R}}^{\leftarrow} \geq \mathcal{D}_{\mathcal{R}}^{\rightarrow}$, meaning that more quantumness of the studied three-level laser could be captured by performing Gaussian measurements of mode B that emitted during the second transition $|l_2\rangle \rightarrow |l_3\rangle$.

In Fig. 2, we plot simultaneously $\mathcal{E}_{\mathcal{R}}$, $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ against the population inversion η using $n_{\mathrm{th}} = 5$ as value of the mean thermal phonon number. For the linear gain coefficient, we used $\mathcal{A} = 100$ kHz in Fig. 2a, $\mathcal{A} = 1$ MHz in Fig. 2b, and $\mathcal{A} = 50$ MHz in Fig. 2c. Quite remarkably, for $\eta = 1$, neither entanglement nor discord in both directions could be detected ($\mathcal{E}_{\mathcal{R}} = \mathcal{D}_{\mathcal{R}}^{\leftarrow} = \mathcal{D}_{\mathcal{R}}^{\rightarrow} = 0$) irrespective of the values of $\mathcal{A}$. This can be understood as follows: since $\eta = 1$ corresponds to the situation in which all the three-level atoms are populated in the ground level $|l_3\rangle$, then it will be no possibility for laser emission by the atoms via the cascade transition $|l_1\rangle \rightarrow |l_2\rangle \rightarrow |l_3\rangle$. Consequently, correlated laser modes in the cavity are not expected. On the other hand, by preparing the atoms in a quantum coherent superposition of the upper $|l_1\rangle$ and ground $|l_3\rangle$ levels, i.e., $\eta \neq 1$, quantum features of the cavity radiation of the nondegenerate three-level laser could be well detected. In particular, when the atoms are initially prepared in a maximum coherent superposition, i.e., $\eta = 0$, we remark that, solely, the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ can capture the quantumness of correlations of the cavity light.

Figure 2a illustrates that, for $\mathcal{A} = 100$ kHz, the Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ is always zero regardless of the values of η. In contrast, the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ are always nonzero for all values of the population inversion η except for $\eta \neq 1$. However, by augmenting the values of the coefficient $\mathcal{A}$, we remark that, in addition to quantum discord, the entanglement $\mathcal{E}_{\mathcal{R}}$ could be also detected. As clearly indicated in Figs. 2b and c, high values of $\mathcal{A} = 2rv^2/\gamma^2$ allow to generate maximum entanglement and discord provided that the three-level atoms are initially prepared in a relatively stronger coherent superposition ($\eta \rightarrow 0$). This entails that quantum correlations can be enhanced via augmenting the rate r at which the atoms are injected into the cavity or augmenting the coupling strength v, or using the two mechanisms at the same time. Manifestly, Fig. 2c shows an interesting situation in which the entanglement $\mathcal{E}_{\mathcal{R}}$ exceeds the quantum discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ in contrary to the predictions based on the assumption that entanglement is only a part of quantum discord [63].

The situation in which quantum discord is found less than entanglement reminds us of the analogous feature observed within two-qubit states in [23]. This can be explained as follows: purely quantum correlations captured by means of quantum discord often emerge as a consequence of quantum coherence property [12]. Whereas, entangled states may involve more than purely quantum correlations, that is, entangled states usually carry classical ones [3]. On the other hand, inspired by the results [13, 61] postulated that classical correlations should not be less than quantum ones. Therefore the case corresponds to $\mathcal{E}_{\mathcal{R}} > \mathrm{Max}[\mathcal{D}_{\mathcal{R}}^{\leftarrow}, \mathcal{D}_{\mathcal{R}}^{\rightarrow}]$ can be viewed as a situation in which entanglement is a certain mixture of purely quantum and purely classical correlations. Overall, Fig. 2 shows that when the Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ failed to capture quantum correlations in the state ρ_{AB}, the Gaussian Rényi-2 discord can capture them even when less coherence are injected in the cavity (small values of the linear gain coefficient $\mathcal{A}$ with $\eta \rightarrow 1$).

Now, we focus our attention on the behavior of the entanglement $\mathcal{E}_{\mathcal{R}}$ and the discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ under influence of thermal effect. In Fig. 3, we plot simultaneously $\mathcal{E}_{\mathcal{R}}$, $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ against the mean thermal phonon number n_{th} using $\mathcal{A} = 200$ kHz in Fig. 3a and $\mathcal{A} = 20$ MHz in Fig. 3b. In both panels, we used $\eta = 0.35$. As shown, the three measures of nonclassicality $\mathcal{E}_{\mathcal{R}}$, $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ are maximum for $n_{\rm th} = 0$ and decrease with increasing $n_{\rm th}$. Obviously, the entanglement $\mathcal{E}_{\mathcal{R}}$ is found more affected by thermal noise than quantum discord, having a tendency to vanish quickly. However, by using high values of the linear gain coefficient $\mathcal{A}$, entangled states could be preserved over a wide range of $n_{\rm th}$. This means that by injecting more and more quantum coherence into the cavity, it is possible to overcome the decoherence effect induced by thermal noise. Besides, the discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ are found more robust against thermal noise, where they decrease monotonically with increasing $n_{\rm th}$, but never vanish.

Strikingly, the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ still nonzero up to $n_{\rm th} = 100$ using $\mathcal{A} = 200$ kHz in Fig. 3a and up to $n_{\rm th} = 5 \times 10^3$ using $\mathcal{A} = 20$ MHz in Fig. 3b. These results assert that the quantumness of correlations in a nondegenerate three-level laser could be captured even in high environmental temperature provided that the atoms are initially prepared in the cavity with a high degree of quantum coherence, i.e., $\mathcal{A} \gg 1$ and $\eta \to 0$. Also, it is interestingly to observe from Fig. 3 that within separable states ($\mathcal{E}_{\mathcal{R}} = 0$), the discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ remain almost constant, reaching an asymptotic regime over a wide range of $n_{\rm th}$. Such behavior is commonly known as the freezing behavior of quantum discord beyond entanglement, where an insightful physical interpretation of this phenomenon is given in [11, 32].

It is not hard to remark from Figs. 2 and 3 that the two discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ reveal rather different trends in similar conditions. This means that inferring on the laser mode A based on the measurements implemented on the laser mode B is completely different from the reverse process, which is an important example of the role played by the observer in quantum mechanics [44]. Quite remarkably, in various circumstances, the discord $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ obtained by performing Gaussian measurements on mode A, emitted during the first transition $|l_1\rangle \to |l_2\rangle$, remains less or equal to the discord $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ obtained by performing Gaussian measurements on mode B, emitted during the second transition $|l_2\rangle \to |l_3\rangle$. In what follows, we show that such behavior is independent of the values of physical and environmental parameters (η, κ, $\mathcal{A}$, $n_{\rm th}$) of the state ρ_{AB}.

In Fig. 4a, we plot $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ using a density values of the parameters η and $\mathcal{A}$ for $n_{\rm th} = 5$ and $\kappa = 3.85$ kHz. While, in Fig. 4b we plot $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ using a density values of the parameters $n_{\rm th}$ and $\mathcal{A}$ for $\eta = 0.35$ and $\kappa = 3.85$ kHz. As vividly illustrated the difference $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ is always superior or equal to zero in various circumstances, which implies that $\mathcal{D}_{\mathcal{R}}^{\leftarrow} \geq \mathcal{D}_{\mathcal{R}}^{\rightarrow}$. Furthermore, employing Eqs. (14) and (23), we obtain

$$\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow} = \ln\left[\frac{ab+b}{ab+a}\frac{ab+a-c^2}{ab+b-c^2}\right], \tag{25}$$

where the discord $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ can be obtained from the expression of $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ by performing the exchange $a \leftrightarrow b$. With some algebra, one can show that the difference $\mathcal{D}_{\mathcal{R}}^{\leftarrow} - \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ given by Eq. (25) and the difference $a - b$ have the same-sign. On the other hand, using the expressions of a and b defined from the covariance matrix (14), we get

$$a - b = \langle \varsigma_1^\dagger \varsigma_1 \rangle - \langle \varsigma_2^\dagger \varsigma_2 \rangle = \frac{\mathcal{A}\,(1 - \eta + 2n_{\rm th})}{2\,(\kappa + \mathcal{A}\eta)}, \tag{26}$$

which is always positive or equal to zero since κ, $\mathcal{A}$ and $n_{\rm th}$ are positive, and $0 \leqslant \eta \leqslant 1$. Therefore, we conclude that the situation $\mathcal{D}_{\mathcal{R}}^{\leftarrow} \geq \mathcal{D}_{\mathcal{R}}^{\rightarrow}$ is always fulfilled by the state ρ_{AB}, which asserts that more quantumness of correlations, in the state ρ_{AB}, can be captured by performing Gaussian measurements on the laser mode B that emitted during the second transition $|l_2\rangle \to |l_3\rangle$. Finally, with the cavity-quantum electrodynamics approaches [64, 65] and the strategy of homodyne measurement [15], our results may be verified experimentally.

4 Conclusion

In a two-mode Gaussian state ρ_{AB}, coupled to a common two-mode thermal bath, a comparative study between two indicators of nonclassicality (entanglement and discord) is presented. The mode $A(B)$ is generated within the first(second) transition of a nondegenerate three-level cascade laser. The stationary covariance matrix of the state ρ_{AB} is evaluated within the good cavity limit and the linear-adiabatic approximation. The Gaussian Rényi-2 entanglement $\mathcal{E}_{\mathcal{R}}$ is used to quantify entanglement, while the Gaussian Rényi-2 discords $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ and $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$ are employed for capturing the quantumness of correlations. It is found that both entanglement and discord could be generated and enhanced via controlling the physical and environmental parameters of the state ρ_{AB}. Optimal entanglement and discord can be achieved when the atoms are initially prepared in a relatively strong coherent superposition, i.e., $\mathcal{A} \gg 1$ and $\eta \to 0$. Quite remarkably, the entanglement $\mathcal{E}_{\mathcal{R}}$ is found more fragile against thermal effect, suffering a sudden death-like behavior. In contrast, quantum discord is found more robust, seeming to be captured in a wide range of environmental temperatures. Numerical as well as analytical analysis showed that the discord $\mathcal{D}_{\mathcal{R}}^{\leftarrow}$ remains always superior or equal to the discord $\mathcal{D}_{\mathcal{R}}^{\rightarrow}$, meaning that more quantumness of correlations can be captured by performing Gaussian measurements on the mode B that generated during the second transition.

These results fairly indicate that, over lossy-noisy channels, nondegenerate three-level lasers can be useful in implementing some quantum information tasks, especially for that do need entanglement. Finally, it would be interesting to investigate the conditions under which the quantum correlations of the two laser modes A and B can be transferred to two mechanical modes for generating, e.g., quantum steering between them. We hope to report on this in a forthcoming work.

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Conflict of Interest

The author declares no conflict of interest.

Data Availability Statement

This manuscript has no associated data.

References

[1] Nielsen, M. A., Chuang, I. L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2019)

[2] Werner, R. F.: Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model. Phys. Rev. A **40**, 4277 (1989)

[3] Horodecki, R., Horodecki, P., Horodecki, M., Horodecki, K.: Quantum entanglement. Rev. Mod. Phys. **81**, 865 (2009)

[4] Pirandola, S., Eisert, J., Weedbrook, C., Furusawa, A., Braunstein, S. L.: Advances in quantum teleportation. Nature Photonics **9**, 641 (2015)

[5] Chen, Y., Liu, S., Lou, Y., Jing, J.: Orbital angular momentum multiplexed quantum dense coding. Phys. Rev. Lett. **127**, 093601 (2021)

[6] O'Brien, J. L.: Optical quantum computing. Science **318**, 1567 (2007)

[7] Harrow, A. W., Hassidim, A., Lloyd, S.: Quantum algorithm for linear systems of equations. Phys. Rev. Lett. **103**, 150502 (2009)

[8] Grosshans, F., Van Assche, G., Wenger, J., Brouri, R., Cerf, N. J., Grangier, P.: Quantum key distribution using Gaussian-modulated coherent states. Nature (London) **421**, 238 (2003)

[9] Datta, A., Shaji, A., Caves, C. M.: Quantum discord and the power of one qubit. Phys. Rev. Lett. **100**, 050502 (2008)

[10] Madsen, M. L. S., Berni, A., Lassen, M., Andersen, U. L.: Experimental investigation of the evolution of Gaussian quantum discord in an open system. Phys. Rev. Lett. **109**, 030402 (2012)

[11] Modi, K., Brodutch, A., Cable, H., Paterek, T., Vedral, V.: The classical-quantum boundary for correlations: Discord and related measures. Rev. Mod. Phys. **84**, 1655 (2012)

[12] Ollivier, H., Zurek, W. H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. **88**, 017901 (2001)

[13] Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A **34**, 6899 (2001)

[14] Ali, M., Rau, A. R. P., Alber, G.: Quantum discord for two-qubit X states. Phys. Rev. A **81**, 042105 (2010)

[15] Weedbrook, C., Pirandola, S., García-Patrón, R., Cerf, N. J., Ralph, T. C., Shapiro, J. H., Lloyd, S.: Gaussian quantum information. Rev. Mod. Phys. **84**, 621 (2012)

[16] Giedke, G., Cirac, J. I.: Characterization of Gaussian operations and distillation of Gaussian states. Phys. Rev. A **66**, 032316 (2002)

[17] Adesso, G., Datta, A.: Quantum versus classical correlations in Gaussian states. Phys. Rev. Lett. **105**, 030501 (2012)

[18] Adesso, G., Girolami, D., Serafini, A.: Measuring Gaussian quantum information and correlations using the Rényi entropy of order 2. Phys. Rev. Lett. **109**, 190502 (2012)

[19] Madhok, V., Datta, A.: Interpreting quantum discord through quantum state merging. Phys. Rev. A **83**, 032323 (2011)

[20] Dakic, B., Lipp, Y. O., Ma, X., Ringbauer, M., Kropatschek, S., Barz, S., Paterek, T., Vedral, V., Zeilinger, A., Brukner, C., Walther, P.: Quantum discord as resource for remote state preparation . Nature Phys. **8**, 666 (2012)

[21] Scarani, V., Bechmann-Pasquinucci, H., Cerf, N. J., Dušek, M., Lütkenhaus, N., Peev, M.: The security of practical quantum key distribution. Rev. Mod. Phys. **81**, 1301 (2009)

[22] Salek, F., Hayashi, M., Winter, A.: Usefulness of adaptive strategies in asymptotic quantum channel discrimination. Phys. Rev. A **105**, 022419 (2022)

[23] Luo, S.: Quantum discord for two-qubit systems. Phys. Rev. A **77**, 042303 (2008)

[24] Freitas, J. N., Paz, J. P.: Dynamics of Gaussian discord between two oscillators interacting with a common environment. Phys. Rev. A **85**, 032118 (2012)

[25] Lü, Y. -Q., An, J.-H., Chen, X. -M., Luo, H. -G., Oh, C. H.: Frozen Gaussian quantum discord in photonic crystal cavity array system. Phys. Rev. A **88**, 012129 (2013)

[26] Qars, J. E., Daoud, M., Laamara, R. A.: Quantifying quantumness of correlations using Gaussian Rényi-2 entropy in optomechanical interfaces. J. Mod. Opt. **65,** 1584 (2018)

[27] Uola, R., Costa, A.C.S., Nguyen, H.C., Gühne, O.: Quantum steering. Rev. Mod. Phys. **92**, 15001 (2020)

[28] Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V., Wehner, S.: Bell nonlocality. Rev. Mod. Phys. **86**, 419 (2014)

[29] Xiang, Y., Kogias, I., Adesso, G., He, Q.: Multipartite Gaussian steering: Monogamy constraints and quantum cryptography applications. Phys. Rev. A **95**, 010101(R) (2017)

[30] Gisin, N., Ribordy, G., Tittel, W., Zbinden, H.: Quantum cryptography. Rev. Mod. Phys. **74**, 145 (2002)

[31] Streltsov, A., Adesso, G., Plenio, M.B.: Quantum coherence as a resource. Rev. Mod. Phys. **89**, 041003 (2017)

[32] Bromley, T. R., Cianciaruso, M., Adesso, G.: Frozen quantum coherence. Phys. Rev. Lett. **114,** 210401 (2015)

[33] Li, C.-M., Lambert, N., Chen, Y.-N., Chen, G.-Y., Nori, F.: Witnessing quantum coherence: from solid-state to biological systems. Sci. Rep. **2**, 885 (2012)

[34] Lostaglio, M., Jennings, D., Rudolph, T.: Description of quantum coherence in thermodynamic processes requires constraints beyond free energy. Nat. Commun. **6**, 6383 (2015)

[35] Scully, M. O., Zubairy, M. S.: Quantum Optics. Cambridge University Press, Cambridge (1997)

[36] Scully, M. O., Wodkiewicz, K., Zubairy, M. S., Bergou, J., Lu, N., Meyer ter Vehn, J.: Two-photon correlated spontaneous emission laser: Quantum noise quenching and squeezing. Phys. Rev. Lett. **60,** 1832 (1988)

[37] Ansari, N. A., Gea-Banacloche, J., Zubairy, M. S.: Phase-sensitive amplification in a three-level atomic system. Phys. Rev. A **41**, 5179 (1990)

[38] Tan, H. -T., Zhu, S. -Y., Zubairy, M. S.: Continuous-variable entanglement in a correlated spontaneous emission laser. Phys. Rev. A **72,** 022305 (2005)

[39] Kapale, K. T., Scully, M. O., Zhu, S. -Y., Zubairy, M. S.: Quenching of spontaneous emission through interference of incoherent pump processes. Phys. Rev. A **67**, 023804 (2003)

[40] Tesfa, S.: Entanglement amplification in a nondegenerate three-level cascade laser. Phys. Rev. A **74,** 043816 (2006)

[41] Sete, E. A., Eleuch, H.: Anomalous optical bistability and robust entanglement of mechanical oscillators using two-photon coherence. J. Opt. Soc. Am. B **32**, 971 (2015)

[42] Ping, Y. -X., Zhang, B., Cheng, Z., Zhang, Y. -M.: Entanglement in the steady state of a two-mode three-level atomic system. Phys. Lett. A **362,** 128 (2007)

[43] Sete, E. A.: Violation of classical inequalities and EPR correlations in a two-mode three-level atomic system. Int. J. Quant. Inform. **6**, 885 (2008)

[44] Qars, J. E.: Unidirectional Gaussian one-way steering. Ann. Phys. (Berlin) **534,** 2100386 (2022)

[45] Xiong, H., Scully, M. O., Zubairy, M. S.: Correlated spontaneous emission laser as an entanglement amplifier. Phys. Rev. Lett. **94,** 023601 (2005)

[46] Alebachew, E.: Continuous-variable entanglement in a nondegenerate three-level laser with a parametric oscillator. Phys. Rev. A **76,** 023808 (2007)

[47] Bekele, M., Yirgashewa, T., Tesfa, S.: Entanglement of mechanical modes in a doubly resonant optomechanical cavity of a correlated emission laser. Phys. Rev. A **107**, 012417 (2023)

[48] Simon, R.: Peres-Horodecki separability criterion for continuous variable systems. Phys. Rev. Lett. **84,** 2726 (2000)

[49] Duan, L. -M., Giedke, G., Cirac J. I., Zoller, P.: Inseparability criterion for continuous variable systems. Phys. Rev. Lett. **84**, 2722 (2000)

[50] Fesseha, F.: Three-level laser dynamics with squeezed light. Phys. Rev. A **63,** 033811 (2001)

[51] Lu, N., Zhu, S. Y.: Quantum theory of two-photon correlated-spontaneous-emission lasers: Exact atom-field interaction Hamiltonian approach. Phys. Rev. A **40,** 5735 (1989)

[52] Louisell, W. H.: Quantum Statistical Properties of Radiation. Wiley, New York (1973)

[53] Akramine, O. E., Makhoute, A., Zitane, M., Tij, M.: Theoretical study of quantum dissipation and laser-noise effects on the atomic response. Phys. Rev. A **58,** 4892 (1998)

[54] Sargent, M. III., Scully, M. O., Lamb, Jr. M. E.: Laser Physics. Addison-Wesley, Reading, Mass (1974)

[55] Qars, J. E.: Total versus quantum correlations in a two-mode Gaussian state. Commun. Theor. Phys. **73,** 055103 (2021)

[56] Rényi, A.: in Proceedings of the 4th Berkeley Symposium on Mathematics, Statistics and Probability. University of California Press, Berkeley (1960)

[57] Vedral, V.: The role of relative entropy in quantum information theory. Rev. Mod. Phys. **74**, 197 (2002)

[58] Rastegin, A. E.: Rényi formulation of entanglement criteria for continuous variables. Phys. Rev. A **95**, 042334 (2017)

[59] Adesso, G., Illuminati, F.: Gaussian measures of entanglement versus negativities: Ordering of two-mode Gaussian states. Phys. Rev. A **72**, 032334 (2005)

[60] Schumacher, B., Westmoreland, M. D.: Sending classical information via noisy quantum channels. Phys. Rev. A **74,** 042305 (2006)

[61] Groisman, B., Popescu, S., Winter, A.: Quantum, classical, and total amount of correlations in a quantum state. Phys. Rev. A **72,** 032317 (2005)

[62] Meschede, D., Walther, H., Muller, G.: One-atom maser. Phys. Rev. Lett. **54,** 551 (1985)

[63] He, Q. Y., Gong, Q. H., Reid, M. D.: Classifying directional Gaussian entanglement, Einstein-Podolsky-Rosen steering, and discord. Phys. Rev. Lett. **114,** 060402 (2015)

[64] Reiserer, A., Rempe, G.: Cavity-based quantum networks with single atoms and optical photons. Rev. Mod. Phys. **87,** 1379 (2015)

[65] McKeever, J., Boca, A., Boozer, A. D., Miller, R., Buck, J. R., Kuzmich, A., Kimble, H. J.: Deterministic generation of single photons from one atom trapped in a cavity. Science **303**, 1992 (2004)